OPTIMAL LOT SIZING WITH SCRAP AND RANDOM BREAKDOWN OCCURRING IN BACKORDER REPLENISHING PERIOD

Chia-Kuan Ting, Yuan-Shyi Peter Chiu*, Chu-Chai Henry Chan
Department of Industrial Engineering and Management,
Chaoyang University of Technology, Wufong, Taichung 413, Taiwan
*ypchiu@cyut.edu.tw

Abstract-This paper is concerned with determination of optimal lot size for an economic production quantity model with scrap and random breakdown occurring in backorder replenishing period. In most real-life manufacturing systems, generation of defective items and random breakdown of production equipment are inevitable. To deal with the stochastic machine failures, production planners practically calculate the mean time between failures (MTBF) and establish the robust plan accordingly, in terms of optimal lot size that minimizes total production-inventory costs for such an unreliable system. Random scrap rate is considered in this study, and breakdown is assumed to occur in the backorder filling period. Mathematical modeling and analysis is used and the renewal reward theorem is employed to cope with the variable cycle length. An optimal manufacturing lot size that minimizes the long-run average costs for such an imperfect system is derived. Numerical example is provided to demonstrate its practical usages.

Key Words- Production, Lot size, Machine breakdown, Backordering, Scrap

1. INTRODUCTION

Becoming a low cost producer is one of the main operation strategies and goals of most manufacturing firms. To accomplish this goal, the company must be able to effectively use its resources and minimize its operating costs. In the field of inventory management, Harris [1] first introduced the economic order quantity (EOQ) model to assist corporations in reducing total inventory costs. EOQ model uses mathematical techniques to balance the setup cost and holding cost, and derives an optimal ordering size that minimizes overall inventory costs. In the manufacturing sector, the economic production quantity (EPQ) model is often utilized for determining the optimal production lot-size that minimizes overall production-inventory costs [2-3]. Regardless of the simplicity of EOQ and EPQ models, they are still applied industry- wide today [4-5].The classic EPQ model implicitly assumes that items produced are of perfect quality. But in real-life production systems, due to many reasons generation of defective items is inevitable. Hence, studies have been carried out to enhance the classic EPQ model by addressing the issue of imperfection quality items produced [6-26].

Boone et al. [12] investigated the impact of imperfect processes on the production run time. They built a model in an attempt to provide managers with guidelines to choose the appropriate production run times to cope with both the defective items and stoppages occurring due to machine breakdowns. Lee and Rosenblatt [16] studied an EPQ model with joint determination of production cycle time and inspection schedules, and they derived a relationship that can be used to determine the effectiveness of maintenance by inspection. Zhang and Gerchak [18] considered joint lot sizing and

inspection policy in an EOQ model with random yield. Hayek and Salameh [25] assumed that all of the defective items produced are repairable and derived an optimal operating policy for EPQ model under the effect of reworking of imperfect quality items. Stock-out situations may also occur due to the excess demand. Sometimes, these shortages can be backordered and satisfied at a future time, hence the overall production-inventory costs can be reduced significantly [19-20,24-25].

Random breakdown of production equipment is another common and inevitable reliability factors that trouble the production planners and practitioners most. To effectively manage and control the disruption and minimize overall production costs, become the primary task of most manufacturing firms. It is no wonder that determining optimal lot-size (or production uptime) for systems with machine failures has received attention from researchers in recent decades (see, for instance [27-38]).

