Monte Carlo Simulations for Real Estate Valuation

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Abstract

We use the Adjusted Present Value (APV) method with Monte Carlo simulations for real estate valuation purposes. Monte Carlo simulations make it possible to incorporate the uncertainty of valuation parameters, in particular of future cash flows, of discount rates and of terminal values. We use empirical data to extract information about the probability distributions of the various parameters and suggest a simple model to compute the discount rate. We forecast the term structure of interest rates using a Cox et al. (1985) model, and then add a premium that is related to both the real estate market and selected property-specific characteristics. Our empirical results suggest that the central values of our simulations are in most cases slightly less than the hedonic values. The confidence intervals are found to be most sensitive to the long-term equilibrium interest rate being used and to the expected growth rate of the terminal value.

Keywords: Real estate valuation; Monte Carlo simulations; Adjusted Present Value (APV)
JEL codes: R32, G12, G23

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Executive Summary

The discounted cash flow method is now widely used as a valuation method for income-producing real estate in many countries. In fact, it is generally accepted that the method yields a fair value estimate in the spirit of the new accountancy standards. This method is very useful indeed, but does suffer from some limitations. These include the facts that (1) the value of the property is needed to compute the discount rate, (2) the discount rate is assumed to be constant during the entire holding period, and (3) uncertainty is not explicitly taken into account.

The main contribution of this paper is in incorporating uncertainty in the valuation process. This is achieved by using the distributions of the various input parameters rather than point estimates as is customary in DCF valuations. We also deal with the two other limitations of the DCF approach in that we develop a method to estimate the discount rate which does not require prior knowledge of a property’s value, and use a time-varying discount rate. The distributions of the parameters are constructed using various data from the Swiss financial and real estate markets, and the empirical analysis is performed using an institutional portfolio of 30 properties located in Geneva.

Our results show that the estimated values are on average 6.7% below hedonic values. This would be expected in the current environment. As our discount rate estimates contain a mean-reversion component, they will overshoot market interest rates during periods of very low interest rates, and hence values will be somewhat conservative during such periods. Also, the standard deviation of our present values is in most cases approximately 10% and is positively related to the percentage of the property which is devoted to commercial uses. The sensitivity analyses suggest that the value estimates react most to changes in the long-term interest rate (one of the components of the discount rate) and to changes in the growth rate of the terminal value. Hence, valuers should take great care in choosing what values to assign to these parameters.

The appeal of incorporating uncertainty in the valuation process is that the analysis does not merely yield a point estimate of the entire distribution of values, but rather the distribution of values. Hence, the probability of the true value of a property being less than various thresholds can be ascertained. This should certainly prove useful to many investors. The
Royal Institution of Chartered Surveyors (RICS) in the U.K., for instance, is currently examining how uncertainty can be used together with the value estimate, which highlights the importance of uncertainty for the valuation profession. Also, there has been some debate in the literature about valuation variation and the margin of error in valuing properties. The approach which is advocated in this paper should constitute a contribution to this debate.
Monte Carlo Simulations for Real Estate Valuation

1. Introduction

Among the various approaches to valuing real estate, the discounted cash flow (DCF) method, using the weighted average cost of capital (WACC) as the discount rate, is well accepted by academics and broadly used by practitioners. The consensus derives from the model’s advantages, in particular its economic rationality. The DCF method takes into account the time value of money and has a unique result regardless of investors’ risk preferences (Mun, 2002). In addition, the procedure is clearly defined and can easily be used by valuers.

Although the DCF method plays a crucial role in valuation, it suffers from at least three pitfalls. First, the traditional DCF analysis is performed under deterministic assumptions (for a discussion, see Wofford, 1978; Mollart, 1988; French and Gabrielli, 2004). In other words, one does not take into account uncertainty in the estimated cash flows; the entire process is therefore devalued when forecasts do not materialise or even when inputs are slightly manipulated (Kelliher and Mahoney, 2000; Weeks, 2003). This criticism is particularly severe in real estate valuation since the terminal value, which is dependent on the last forecasted free cash flow, the perpetual rate of growth and on the discount rate, is in most cases the largest component of the present value. If such parameters are not determined very rigorously, the estimated value of a property can be very far off its market value. When the latter value is known, one can also say that it is easy to set parameters so as to obtain a present value that is close to it.

Another drawback of the DCF method is that there is a circularity problem when part of the asset is financed by debt. Indeed, the value of the asset is required to compute the WACC, but the value of the asset is precisely what we are looking for. Finally, the discount rate is assumed to be constant through time though research has shown that prices and returns on financial assets are related more to changes in the required rate of return than to changes in expected cash flows (Fama and French, 1989; Ferson and Campbell, 1991). To model the time-varying nature of the required rate of return, Geltner and Mei (1994) and Clayton (1996) use a vector autoregressive procedure to analyse returns on private real estate. The latter
author, for instance, finds that the risk premium on direct unsecuritised commercial real estate varies over time and is strongly related to general economic conditions.

In this research, we use the Adjusted Present Value (APV) methodology, developed by Myers (1974), but by adding Monte Carlo simulations. Under some assumptions, the APV method yields the same results as the widely used DCF technique (Fernandez, 2005), but it solves the circularity problem created by debt financing (Achour-Fischer, 1999). In addition, with Monte Carlo simulations, which are based on statistical measures and probability distributions of the variables that enter in the APV method, we address the uncertainty issue.

With the APV methodology, the discount rate represents the required rate of return for fully equity-financed properties. Many data analyses have lead us to conclude that the Capital Asset Pricing Model (CAPM) is in most cases not applicable to estimate this required rate of return\(^1\). First, there are usually not sufficient historical data for direct real estate investments. Second, an appropriate definition of the market portfolio and in particular of the relative weight of real estate in such portfolio is difficult. Third, the returns on indirect real estate investments may be poor proxies for direct real estate returns (Lizieri and Ward, 2000). This problem is exacerbated when one attempts to remove the effect of leverage. Further, historical returns may be poor proxies for expected future returns (Geltner and Miller, 2001). Finally, as mentioned previously, most such models assume that risk is constant over time.

