SOME RELATIONSHIPS INCLUDING p-ADIC GAMMA FUNCTION AND q-DAEHEE POLYNOMIALS AND NUMBERS

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Abstract. In this paper, we investigate p-adic q-integral (q-Volkenborn integral) on $\mathbb{Z}_p$ of p-adic gamma function via their Mahler expansions. We also derived two q-Volkenborn integrals of p-adic gamma function in terms of q-Daehee polynomials and numbers and q-Daehee polynomials and numbers of the second kind. Moreover, we discover q-Volkenborn integral of the derivative of p-adic gamma function. We acquire the relationship between the p-adic gamma function and Stirling numbers of the first kind. We finally develop a novel and interesting representation for the p-adic Euler constant by means of the q-Daehee polynomials and numbers.

1. Introduction

The p-adic numbers are a counterintuitive arithmetic system, which were firstly introduced by the Kummer in 1850. In conjunction with the introduction of these numbers, some mathematicians and physicists started to investigate new scientific tools utilizing their useful and positive properties. Firstly Kurt Hensel, the German mathematician, (1861-1941) improved the p-adic numbers in a study concerned with the development of algebraic numbers in power series in circa 1897. Some effects of these researches have emerged in mathematics and physics such as p-adic analysis, string theory, p-adic quantum mechanics, QFT, representation theory, algebraic geometry, complex systems, dynamical systems, genetic codes and so on (*cf.* [1-10, 12-18]; also see the references cited in each of these earlier studies). The one important tool of these investigations is p-adic gamma function which is firstly described by Yasou Morita [15] in about 1975. Intense research activities in such an area as p-adic gamma function is principally motivated by their importance in p-adic analysis. Therefore, in recent fourty years, p-adic gamma function and its generalizations have been investigated and studied extensively by many mathematicians, *cf.* [2, 4-8, 12, 14-16, 18]; see also the related references cited therein.

Kim *et al.* [11] defined Daehee polynomials $D_n(x)$ by means of the following exponential generating function:

$$\sum_{n=0}^{\infty} D_n(x)\frac{t^n}{n!} = \frac{\log(1+t)}{t}(1+t)^x . \tag{1.1}$$

In the case $x=0$ in the Eq. (1.1), one can get $D_n(0) := D_n$ standing for n-th Daehee number, see [1, 9, 11] for more detailed information about these related issues.

Let $p \in \{2, 3, 5, 7, 11, 13, 17, \cdots\}$ be a prime number. For any nonzero integer a, let $ord_p a$ be the highest power of p that divides a, i.e., the greatest m such that $a \equiv 0 \,(\mathrm{mod}\, p^m)$ where we used the notation $a \equiv b \,(\mathrm{mod}\, c)$ meant c divides $a-b$. Note that $ord_p 0 = \infty$. The p-adic absolute value (norm) of x is given by $|x|_p = p^{-ord_p x}$ for $x \neq 0$ and $|0|_p = 0$.

Now we provide some basic notations: $\mathbb{N} = \{1, 2, 3, \cdots\}$ denotes the set of all natural numbers, $\mathbb{Z} = \{\cdots, , -1, 0, 1, \cdots\}$ denotes the ring of all integers, $\mathbb{C}$ denotes the field of all complex numbers, $\mathbb{Q}_p = \left\{x = \sum_{n=-k}^{\infty} a_n p^n : 0 \leqq a_i \leqq p-1\right\}$ denotes the field of all p-adic numbers, $\mathbb{Z}_p = \left\{x \in \mathbb{Q}_p : |x|_p \leqq 1\right\}$ denotes the ring of all p-adic integers and $\mathbb{C}_p$ denotes the completion of the algebraic closure of $\mathbb{Q}_p$.

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For more information about p-adic analysis, see [1-10, 12-18] and related references cited therein.

The q-number is defined by $[n]_q = \frac{q^n-1}{q-1}$. The symbol q can be variously considered as indeterminates, complex number $q \in \mathbb{C}$ with $0 < |q| < 1$, or p-adic number $q \in \mathbb{C}_p$ with $|q-1|_p < p^{-\frac{1}{p-1}}$ so that $q^x = \exp(x \log q)$ for $|x|_p \leqq 1$.