Example of studies that addressed the machine breakdown issues are surveyed below. Groenevelt, Pintelon, and Seidmann [27] studied two production control policies to deal with the machine failures. The first one assumes that the production of the interrupted lot is not resumed (called no resumption (NR) policy) after a breakdown. While the second policy considers that the production of the interrupted lot will be immediately resumed (called abort/ resume (AR) policy) after the breakdown is fixed and if the current on-hand inventory falls below a certain threshold level. Both of their proposed policies assume that the repair time is negligible and they studied the effects of machine breakdowns and corrective maintenance on the economic lot sizing decisions. Chiu et al. [30] investigated the optimal run time for EPQ model with scrap, rework and random breakdown. They proposed and proved theorems on conditional convexity of the integrated cost function and on bounds of the production run time. Then, an optimal run time was located by the use of the bisection method based on the intermediate value theorem. Makis and Fung [33] studied effects of machine failures on the optimal lot size as well as on optimal number of inspections. Formulas for the long-run expected average cost per unit time was obtained. Then the optimal production/inspection policy that minimizes the expected average costs was derived. Abboud [38] considered an EMQ model with Poisson machine failures and random machine repair time. A simple approximation model was developed to describe the behavior of such systems, and specific formulations were derived for the cases where the repair times are exponential and constant. This study is concerned with determination of optimal lot size for an EPQ model with scrap, shortages allowed and backordered, and random breakdown occurring in backorder-filling period. Since little attention was paid to the aforementioned area, this paper intends to bridge the gap.

2. ASSUMPTION AND MATHEMATICAL MODELING

This paper considers a manufacturing process with the following features:
(1) It may randomly produce x portion of defective items at a rate d.
(2) All imperfect quality items are assumed not repairable, are treated as scrap.
(3) The production rate P is much larger than the demand rate λ and the production rate of scrap items d can be expressed as $d=Px$.
(4) Shortages are allowed and backordered, they will be satisfied first when the next replenishment production cycle begins.
(5) According to the mean time between failures (MTBF) data, a single machine

breakdown occurs at only backorder replenishing period with random occurrence times (refer to Figure 1).
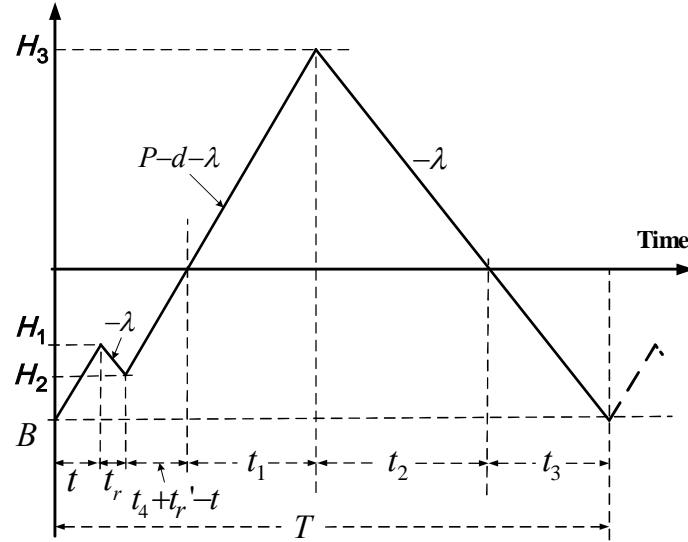

Figure 1 : On-hand inventory of perfect quality items in EPQ model with scrap and breakdown occurring in backorder-filling period

The abort/resume (AR) inventory control policy is adopted in this study and under such policy, when a breakdown takes place the machine is under corrective maintenance immediately, and the repair time is assumed to be constant. The interrupted lot will be resumed right after the restoration of machine. Cost parameters considered in the proposed model include setup cost K, unit holding cost h, unit production cost C, disposal cost per scrap item C_S, unit shortage/backordered cost b, and cost for repairing and restoring machine M. Additional notations are listed below.

t = production time before a random breakdown occurs,
t_r = time required for repairing and restoring the machine,
t_r' = time required for producing sufficient stocks to satisfy the demand during machine repair time t_r,
t_4 = time required for filling the backorder quantity B (excluding t_r and t_r'),
t_1 = time for piling up stocks during the production uptime in each cycle,
t_2 = time required for depleting all available perfect quality on-hand items,
t_3 = shortage permitted time,
T_1 = the optimal production uptime to be found for the proposed EPQ model,
H_1 = the level of backorder quantity when machine breakdown occurs,
H_2 = the level of backorder quantity when machine is repaired and restored,
H_3 = the maximum level of on-hand inventory for each production cycle,
Q = production lot size for each cycle,
B = the maximum backorder level allowed for each cycle,
T = the production cycle length,
$TC(T_1,B)$ = total production-inventory costs per cycle,
$TCU(T_1,B)$ = total production-inventory costs per unit time (e.g. annual),
$E[TCU(T_1,B)]$ = the expected total production-inventory costs per unit time.