The contributions of the paper are as follows. First, we address formally the issue of uncertainty in valuing real estate. This is achieved by using a Monte Carlo approach within an APV framework. Further, our approach prevents subjective changes of the values of the parameters used to compute the terminal value, as these are obtained by clearly defined models or procedures. Finally, we model the discount rate by considering that it has two components: a risk free interest rate and a risk premium. We model the interest rate by using the Cox et al. (1985) model. Such model allows us to assume that the discount rate is not constant through time and that it depends on the present level of interest rates and their volatility. We suggest an innovative solution to estimate the risk premium which is assumed to depend on a real estate market premium and on property specific attributes. The attributes are measured by selected hedonic attributes which include the quality of location, the age and

\(^1\) A notable exception to this is Baroni et al. (2001).
the quality of buildings. Hence our method considers that risk is multidimensional and is not only related to covariance with the market as posited by the CAPM. In that sense it is more closely related to Arbitrage Pricing Theory (APT).

The Monte Carlo technique, whose name comes from the famous casino in Monaco\textsuperscript{2}, was developed by famous scientists, such as Enrico Fermi, in the 1930s when calculating the neutron diffusion, or John von Neumann and Stanislaw Ulam who established the mathematical basis for probability density functions (Fishman, 1999). It has been subsequently used to solve problems related to the atomic bomb, medicine, chemistry, astronomy or agriculture. In finance, Monte Carlo simulations have also been largely used for many years, in particular to price derivatives, to forecast stock prices or interest rates, as well as in capital budgeting (Dixit and Pindyck, 1994). In real estate research, authors like Pellat (1972) and Pyhrr (1973) have used simulations – but not Monte Carlo simulations - to analyse uncertainties related to investment forecasting. In the same vein, Mallison and French (2000) analyse the uncertainty issues related to any valuation. The Monte Carlo simulation technique has also been applied to forecast future cash flows in order to improve long-term decisions in real estate (Kelliher and Mahoney, 2000; Tucker, 2001; French and Gabrielli, 2004). Our approach differs from previous research in that we forecast a time-varying discount rate that also includes a premium related to selected hedonic characteristics. In practice, the use of the Monte Carlo simulation technique is quite limited, probably partly due to the mathematical and statistical dimension of this approach\textsuperscript{3}.

We apply our approach to an institutional real estate portfolio for which we have the estimated hedonic value for each of 30 properties. This allows us to compare our simulated values with the hedonic estimates. Overall, we find that the central values of the simulations are quite similar (albeit lower) to the hedonic values, but the standard deviation of the present value estimates provides for an interesting measure of risk. In addition, the sensitivity analysis clearly shows the crucial role played by the growth rate in calculating the terminal values, but also of long-term interest rates.

\textsuperscript{2} The mathematician Stanislaw Martin Ulam tells in an autobiography that the method was called Monte Carlo to honour his uncle who was a tenacious gambler at the Monaco casino.

\textsuperscript{3} In Switzerland, the CIFI (Centre d'Information et de Formation Immobilières) uses this approach for the valuation of real estate portfolios.
The remainder of the paper is organised as follows. In section 2, we briefly present the APV methodology and highlight how it addresses some of the pitfalls of the traditional DCF technique. Section 2 also contains a discussion of how we estimate the various components of the APV and of the hypotheses that are made concerning the probability distributions of variables. The data and some descriptive statistics are presented in section 3, while section 4 contains the results of Monte Carlo simulations and of their sensitivity. Section 5 concludes.

2. Method

The APV methodology postulates that an asset has a value under perfect market conditions plus, possibly, an additional value resulting from market imperfections. Considering among market imperfections only the debt financing and using forecasted cash flows for a finite time horizon, the value of a property can be written as follows:

\[
P V_0 = \sum_{t=1}^{T} \frac{FCF_t}{(1 + k_u)^t} + \sum_{t=1}^{T} \frac{k_i \cdot \tau \cdot D_t - 1}{(1 + k_i)^t} + \frac{TV_T}{(1 + k_u)^T}
\]

where

- \( PV_0 \) = value of the property at time \( t=0 \)
- \( FCF_t \) = free cash-to-property at time \( t \) (\( t = 1 \) to \( T \))
- \( D_t \) = value of debt at time \( t \)
- \( TV_T \) = terminal value at time \( T \)
- \( k_u \) = cost of capital for a fully equity-financed property
- \( k_i \) = pre-tax cost of debt
- \( \tau \) = tax rate

The advantage of equation (1) above the standard DCF formula with the average cost of capital as the discount rate is that it considers the debt financing effects separately and consequently resolves the circularity problem. Moreover, the free cash flows are discounted at a rate that can be obtained from pension funds, as such investors in many countries (including Switzerland) buy properties without any leverage. When institutional investors are tax-exempt, which is the case in Switzerland but in many other countries as well, the present value of the tax shield is zero and equation (1) reduces to:

\[
P V_0 = \sum_{t=1}^{T} \frac{FCF_t}{(1 + k_u)^t} + \frac{TV_T}{(1 + k_u)^T}
\]
As the focus of this paper is the valuation of an institutional portfolio, we use equation (2) to compute the present value of a property. This requires that the behaviour of the parameters that enter into the formula be modelled: (1) the annual free cash flows during the forecasting period, (2) the terminal value at the end of the forecasting period and (3) the discount rate. For the sake of simplicity we will use the same model regardless of whether the properties are entirely residential or whether some fraction of the property is devoted to other uses. Swiss institutional investors predominantly purchase residential properties, with such use accounting for approximately 85% of their real estate holdings.

2.1 Free cash flows (FCF)

For tax-exempt investors, the free cash flow to property for year \( t \) can be written as:

\[
FCF_t = (1 - \nu_t) PGI_t - C_t - CAPEX_t
\]

where

- \( \nu_t \) = vacancy rate in year \( t \)
- \( PGI_t \) = potential gross income in year \( t \)
- \( C_t \) = operating cash expenses in year \( t \)
- \( CAPEX_t \) = additional investment (ie capital expenses) in year \( t \)

Rents are the major source of cash inflows and they depend on future market conditions, the characteristics of the properties, but also on various legal constraints. The potential gross income (PGI) for the first year (Year 1) is assumed to be known for the various components of the property (apartments, underground garages, shops, etc.). We then assume that the growth of the PGI over time is normally distributed. The choice of the mean and the standard deviation of the growth rate is crucial. Growth will depend not only on macroeconomic factors such as expected GDP growth, expected inflation or demographic phenomena, but also on property-specific characteristics such as the quality or the age of building, but also the quality of location. The actual level of rents partly captures these variables, but we have to recognise that the appropriate future growth rate for a well located and well constructed new building might be quite different from the rate applicable to a low quality and poorly located old building. From a theoretical point of view, it would be better if various growth rates could be considered, but in practice these are very difficult to estimate. The growth rate of rents is one of the key drivers of property values and therefore its estimation should rely on a
procedure that is as objective as possible. In this paper, we use historical data to proxy for future growth rates.

