

# MULTIVARIATE TECHNIQUE FOR CLASSIFICATION RULE SEARCHING

## *Exemplified by CT Data of Patient*

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**Keywords:** Correspondence Analysis, Fisher's linear Discriminant function, multivariate categorical data, parallel coordinate plots.

**Abstract:** In the process of searching classification rules for multivariate categorical data, it is crucial to find a quick start to locate the combination of levels of input and response variables which can contribute to the most correct classification rate for the response variable. Fisher's linear discriminant function is proposed to select some important input-variable candidates; then, correspondence analysis is used to ascertain that the level of candidates is closely related to the appropriate level of response variable. The closest linkage between input variable and response variables is chosen as the rule for each input-variable candidate. The algorithm is applied to the hospital data of patients whose CT scan diagnosis awaits a decision. The result shows that my algorithm is not only quicker than an exhaustive search but the result is also identical to the optimum solution by exhaustive search in terms of the correct classification rate. The correct classification rate is about 80%. Finally, two parallel coordinate plots of the 20% mistakenly classified data and the corresponding correctly classified data are compared, showing their mutual confounding and explaining why the correct classification rate cannot be further improved.

## 1 INTRODUCTION

Patients sent to the emergency unit of a hospital need immediate care to save their life. To attend to patients in an appropriate manner, correct treatment is crucial, leading to the issue of making a correct diagnosis. In this investigation a data set on 959 patients sent to a local hospital within a certain period of time for emergency care is collected. The on duty physicians face the decision whether the patients need head-computed tomography (HCT), more commonly known as *computed axial tomography* (CAT or CT scan). Since each patient differs, a rule based on the physical characteristics (such as blood pressure, breath, mental status, and triage level, etc.) of the patient should be formulated to help the physician make an appropriate decision on the need for HCT. The first 80% (767) of the original data is used as a training set to establish the rule; whereas, the second 20% (192) is used as test data to demonstrate the effectiveness of the rule.

The data set contains seven independent variables, A1-A7 (such as sex, age, triage level,

mental status, breathing rate, diastolic blood pressure and pulse rate) and one response variable, HCT (D). The medical data are further classified by the physician as in the contingency table shown in Table 1 in Appendix A. Note that D has two meanings: (1) the response variable of the HCT, being the highest standard determined by the on-duty physician, and (2) the result determined by the classification rule. At first the double meaning might be somewhat confusing, but it eliminates the need to define another variable, as should be clear from the context as this report proceeds.

A simple and direct way to solve the problem is enumeration, an exhaustive method. In the single-variable rule search, there will be  $2^p$  ways to classify a categorical input variable with  $p$  levels into a response variable of 2 levels. For example, to find the best rule for variable A3 (with three levels) which will provide the most correct classification via HCT (with two levels), the correct rate for the following six rules must be computed: (A3=1, D=1), (A3=1, D=2), (A3=2, D=1), (A3=2, D=2), (A3=3, D=1), (A3=3, D=2). Since there are seven

independent variables and each is classified as 2, 5, 3, 4, 3, 3 and 4 levels,  $2*(2+5+3+4+3+3+4)=48$  rules need to be evaluated. The classification rule (A3=1, D=1) means that if the patient's triage level is 1, he/she needs to have the HCT. This decision is applied to the training data; however, it may be incorrect according to the criterion of response variable D. Thus, a correct rate can be calculated, and the highest correct rate chosen. In this case, when dealing with only one independent variable, the obtained rule is applied to the test data set to determine whether this rule can render a similar correct classification rate. This type of selection procedure can be applied to the two independent variables. When variable  $x_1$  has  $\ell_1$  levels and variable  $x_2$  has  $\ell_2$  levels, then there are  $2 \ell_1 \ell_2$  rules to be evaluated. In this case study, there will be  $2*(2*5+2*3+2*4+2*3+2*3+2*4+5*3+5*4+5*3+5*3+5*4+4*3+4*3+4*4+3*3+3*4+3*4)=404$  rules altogether. Clearly, evaluating each in succession is very time consuming; therefore, a faster, more reliable method should be sought. For this purpose, two multivariate techniques are used to solve the problem in sequence. First, Fisher's linear discriminant function is built and important input variables selected. By following the correspondence analysis, the level from the input variable closest to the level of the response variable can be selected as the classification rule. For example, variable triage (A3) is considered to be one of the most important of the seven independent variables. Then, a Euclidean distance can be derived between the level of independent variable A3 and the level of decision variable D. The shortest distance between them is chosen as the classification rule for input variable A3. By extending the aforementioned procedure for a single variable to two variables, a combination of composite rules to determine the optimum correct classification rate can be established, in this case about 80%. Finally, two parallel coordinate plots of the 20% mistakenly classified data and the corresponding correctly classified data are compared, showing their mutual confounding and explaining why the correct classification rate cannot be further improved. Although Fisher's linear discriminant function and correspondence analysis are two well known techniques in multivariate analysis, using them together to find the classification rule for multivariate categorical data is unusual. Fisher's function is mostly used to classify multivariate continuous data into various categories. In the literature, one can find its application to face detection (Yang, Kriegman and Ahuja, 2001) and its