For $f \in UD(\mathbb{Z}_p) = \{f \,|\, f$ is uniformly differentiable function at a point $a \in \mathbb{Z}_p\}$, Kim defined the q-Volkenborn integral or p-adic q-integral on $\mathbb{Z}_p$ of a function $f \in UD(\mathbb{Z}_p)$ in [10] as follows:

$$I_q(f) = \int_{\mathbb{Z}_p} f(x)\, d\mu_q(x) = \lim_{N\to\infty} \frac{1}{[p^N]_q} \sum_{x=0}^{p^N-1} f(x)\, q^x. \tag{1.2}$$

Suppose that $f_1(x) = f(x+1)$. Then, we see that

$$qI_q(f_1) = I_q(f) + (q-1)\, f(0) + \frac{q-1}{\log q} f'(0). \tag{1.3}$$

For these related issues, see [1, 3, 9-11, 17] and related references cited therein.

The q-Daehee numbers $D_{n,q}$ and q-Daehee polynomials $D_{n,q}(x)$ are defined by means of q-Volkenborn integrals:

$$D_{n,q} = \int_{\mathbb{Z}_p} (x)_n\, d\mu_q(x) \quad (n \geq 0) \tag{1.4}$$

$$D_{n,q}(x) = \int_{\mathbb{Z}_p} (x+y)_n\, d\mu_q(y) \quad (n \geq 0), \tag{1.5}$$

where the symbol $(x)_n$ denotes the falling factorial given by

$$(x)_n = x(x-1)(x-2)\cdots(x-n+1). \tag{1.6}$$

The falling factorial $(x)_n$ satisfies the following identity:

$$(x)_n = \sum_{k=0}^{n} S_1(n,k)\, x^k, \tag{1.7}$$

where $S_1(n,k)$ denotes the Stirling number of the first kind (see [9, 11]).

It is obvious that $\lim_{q\to 1} D_{n,q} := D_n$ and $\lim_{q\to 1} D_{n,q}(x) := D_n(x)$.

The q-Daehee numbers and polynomials of the second kind are introduced by the following q-Volkenborn integrals:

$$\widehat{D}_{n,q} = \int_{\mathbb{Z}_p} (-x)_n\, d\mu_q(x) \quad (n \geq 0) \tag{1.8}$$

$$\widehat{D}_{n,q}(x) = \int_{\mathbb{Z}_p} (-x-y)_n\, d\mu_q(y) \quad (n \geq 0). \tag{1.9}$$

The q-Daehee polynomials and numbers and their various generalizations have been studied by many mathematicians, *cf.* [1, 9, 11]; see also the related references cited therein.

The p-adic gamma function is defined as follows

$$\Gamma_p(x) = \lim_{n\to x} (-1)^n \prod_{\substack{j<n \\ (p,j)=1}} j \qquad (x \in \mathbb{Z}_p), \tag{1.10}$$

where n approaches x through positive integers.

The p-adic Euler constant γ_p is defined by the following formula

$$\gamma_p := -\frac{\Gamma_p'(1)}{\Gamma_p(0)} = \Gamma_p'(1) = -\Gamma_p'(0). \tag{1.11}$$

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The p-adic gamma function in conjunction with its several extensions and p-adic Euler constant have been developed by many physicists and mathematicians, *cf.* [2, 4-8, 12, 14-16, 18]; see also the references cited in each of these earlier works.

For $x \in \mathbb{Z}_p$, the symbol $\binom{x}{n}$ is given by $\binom{x}{0} = 1$ and $\binom{x}{n} = \frac{x(x-1)\cdots(x-n+1)}{n!}$ $(n \in \mathbb{N})$.

Proposition 1. *(Kim et al. [9]) The following relation holds true for $n \geq 0$:*

$$\int_{\mathbb{Z}_p} \binom{x}{n} d\mu_q(x) = \frac{q^n}{(1-q)^n} - \frac{1}{\log q} \sum_{j=0}^{n} \frac{(-1)^j q^{n-j}}{j(1-q)^{n-j}}. \tag{1.12}$$

Let $x \in \mathbb{Z}_p$ and $n \in \mathbb{N}$. The functions $x \to \binom{x}{n}$ form an orthonormal base of the space $C(\mathbb{Z}_p \to \mathbb{C}_p)$ with respect to the euclidean norm $\|\cdot\|_\infty$. The mentioned orthonormal base satisfy the following equality:

$$\binom{x}{n}' = \sum_{j=0}^{n-1} \frac{(-1)^{n-j-1}}{n-j} \binom{x}{j} \quad \text{(see [13, 16, 18])}. \tag{1.13}$$