The level of the cash inflow is also a function of a specific type of risk related to real estate investment, ie the vacancy rate ($\nu$). We will assume that the latter is uniformly distributed between the historical minimum and maximum vacancy rates for similar properties. By multiplying the PGI by $(1-\nu)$, we obtain the rent or total rent, ie the amount of cash inflow that is expected from renting out the property. For the sake of simplicity, we omit to explicitly consider the rate of unpaid rent (ie tenants who do not pay their rent), which implies that the PGI is net of unpaid rent.

Cash outflows include operating expenses, property taxes, insurance, and utilities. These are largely fixed, ie they will occur whether the property is or is not fully occupied. The variable component of these expenses is largely dependent on the age of the building, such that we will model the uncertain part of these expenses as a function of both age and rent. Historical data and professional expertise can help determine the level of annual fixed expenses as a percentage of rents and be useful in creating a model to estimate variable expenses. If sufficient data were available, one could also model the level of operating expenses by including other independent variables, such as the building quality or the quality of recent improvements.

Additional investments have to be forecasted to maintain or to improve the quality of the properties, or in some cases to increase their size. The amounts taken into consideration should be those that are forecasted by the owner, preferably with the help of an architect who has received a clear mandate to estimate the future investments required to reach the goals set above. In some countries or cities, due to legal restrictions to rent increases, one difficulty will then be to model future cash flows which depend on such additional investments.

2.2 Terminal value
The terminal value should be a proxy for the market value of the property at the end of the forecasting period under normal market conditions. We use Gordon’s growth model which is often used both by academics (Damodaran, 2003; Geltner and Miller, 2001) and professionals. To avoid obtaining aberrant terminal value estimates, it is important to first “normalize” the free cash flow of the last year of the horizon period. As we rely on a model
to forecast future cash flows, we will use the arithmetic mean of the free cash flows of the last three years to proxy for the normalized free cash to property of the last year. As is the case for the cost of capital, the perpetual rate of growth is highly related to the inflation rate. The residual life of a building is limited, however, such that the rate of growth will sooner or later become negative. Consequently, in countries where inflation is low, the rate of growth is low or even set equal to zero. If not, the resulting estimated terminal value is too high, considering the level of PGI at the end of the forecasted period. In other words, we argue that it is possible, and in some cases preferable, to estimate the terminal value by using the gross income multiplier that prevails under normal market conditions.

We calculate the terminal value as:

\[
TV_T = \frac{FCF_{T+1}}{k_u - \bar{g}} = \frac{(FCF_T + FCF_{T-1} + FCF_{T-2})}{3} \frac{1}{k_u - \bar{g}} \frac{1}{1 + \bar{g}}
\]

where

- \( FCF_{T+1} \) = free cash flow of period \( T+1 \)
- \( k_u \) = discount rate
- \( \bar{g} \) = perpetual growth rate of the free cash flows

### 2.3 Discount rate

To forecast the expected return on real estate, we assume that the discount rate is time-varying and dependent on market interest rates. We first assume that the discount rate for a fully equity-financed property is higher than the risk-free interest rate (thereafter interest rate) observed on the market, but lower than the historical return of stocks. Thus, the following inequality is assumed to hold:

\[
i_r < k_u < k_s
\]

where

- \( i_r \) = interest rate observed on the market
- \( k_u \) = required rate of return for a fully equity-financed property
- \( k_s \) = historical rate of return of the stock market

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\(^4\) Baroni et al. (2001) simulate the paths of the terminal value using a geometric Brownian motion. This method, however, requires that the initial value be known. We cannot use this method as the initial value (i.e., the estimated value) is precisely what we are looking for.
We then compute the discount rate, $k_u$, as the sum of the interest rate plus a risk premium that is required by investors. Thus:

$$i_r < (k_u = i_r + P) < k_s$$

(6)

where $P$ is the risk premium. The procedure used to set the interest rate and the risk premium is discussed next.

2.3.1 Interest rate model

The interest rate used should be highly correlated to the mortgage interest rate. In Switzerland, until the 1990s, the reference for mortgage rates was the savings deposit rate paid to customers plus a margin (Bruand, 1998)\(^5\). During the 1990s, some banks, in particular large banks, shifted toward another strategy, using the money market rates, such as the 3- or 6-month Libor, as the reference for the mortgage rate.

There exist various models to forecast interest rates and, in general, these have two components: the drift and the volatility. One of the most widely used model is that of Cox et al. (1985), thereafter CIR:

$$dr_t = a(b - r_t)dt + \sqrt{r_t} \sigma dW_t$$

(7)

where

- \(dr_t\) = increment in the interest rate at time \(t\)
- \(a\) = a non-negative constant (the mean-reversion speed)
- \(b\) = a constant (the long-term equilibrium interest rate)
- \(\sigma\) = the volatility of the interest rate
- \(dW_t\) = the Wiener increment, \(dW_t = W_{t+\Delta t} - W_t\)

The drift term implies that the interest rate normally will rise when it is below the long-term mean, and that it will normally fall when it is above the mean. The discrete approximation of the CIR model is as follows:

$$\Delta r = \alpha (b - r) \Delta t + \sqrt{r \sigma} \varepsilon \sqrt{\Delta t}$$

(8)

where \(\varepsilon \to N(0,1)\).