combination with linear programming (Lam and Moy, 2003). Used to detect the root of two categorical variables. Correspondence analysis can be used in the ecological study of animal populations (Allombert, Gaston and Martin, 2005).

## 2 FISHER'S LINEAR DISCRIMINANT FUNCTION

Suppose that there is a sample data set  $X$  of multivariate variable  $x$  composed of samples  $X_j$  with sample size of  $n_j$ ,  $j=1,2,\dots,J$ , from  $J$  populations. To obtain the optimum classification rule for multivariate sample  $X$ , Fisher suggests finding the linear combination of  $a^T x$  which maximizes the ratio of between-group-sum of squares to the within-group-sum of squares (Hardle and Simar, 2003),

$$\frac{a^T B a}{a^T W a}, \tag{1}$$

where  $B$  is the between-group-sum of squares, defined as (Johnson and Wichern, 2003)

$$B = \sum_{i=1}^J n_i (\bar{x}_i - \bar{x})(\bar{x}_i - \bar{x})^T; \tag{2}$$

whereas,  $W$  is the within-group-sum of squares, defined as

$$W = \sum_{i=1}^J \sum_{j=1}^{n_i} (x_{ij} - \bar{x}_i)(x_{ij} - \bar{x}_i)^T. \tag{3}$$

Note that  $x_{ij}$  represents the  $j^{th}$  sample from population  $i$ ;  $\bar{x}_i$ , the sample mean of population  $i$ ;  $\bar{x}$ , the grand average of the total samples. The solution of vector  $a$  is found in Theorem 1 (Hardle and Simar, 2003).

**Theorem 1.** The vector  $a$  that maximizes (1) is the eigenvector of  $W^{-1}B$  corresponding to the largest eigenvalue.

Now, the pertinent discrimination rule is as follows: Classify  $x$  into group  $j$  where  $a^T \bar{x}_j$  is closest to  $a^T x$ . When  $J=2$  is grouped, the discriminant rule is computed as follows: The corresponding eigenvector in Theorem 1 is  $a = W^{-1}(\bar{x}_1 - \bar{x}_2)$ . The corresponding discriminant rule is

$$\begin{aligned} x &\rightarrow \Pi_1 && \text{if } a^T(x - \bar{x}) > 0 \\ x &\rightarrow \Pi_2 && \text{if } a^T(x - \bar{x}) \leq 0 \end{aligned} \quad (4)$$

where  $\Pi_i$  represents population  $i$ .

Note that all the data are not assumed to be normal; the only assumption is that they are real numbers.

After this short introduction to Fisher's linear discriminant function, the focus is now directed toward its application to the hospital data. To sketch the eight variables of 767 data set, parallel coordinate plots are used. The result is shown in Appendix B. Since variable A1 is nominal and the analysis in Appendix B indicates that variable A1 has no strong correlation with variable D. Fisher's linear discriminant function with input variables (A2-A7) is found. Vector  $a$  is [0.00023445, -0.00079367, 0.00062467, 0.0020182, -2.9766e-06, -0.00035162]<sup>T</sup>; the grand average  $\bar{x}$  is [3.4433, 1.7901, 1.1917, 1.9974, 2.9126, 2.3051], the values in  $a$  and  $\bar{x}$  corresponding to variables A2-A7. Then, the number of individuals coming from  $\Pi_j$ , which have been classified into  $\Pi_i$  by  $n_{ij}$ , are denoted. By applying the discriminant rule in Eq. (1) to the test data set, one has  $n_{12}=0$  and  $n_{21}=48$ , the correct classification rate being 0.75. By examining the magnitude of the coefficients in  $a$ , it is clear that the three most important variables are A5 (breathing), A3 (triage) and A4 (mental state); whereas, the least important is A6 (blood pressure). Thus, the correlation between the level of these variables and the CT level is investigated and the closest relationship between them in terms of the Euclidean distance searched. The closest is chosen as the rule to classify the patients who need HCT. A detailed explanation follows.