Mahler investigated a generalization for continuous maps of a p-adic variable utilizing the special polynomials as binomial coefficient polynomial [13] in 1958. It implies that for any $f \in C(\mathbb{Z}_p \to \mathbb{C}_p)$, there exist unique elements $a_o, a_1, a_2, \ldots$ of $\mathbb{C}_p$ such that

$$f(x) = \sum_{n=0}^{\infty} a_n \binom{x}{n} \quad (x \in \mathbb{Z}_p). \tag{1.14}$$

The base $\left\{\binom{*}{n} : n \in \mathbb{N}\right\}$ is named as Mahler base of the space $C(\mathbb{Z}_p \to \mathbb{C}_p)$, and the components $\{a_n : n \in \mathbb{N}\}$ in $f(x) = \sum_{n=0}^{\infty} a_n \binom{x}{n}$ are called Mahler coefficients of $f \in C(\mathbb{Z}_p \to \mathbb{C}_p)$. The Mahler expansion of the p-adic gamma function Γ_p and its Mahler coefficients are discovered in [16] as follows.

Proposition 2. *For $x \in \mathbb{Z}_p$, let $\Gamma_p(x+1) = \sum_{n=0}^{\infty} a_n \binom{x}{n}$ be Mahler series of Γ_p. Then its coefficients satisfy the following identity:*

$$\sum_{n \geqq 0} (-1)^{n+1} a_k \frac{x^k}{k!} = \frac{1-x^p}{1-x} \exp\left(x + \frac{x^p}{p}\right). \tag{1.15}$$

The outlines of this paper are as follows: the first part is introduction which provides the required information, notations, definitions and motivation; in part 2, we are interested in constructing the correlations between the p-adic gamma function and the q-Daehee polynomials and numbers by using the methods of the q-Volkenborn integral and Mahler series expansion; in the last part, we examine the results derived in this paper.

2. Main Results

This section provides some properties, identities and correlations for the mentioned gamma function, q-Daehee polynomials and numbers, Stirling numbers of the first kind and p-adic Euler constant.

The q-Volkenborn integral on $\mathbb{Z}_p$ of the p-adic gamma function via Proposition 1 and Proposition 2 is as follows.

Theorem 1. *The following identity holds true for $n \in \mathbb{N}$:*

$$\int_{\mathbb{Z}_p} \Gamma_p(x+1) d\mu_q(x) = \sum_{n=0}^{\infty} a_n \left(\frac{q^n}{(1-q)^n} - \frac{1}{\log q} \sum_{j=0}^{n} \frac{(-1)^j q^{n-j}}{j(1-q)^{n-j}} \right),$$

where a_n is given by Proposition 2.

Proof. For $x, y \in \mathbb{Z}_p$, by Proposition 2, we get

$$\int_{\mathbb{Z}_p} \Gamma_p(x+1) d\mu_q(x) = \sum_{n=0}^{\infty} a_n \int_{\mathbb{Z}_p} \binom{x}{n} d\mu_q(x) = \frac{q^n}{(1-q)^n} - \frac{1}{\log q} \sum_{j=0}^{n} \frac{(-1)^j q^{n-j}}{j(1-q)^{n-j}}$$

and using the formula (1.12), we acquire

$$\int_{\mathbb{Z}_p} \Gamma_p(x+1)\, d\mu_q(x) = \sum_{n=0}^{\infty} a_n \left(\frac{q^n}{(1-q)^n} - \frac{1}{\log q} \sum_{j=0}^{n} \frac{(-1)^j q^{n-j}}{j(1-q)^{n-j}} \right),$$

which implies the desired result. □

We present one other q-Volkenborn integral of the p-adic gamma function via q-Daehee polynomials by Theorem 2.

Theorem 2. *Let $x, y \in \mathbb{Z}_p$. We have*

$$\int_{\mathbb{Z}_p} \Gamma_p(x+y+1)\, d\mu_q(x) = \sum_{n=0}^{\infty} a_n \frac{D_{n,q}(y)}{n!}, \tag{2.1}$$

where a_n is given by Proposition 2.

Proof. For $x, y \in \mathbb{Z}_p$, by the relation $\binom{x+y}{n} = \frac{(x+y)_n}{n!}$ and Proposition 2, we get

$$\int_{\mathbb{Z}_p} \Gamma_p(x+y+1)\, d\mu_q(x) = \int_{\mathbb{Z}_p} \sum_{n=0}^{\infty} a_n \frac{(x+y)_n}{n!} d\mu_q(x) = \sum_{n=0}^{\infty} a_n \frac{1}{n!} \int_{\mathbb{Z}_p} (x+y)_n\, d\mu_q(x),$$

which is the desired result (2.1) via Eq. (1.5). □

We now examine a consequence of the Theorem 2 as follows.