\(^5\) Bruand (1998) created a model to determine the mortgage rate for Switzerland using 6-month Eurofranc money market rates. The adjustment process, which is undertaken at most twice yearly, occurs when the trend in the movement of the interest rate is confirmed. By adopting a filter to 6-month Eurofranc rates, Bruand obtains the series of mortgage rate changes. Our objective being to forecast the interest rate levels, we will not use any filter.
In the CIR model, there exists a linear relationship between the long rates, $R(t,T)$, and the short rates, $r_t$. This relationship is as follows:

$$R(t,T) = A(t,T) + B(t,T)r_t$$

where

$$A(t,T) = \frac{2\gamma \left( \alpha + \gamma \right) \left( e^{(T-t)} - 1 \right)}{\left( \gamma + \alpha \right) \left( e^{(T-t)} - 1 \right) + 2\gamma} e^{2\gamma T}$$

$$B(t,T) = \frac{2 \left( e^{\gamma (T-t)} - 1 \right)}{\left( \gamma + \alpha \right) \left( e^{(T-t)} - 1 \right) + 2\gamma}$$

$r_t =$ short-term interest rate at time $t$

$$\gamma = \sqrt{\alpha^2 + 2\sigma^2}$$

2.3.2 The premium

The risk premium, $P$, that investors require is assumed to vary between two boundaries and to be always positive\(^6\). The size of this premium varies across countries and is also dependent on the characteristics of properties as proxied by selected hedonic attributes. This premium can therefore be divided into two parts:

$$P = p_1 + p_2$$

The first component, $p_1$, stems from the participation in the real estate market. The second component, $p_2$, is a function of property characteristics such as the quality of location, and the quality and the age of the property. To compute the $p_2$ premium, we can construct a linear rating system whose quality will depend on the set of qualitative data that are available. If hedonic characteristics are available, as is the case in this research, those can be used\(^7\). The level of $p_2$ will most likely vary across regions. From a theoretical perspective, our approach is thus close in spirit to an Arbitrage Pricing Theory (APT) set up as we consider that several sources of risk are priced.

The following procedure is suggested when the hedonic characteristics concerning the quality of construction and that of location as well as the age of buildings are available. For the first two characteristics, the ratings are 1-excellent; 2-very good; 3-normal; 4-bad. For the age of the buildings, we use the following criteria: age between 0-5 years (assigned a grade of 1), age between 5-15 years (grade of 2), age between 15-40 years (grade of 3) and age greater

\(^6\) This is because we assume that the lower boundary for the required rate of return is the interest rate.

\(^7\) It is customary for Swiss institutions to value their residential properties using the hedonic approach, and hence attributes are in most cases readily available.
than 40 years (grade of 4). We then assume that the quality of the building and that of location are more valuable features for an investor than the age of the building, so that we assign a 40% weight to each of the first two characteristics and a weight of 20% to age. We assign 100 points for a grade of 1; 75 points for a grade of 2; 50 points for a grade of 3 and 25 points for a grade of 4. The total number of points (TP) is given by:

\[ TP = w(\text{building quality}) \times P(\text{building quality}) + w(\text{location}) \times P(\text{location}) + w(\text{age}) \times P(\text{age}) \]  

(14)

where \( w \) is the weight and \( P \) the number of points.

The value of \( p_2 \) is then calculated as:

\[ p_2 = (100 - TP) / 100 \]  

(15)

To illustrate, consider a building of high quality (grade of 1), with an average quality location (grade of 3) and constructed 18 years ago (grade of 3). Therefore, \( TP = 40\% \times 100 + 40\% \times 50 + 20\% \times 25 = 65 \) points. Then, the premium \( p_2 \) would be equal to \( (100-65)/100 = 0.45\% \). In contrast, the \( p_2 \) premium for a luxurious new building with an excellent location will be zero. Although this system is somewhat arbitrary, it makes sense and is consistent with the hedonic approach. As a general rule, high quality properties are likely to be occupied by more secure tenants and are viewed as less risky by investors (Gunnelin et al., 2004).

2.4 Correlations and other considerations

In addition to being uncertain, the variables used in the valuation process are not completely independent from each other. For example, rents and property prices may be correlated such that we should take into consideration their co-movements when performing our simulations. Historical data on the evolution of property prices and rents should constitute a good indicator for future correlations. What about the correlations between interest rates and property prices or between interest rates and rents? From a theoretical point of view, an interest rate increase should induce a decrease in property prices, and vice versa. However, an interest rate increase not only induces a rise of interest expenses, but also of the cost of equity, and therefore there will be a pressure to increase rents. We hypothesise a positive correlation between mortgage rates and rents, and a negative correlation between vacancy rates and rents. Further, it seems reasonable to assume that \( p_1 \) is higher when interest rates are low and vice versa, which means that there is a negative correlation between the two series.
In many countries, rent adjustments are possible to compensate for changes in interest rates, though we observe that the adjustments for interest increases are in most cases more systematic than adjustments for interest decreases. However, as many laws do not allow adjustment beyond certain limits, rent adjustments are constrained (for Geneva, see Aziz et al., 2005). The same is often true when major capital expenditures are undertaken, ie rents cannot be increased for the return on the invested capital to remain constant.

In summary, we perform the following steps to run the Monte Carlo simulations: (1) we estimate the free cash flows by means of equation (3) and calculate single point estimates of future free cash flows using a probability density function for each of the components of the free cash flow; (2) we calculate a term structure of interest rates using equations (9), (10), (11) and (12); (3) we compute the premium \( P = p_1 + p_2 \) and add it to the rate calculated in step (2); and (4) we estimate the terminal value using equation (4). This procedure yields a single point estimated value. The procedure is then repeated 50,000 times to yield a distribution of possible property values.

### 3. Data

We apply our valuation methodology to the real estate portfolio of a tax-exempt Swiss institutional investor. The portfolio contains 30 properties with an estimated market value in excess of CHF 237 million (Euros 160 million) as of the end of 2004. Most of the properties are residential buildings, but some also contain office or retail uses. The time horizon for the forecasting period is set at ten years. We provide the detailed computations for a 50-year old and well constructed building which has a good location (Building “Edelweiss”). A capital expenditure of CHF 150,000 is forecasted in year 3. From the first year pro formas, it appears that the annual rent for residential use is CHF 4,500 per room\(^8\). In addition, the building contains retail premises yielding a first-year rent of CHF 150,000. Table I reports selected building and financial characteristics.

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\(^8\) It would be better to use the rent per square meter criterion, but in Geneva the rent per room criterion is commonly used even in laws and in administrative documents (note that a kitchen is considered as being a room).
Statistical data available in Switzerland concerning rent levels and changes are unfortunately not very useful for our study, as there is only one global index for the whole country. Instead, we rely on real estate price indices, which are calculated for various regions of the country, to proxy for the mean and standard deviation of rental growth. By doing so, we have estimates concerning the Geneva area for the period 1971-2004, for both old and new buildings and for several property uses. Table II contains summary statistics for real estate capital returns in Geneva. All data, computed by Wuest & Partners, are obtained from the Swiss Federal Office of Statistics. New residential buildings exhibit higher return and risk characteristics than old buildings, and commercial properties appear to command lower capital returns but higher risk than residential properties. The volatility of workshop returns is particularly high.