### 3 CORRESPONDENCE ANALYSIS

The aim of correspondence analysis is to develop simple indices showing relationships between the rows and columns of a contingency table, wherein row and column represents one category of the corresponding variables. The entry  $x_{ij}$  in table X (with dimension (nxp)) represents the number of observations in a sample which simultaneously fall in the  $i^{th}$  row category and the  $j^{th}$  column category, for  $i = 1, 2, \dots, n$  and  $j = 1, 2, \dots, p$ . Then the

association between the row and column categories can be measured by an  $\chi^2$ -test statistic defined as

$$\chi^2 = \sum_{i=1}^n \sum_{j=1}^p (x_{ij} - E_{ij})^2 / E_{ij} \quad (5)$$

where  $E_{ij} = \frac{x_i \cdot x_j}{x_{..}}$  with  $x_i$  represents the sum in the  $i^{th}$  row;  $x_j$ , the sum in the  $j^{th}$  column; and

$x_{..} = \sum_{i=1}^n x_i$ , the grand total. Under the hypothesis of

independence,  $\chi^2$  has an  $\chi^2_{(n-1)(p-1)}$  distribution. If the test statistic  $\chi^2$  is significant at the 5% level, investigating the special reasons for the departure from independence is worthwhile. To extract the elements of dependence, the principle of correspondence analysis (CorrAna) is brought into play. The CorrAna procedure first determines the SVD (singular value decomposition) of matrix  $C$  (nxp) with elements (Hardle and Simar, 2003)

$$c_{ij} = (x_{ij} - E_{ij}) / E_{ij}^{1/2} \quad (6)$$

When assuming that the rank of  $C$  is  $R$ , the SVD of  $C$  yields

$$C = \Gamma \Lambda \Delta^T \quad (7)$$

where  $\Gamma$  contains the eigenvectors  $\gamma_k$  (nx1) of  $CC^T$ ,  $\Delta$  the eigenvectors  $\delta_k$  (px1) of  $C^T C$  where  $k = 1, 2, \dots, R$  and  $\Lambda = \text{diag}(\lambda_1^{1/2}, \dots, \lambda_R^{1/2})$  (where  $\text{diag}$  represents the diagonal matrix) with  $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_R$  (the eigenvalues of  $CC^T$ ). By defining the matrices  $A$  (nxn) and  $B$  (pxp) as

$$A = \text{diag}(x_i) \text{ and } B = \text{diag}(x_j) \quad (8)$$

one can calculate  $r_k$  (nx1) and  $s_k$  (px1),  $k = 1, 2, \dots, R$  as

$$\begin{aligned} r_k &= \sqrt{\lambda_k} A^{-1/2} \gamma_k, \\ s_k &= \sqrt{\lambda_k} B^{-1/2} \delta_k, \end{aligned} \quad (9)$$

where point vectors  $[r_1, r_2]$  and  $[s_1, s_2]$  are plotted onto a two-dimensional graph, called biplot, with  $n$  points in point vector  $[r_1, r_2]$  representing the  $n$  rows and  $p$  points in  $[s_1, s_2]$  representing the  $p$  columns. Thus, the entire contingency table can be simplified as  $n+p$  points on the 2D graph. The

relationship between  $n$  points  $[r_1, r_2]$  and  $p$  points  $[s_1, s_2]$  explains why rows and columns are not independent.

Now, the correspondence analysis (CorrAna) is applied to the contingency table of variables A5 and D as an illustration, shown in Table 1.

Table 1: Contingency table of variables A5 and D for 767 Data.

		HCT (D)	
		1	2
Breath (A5)	<10/min	1	4
	10~24/min	249	510
	>24/min	3	0

The corresponding  $R$  of  $C$  for Table 1 is 1; moreover, vector  $r_1 = [-0.2762, -0.0038143, 1.4253]$  and vector  $s_1 = [0.13109, -0.064523]$ . Since  $R = 1$ , there are no  $r_2$  and  $s_2$ . A zero vector is substituted for  $r_2$  and  $s_2$  to make a 2D biplot, shown in Fig. 1.