Corollary 1. *Choosing $y = 0$ in Theorem 2 gives the fo llowing relation including Γ_p and $D_{n,q}$:*

$$\int_{\mathbb{Z}_p} \Gamma_p(x+1)\, d\mu_q(x) = \sum_{n=0}^{\infty} a_n \frac{D_{n,q}}{n!}, \tag{2.2}$$

where a_n is given by Proposition 2.

Here is the p-adic q-integral of the derivative of the p-adic gamma function.

Theorem 3. *For $x, y \in \mathbb{Z}_p$, we have*

$$\int_{\mathbb{Z}_p} \Gamma_p'(x+y+1)\, d\mu_q(x) = \sum_{n=0}^{\infty} \sum_{j=0}^{n-1} a_n \frac{(-1)^{n-j-1} D_{j,q}(y)}{(n-j)\, j!}.$$

Proof. In view of (1.3), we obtain

$$\int_{\mathbb{Z}_p} \Gamma_p'(x+y+1)\, d\mu_q(x) = \int_{\mathbb{Z}_p} \sum_{n=0}^{\infty} a_n \binom{x+y}{n}' d\mu_q(x) = \sum_{n=0}^{\infty} a_n \int_{\mathbb{Z}_p} \binom{x+y}{n}' d\mu_q(x)$$

and using (1.13), we derive

$$\begin{aligned} \int_{\mathbb{Z}_p} \Gamma_p'(x+y+1)\, d\mu_q(x) &= \sum_{n=0}^{\infty} \sum_{j=0}^{n-1} a_n \frac{(-1)^{n-j-1}}{n-j} \int_{\mathbb{Z}_p} \binom{x+y}{j} d\mu_q(x) \\ &= \sum_{n=0}^{\infty} \sum_{j=0}^{n-1} a_n \frac{(-1)^{n-j-1}}{n-j} \frac{D_{n,q}(y)}{j!}. \end{aligned}$$

□

The immediate result of Theorem 3 is given as follows.

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Corollary 2. *For $x \in \mathbb{Z}_p$, we have*

$$\int_{\mathbb{Z}_p} \Gamma_p'(x+1)\, d\mu_q(x) = \sum_{n=0}^{\infty} \sum_{j=0}^{n-1} a_n \frac{(-1)^{n-j-1} D_{j,q}}{(n-j)\, j!}.$$

The p-adic gamma function can be determined by means of the Stirling numbers of the first kind as follows.

Theorem 4. *For $x, y \in \mathbb{Z}_p$, we obtain*

$$\Gamma_p(x+y+1) = \sum_{n=0}^{\infty} \sum_{k=0}^{n} a_n \frac{S_1(n,k)}{n!} (x+y)^k .$$

Proof. From (1.7) and Proposition 2, we have

$$\Gamma_p(x+y+1) = \sum_{n=0}^{\infty} a_n \frac{(x+y)_n}{n!} = \sum_{n=0}^{\infty} \frac{a_n}{n!} \sum_{k=0}^{n} S_1(n,k)(x+y)^k ,$$

which gives the desired result. □

As a result of Theorem 4, one other q-Volkenborn integral of the p-adic gamma function via the q-Bernoulli polynomials is stated below.

Corollary 3. *For $x, y \in \mathbb{Z}_p$, we acquire*

$$\int_{\mathbb{Z}_p} \Gamma_p(x+y+1)\, d\mu_q(x) = \sum_{n=0}^{\infty} \sum_{k=0}^{n} a_n \frac{S_1(n,k)}{n!} B_{k,q}(y) ,$$

where $B_n(x)$ denotes n-th q-Bernoulli polynomial defined in [9] *by the following p-adic q-integral on $\mathbb{Z}_p$:*

$$B_{n,q}(x) = \int_{\mathbb{Z}_p} (x+y)^n\, d\mu_q(y) \qquad (n \geqq 0) .$$

We now provide a new and interesting representation of the p-adic Euler constant by means of q-Daehee polynomials and numbers.

