Active Queue Management: Comparison of Sliding Mode Controller and Linear Quadratic Regulator

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Abstract: - Active Queue Management (AQM) takes a trade-off between link utilization and delay experienced by data packets. From control point of view, it is rational to regard AQM as a typical regulation system. In this paper, two types of controllers, i.e. Sliding Mode Variable Structure (SMVS) controller and optimal Linear Quadratic Regulator (LQR) are designed for AQM. Simulation results conform the robust performance of SMVS controller against the disturbance. At the same time, a complete comparison between SMVS and LQR controllers is made. The conclusion is that both transient and steady state performance of SMVS controller is better than that of LQR one.

Key-Words: - Active queue management, congestion control, robust control, sliding mode control, linear quadratic regulator.

1 Introduction
TCP congestion control mechanism, while necessary and powerful, are not sufficient to provide good service in all circumstances, specially with the rapid growth in size and the strong requirements to QoS support, because there is a limit to how much control can be accomplished at end system. It is needed to implement some measures in the intermediate nodes to complement the end system congestion avoidance mechanisms. Active Queue Management (AQM), as one class of packet dropping/marking mechanism in the router queue, has been recently proposed to support the end-to-end congestion control in the Internet [1]. It has been a very active research area in the Internet community. The goals of AQM are (1) reduce the average length of queue in routers and thereby decrease the end-to-end delay experimented by packets, and (2) ensure the network resources to be used efficiently by reducing the packet loss that occurs when queues overflow. AQM highlights the tradeoff between delay and throughput. By keeping the average queue size small, AQM will have the ability to provide greater capacity to accommodate nature-occurring burst without dropping packets, at the same time, reduce the delays seen by flow, this is very particularly important for real-time interactive applications. RED [2] was originally proposed to achieve fairness among sources with different burst attributes and to control queue length, which just meets the requirements of AQM. However, many subsequent studies verified that RED is unstable and too sensitive to parameter configuration, and tuning of RED has been proved to be a difficult job [3,4]. The variable structure controller is known to be robust to parameter uncertainty and external disturbance because of the sliding motion on a predefined hyperplane [5,6]. The most distinguishing property of variable structure control systems (VSCS) is that the closed loop system is completely insensitive to system uncertainties and external disturbances. The increased interest in the use of sliding mode strategy in the control algorithms is explained by the fact that robustness has become a major requirement in modern control applications. A great deal of efforts has been put on establishing both theoretical VSCS concepts and practical applications [7,8,9,10]. Due to its excellent invariance and robustness properties, variable structure control has been developed into a general design method and extended to a wide spectrum of system types including multivariable, large-scale, infinite-dimensional and stochastic systems.

The intuition and heuristic design is not always scientific and reasonable under any conditions. Of course, since Internet is a rather complex huge system, it is very difficult to have a full-scale and systematic comprehension, but importance has been considerably noted. The mathematical modeling of the Internet is the first step to have an in-depth understanding, and the algorithms designed based on the rational model should be more reliable than one original from intuition. In
some of the references, the nonlinear dynamic model for TCP flow control has been utilized and some controllers like LQR and Adaptive Virtual Queue Algorithm have been designed for that [11,12,13,14,15]. Although LQR controller successfully related some limitations of RED, for instance, the queue length and dropping/marking probability are decoupled, whenever the queue length can be easily controlled to the desired value; the system has relatively high stability margin. The shortcomings of LQR controller are also obvious. The modification of probability excessively depends on buffer size. As a result, for small buffer the system exhibits sluggishness. Secondly, for small reference queue length, the system tends to performance poorly, which is unfavorable to achieve the goal of AQM because small queue length implies small queue waiting delay. Thirdly, the status of actual network is rapidly changeable, so we believe that it is problematic and unrealistic, at least inaccurate, to take the network as a linear and constant system just like the designing of LQR controller. Affirmatively, the algorithm based on this assumption should have limited validity, such as inability against disturbance or noise. We need more robust controller to adapt complex and mutable network environment, which will be our motivation and aim in this study. In the research, we will apply one of the advanced robust control theory, variable structure sliding mode control, to design the AQM controller, and expect it to have the perfect performance and be better suited for AQM than LQR controller.