The production rate P of perfect quality items must always be greater than or

equal to the sum of the demand rate λ and the production rate of defective items d. Hence, the following condition must hold: $(P-d-\lambda)>0$ or $(1-x-\lambda/P)>0$. Because t denotes production time before a breakdown taking place in the backorder replenishing period t_4, that is $t < t_4$. Let g be the constant machine repair time, hence $t_r = g$. The following derivation procedure is similar to what was used by prior studies [20,25].

From Figure 1, one can obtain the following: the level of backorder H_1 (when machine breakdown occurs); the level of backorder H_2 (when machine is repaired and restored); the maximum level of on-hand inventory H_3; the production uptime T_1; the cycle length T; t_r'; time for piling up stocks t_1; time required for depleting all available on-hand items t_2; t_3; time required for filling B (the maximum backorder quantity) t_4; and the production lot size Q.

$$H_1 = B - (P - d - \lambda)t \tag{1}$$

$$H_2 = H_1 + t_r\lambda = \left[B - (P - d - \lambda)t\right] + g\lambda \tag{2}$$

$$H_3 = (P - d - \lambda)t_1 \tag{3}$$

$$T_1 = t_4 + t_r' + t_1 = \frac{Q}{P} \tag{4}$$

$$T = t_4 + t_r + t_r' + t_1 + t_2 + t_3 \tag{5}$$

$$t_r' = \frac{g\lambda}{P - d - \lambda} \tag{6}$$

$$t_1 = \frac{H_3}{P - d - \lambda} \tag{7}$$

$$t_2 = \frac{H_3}{\lambda} \tag{8}$$

$$t_3 = \frac{B}{\lambda} \tag{9}$$

$$t_4 = \frac{B}{P - d - \lambda} \tag{10}$$

$$Q = P \cdot T_1 = P\left[t_4 + t_r' + t_1\right] \tag{11}$$

where $d=Px$.

As depicted in Figure 2, the total scrap items produced during production uptime T_1 can be obtained as shown in equation (12).

$$d \cdot T_1 = x \cdot Q = P \cdot x \cdot \left[t_4 + t_r' + t_1\right] \tag{12}$$

Total production-inventory cost per cycle $TC_1(T_1,B)$ is:

$$
\begin{aligned}
TC_1(T_1, B) &= K + C \cdot (P \cdot T_1) + Cs \cdot (T_1 \cdot P \cdot x) + M + h \cdot \left[H_3 \cdot (t_1)/2 + H_3 \cdot (t_2)/2\right] \\
&+ h \cdot \left[(d \cdot t) \cdot (t)/2 + (d \cdot t) \cdot t_r + (d \cdot t + d \cdot T_1) \cdot (t_4 + t_r' - t + t_1)/2\right] \\
&+ b \cdot \left[(B + H_1) \cdot t/2 + (H_1 + H_2) \cdot t_r/2 + H_2 \cdot (t_4 + t_r' - t)/2 + B(t_3)/2\right]
\end{aligned}
\tag{13}
$$

Substituting all related parameters from equations (1) to (12) in equation (13), one obtains $TC_1(T_1,B)$ as follows.