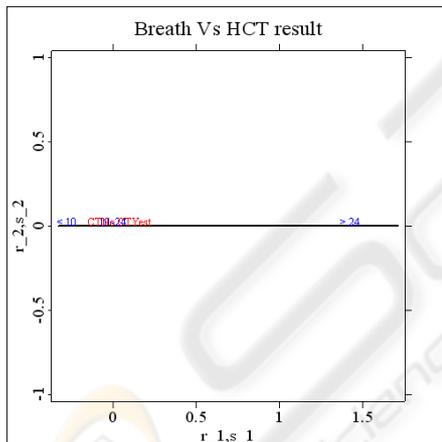


Figure 1: Biplot of variables A5 and D.

Note that the two levels of response variable D (CTYes corresponding to HCT [D=1] and CTNo, HCT [D=2]), are close to the second level of independent variable A5. Examining the value of  $r_1$  (representing A5) and  $s_1$  (representing D), one may clearly observe that HCT (D=2) is closer to (A5=2) than HCT (D=1). The value of HCT (D=2) in Figure 1 is -0.064523, being the second value in  $s_1$ ; the value of (A5=2), -0.0038143, being the second value in  $r_1$ ; the value of HCT (D=1), 0.13109, being the

first value in  $s_1$ . The rule is then derived as if (A5=2); therefore, HCT should not be administered. Thus when the breathing level is normal, the patient does not need HCT. By applying this rule to the remaining test data, one has  $n_{12}=0$  and  $n_{21}=47$ , the correct rate being 0.75521. Table 1 clearly indicates that the rule is optimum; hence, any other rule will yield a worse correct rate. The effectiveness of applying this rule to the test data can be clearly observed by scrutinizing the contingency table of variables A5 and D for the test data set, shown in Table 2.

Table 2: Contingency table of variables A5 and D for 192 Data.

		HCT (D)	
		1	2
Breath (A5)	<10/min	1	0
	10~24/min	47	144
	>24/min	0	0

The rule can be applied to Table 2 to obtain Figure 2, where the correct result from the main rule (if [A5=2] then do not administer HCT) is marked in red; whereas, the correct result from the congruent rule (if [A5≠ 2] then administer HCT) is marked in blue. The correct rate is  $(1+0+144)/192=0.75521$ . No other classification rule will result in a better correct rate.

		HCT (D)	
		1	2
Breath (A5)	<10/min	1	0
	10~24/min	47	144
	>24/min	0	0

Figure 2: Classification rule with contingency table.

By applying the same procedure to the contingency tables of variables A3 and D, and A4 and D, the correct rates of 0.70312 and 0.72396, respectively, are obtained. The rules derived are if (A3=1), then HCT (D=1); and if (A4=2), then HCT (D=1), respectively. These values indicate that when the triage level is 1, the patient needs HCT; and when the mental status is 'to call', HCT, respectively. The corresponding biplots are shown in Figures 3 and 4. For the sake of analytical completeness, the contingency tables of variables A1-A7 vs D for both the training and the test data sets are listed in Appendix C.

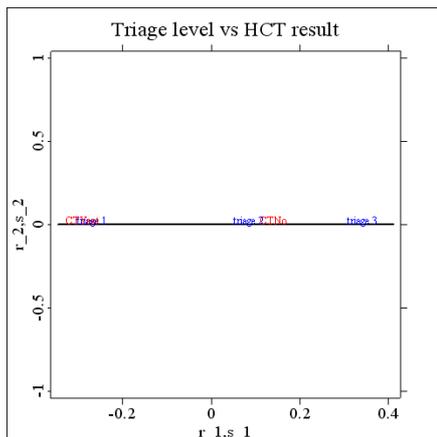


Figure 3: Biplot of variables A3 and D.

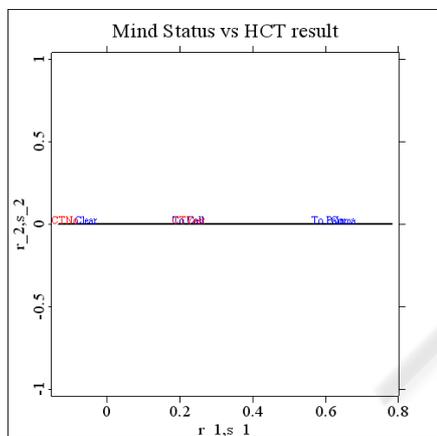


Figure 4: Biplot of variables A4 and D.