2 TCP flow control model

In [11], a nonlinear dynamic model for TCP flow control has been developed based on fluid-flow theory. This model can be stated as follows

\[
\begin{align*}
\frac{dW(t)}{dt} &= \frac{1}{R(t)} - \frac{W(t)W(t-R(t))}{2R(t)} p(t-R(t)) \\
\frac{dq(t)}{dt} &= \frac{N(t)}{R(t)} W(t) - C(t)
\end{align*}
\]

The above nonlinear and time-varying system was approximated as a linear constant system by small-signal linearization about an operating point [16] (Fig. 1). In the block diagram, \( C(s) \) and \( G(s) \) are the controller and the plant, respectively. The meaning of parameters presented in Fig. 1 are as following

\[
K(t) = \left[ \frac{R(t)C(t)}{2N(t)} \right]^3, \quad T_1(t) = R(t), \quad T_2(t) = \frac{R^2(t)C(t)}{2N(t)}
\]

where

- \( C(t) \) : Link capacity (packets/sec)
- \( q_o \) : Queue reference value
- \( N(t) \) : Load factor, i.e., number of active sessions
- \( R(t) \) : Round-trip time (RTT), \( R(t) = 2 \left( \frac{q(t)}{C(t)} + T_p \right) \), \( T_p \) is the fixed propagation delay
- \( p(t) \) : Dropping/marking probability
- \( q(t) \) : Instantaneous queue

We believe that the AQM controller designed with the simplified and inaccurate linear constant model should not be optimal, because the actual network is very changeful; the state parameters are hardly kept at a constant value for a long time. Moreover, the equations (1) only take consideration into the fast retransmission and fast recovery, but ignore the timeout mechanism caused by lacking of enough duplicated ACK, which is very usual in burst and short-lived services. In addition to, there are many non-respective UDP flows besides TCP connections in networks; they are also not included in equations (1). These mismatches in model will have negative impact on the performance of controller designed with the approach depending with the accurate model. For the changeable network, the robust control should be an appropriate choice to design controller for AQM. The variable structure sliding mode control action is one of the best that can help us.

3 Sliding mode control design

In this section, we will discuss of SMVS [5,6] controller design for AQM. Firstly, suppose that \( x_1 = e \), \( x_2 = de/dt \), so the plant depicted in Fig. 1 is described by a second order system as

\[
\begin{align*}
\frac{dx_1}{dt} &= x_2 \\
\frac{dx_2}{dt} &= -a_1(t)x_1 - a_2(t)x_2 - b(t) + F(t)
\end{align*}
\]
\[ a_{1\text{min}} \leq a_1 \leq a_{1\text{max}} \]
\[ a_{2\text{min}} \leq a_2 \leq a_{2\text{max}} \]
\[ 0 < b_{\text{min}} \leq b \leq b_{\text{max}} \]

where
\[ a_1(t) = \frac{1}{T_1(t)T_2(t)}, \quad a_2(t) = \frac{T_1(t) + T_2(t)}{T_1(t)T_2(t)}, \quad b(t) = \frac{K(t)}{T_1(t)T_2(t)} \]

\[ F(t) = \frac{d^2}{dt^2} q_o + \frac{T_1(t) + T_2(t)}{T_1(t)T_2(t)} \frac{d}{dt} q_o + \frac{1}{T_1(t)T_2(t)} q_o \]

(4)

\[ F(t) \] is regarded as the system disturbance. For the convenience of implementation, the following control law is adopted

\[ p = \psi x_1. \] (5)

where

\[ \psi = \begin{cases} \alpha, & \text{if } x_1 \sigma > 0 \\ \beta, & \text{if } x_1 \sigma < 0 \end{cases} \] (6)

where \( \sigma \) is the switching function [5,6] as

\[ \sigma = c x_1 + x_2 = 0; \quad c > 0 \text{ is scalar} \] (7)

According to the existence condition for a sliding line: \( \lim_{\sigma \to 0} \sigma \frac{d\sigma}{dt} < 0 \) [5], we have

\[ \alpha \geq \max_{a_1,a_2,b} \left\{ \frac{1}{b(t)} \left[ ca_2(t) - c^2 - a_1(t) \right] \right\} \]

\[ \beta \leq \min_{a_1,a_2,b} \left\{ \frac{1}{b(t)} \left[ ca_2(t) - c^2 - a_1(t) \right] \right\} \] (8)

Here, if conditions (8) are satisfied, the system must have a sliding regime on the switching line \( \sigma = 0 \), but we are unable to determine if the system could hit this sliding line. The following theorem can answer this question