$$
\begin{aligned}
TC_1(T_1, B) &= K + M + P \cdot T_1 \cdot \left[C + C_s \cdot x\right] + gPt \cdot \left[hx - b \cdot (1-x)\right] - hT_1 P(1-x) \cdot \left[(B/\lambda) + g\right] \\
&+ \frac{hT_1^2 P}{2}\left[\frac{P(1 - 2x + x^2)}{\lambda} + 2x - 1\right] + \frac{(b+h)}{2} \cdot \left\{g(2B + g\lambda) + \frac{B^2}{\lambda}\right\} \cdot \left(\frac{1-x}{1 - x - (\lambda/P)}\right)
\end{aligned}
\tag{14}
$$

The production cycle length is not constant due to the assumption of random scrap rate and a uniformly distributed random breakdown is assumed to occur in the backorder filling period. Thus, to take the randomness of scrap and breakdown into account, one can use the renewal reward theorem in inventory cost analysis to cope with the variable cycle length and the integration of $TC_1(T_1,B)$ to deal with the random breakdown happening in period t_4. The expected total production- inventory costs per unit time can be calculated as follows.
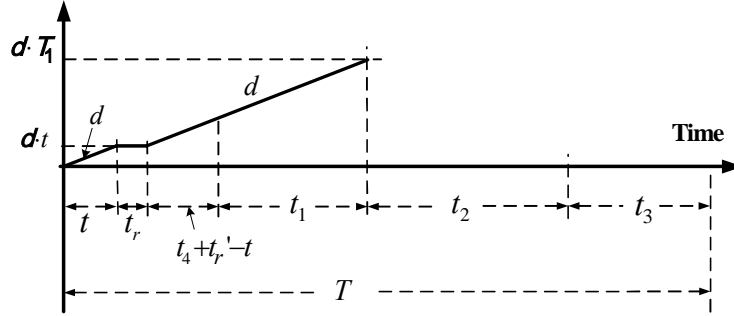

Figure 2 : On-hand inventory of scrap items in EPQ model with scrap and breakdown occurring in backorder-filling period

$$E\left[TCU\left(T_1,B\right)\right]=\frac{E\left[\int_0^{t_4}TC_1\left(T_1,B\right)\cdot f\left(t\right)dt\right]}{E[T]}=\frac{E\left[\int_0^{t_4}TC_1\left(T_1,B\right)\cdot\left(1/t_4\right)dt\right]}{T_1P\left(1-E[x]\right)/\lambda}\tag{15}$$

$$\therefore E\left[TCU\left(T_1,B\right)\right]=\lambda\cdot\left[\frac{\left(K+M\right)}{T_1P}+C\right]\cdot\frac{1}{\left(1-E[x]\right)}+C_S\lambda\cdot\frac{E[x]}{\left(1-E[x]\right)}+\frac{h}{2}\left[-2B+T_1\cdot\left(P-\lambda\right)\right]\cdot\frac{1}{\left(1-E[x]\right)}$$
$$-hg\lambda+\frac{1}{2T_1P}\left\{g\lambda\cdot\left[\left(b+h\right)\left(2B+g\lambda\right)-Bb\right]+\left(b+h\right)B^2\right\}\cdot\frac{1}{\left(1-E[x]\right)}\cdot E\left[\frac{1-x}{1-x-\left(\lambda/P\right)}\right]\tag{16}$$
$$+h\cdot\left[B-T_1\cdot\left(P-\lambda\right)\right]\cdot\frac{E[x]}{\left(1-E[x]\right)}+\frac{T_1Ph}{2}\cdot\left[\frac{E\left[x^2\right]}{\left(1-E[x]\right)}\right]+\frac{1}{2}\frac{Bhg\lambda}{T_1P}\frac{1}{\left(1-E[x]\right)}\cdot E\left[\frac{x}{1-x-\left(\lambda/P\right)}\right]$$

Let $E_0=\dfrac{1}{1-E[x]};\ E_1=\dfrac{E[x]}{1-E[x]};\ E_2=\dfrac{E\left[x^2\right]}{1-E[x]};\ E_3=\dfrac{1}{1-E[x]}E\left[\dfrac{1-x}{1-x-\dfrac{\lambda}{P}}\right];\ E_4=\dfrac{1}{1-E[x]}E\left[\dfrac{x}{1-x-\dfrac{\lambda}{P}}\right]$