From the data reported in Appendix C and previous results, one may assert that analysis by Fisher's linear discriminant function and CorrAna provides a quick start for identifying the important input variables for searching the classification rule. In this case the important input variables are A3, A4 and A5, being in agreement with the Chi-squares test on the contingency table shown in Appendix C. Furthermore, the rule provided by CorrAna is in very strong agreement with the exhaustive search, with only a small difference in the rule for A4. The rules by CorrAna for each important variable are: if A3=1 then D=1; if A4=2, then D=1; if A5=2 then D=2. These rules differ from the ones obtained by exhaustive search only in variable A4, where if A4=1 then D=2; however, there is no difference in the correct classification rate.

The optimum correct classification rate by the single-variable classification rule is 0.75521, provided by the rule of input variable A5, where if (A5=2) then D=2. Further research is needed to find

composite classification rules for two variables which, perhaps, would render a higher correct rate. The result of two-variable classification is explained below and listed in the combination of composite rules.

#### 4 OPTIMUM RULE OF COMBINATION FOR TWO VARIABLES

Following the previous arguments, the combination rule for two variables is straightforward. For the sake of simplicity, only the procedure for finding the optimum correct rate is illustrated. Variables A3 and A4 are used to form a new variable, A9, where

$$A9=(A3-1)*4+A4. \quad (10)$$

Here A9, A3 and A4, in addition to designating the composite variable name, triage and mental status, also represent the level of the corresponding variables. Since the level of variable A3 is three and that of A4 is 4, the level of A9 indicates the combination level of A3 and A4, as shown in equation (10). For example, when A9=9 the equation denotes that A3=3 and A4=1. Theoretically, the total level of A9 is 12; however, in this case study it is only 10 because levels 11 and 12 of A9 are missing. Therefore, the frequencies of (A3=3 and A4=3) and (A3=3 and A4=4) are zero with regard to both levels of HCT response variable (D). Applying CorrAna to A9 with regard to response D, the biplot shown in Figure 5 is obtained.

For a clearer display, the labels of the levels of response D are changed from 'CTYes' and 'CTNo' to 'Y' and 'N'. It is clear from Figure 5 that levels 8, 3, 4 and 2 are located to the left of 'Y' and levels 10 and 9 to the right of 'N'; whereas, the other levels remain between 'Y' and 'N.' Thus, it can be said that when (A3=2 and A4=4) or (A3=1 and A4=3 or 4 or 2), then one should judge D=1; whereas, when (A3=3 and A4=2) or (A3=3 and A4=1), then D=2. Therefore, when the triage level is 2 and the mental status level is a coma, or when the triage level is 1 and the mental status is unclear, one should judge HCT to be necessary. In such cases, the patients are in serious conditions; thus, administering HCT is appropriate and useful in diagnosing the root problem. However, when patient is in a level 3 triage and the mental status is clear or one capable of responding to a verbal stimulus, HCT is unnecessary. Note that in these two cases, the patients are in better condition, but that many patients do not fall into either of these stated categories. To overcome this

problem, the single rule of ( if  $A5=2$ ,  $D=2$  ) is applied to the remainder.

It is worth noting that the single variable rule, the shortest Euclidean distance between the levels of the independent and the response variables is chosen as the classification rule; whereas, in the two-variable rule, the levels of the composite formed by the two variables on each side of the response variable is chosen as the composite rule. These principles are true because in the single-variable case, once the main classification rule has been set, its complement (congruent) is automatically determined. For example, in the case of  $A5$  and  $D$ , as shown in Figure 2, once the main rule, if ( $A5=2$ ) then  $D=2$ , is set, its complement, if ( $A5 \neq 2$ ) then  $D=1$ , is automatically determined. Note that ( $A5 \neq 2$ ) means that ( $A5=1$  or  $3$ ). Therefore, once the destiny of ( $A5=2$ ) is determined as  $D=2$ , the other choices of  $A5$  have a pre-determined result. Thus, a reasonable choice for the classification rule can be based on the shortest linkage between the levels from the independent and the response variables.