Theorem 5. *We have*

$$\gamma_p = \sum_{n=0}^{\infty} \sum_{j=0}^{n-1} a_n \frac{(-1)^{n-j-1}}{(n-j)\, j!} \frac{(qD_{n,q} - D_{n,q}(-1))}{1-q} + \frac{\Gamma_p^{(2)}(0)}{\log q}.$$

Proof. Taking $f(x) = \Gamma_p'(x)$ in Eq. (1.3) yields the following result

$$q \int_{\mathbb{Z}_p} \Gamma_p'(x+1)\, d\mu_q(x) - \int_{\mathbb{Z}_p} \Gamma_p'(x)\, d\mu_q(x) = (q-1)\,\Gamma_p'(0) + \frac{q-1}{\log q} \Gamma_p^{(2)}(0)$$

where $\Gamma_p^{(2)}(0)$ is the second derivative of the p-adic gamma function at $x = 0$, and with some basic calculations and using Theorem 3, we have

$$\begin{aligned} LHS &= q \int_{\mathbb{Z}_p} \Gamma_p'(x+1)\, d\mu_q(x) - \int_{\mathbb{Z}_p} \Gamma_p'(x)\, d\mu_q(x) \\ &= q \sum_{n=0}^{\infty} \sum_{j=0}^{n-1} a_n \frac{(-1)^{n-j-1}}{n-j} \frac{D_{n,q}}{j!} - \sum_{n=0}^{\infty} \sum_{j=0}^{n-1} a_n \frac{(-1)^{n-j-1}}{n-j} \frac{D_{n,q}(-1)}{j!} \\ &= \sum_{n=0}^{\infty} \sum_{j=0}^{n-1} a_n \frac{(-1)^{n-j-1}}{n-j} \frac{(qD_{n,q} - D_{n,q}(-1))}{j!} \end{aligned}$$

and

$$RHS=(1-q)\gamma_p+\frac{q-1}{\log q}\Gamma_p^{(2)}(0).$$

Finally,

$$\sum_{n=0}^{\infty}\sum_{j=0}^{n-1}a_n\frac{(-1)^{n-j-1}}{(n-j)\,j!}\frac{(qD_{n,q}-D_{n,q}(-1))}{1-q}+\frac{\Gamma_p^{(2)}(0)}{\log q}=\gamma_p,$$

whence the asserted result. □

We state the following theorem including a relation between $\Gamma_p(x)$ and $\widehat{D}_{n,q}(x)$.

Theorem 6. *For $x,y\in\mathbb{Z}_p$, we have*

$$\int_{\mathbb{Z}_p}\Gamma_p(-x-y+1)\,d\mu_q(x)=\sum_{n=0}^{\infty}a_n\frac{\widehat{D}_{n,q}(y)}{n!},$$

where a_n is given by Proposition 2.

Proof. For $x,y\in\mathbb{Z}_p$, by the relation $\binom{-x-y}{n}=\frac{(-x-y)_n}{n!}$ and Proposition 2, we get

$$\int_{\mathbb{Z}_p}\Gamma_p(-x-y+1)\,d\mu_q(x)=\int_{\mathbb{Z}_p}\sum_{n=0}^{\infty}a_n\frac{(-x-y)_n}{n!}d\mu_q(x)=\sum_{n=0}^{\infty}a_n\frac{1}{n!}\int_{\mathbb{Z}_p}(-x-y)_n\,d\mu_q(x),$$

which is the desired result with Eq. (1.9). □

A consequence of Theorem 6 is given by the following corollary.

Corollary 4. *Upon setting $y=0$ in Theorem 6 gives the following relation for Γ_p and $D_{n,q}$:*

$$\int_{\mathbb{Z}_p}\Gamma_p(-x+1)\,d\mu_q(x)=\sum_{n=0}^{\infty}a_n\frac{\widehat{D}_{n,q}}{n!},$$

where a_n is given by Proposition 2.

3. Conclusion

In this paper, we have discovered three q-Volkenborn integrals (p-adic q- integrals) of p-adic gamma function via their Mahler expansions. The first has been obtained directly as a result of q-Volkenborn integral of $\Gamma_p(x)$. The others have been derived in terms of the q-Daehee polynomials and numbers and q-Daehee polynomials and numbers of the second kind. Then, we have investigated q-Volkenborn integral of the derivative of p-adic gamma function. Furthermore, we have attained the correlationship involving the p-adic gamma function and Stirling numbers of the first kind. We have finally given a novel representation for the p-adic Euler constant by means of the q-Daehee polynomials and numbers.

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