Vacancy rates (Table III), calculated by the Swiss National Bank, are available for 1975-2004 but unfortunately only at the national level. They were low during the whole period, which is typical of the Swiss residential real estate market. More refined statistics would probably highlight differences between rural areas and urban agglomerations such as Zurich, Geneva or Basel.

Cash operating expenses are set as a percentage of gross potential income, based on historical data taken from the real estate portfolio which is used for the empirical investigation in this paper, as well as on estimates that were provided by professionals for residential buildings in the Geneva area. The fixed component is set at 10% of rents. The variable component, which is between 5-20% of rents, is related to age in the following manner: 15-20% for buildings older than 40 years, 10-15% for buildings between 10 and 40 years old and 5-10% for buildings less than 10 years old. These cut-offs are obviously somewhat arbitrary, but at least they show that operating expenses and therefore operating cash flows are a function of the age of the buildings. Operating expenses are assumed to follow a triangular distribution, with a most likely value of 22.5%. As far as capital expenditures are considered, we obtained detailed information about their date of occurrence in the future and their forecasted amounts. The modelling of such behaviour is beyond the scope of this paper however. Rent rises subsequent to capital expenses are also assumed to follow triangular distributions. Note that we distinguish between two types of capital expenditures (ie minor and major expenditures). An expense is defined as major when it exceeds the annual rental income in a given year. Distributional assumptions appear in Table I.
We use 6-month Eurofranc rates for the 1974-2004 period to apply the CIR model and the Swiss Market Equity Index at the end of each quarter to determine the maximum level of the risk premium ($P$). Both sets of data are taken from Datastream (see Table III). The Datastream Market Index, which is a capital weighted index which includes all firms traded on the Swiss exchange (SWX), exhibited strong returns during the period with a mean annual return of approximately 10%. The average interest rate was 3.1% during the 1974-2004 period (the rate in table III is an annualised rate).

The premium $P$ is assumed to vary between 0 and 2.5%. The first component ($p_1$) is assumed to vary between 0 and 1.5%. We opt for a truncated normal distribution with a mean of 0.075%, a standard deviation of 1%, a minimum of 0% and a maximum of 1.5%. The second component ($p_2$), which is a function of the hedonic characteristics of properties, varies between 0-1%. For each property, we compute $p_2$ according to the procedure described in section 2.

The central value of the 0-2.5% range is consistent with the premium required for real estate investments by pension funds in Geneva. It is worthwhile to dig deeper into the required risk premium to examine its relation with historical risk premia in Switzerland. At the portfolio level (and a fortiori at the market level), the premium $P$ will be comprised of $p_1$ and a level of $p_2$ in line with the portfolio attributes. As buildings in a portfolio cannot all be new, of excellent quality and located in excellent areas, the average value of $p_2$ will be in the 0.5-0.75% range. If we consider the average of the 0-1.5% range for $p_1$ (ie 0.75%), the average risk premium is approximately 1.25-1.5%. A comparison of that figure with the historical return on real estate at the national level net of the interest rate level provides for a useful check of our assumptions. Hoesli and Hamelink (2004) find that real estate in Switzerland yielded an average return of 5.3% for the period 1979-2002. Considering that 6-month Eurofranc rates have exhibited an average of 3.1% over the last 30 years, our assumptions seem plausible.

The correlations between variables are impossible to obtain due to lack of data. Based on good judgement, we consider a negative correlation of −0.5 between the premium $p_1$ and the interest rates and a negative correlation of −0.75 between the rental growth rate and the vacancy rate. In addition, we consider a positive correlation of 0.5 between rents and operating expenses.
4. Results

In this section, we present our results both for the portfolio of 30 properties and for building “Edelweiss”. The starting point for any valuation using our method is to estimate the term structure of interest rates. Using conditional maximum likelihood estimation on historical 6-month Eurofranc interest rates to compute the various parameters of equation (9), we obtain the results that appear in Table IV. These parameters (pullback, long-term equilibrium and instantaneous standard deviation) are significantly different from zero and are close to those obtained by Bruand (1998) for the period 1975-1995. We do not check for the stability of these parameters for sub-periods as this would be beyond the objective of this paper. We then calculate the term structure of interest rates with the help of equations (9), (10), (11) and (12) (see Figure I). As the initial interest rate is very low (less than 1%) and the equilibrium long-term rate is 4% (b parameter in Table IV), the term structure is upward slopping.

To each estimated interest rate we add the risk premium, \( P = p_1 + p_2 \). As already mentioned, \( p_1 \) is assumed to follow a truncated normal distribution with a mean of 0.05%, a standard deviation of 1%, a minimum of 0 and a maximum of 1.5%. The property-specific premium \( p_2 \) for building “Edelweiss” is obtained as follows: as the building has a very good location (grade of 2), is of excellent quality (grade of 1) and is older than 40 years (grade of 4), we assign \( 40\% \times 100 + 40\% \times 75 + 20\% \times 25 = 75 \) points to the building. The \( p_2 \) premium is thus equal to \( (100-75)/100 = 0.25\% \).

Knowing the discount rates and all components of the free cash flows, we run the simulations (50,000 iterations) and obtain for each building the distribution of present values. The interpretation of the distribution of present values is straightforward. The range of possible present values and the shape of the distribution reflect the uncertainty issues related to the valuation of each property. For building “Edelweiss”, the distribution and the statistics of that distribution are given in Figure II. We observe that the mean present value for the building is CHF 5.67 million and the standard deviation CHF 0.6 million. The results span from a low of CHF 3.3 million to a high of CHF 7.9 million, which represents a range of more than CHF 4.5 million. However, 90% of the present values are between CHF 4.58-6.75 million, which is a much tighter range, ie about half the range of all possible outcomes.
The importance of the shape and of the range of possible outcomes varies according to the objectives of the persons using the real estate valuations. Banks granting mortgage loans may be more interested by the whole range of present values on the left hand side of the distribution to analyse the likelihood that the value of the building would fall below some threshold. Knowing this likelihood should help them set the interest rate that they will charge. In contrast, the borrower may be more inclined not to argue with the banker on the basis of the whole range of values to the left of the mean value, but rather to advocate using only part of the range of values as it is unlikely that the market value of the property will decline drastically over some given time horizon. The same behaviour may be observed for the seller of a property or a real estate portfolio manager. In this respect, the concept of Value-at-Risk (VaR) would be useful as it would provide an estimate of the maximum loss for various time horizons (see Baroni et al., 2001).