If the same argument is followed in the two-variable case, then there is only one level in the composite variable  $A9$  which can be associated with one of the two levels of  $D$ . The other values of  $A9$  will be assigned with the alternative value of  $D$ , a procedure which does not make good sense since the other values are not necessarily exclusive with the chosen value in the main rule. To clarify this point, an illustrative example is given as follows. If the shortest distance between level 5 of  $A9$  and level 2 of  $D$  is chosen as the classification rule, then by the same argument in single-variable, level 5 of  $A9$  should be associated with level 2 of  $D$  and other values (these include level 9, of course) of  $A9$  should be with level 1 of  $D$ . However, Figure 5 clearly shows that level 9 of  $A9$  should be associated with level 2 of  $D$  since it is closely associated with level 2 of  $D$  by the interpretation of correspondence analysis (Hardle and Simar, 2003). Thus, the only reasonable classification rule is to divide the levels of the composite variable into three regions with the levels of  $D$  as the demarcation points. With the levels in the middle region undecided, the levels in the left region are associated with the left demarcation point; whereas, the levels in the right region are assigned to the right extremity. Note also that the levels in the middle region can be classified later by the rule derived from the single variable. The optimum correct classification rate by these two-variable classification rules in addition to the

single rule is 0.76562, with  $n_{12}=2$  and  $n_{21}=43$ . A slightly better result is achieved than from the single variable rule where the correct classification rate is 0.75521, with  $n_{12}=0$  and  $n_{21}=47$ .

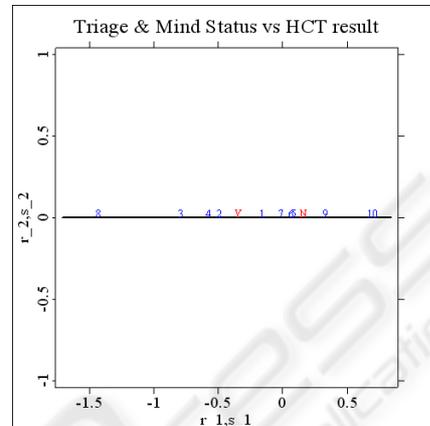


Figure 5: Biplot of variables  $A9$  and  $D$ .

By examining the two misclassifications of  $n_{12}$ , one finds an additional rule to eliminate  $n_{12}$ : When ( $A3=1$ ,  $A4 \geq 3$ ,  $A6=3$ ) then  $D=2$ . This means that when the triage level is 1, the mental status is 'to pain' or 'coma', and the diastolic blood pressure is above 110 mm Hg (very serious high blood pressure), the patient should not be administered HCT because the situation is probably too dangerous. This is a special provision under the rule of stating that when ( $A3=1$ ,  $A4 \geq 3$ ) then  $D=1$ , thereby indicating the importance of abnormally high diastolic blood pressure, a strong indicator to overrule the HCT decision under serious health conditions.

At this point, the correct classification rate is 0.77604, with  $n_{12}=0$  and  $n_{21}=43$ . Note that  $n_{21}$  means the number of misclassified members, thereby these members are treated as not administering HCT ( $D=2$ ) when in fact they need for administering HCT ( $D=1$ ). Misclassifying  $D=1$  as  $D=2$  is more serious than that of  $D=2$  as  $D=1$  since the penalty for the former error is life or death; whereas, the consequence of the latter is merely a waste of CT resource utilization. Note that of 192 patients only 48 patients were classified as  $D=1$ ; moreover, of these 48, the classification was correct only five times. Since correct classification rate for the 48 patients was very low, it is worthwhile to investigate why  $n_{21}$  cannot be reduced. By examining the sorted data of  $n_{21}=43$ , one notices

that the age factor has been overlooked. When considering the age factor ( $A_2$ ), a new rule is formulated to reduce  $n_{21}$ : when ( $A_2=5$ ,  $A_3=1$ ,  $A_6=3$ ) then  $D=1$ . This means that when the patient's age is above 65 years, the triage level is 1, and the diastolic blood pressure is above 110 mm Hg, the patient should be administered HCT. When this rule is applied in addition to the previous composite rules, the correct classification rate is increased to 0.79167, with  $n_{12}=1$  and  $n_{21}=39$ . The result of  $n_{12}=1$  is an exception to the previous rule. Apparently, nothing can be done to further reduce the  $n_{21}$ . The parallel coordinate plot (PCP) of seven variables ( $A_1$ - $A_7$ ) for the data of  $n_{21}=39$  is shown in Figure 6.