Then equation (16) becomes:

$$\therefore E\left[TCU\left(T_1,B\right)\right]=\lambda\cdot\left[\frac{\left(K+M\right)}{T_1P}+C\right]\cdot E_0+C_S\lambda\cdot E_1+\frac{h}{2}\left[-2B+T_1\cdot\left(P-\lambda\right)\right]\cdot E_0-hg\lambda+\frac{1}{2}\frac{Bhg\lambda}{T_1P}\cdot E_4$$
$$+\frac{1}{2T_1P}\left\{g\lambda\cdot\left[\left(b+h\right)\left(2B+g\lambda\right)-Bb\right]+\left(b+h\right)B^2\right\}\cdot E_3+h\cdot\left[B-T_1\cdot\left(P-\lambda\right)\right]\cdot E_1+\frac{T_1Ph}{2}\cdot E_2\tag{17}$$

2.1 Convexity of the expected cost function E[TCU(T₁,B)]

The optimal inventory operating policy can be obtained by minimizing the expected cost function. For the proof of convexity of $E[TCU(T_1,B)]$, one can utilize the Hessian matrix equation [39] and verify the existence of the following:

$$[T_1 \quad B] \cdot \begin{pmatrix} \dfrac{\partial^2 E[TCU(T_1,B)]}{\partial T_1^2} & \dfrac{\partial^2 E[TCU(T_1,B)]}{\partial T_1 \partial B} \\ \dfrac{\partial^2 E[TCU(T_1,B)]}{\partial T_1 \partial B} & \dfrac{\partial^2 E[TCU(T_1,B)]}{\partial B^2} \end{pmatrix} \cdot \begin{bmatrix} T_1 \\ B \end{bmatrix} > 0 \tag{18}$$

$E[TCU(T_1,B)]$ is strictly convex only if equation (18) is satisfied, for all T_1 and B different from zero. From equations (17) and (18), by computing all the elements of the Hessian matrix equation, one obtains:

$$[T_1 \quad B] \cdot \begin{pmatrix} \dfrac{\partial^2 E[TCU(T_1,B)]}{\partial T_1^2} & \dfrac{\partial^2 E[TCU(T_1,B)]}{\partial T_1 \partial B} \\ \dfrac{\partial^2 E[TCU(T_1,B)]}{\partial T_1 \partial B} & \dfrac{\partial^2 E[TCU(T_1,B)]}{\partial B^2} \end{pmatrix} \cdot \begin{bmatrix} T_1 \\ B \end{bmatrix} \tag{19}$$

$$= \frac{1}{1-E[x]} \cdot \frac{\lambda}{T_1 P} \cdot \left\{ 2(K+M) + (b+h)\lambda g^2 \cdot E\left(\frac{1-x}{1-x-(\lambda/P)}\right) \right\} > 0$$

Equation (19) is resulting positive because all parameters are positive. Hence, $E[TCU(T_1,B)]$ is a strictly convex function. It follows that for the optimal production uptime T_1 and the maximal backorder level B, one can differentiate $E[TCU(T_1,B)]$ with respect to T_1 and with respect to B, and solve linear systems of equations (20) and (21) by setting these partial derivatives equal to zero.