The same analysis is performed for all 30 properties in our sample. Table V contains the simulation results for each property. As all properties are owned by the same institution it is not possible to disclose the value of the properties, but rather all values have been standardised and statistics are given in percentage terms. The table provides the standard deviation of the present value distribution for each building as well as the percentage difference between the estimated hedonic value and the value computed with our method. The distribution standard deviations vary somewhat across buildings with a low of 9.91% (building #18) and a high of 14.53% (Building #21). This is not a surprise as the parameters we use are not the same for all properties. Some properties are older and we use accordingly lower mean and standard deviations in the growth rates. Also, the weight of commercial uses is not the same across buildings (last column of Table V), and as commercial real estate is traditionally more volatile than residential real estate, the standard deviation of present values will ceteris paribus be positively related with the weight of commercial uses. Building #21, for instance, has 95% of its revenues which stem for commercial real estate.

Unfortunately, we are not able to get correlations across properties in order to conduct a detailed portfolio analysis. However, we can compare the sum of the 30 present values with the estimated hedonic value for the entire portfolio. This yields that our estimated values lie on average 6.7% (\(=\frac{100}{107.15}-1\)) below the hedonic values. This is not surprising for at least two reasons. First, we use actual rents as a starting point in our simulations, and those will in
In many cases be less than market rents as rents charged to current tenants only adjust imperfectly to market rents. Second, our estimated discount rates constitute long-term trending rates and as such should yield more conservative value estimates than if current (historically very low) rates were to be used.

In a next step, we perform a sensitivity analysis to gauge the impact on the present value of changes in input parameters for building “Edelweiss” (Table VI). For all uncertain parameters, we examine the impact of a 25% decrease and 25% increase, respectively, in the initial values. These include the initial interest rate, the volatility and the long term equilibrium in the CIR model, both components of the risk premium (ie $p_1$ and $p_2$), the mean and standard deviation of the potential gross income and the terminal value growth rate. For instance, if the initial interest rate in the CIR model is 0.8%, then we perform our sensitivity analysis using values of 0.8%*1.25 = 1% and 0.8%*0.75= 0.6%, respectively.

The sensitivity analysis results show that a change in the long-term equilibrium rate has a strong impact on the estimated value due to the impact on the discount rate. For example, if the long-term rate in the CIR model decreases to 3% (a decrease of 25% from the initial value of 4%), this translates into an increase to CHF 7 million of the mean present value of the building (CHF 5.7 million in the original simulation). The components of the premium above interest rates also have an important impact on the present value, albeit less important than the long-term equilibrium rate.

The estimated growth rate of the terminal value also has a substantial impact on the estimated values. Recall that in our base scenario we adopted a zero growth model after the 10th year of the horizon period. When we drop this hypothesis, and consider truncated normal distributions for the growth rate, we obtain insightful results. Three hypotheses are considered and the impact of these is reported in the bottom part of Table VI. As the mean of the growth rate increases, so does the mean of the present value. Not surprisingly, we observe higher maximum present values (CHF 13 million) when the growth rate of the cash flows beyond period $t = 10$, is assumed to have a mean of 1% per year and the rate is truncated between 0 and 2%. If the upper limit for the growth rate is set at 3%, then the maximum present value exceeds CHF 22 million. Valuers would be very hard pressed however using such a growth rate in Switzerland given the structure and constraints of the residential market.
Interestingly, our present values appear to be less sensitive to changes in the risk premia and even less so to changes in the potential gross income rate of growth.

5. Conclusions

The discounted cash flow method is now widely used as a valuation method for income-producing real estate in many countries. In fact, it is generally accepted that the method yields a fair value estimate in the spirit of the new accountancy standards. This method is very useful indeed, but does suffer from some limitations. These include the facts that (1) the value of the property is needed to compute the discount rate, (2) the discount rate is assumed to be constant during the entire holding period, and (3) uncertainty is not explicitly taken into account.

The main contribution of this paper is in incorporating uncertainty in the valuation process. This is achieved by using the distributions of the various input parameters rather than point estimates as is customary in DCF valuations. We also deal with the two other limitations of the DCF approach in that we develop a method to estimate the discount rate which does not require prior knowledge of a property’s value, and use a time-varying discount rate. The distributions of the parameters are constructed using various data from the Swiss financial and real estate markets, and the empirical analysis is performed using an institutional portfolio of 30 properties located in Geneva.

Our results show that the estimated values are on average 6.7% below hedonic values. This would be expected in the current environment. As our discount rate estimates contain a mean-reversion component, they will overshoot market interest rates during periods of very low interest rates, and hence values will be somewhat conservative during such periods. Also, the standard deviation of our present values is in most cases approximately 10% and is positively related to the percentage of the property which is devoted to commercial uses. The sensitivity analyses suggest that the value estimates react most to changes in the long-term interest rate (one of the components of the discount rate) and to changes in the growth rate of the terminal value. Hence, valuers should take great care in choosing what values to assign to these parameters.
The appeal of incorporating uncertainty in the valuation process is that the analysis does not merely yield a point estimate of the entire distribution of values, but rather the distribution of values. Hence, the probability of the true value of a property being less than various thresholds can be ascertained. This should certainly prove useful to many investors. The Royal Institution of Chartered Surveyors (RICS), for instance, is currently examining how uncertainty can be used together with the value estimate, which highlights the importance of uncertainty for the valuation profession. Also, there has been some debate in the literature about valuation variation and the margin of error in valuing properties (Adair et al., 1996; Crosby et al., 1998). The approach which is advocated in this paper should constitute a contribution to this debate.