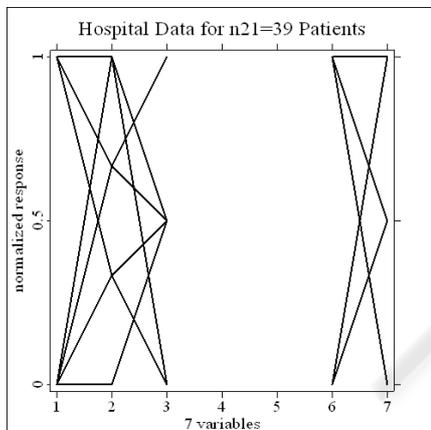


Figure 6: Parallel coordinate plots of 39 hospital data.

Several points are noteworthy. First, in comparison with Figure B1, Figure 6 has only one colour (black) since the data of  $n_{21}=39$  are members of  $D=1$  only. Second, there is no connection between variables  $A_3$  through  $A_6$ , thereby, indicating that the levels of  $A_4$  and  $A_5$  are of single value. Indeed,  $A_4=1$  and  $A_5=2$ , thus indicating that patients having a clear mental status and a normal breathing rate are easily misclassified as not needing HCT, an understandable error. Third, there are four age levels (2-5) instead of the five in the original setting. Each age level is connected with two triage levels except for age level 2 of which is connected to only triage level 2. In comparison with the PCP in Figure B1, the pattern in Figure 6 is quite different, wherein each age level is connected to almost every triage level. Fourth, the diastolic blood pressure is shown only for levels 2 and 3, thereby indicating that none of the patients has normal blood pressure. Moreover, the pulse levels are at 1-3, thus indicating that none of the patients has an unusually high pulse rate (greater

than 120/min). Furthermore, there is no connection between  $A_6=2$  and  $A_7=1$ , thereby indicating that no patient has blood pressure in the range of 80 to 110 mm Hg and a pulse rate lower than 60/min.

After a close examination of the sorted 39 data sets, another rule is discovered: if ( $A_2=5$ ,  $A_3=1$ ,  $A_4=1$ ) then ( $D=1$ ). This rule indicates that when the patient is very old (more than 65 years) and has a triage level of 1 and a clear mental status, HCT should be administered. This rule will reduce one mistake in  $n_{21}$ , thereby rendering the correct classification rate of 0.79688 with  $n_{12}=1$  and  $n_{21}=38$ , the optimum discoverable solution. The PCP of the  $n_{21}=38$  data set is shown in Figure 7. When comparing Figures 6 and 7, one notices that the line connecting the normalized value of  $A_2=1$  to  $A_3=0$  in Figure 6 has been deleted from Figure 7.

There is no observable distinction between  $D=1$  from 38 patients and  $D=2$  from 122 patients, extracted from 144 data sets, wherein  $D=2$  in the test data has the response variable  $D=2$  with  $A_2>1$  and  $A_4=1$  and  $A_5=2$  and  $A_6>1$  and  $A_7<4$ . The aforementioned conditions set for  $D=2$  are exactly the same as for the 38 sets except for  $D=1$ . The PCP of the 122 sets is shown in Figure 8. When comparing Figures 7 and 8, it is clear that if the line segments in each Figure are treated as elements in a set, then Figure 7 can be regarded as contained in Figure 8 in terms of the set concept, thereby demonstrating that since the 38 sets are prominently involved with the corresponding 122 sets, the two cannot be separated by any rule. For the sake of completeness, part of the XploRe (Hardle, Klinke and Muller, 2000) code is listed to illustrate the formulation of the composite rule in Appendix D. The self-explanatory code is similar to the c code.

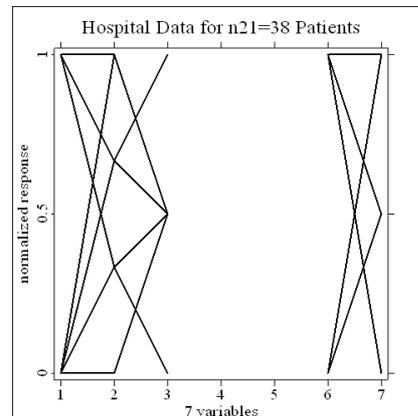


Figure 7: Parallel coordinate plots of 38 hospital data.

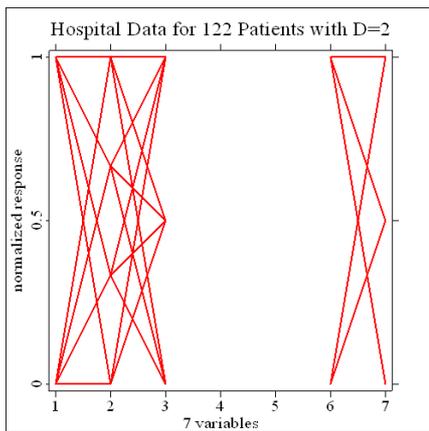


Figure 8: Parallel coordinate plots of 122 hospital data.