$$\frac{\partial E[TCU(T_1,B)]}{\partial T_1} = \frac{-\lambda(K+M)\cdot E_0}{T_1^2 P} + \frac{h(P-\lambda)\cdot E_0}{2} - \frac{1}{2}\frac{Bhg\lambda \cdot E_4}{T_1^2 P} - h(P-\lambda)\cdot E_1 \tag{20}$$

$$+ \frac{Ph}{2}\cdot E_2 - \frac{1}{2T_1^2 P}\left\{ g\lambda\cdot\left[(b+h)(2B+g\lambda) - Bb\right] + (b+h)B^2 \right\}\cdot E_3$$

$$\frac{\partial E[TCU(T_1,B)]}{\partial B} = -h + \frac{1}{T_1 P}\left\{ g\lambda\cdot(b/2+h) + (b+h)B \right\}\cdot E_3 + \frac{hg\lambda}{2T_1 P}\cdot E_4 \tag{21}$$

$$T_1^* = \frac{1}{P}\sqrt{\frac{2(K+M)\lambda E_0 + (b+h)\lambda^2 g^2 E_3 - \dfrac{\left[\left(\dfrac{b}{2}+h\right)\lambda g E_3 + \dfrac{h\lambda g}{2}E_4\right]^2}{(b+h)E_3}}{h(1-\lambda/P)E_0 - 2h(1-\lambda/P)E_1 - h^2/(b+h)E_3 + hE_2}} \tag{22}$$

Therefore, one has:

$$\therefore T_1^* = \frac{1}{P}\sqrt{\frac{2(K+M)\lambda E_0 + (b+h)\lambda^2 g^2 E_3 - \dfrac{\lambda^2 g^2}{(b+h)E_3}\cdot\left[\left(\dfrac{b}{2}+h\right)\cdot E_3 + \dfrac{h}{2}\cdot E_4\right]^2}{h(1-\lambda/P)E_0 - 2h(1-\lambda/P)E_1 + hE_2 - h^2/(b+h)E_3}} \tag{23}$$

$$B^* = \frac{hP}{(b+h)E_3}T_1^* - \frac{(b+2h)\lambda g E_3 + h\lambda g E_4}{2(b+h)E_3} \tag{24}$$

or

$$\therefore B^* = \left(\frac{h}{b+h}\right)\left(\frac{P}{E_3}\right)T_1^* - \frac{\lambda g}{2}\left[1 + \left(\frac{h}{b+h}\right)\cdot\left(1 + \frac{E_4}{E_3}\right)\right] \tag{25}$$

From equations (4), (23), and (25) one can obtain the optimal lot-size Q^* and optimal backorder level B^* as shown in equations (26) and (27). The expected total production- inventory cost $E[TCU(T_1,B)]$ can be obtained by substituting T_1 and B^* from equations (22) and (23) into equation (16).

$$Q^* = \sqrt{\frac{2(K+M)\lambda + (b+h)\lambda^2 g^2 \cdot E\left[\frac{1-x}{1-x-\lambda/P}\right] - \frac{\lambda^2 g^2 \cdot [1-E[x]]}{(b+h)E_3} \cdot \left[\left(\frac{b}{2}+h\right)\cdot E_3 + \frac{h}{2}\cdot E_4\right]^2}{h\left(1-\frac{\lambda}{P}\right) - 2h\left(1-\frac{\lambda}{P}\right)\cdot E[x] + h\cdot E[x^2] - \frac{h^2[1-E[x]]}{(b+h)\cdot E_3}}} \tag{26}$$

$$B^* = \left(\frac{h}{b+h}\right)\left(\frac{1}{E_3}\right)Q^* - \frac{\lambda g}{2}\left[1+\left(\frac{h}{b+h}\right)\cdot\left(1+\frac{E_4}{E_3}\right)\right] \tag{27}$$

2.2 Results and verification

Suppose that the breakdown factor is not considered, then the cost and time for repairing failure machine $M=0$ and $g=0$, equations (26) and (27) become the same equations as were given by Chiu and Chiu [19]:

$$Q^* = \sqrt{\frac{2K\lambda}{h\left(1-\frac{\lambda}{P}\right) - 2h\left(1-\frac{\lambda}{P}\right)\cdot E[x] + h\cdot E[x^2] - \frac{h^2[1-E[x]]^2}{(b+h)\cdot E\left[\frac{1-x}{1-x-\lambda/P}\right]}}} \tag{28}$$

$$B^* = \left(\frac{h}{b+h}\right)\cdot\frac{1-E[x]}{E\left[\frac{1-x}{1-x-\lambda/P}\right]}\cdot Q^* \tag{29}$$