As is the case of all techniques, the quality of the outputs from a Monte Carlo simulation largely depends on the quality of the inputs (Li, 2000). With this in mind, further research should focus on the stability of the model that we use when other portfolios are used and for different periods of the real estate cycle. In particular, when the real estate market will be bearish again, it will be of interest to compare estimated values to hedonic values or to estimates generated using other valuation methods. Also, further investigation of which property attributes should be considered when constructing the property-specific risk premium and of how these mostly qualitative attributes should be weighted is warranted. Finally, it would seem fruitful to dig deeper in the relation between capital expenses and property values. This could suggest optimal time windows for undertaking such expenses during the life cycle of buildings. In doing so, a better understanding of the linkages between capital expenses and various other variables should emerge.
Table I. Building and financial characteristics and parameter probability distributions for building “Edelweiss”

<table>
<thead>
<tr>
<th></th>
<th>2 rooms</th>
<th>3 rooms</th>
<th>3.5 rooms</th>
<th>5.5 rooms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of flats</td>
<td>6</td>
<td>2</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>Total number of rooms</td>
<td>12</td>
<td>6</td>
<td>14</td>
<td>33</td>
</tr>
<tr>
<td>Price per room</td>
<td>CHF 4,500</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>City</td>
<td>Geneva</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age</td>
<td>&gt;50 years</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quality of location</td>
<td>Good</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Building quality</td>
<td>Excellent</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Residential Use**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Distribution</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rental growth rate</td>
<td>Normal</td>
<td>Historical mean and volatility for real estate capital returns in the Geneva area</td>
</tr>
<tr>
<td>Vacancy rate</td>
<td>Uniform</td>
<td>Historical minimum and maximum for vacancy rates in Switzerland</td>
</tr>
<tr>
<td>Operating expenses</td>
<td>Triangular</td>
<td>Minimum 15% of rents, maximum 30%, most likely value 23%</td>
</tr>
<tr>
<td>Rent rise when major CAPEX</td>
<td>Triangular</td>
<td>Minimum 0, maximum 10%, most likely value 7.5%</td>
</tr>
<tr>
<td>Rent rise when minor CAPEX</td>
<td>Triangular</td>
<td>Minimum 0, maximum 5%, most likely value 3.5%</td>
</tr>
</tbody>
</table>

**Commercial Use**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Distribution</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Potential Gross Income (annual)</td>
<td>CHF 150,000</td>
<td></td>
</tr>
<tr>
<td>Commercial rental growth rate</td>
<td>Normal</td>
<td>Historical mean and volatility for real estate capital returns in Switzerland</td>
</tr>
<tr>
<td>Vacancy rate</td>
<td>Uniform</td>
<td>Minimum of 4% and maximum of 8%</td>
</tr>
<tr>
<td>Operating expenses</td>
<td>Triangular</td>
<td>Minimum 15% of rents, maximum 30%, most likely value 23%</td>
</tr>
<tr>
<td>Rent rise when major CAPEX</td>
<td>Triangular</td>
<td>Minimum 0, maximum 15%, most likely value 10%</td>
</tr>
<tr>
<td>Rent rise when minor CAPEX</td>
<td>Triangular</td>
<td>Minimum 0, maximum 10%, most likely value 7.5%</td>
</tr>
</tbody>
</table>
Table II. Descriptive data for real estate capital returns in Geneva, 1971-2004

<table>
<thead>
<tr>
<th>Panel A. Residential</th>
<th>Mean</th>
<th>Std</th>
<th>Min</th>
<th>Max</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Old buildings</td>
<td>0.032</td>
<td>0.086</td>
<td>-0.115</td>
<td>0.22</td>
<td>34</td>
</tr>
<tr>
<td>New buildings</td>
<td>0.048</td>
<td>0.094</td>
<td>-0.086</td>
<td>0.272</td>
<td>34</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B. Commercial</th>
<th>Mean</th>
<th>Std</th>
<th>Min</th>
<th>Max</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Offices</td>
<td>0.023</td>
<td>0.096</td>
<td>-0.193</td>
<td>0.352</td>
<td>34</td>
</tr>
<tr>
<td>Workshops</td>
<td>0.028</td>
<td>0.209</td>
<td>-0.718</td>
<td>0.534</td>
<td>34</td>
</tr>
<tr>
<td>Retail</td>
<td>0.016</td>
<td>0.096</td>
<td>-0.147</td>
<td>0.293</td>
<td>34</td>
</tr>
</tbody>
</table>

Table III. Descriptive statistics for the vacancy rate, 6-month Eurofranc rate and Datastream Stock Market Index for Switzerland, for the period 1974-2004 (1975-2004 for vacancy rates), various frequencies

<table>
<thead>
<tr>
<th>Data type</th>
<th>Frequency</th>
<th>Mean</th>
<th>Std</th>
<th>Min</th>
<th>Max</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vacancy rate</td>
<td>Yearly</td>
<td>0.010</td>
<td>0.005</td>
<td>0.004</td>
<td>0.018</td>
<td>30</td>
</tr>
<tr>
<td>6-month Eurofranc rate</td>
<td>Monthly</td>
<td>0.031</td>
<td>0.006</td>
<td>0.013</td>
<td>0.043</td>
<td>372</td>
</tr>
<tr>
<td>DS stock returns</td>
<td>Quarterly</td>
<td>0.025</td>
<td>0.112</td>
<td>-0.420</td>
<td>0.208</td>
<td>124</td>
</tr>
</tbody>
</table>

Table IV. Conditional maximum likelihood estimation for interest rates (CIR model)

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Log L</th>
<th>a</th>
<th>b</th>
<th>s</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monthly</td>
<td>841</td>
<td>0.480</td>
<td>0.040</td>
<td>0.021</td>
<td>372</td>
</tr>
</tbody>
</table>

(2.89) (4.18) (9.51)

Note: a is the pullback, b is the long term equilibrium and s the instantaneous standard deviation, t-stats in parentheses
Table V. Standard deviation of present values, percentage difference with hedonic values and property uses (portfolio of 30 properties)