## 5 CONCLUSIONS

Two multivariate techniques have been proposed to clarify patients sent to an emergency room to wait for a decision on the administration of HCT. The 959 data set were segmented into two portions, a 767 training data set and a 192 test data set, after which Fisher's linear discriminant function was used to find linear rule vector  $a$ . Since classification using rule vector  $a$  in equation (4) is not practical for on-duty physicians, three important variables, such as triage (A3), mental status (A4) and breathing rate (A5), were chosen on the basis of the magnitude of the coefficients of  $a$ . Next, correspondence analysis was used to determine the simple classification rule most suitable for each variable to classify the need for administering HCT. The simple classification rule has the format of ( if A5=x then D=y ) where x and y are the levels of the input (A5) and output (D) variables, respectively. The selection of the rule is based on the shortest Euclidean distance between the levels of the input variable (e.g., A5) and response variable D located on the x-axis of a biplot. The case study demonstrated that output from the joint effort of the multivariate technique coincided with the exhaustive search, a promising result. The optimum correct rate is only 0.75521 with  $n_{12}=0$  and  $n_{21}=47$  for the rule of (if A5=2 the D=2), meaning that if the patient's breathing rate lies within the normal range of 10~24/min, HCT is not needed.

The extension of a single-variable classification rule to a two-variable one is straightforward, yet requiring a small modification for choosing the rule. First, a composite variable (e.g., A9) is formed by a

linear combination of the two variables based on equation (10) so that each combination of the levels from the two maps into an integer level of the composite variable. Then typical CorrAna is applied to the contingency table formed by variables A9 and D, wherein a biplot is produced with points representing both the levels of the composite variable and response variable D. By taking the two points of the levels of D as the demarcation points, the x-axis can be cut into three regions: one to the left of the left extremity, the second between the demarcations, and the third to the right of the right extremity. Moreover, the levels in the left regions are assigned to the level of D at the left demarcation point, the levels in the right regions to the level of D at the right demarcation point, and the levels of the composite variable between to the level of D on the basis of the optimum classification rule from the single variable. The two variables selected are triage (A3) and mental status (A4), which render the highest correct classification rate among all combinations of two variables. The rules state that when (A3=2 and A4=4) or (A3=1 and A4=3 or 4 or 2) D=1; whereas, when (A3=3 and A4=2) or (A3=3 and A4=1), D=2. Thus, the correct classification rate is 0.76562 with  $n_{12}=2$  and  $n_{21}=43$ .

The correct classification rate can be further increased by examining the structure of the sorted but misclassified items in the test data set. The formulation of the composite rule for the case study is listed in Appendix D, with the correct classification rate of 0.79688 with  $n_{12}=1$  and  $n_{21}=38$ . The composite rules may be generally summarized as (1) when the triage level is 2 and the mental status is a coma, or when the triage level is 1 and the mental status is unclear, HCT should be administered, and (2) when the patient is in level 3 of triage and the mental status is one capable of responding to a verbal stimulus, HCT is unnecessary. Exceptional rules should be applied to patients older than 65 years (A2) and those with high diastolic blood pressure (A6). For example, (1) when the triage level is 1, the mental status is 'to pain' or 'coma', and the diastolic blood pressure is above 110 mm Hg (seriously high), the patient should not be administered HCT; (2) when the patient's age is older than 65 years, the triage level is 1, and the diastolic blood pressure is above 110 mm Hg, the patient must be administered HCT. It is noteworthy that the variables of sex (A1) and pulse rate (A7) are not considered in the composite rules.

To show why the correct classification rate cannot be increased, two parallel coordinate plots of the  $n_{21} = 38$  data set being in  $D=1$  and the corresponding 122 data set being in  $D=2$  were compared. The two data sets had the same domains for variables A1-A7. The comparison indicated that since both are prominently involved (highly similar), they cannot be separated by any rule. Thus, no improvement can be made in the correct classification rate.

## ACKNOWLEDGEMENTS

I gratefully thank my colleague Prof. Paul Chen for both encouraging me to pursue this research and generally sharing his hospital data. I also express appreciation to Dr. Cheryl Rutledge, Department of English, DaYeh University, for her editorial assistance.

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