Further, suppose that the regular production process produces no defective items, i.e. $x = 0$, then equations (28) and (29) become the same equations as were presented by the classic EPQ model with backordering permitted [40]:

$$Q^* = \sqrt{\frac{2K\lambda}{h\left(1-\frac{\lambda}{P}\right)}}\cdot\sqrt{\frac{b+h}{b}} \tag{30}$$

$$B^* = \left[\frac{h}{(b+h)}\left(1-\frac{\lambda}{P}\right)\right]\cdot Q^* \tag{31}$$

3. NUMERICAL EXAMPLE AND DISCUSSION

Suppose that annual demand of a manufactured product is 3,600 units and the production rate of this item is 9,000 units per year. According to the MTBF data from the maintenance department a uniformly distributed breakdown is assumed to occur in the backorder filling period. When a breakdown happens, the abort/resume policy is used. The percentage of scrap items produced x, follows a uniform distribution over the interval [0, 0.2]. Other parameters are summarized as follows.

C_S = \$0.3 disposal cost for each scrap item,
C = \$1 per item,
M = \$500 repair cost for each breakdown,
K = \$450 for each production run,
h = \$0.6 per item per unit time,
b = \$0.2 per item backordered per unit time,
g = 0.018 years, time needed to repair and restore the machine.

A demonstration of the convexity of the long-run average costs $E[TCU(T_1,B)]$ is depicted in Figure 3. From equations (23), (26), (27) and (17), one can obtain the optimal production uptime $T_1^* =0.9443$ years (or the optimal production batch size

Q^*=8,499), the optimal backorder level B^* =3,108, and the optimal long run expected costs $E[TCU(T_1^*,B^*)]=$ \$5,011.<u>30.</u>

3.1 Sensitivity analyses

Figure 4 shows the behavior of the optimal production lot size Q^* with respect to random percentage of defective items x, where each x-value represents a uniform distributed random variable over the interval $[0, x]$. It may be seen that as the random percentage of defective items x increases, the optimal production lot size Q^* decreases significantly.
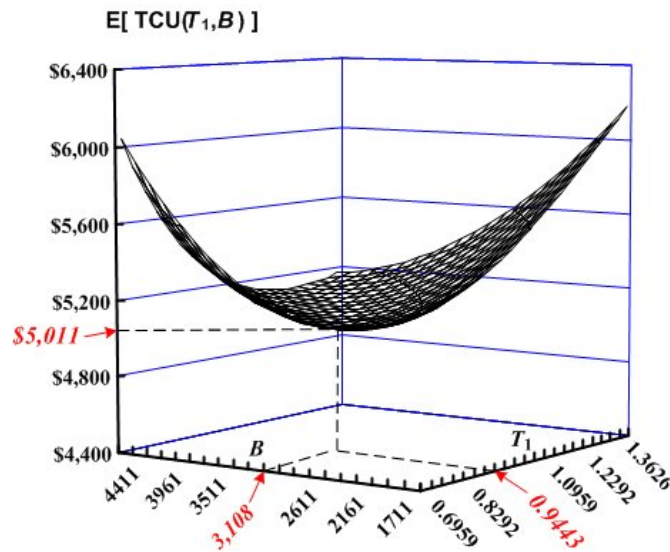