<table>
<thead>
<tr>
<th>Building #</th>
<th>Std deviation of PV (%)</th>
<th>Hedonic/Mean of Present Value (%)</th>
<th>Residential (%)</th>
<th>Commercial (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11.36</td>
<td>4.37</td>
<td>62</td>
<td>38</td>
</tr>
<tr>
<td>2</td>
<td>11.12</td>
<td>11.23</td>
<td>76</td>
<td>24</td>
</tr>
<tr>
<td>3</td>
<td>9.94</td>
<td>17.12</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>10.51</td>
<td>2.06</td>
<td>94</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>10.15</td>
<td>7.94</td>
<td>99</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>10.33</td>
<td>-5.81</td>
<td>98</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>11.18</td>
<td>-3.44</td>
<td>76</td>
<td>24</td>
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<tr>
<td>8</td>
<td>10.88</td>
<td>15.33</td>
<td>86</td>
<td>14</td>
</tr>
<tr>
<td>9</td>
<td>9.94</td>
<td>-7.15</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>11.00</td>
<td>6.15</td>
<td>85</td>
<td>15</td>
</tr>
<tr>
<td>11</td>
<td>9.94</td>
<td>-18.08</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>10.57</td>
<td>11.22</td>
<td>92</td>
<td>8</td>
</tr>
<tr>
<td>13</td>
<td>11.06</td>
<td>22.41</td>
<td>78</td>
<td>22</td>
</tr>
<tr>
<td>14</td>
<td>10.45</td>
<td>-6.75</td>
<td>95</td>
<td>5</td>
</tr>
<tr>
<td>15</td>
<td>11.53</td>
<td>9.26</td>
<td>61</td>
<td>39</td>
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<tr>
<td>16</td>
<td>11.30</td>
<td>3.34</td>
<td>67</td>
<td>33</td>
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<tr>
<td>17</td>
<td>11.24</td>
<td>10.67</td>
<td>73</td>
<td>27</td>
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<tr>
<td>18</td>
<td>9.91</td>
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<td>0</td>
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<tr>
<td>19</td>
<td>10.27</td>
<td>6.70</td>
<td>98</td>
<td>2</td>
</tr>
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<td>20</td>
<td>10.70</td>
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<tr>
<td>21</td>
<td>14.53</td>
<td>20.29</td>
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<td>95</td>
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<td>22</td>
<td>10.39</td>
<td>7.14</td>
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<tr>
<td>23</td>
<td>10.94</td>
<td>4.70</td>
<td>85</td>
<td>15</td>
</tr>
<tr>
<td>24</td>
<td>11.66</td>
<td>11.04</td>
<td>61</td>
<td>39</td>
</tr>
<tr>
<td>25</td>
<td>10.63</td>
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<tr>
<td>26</td>
<td>13.37</td>
<td>-5.49</td>
<td>41</td>
<td>59</td>
</tr>
<tr>
<td>27</td>
<td>10.21</td>
<td>0.77</td>
<td>99</td>
<td>1</td>
</tr>
<tr>
<td>28</td>
<td>10.76</td>
<td>5.18</td>
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<td>29</td>
<td>10.09</td>
<td>13.81</td>
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</tr>
<tr>
<td>30</td>
<td>10.82</td>
<td>13.01</td>
<td>86</td>
<td>14</td>
</tr>
<tr>
<td>All properties</td>
<td></td>
<td></td>
<td>7.15</td>
<td></td>
</tr>
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</table>
Table VI. Sensitivity analysis for building “Edelweiss”

<table>
<thead>
<tr>
<th>CIR model</th>
<th>Initial interest rate</th>
<th>Volatility</th>
<th>Long term equilibrium</th>
</tr>
</thead>
<tbody>
<tr>
<td>Change</td>
<td>-25%</td>
<td>+25%</td>
<td>-25%</td>
</tr>
<tr>
<td>Mean</td>
<td>5,852,142</td>
<td>5,500,705</td>
<td>5,669,680</td>
</tr>
<tr>
<td>Std Dev</td>
<td>686,121</td>
<td>622,768</td>
<td>650,150</td>
</tr>
<tr>
<td>Minimum</td>
<td>3,246,587</td>
<td>3,341,477</td>
<td>3,544,892</td>
</tr>
<tr>
<td>Maximum</td>
<td>7,980,917</td>
<td>7,692,362</td>
<td>7,795,851</td>
</tr>
<tr>
<td>Skewness</td>
<td>-7.30E-03</td>
<td>5.12E-02</td>
<td>3.62E-03</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>2.864498</td>
<td>2.78931</td>
<td>2.756194</td>
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<table>
<thead>
<tr>
<th>p1 and p2 premia</th>
<th>p1</th>
<th>p2</th>
</tr>
</thead>
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<tr>
<td>Change</td>
<td>-25%</td>
<td>+25%</td>
</tr>
<tr>
<td>Mean</td>
<td>6,242,395</td>
<td>5,567,414</td>
</tr>
<tr>
<td>Std Dev</td>
<td>624,378</td>
<td>583,718</td>
</tr>
<tr>
<td>Minimum</td>
<td>3,901,586</td>
<td>3,808,312</td>
</tr>
<tr>
<td>Maximum</td>
<td>7,986,258</td>
<td>7,609,934</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.41905</td>
<td>-5.99E-02</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>3.11224</td>
<td>2.717727</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Growth rate of potential gross income</th>
<th>$\mu$</th>
<th>$s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Change</td>
<td>-25%</td>
<td>25%</td>
</tr>
<tr>
<td>Mean</td>
<td>5,865,658</td>
<td>5,909,934</td>
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<tr>
<td>Std Dev</td>
<td>609,986</td>
<td>618,004</td>
</tr>
<tr>
<td>Minimum</td>
<td>3,842,415</td>
<td>3,885,903</td>
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<tr>
<td>Maximum</td>
<td>7,908,025</td>
<td>7,949,194</td>
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<tr>
<td>Skewness</td>
<td>-0.18913</td>
<td>-0.22187</td>
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<tr>
<td>Kurtosis</td>
<td>2.871095</td>
<td>2.912349</td>
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<th>Terminal value growth rate</th>
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<tr>
<td>Distribution parameters</td>
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<tr>
<td>Min</td>
</tr>
<tr>
<td>Max</td>
</tr>
<tr>
<td>Mean</td>
</tr>
<tr>
<td>Std Dev</td>
</tr>
<tr>
<td>Minimum</td>
</tr>
<tr>
<td>Maximum</td>
</tr>
<tr>
<td>Skewness</td>
</tr>
<tr>
<td>Kurtosis</td>
</tr>
</tbody>
</table>
Figure I. Term structure of interest rates

![Term structure of interest rates](image)

Figure II. Distribution of the present value for building “Edelweiss”

![Distribution of the present value for building “Edelweiss”](image)

- **Minimum**: 3335127
- **Mean**: 5672066
- **Maximum**: 7786397
- **Std Dev**: 644320
- **Skewness**: -0.00715
- **Kurtosis**: 2.823
References


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