**Theorem [5]** A necessary and sufficient condition for hitting to occur in the system consisting with (2) and (5) is that the characteristic equation (9) has no non-negative real roots

\[ p^2 + a_{2\text{min}} p + a_{1\text{min}} + \inf_T \{ c \alpha \} = 0 \] (9)

The conservative condition that equation (9) has no non-negative roots is that it has complex roots, so

\[ a_{2\text{min}}^2 - 4(a_{1\text{min}} + \inf_T \{ c \alpha \}) < 0 \] (10)

Namely,

\[ \alpha > \frac{a_{2\text{min}}^2 - 4a_{1\text{min}}}{4b_{\text{min}}} \] (11)

For a variable-parameters and single-input system, the excellent performance could be reached if the conditions (11) and (8) would be satisfied, at the same time, the control law (5) should also be implemented. For the sliding parameter \( c \), only requirement is to keep it more than zero [5]. Theoretically speaking, \( c \) should be relatively large since it is larger the transient process is shorter. However, for the network queue management system, \( c \) is limited. Taking \( \rho(t) \) into consideration, its meaningful value is in section [0,1], i.e., the control variable is limited. Therefore, we need to choose the combination of constant control and proportional control as new control law

\[ p = \begin{cases} \psi x_1, & |x_1| < M \\ \theta, & |x_1| > M \end{cases} \] (12)

where

\[ \theta = \begin{cases} M, & \sigma > 0 \\ m, & \sigma < 0 \end{cases} \] (13)

\( M \) and \( m \) denote the maximum and minimum of control variable, respectively, i.e., \( M = 1 \) and \( m = 0 \). The other parameters are same with (5). For the sake of convenience, suppose that \( k = \alpha > 0 > \beta = -k \). Since the control variable is limited, the sliding mode regime in phase space will be restricted in a certain scope, so that the choice of sliding mode parameter \( c \) is also restricted, otherwise, the controller variable, queue length, would rush out of the sliding mode regime, and sharp overshoot and great oscillation will occur, which degrades the performance of router.

We start to design SMVS controller for AQM based on the above theory and approach. For the purpose of SMVS controller being suitable for most of dynamic systems, the varying scope of parameters in TCP/AQM system is assumed as following

\[ N(t) : 1 \to 300, T_q = 0.02 \text{sec}, q_o : 0 \to 300 \text{packets}, \]
\[ C(t) : 1250 \to 7500 \text{packets/sec} \]

Therefore,

\[ a_{2\text{min}} = 3.8501, a_{2\text{max}} = 1250, a_{1\text{min}} = 0.015, \]
\[ a_{1\text{max}} = 60000, b_{\text{min}} = 2604.2, b_{\text{max}} = 28125000 \]

Let the sliding mode parameter \( c = 2 \).

According to the existence condition (8), we have

\[ \alpha > 0.958, \beta < -0.0021 \]
With the reachable condition of sliding mode (11), we also have
\[ \alpha > 0.0015 \]
Thus, the controller parameters can be determined as
\[ k = \alpha = -\beta = 0.96 \]
So far, a novel AQM algorithm based sliding mode variable structure control theory is obtained.

4 Linear quadratic regulator design
Consider the system described by state space equation as in (2). In order to design the LQR controller, the parameters should not be uncertain, so the following parameters are made:
\[ a_1 = \frac{a_{1\text{max}} + a_{1\text{min}}}{2}, \]
\[ a_2 = \frac{a_{2\text{max}} + a_{2\text{min}}}{2}, \]
\[ b = \frac{b_{\text{max}} + b_{\text{min}}}{2}, \]  \hspace{1cm} (14)
To design the optimal controllers for such system, a cost function is considered as:
\[ J = \frac{1}{2} \int_0^{\infty} \left( q_1 x_1^2 + q_2 x_2^2 + q_3 p^2 \right) \]  \hspace{1cm} (15)
For this system, the following parameters are assumed:
\[ q_1 = q_2 = 10, q_3 = 1 \]  \hspace{1cm} (16)
The corresponding optimal control law can be obtained by solving the Algebraic Riccati Equation (ARE).