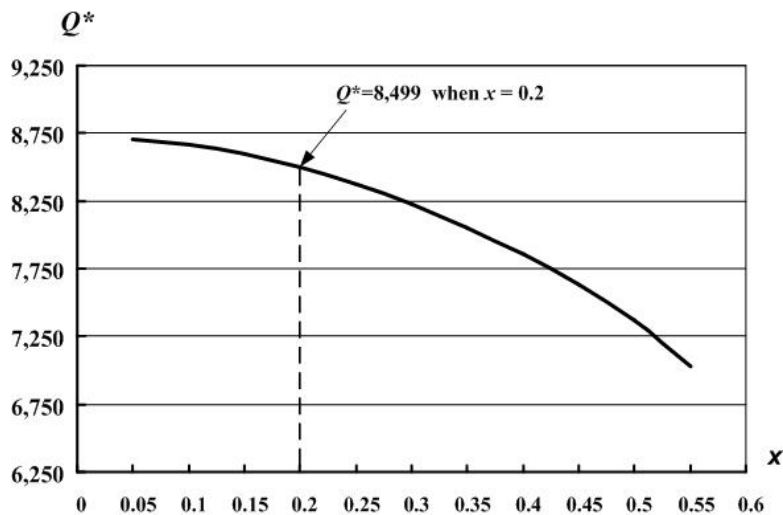Figure 3: Convexity of the expected cost function E[TCU(T$_1$,B)]

Figure 4: Variation of scrap rate effects on the optimal production lot size Q*

The behavior of the optimal expected cost function $E[TCU(T_1^*,B^*)]$ with respect to random percentage of defective items x is depicted in Figure 5. It may be noted that

as random percentage of defective items x increases, the optimal expected cost function $E[TCU(T_1*,B*)]$ increases significantly.

Suppose the result of this investigation is not available, one probably can only use a closely related lot-size solution given by [24] for solving such an unreliable EPQ model and obtaining Q=5,849 (or T_1=0.6498) and B=2,180. Plugging this lot-size solution into Eq. (17), one has $E[TCU(T_1,B)]$=\$5,074.99. It is 4.51% more on total setup and holding costs than the optimal production-inventory costs computed by the result of the present study.

4. CONCLUSION

In most real-life manufacturing systems, generation of defective items and breakdown of production equipment are inevitable. One cannot count on classical EPQ model to determine the optimal replenishment policy for such a practical system, because it does not consider the imperfect quality factors. The effects of these reliability situations on the EPQ model must be specifically investigated in order to minimize overall production-inventory costs. Since little attention was paid to the aforementioned area, this paper intends to fill the gap.
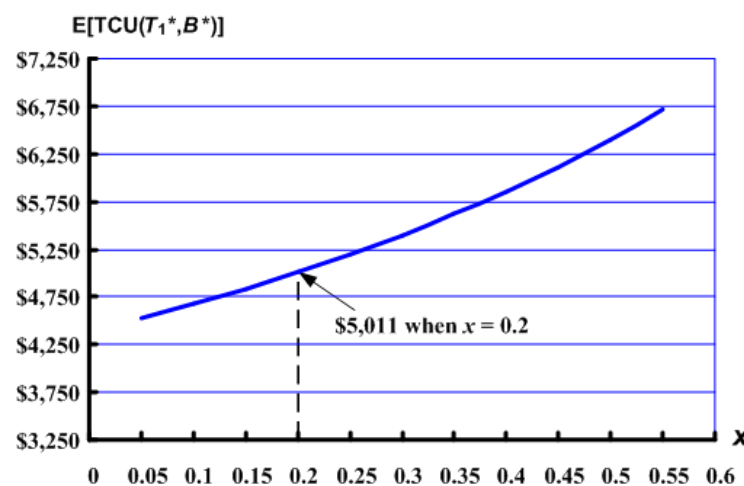

Figure 5: Variation of scrap rate effects on the optimal expected cost function $E[TCU(T_1*,B*)]$

Mathematical modeling is employed in this study. The disposal cost for each scrap item and the repairing cost for the broken-down machine are included in the cost analysis. The renewal reward theorem is utilized to cope with the variable cycle length of the proposed system. An optimal production lot size that minimizes the long-run average costs for such an imperfect quality EPQ model is derived, where shortages are permitted and backordered. A numerical example is provided in Section 3 to demonstrate its practical usage. For future research, one interesting topic will be to consider reworking of the repairable defective items for the same unreliable systems.

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