5 Simulation results
The performance of the proposed controller is simulated using MATLAB. The network topology used for simulation, is depicted in Fig. 2. The only bottleneck link lies between node A and node B. The buffer size of node A is 300 packets, and default size of the packet is 500 bytes. Queue A is SMVS scheme, and the others are Drop Tail. All sources are classed into three groups. The first one includes \( N_1 \) greedy sustained FTP application sources, the second one is composed of \( N_2 \) burst HTTP connections, each connection has 10 sessions, and the number of pages per session is 3. The third one has \( N_3 \) UDP sources, which follow the exponential service model, the idle and burst time are 10000ms and 1000ms, respectively, and the sending rate during "on" duration is 40kbps. We introduced short-lived HTTP flows and non-responsive UDP services into the router in order to generate a more realistic scenario, because it is very important for a perfect AQM scheme to achieve full bandwidth utilization in the presence of noise and disturbance introduced by these flows. The links between node A and all sources have the same capacity and propagation delay pair \( (L_1, \tau_1) \).
The pair \( (L_2, \tau_2) \) and \( (L_3, \tau_3) \) define the parameter of links AB and BC, respectively.

In the first study, we will use the most general network configuration to testify whether the SMVS controller can reach the goals of AQM, and freely control the queue length to stabilize at the arbitrary expected value. Therefore, given that \( (L_1, \tau_1) = (10\text{Mbps} , 15\text{ms}) \), \( (L_2, \tau_2) = (5\text{Mbps} , 15\text{ms}) \), \( (L_3, \tau_3) = (45\text{Mbps} , 15\text{ms}) \). \( N_1 = 270 \), \( N_2 = N_3 = 0 \).
Let the expected queue length equal to 75 packets, the instantaneous queue length is depicted in Fig. 3. After a very short regulating process, the queue settles down its stable operating point. RED algorithm is unable to accurately control the queue length to the desired value. The queue length varies with network loads. The load is heavier the queue length is longer. Attempting to control queue length through decreasing the interval between high and law thresholds, then it is likely to lead queue oscillation. Although LQR controller could regulate the queue to the fixed point, the integrated performance needs to be improved, such as the transient process is too long and the fluctuation in steady state is great, for small queue length, which lows the link utilization. The queue evaluation of router A, controlled by LQR controller \( (q_o = 75\text{packets}) \), is plotted in Fig. 4. Evidently, LQR controller takes the longer time to settle down the reference point. For the sake of clearness, the curves of probability are plotted in Fig. 5.
Considering the requirement of the steady state performance, it is impractical to increase the difference between $a$ and $b$ to speed up the response of LQR controller. With the higher sampling frequency, the computation will be significantly exhausted. The only feasible way is to add the buffer size. In order to illustrate this ability, we redo the above simulation with 600 packets buffer size, which the results are also plotted in Figs. 4 and 5. Indeed, the large buffer is able to enhance the responsibility of LQR controller, but this ability is limited, moreover it seems to be wasteful. Conversely, the SMVS controller has the ideal performance without any additional regulation mechanism. In order to evaluate the performance in steady state, we calculate the average and the standard deviation of the queue length in steady state. For the convenience of comparison, choose the queue length between 40 and 50 seconds as sample data. In this case, the standard deviation of LQR controller (32.3336) is much larger than that of SMVS controller (2.5928). Fig. 6 presents the case of small reference queue length. Except $q_o = 15$, the other parameters are unchangeable.

In this section, Firstly, let $N_1 = 270, N_2 = 400, N_3 = 0$, the evaluation of queue size is shown in Fig. 7. As it can be seen, the proposed SMVS controller has better performance than that of LQR one. Next, given that $N_1 = 270, N_2 = 0, N_3 = 50$, we further investigate performance against the disturbance caused by the non-responsive UDP flows. Fig. 8 shows the results, obviously, the LQR controller is very sensitive to this disturbance, while the SMVS controller operates in a relatively stable state. The queue fluctuation increases with introducing the UDP flows, but the variance is too much smaller comparing with LQR one.
6 Conclusion
In this paper, a robust sliding mode variable structure controller was designed for the objective of active queue management. For this purpose, a linearized model of the TCP flow [12,16] was considered. We applied a robust SMVS control strategy to this system because this advanced robust control methodology is insensitive to system dynamic parameters and is capable of being against disturbance and noise, which is very suitable for the mutable network environment. We took a complete comparison between performance of the proposed SMVS and LQR controllers under various scenarios. The conclusion was that the integrated performance of SMVS controller was superior to that of LQR one. The SMVS controller was very responsive, stable and robust, especially for the small reference queue system, but its performance was inferior when active TCP sessions were relatively small. Thus, it will be very imperious to design the controller suitable for light load, and then integrate it with SMVS controller using adaptive control technology.

References: