Joint Symbol Timing and Frequency Offset Estimation for Wireless OFDM Systems

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Abstract—This work presents a new technique for blind and joint estimation of symbol timing and carrier frequency offset in wireless orthogonal frequency division multiplexing (OFDM) systems. The joint estimation is achieved by sensing the interference introduced at the fast Fourier transform (FFT) output when a carrier frequency or timing offsets exist. The synchronization parameters are selected such that the interference is minimized. The proposed joint estimator is highly efficient because it does not require any overhead or channel state information. Simulation results show that the system is effective and robust even at low signal-to-noise ratios.

I. INTRODUCTION

Due to the recent development of digital signal processing algorithms and very large-scale integrated circuit technologies, the initial implementation obstacles of the orthogonal frequency division multiplexing (OFDM) no longer exist [1]-[3]. Therefore, OFDM has been adopted in several digital communication systems such as digital audio broadcasting [4], terrestrial digital television and HDTV broadcasting [5]. Moreover, various projects and prototypes of OFDM systems are widely used, including digital video broadcasting for digital terrestrial television (DVB-T) by the European Broadcasting Union, WLAN, and WiMAX, etc.

OFDM is a modulation scheme in which $N$ parallel data streams modulate $N$ orthogonal subcarriers. Such configuration is remarkably effective in combating high levels of multipath propagation, with a wide spread of delays between the received multipath signals [6]. The superior performance of OFDM with respect to multipath interference effects is only achieved via a careful selection of the system parameters. Moreover, the existence of a large number of adjacent orthogonal subcarriers has made the OFDM highly sensitive to synchronization errors.

OFDM system synchronization, which includes symbol timing (ST) recovery and carrier frequency offset (CFO) estimations, is a critical issue that has attracted a considerable attention by researchers. The ST and the CFO have to be accurately estimated and compensated before the fast Fourier transform (FFT) to avoid loss of orthogonality, and thus interference. While OFDM systems can tolerate some errors in the ST estimates due to the use of the cyclic prefix (CP) [3], the CFO must be estimated accurately at the receiver to avoid a severe degradation in the signal-to-noise ratio (SNR). OFDM systems are highly sensitive to CFO and they can tolerate offsets which are only a fraction of the frequency spacing between the subcarriers without a large degradation in system performance [7]. As a consequence, an increasing number of research studies have been dedicated to deduce efficient synchronization techniques for OFDM systems. However, most of the work presented in the literature was focused on single parameter estimation, i.e., ST or CFO. In such works, it is usually assumed that one of the parameters is perfectly known and compensated while the other parameter needs to be estimated [8], [9]. Hence, the research work dedicated for joint estimation of ST and CFO is very little compared to the single parameter estimation work. Moreover, a good number of the proposed estimation algorithms are data-aided in the sense that rely on inserting one or more specifically designed training symbols or pilots in the transmitted signal [10]-[14]. Generally speaking, data-aided algorithms have high accuracy, low complexity, and wide estimation range [11]. However, allocating a certain portion of the system resources for synchronization deteriorates the power and bandwidth efficiencies. Blind joint estimation algorithms have better power and bandwidth efficiencies than data-aided techniques [15]-[21], however this comes at the expense of accuracy. Moreover, they usually require more computational power to extract the synchronization parameters. An efficient and well referred-to approach for blind synchronization is presented in [15], [22]. In this approach, the periodic nature of the time-domain OFDM signal caused by using the cyclic prefix is exploited for synchronization.

In this work, we propose a new technique for blind and joint estimation of the ST and CFO of OFDM systems. The proposed technique is based on exploiting the interference that results when the subcarriers’ orthogonality is destroyed due the absence of the correct synchronization information. The synchronization parameters are selected such that the interference is minimized. The proposed algorithm does not require training symbols, channel state parameters, or CP.

The remainder of this paper is organized as follows. A
general OFDM system model is described in Section II. Then the joint estimation algorithm is proposed in Section III. Simulation results are presented in Section IV. Conclusions are given in Section V.

II. OFDM SYSTEM MODEL

In OFDM systems, a sequence of independent complex symbols \( \mathbf{X} = [X_0, X_1, \ldots, X_{2K-1}] \) is used to modulate a \( 2K \) orthogonal subcarriers \( \mathbf{C} = [C_0, C_1, \ldots, C_{2K-1}] \). The elements of the sequence \( \mathbf{X} \) are usually drawn uniformly from a QAM or MPSK constellations. The sequence \( \mathbf{X} \) is zero-padded and applied to an \( N \)-points inverse fast Fourier transform (IFFT) process. The \( l \)th OFDM transmission symbol is given by [7]

\[
x_n(l) = \frac{1}{N} \sum_{i=-K}^{K-1} X_i(l) \exp \left( \frac{j2\pi i n}{N} \right),
\]

\( n = 0, 1, \ldots, N-1 \), \( N \geq 2K+1 \), \( l = 0, 1, \ldots, L-1 \).

To ensure that no inter-symbol-interference (ISI) is introduced between adjacent OFDM symbols and to maintain the orthogonality of all subcarriers, a time-domain guard-band, denoted as the cyclic prefix (CP), is created by copying the last \( N_{CP} \) samples of the IFFT output and appending them at the beginning of the symbol to be transmitted. The value of \( N_{CP} \) should be greater than the channel impulse response. Therefore, the transmitted OFDM block \( \mathbf{x} \) has \( N_T = N + N_{CP} \) samples,

\[
\mathbf{x} = \{x_{N-N_{CP}}, \ldots, x_{N-2}, x_{N-1}, x_0, x_1, \ldots, x_{N-1}\}
\]

(2)

The useful part of the OFDM symbol does not include the \( N_{CP} \) prefix samples and has a duration of \( T \) seconds. At the receiver front-end, the received signal is applied to a matched filter and then it is sampled at a rate \( T_s = T/N \) producing the sequence

\[
\mathbf{Y} = \{y_{N-N_{CP}}, \ldots, y_{N-2}, y_{N-1}, y_0, y_1, \ldots, y_{N-1}\}
\]

(3)

where

\[
y_n(l) = H_k(l)x_n(l) \exp \left( j \left( \frac{2\pi \epsilon (n + \epsilon N_T)}{N} + \phi \right) \right) + w_n(l)
\]

(4)

and where \( H_k(l) \) is the channel transfer function at the frequency of the \( k \)th subcarrier of the \( l \)th OFDM block, \( w_n \) is modeled as a white Gaussian process with zero mean and variance \( \sigma_w^2 = E \left[ |w_n|^2 \right] \), \( \epsilon \) is the CFO normalized to the subcarriers frequency spacing, and \( \phi \) is an initial unknown phase offset. By substituting (1) into (4) we obtain,

\[
y_n(l) = \frac{1}{N} \sum_{i=-K}^{K-1} H_i(l)X_i(l) \exp \left[ j \left( \frac{2\pi \epsilon (n + \epsilon lN_T)}{N} \right) + \phi \right] + w_n(l)
\]

(5)

After removing the \( N_{CP} \) prefix samples, the remaining \( N \) samples are fed to the fast Fourier transform (FFT). However, without symbol timing information, the \( N \) samples fed to the FFT do not necessarily belong to one OFDM block. Therefore, the FFT window will generally contain \( N \) samples that belong to two consecutive OFDM blocks, the \( k \)th subcarrier can be expressed as

\[
Y_k(l) = \frac{1}{N} \sum_{n=0}^{N-1} \sum_{i=-K}^{K-1} H_i(l)X_i(l) \exp \left[ j \left( \frac{2\pi \epsilon (n + \epsilon lN_T)}{N} \right) + \phi \right] + \frac{1}{N} \sum_{n=N}^{N+\delta-1} \sum_{i=-K}^{K-1} H_i(l+1)X_i(l+1)
\]

\[
\cdot \exp \left[ j \left( \frac{2\pi \epsilon (n + \epsilon l + \epsilon (l+1)N_T)}{N} \right) + \phi \right] + W_k(l),
\]

(6)

where \( \delta \) is the timing offset (TO) in samples. As it can be noted from (6), \( Y_k \) consists of the desired signal in addition to interference from other subcarriers and from adjacent OFDM symbols, hence it can be expressed as

\[
Y_k = \alpha_k H_k X_k + W_k + I_k
\]

(7)

where \( \alpha \) is an attenuation factor and \( I_k \) is an additive interference term. In the case that \( \delta \leq N_{CP} \), the FFT output reduces to [7]

\[
Y_k(l) = H_k(l)X_k(l) \sin(\pi \epsilon) \sin(\pi \epsilon \frac{2l + \hat{N}}{\hat{N}})
\]

\[
\cdot \exp \left( j\pi \left( \epsilon (2l + \hat{N}) + \phi \right) \right) + I_k(l) + W_k(l)
\]

(8)

where \( \hat{N} = (N-1)/N \) and where

\[
I_k(l) = \sum_{i=-K}^{K-1} H_i(l)X_i(l) \frac{\sin(\pi \epsilon)}{N \sin(\frac{\pi (i-k+\epsilon)}{\hat{N}})}
\]

\[
\cdot \exp \left( j\pi \left( \epsilon (2l + \hat{N}) - \frac{i-k}{N} + \phi \right) \right).
\]

(9)

Moreover, if \( \epsilon = 0 \) then

\[
Y_k(l) = H_k(l)X_k(l) \exp(j\phi) + W_k(l)
\]

(10)

III. JOINT SYMBOL TIMING AND CFO ESTIMATION

By comparing the output of the FFT in the presence and absence of ST and CFO which is given in (7) and (10), respectively, we observe that (10) contains an additional term which corresponds to the interference. This interference was exploited in [23] to estimate the CFO by minimizing the variance of the sequence \( |Y_k|^2 \), \( \mathbf{Y} = \{Y_0, Y_1, ..., Y_{N-1}\} \). The cost function for this estimator is defined as

\[
J_1(\epsilon) = \text{var}(|\mathbf{Y}|^2), \quad (11)
\]
and the estimated CFO \( \hat{\epsilon} \) is given by

\[
\hat{\epsilon} = \arg \min J_1(\epsilon) |_{\delta = 0} \quad (12)
\]

where \( \epsilon \) is the trial values of \( \epsilon \). Similarly, the ST was estimated in [8] using the same principle, however, the cost function is

\[
J_2(\Delta) = \text{var}[|Y|^2] \quad (13)
\]

where \( \Delta = \delta/N \) is the normalized symbol timing offset. Thus the estimated timing offset \( \hat{\Delta} \) is given by

\[
\hat{\Delta} = \arg \min J_2(\Delta) |_{\epsilon = 0} \quad (14)
\]

where \( \hat{\Delta} \) is the trial values of \( \Delta \). By observing that the cost functions described in (11) and (13) require exactly the same computations, then the same approach can be applied to jointly estimate the ST and the CFO. Thus,

\[
\hat{\epsilon}, \hat{\Delta} = \arg \min J(\hat{\epsilon},\hat{\Delta}). \quad (15)
\]

where

\[
J(\epsilon, \Delta) = \text{var}[|Y|^2] |_{\Delta, \epsilon \neq 0} \quad (16)
\]

The variance in (15) can be approximated by the sample variance, hence

\[
J(\hat{\epsilon}, \hat{\Delta}) \approx \frac{1}{N} \sum_{i=-K}^{K-1} |Y_i|^4 \left[ \frac{1}{N} \sum_{i=-K}^{K-1} |Y_i|^2 \right]^2 \quad (17)
\]

It should be noted that the cost function described by (11) is periodic with respect to CFOs that are multiple of the subcarrier frequency spacing. Thus, the estimation range is limited to half of the subcarrier frequency spacing. The block diagram of the proposed synchronizer is depicted in Fig. 1.

As demonstrated by (15), the joint estimation requires two-dimensional search process which might be complicated and time consuming depending on the search-step resolution and on the targeted system performance. The two-dimensional search can be replaced by two one-dimensional operations, one for \( \hat{\epsilon} \) and the other for \( \hat{\Delta} \). The major limitation for applying the one-dimensional search approach is the coupling-effect [25]. The coupling effect occurs because the uncertainty in one parameter affects the estimation of the other parameter. For example, estimating the ST in the presence of CFO is very challenging due to the interference which causes SNR degradation. Similarly, the presence of ST uncertainty will substantially deteriorate the CFO estimates accuracy.

In this paper, we assume that the integer part of the CFO is estimated and compensated [9]. The fractional CFO, \( |\epsilon| < 1/2 \), is unknown and its value is randomly selected from a finite set that consists of \( M \) elements. Consequently, \( \hat{\epsilon} \) is given by

\[
\hat{\epsilon} = m\epsilon_{\text{step}} \quad m = \{(1-M)/2, \ldots, 0, \ldots, (M-1)/2\} \quad (18)
\]

where \( \epsilon_{\text{step}} \) is the CFO searching step, \( (M-1)\epsilon_{\text{step}} = 1 \). The value of \( \hat{\epsilon} \) is obtained by linearly searching for the minimum variance point through all possible values of \( M \). Then, \( \hat{\epsilon} \) is selected as the trial value that corresponds to the minimum variance point. The value of \( \epsilon_{\text{step}} \) must be selected such that the produced estimates are accurate enough to prevent any significant performance degradation. Obviously, \( \epsilon_{\text{step}} \) depends on the OFDM system parameters, particularly, the modulation type and order. For example, higher order modulations such as 64-QAM are very sensitive to CFOs, hence it requires very small \( \epsilon_{\text{step}} \). A general rule for selecting \( \epsilon_{\text{step}} \) is to set a targeted BER value that the system must achieve. For example, in the case of 8K DVB-T systems, the aim is to obtain a BER after Viterbi decoding that is approximately \( 2.10^{-4} \) over additive white Gaussian noise (AWGN) channels. If 16-QAM and 2/3 code rate are used, then the required SNR to achieve the targeted BER is about 12 dB [5]. Fig. 2 shows the SNR degradation of an 8K DVB-T system using QPSK and 16-QAM subcarriers modulation as a function of CFO, code rates of 1/2 and 2/3 are used. For example, consider an OFDM system using a 16-QAM and 2/3 code rate. If the system can tolerate SNR degradation of 2 dB, then the CFO should not exceed 0.04 and \( \epsilon_{\text{step}} \leq 0.04 \) must be used. A similar scenario can be applied to the ST estimation case, however the SNR degradation is not as severe as in the CFO due to CP guardband.

For the timing offset, it is usually assumed the sampling clocks at the transmitter and the receiver are synchronized [22]. Consequently, the trial values of \( \Delta \) are discrete and the number of trial values is equal to \( N \).

### IV. Numerical Results

Consider an OFDM system with \( N = 8192 \) subcarriers modulated using 16-QAM, the length of the prefix samples \( N_{CP} = 512 \) which corresponds to 6.25\% of the OFDM symbol length. Monte Carlo simulation is used to assess the system performance over AWGN channel. Each simulation point is produced using \( 10^4 \) OFDM symbols. The estimates accuracy is assessed by means of mean squared error (MSE) where

\[
MSE_{ST} = E[(\hat{\Delta} - \Delta)^2], \quad \epsilon \neq 0 \quad (19)
\]

and

\[
MSE_{CFO} = E[(\hat{\epsilon} - \epsilon)^2], \quad \Delta \neq 0 \quad (20)
\]

The system performance is evaluated in two different operation modes, the acquisition mode and the tracking mode.
In the acquisition mode, the initial values of the CFO and the ST are very high. In the tracking mode we assume that both parameters are equal to zero. The \(MSE_{ST}\) is evaluated for various combinations of \(\Delta\) and \(\epsilon\) as a function of SNR over AWGN channel for a SNR range of -5 dB to 15 dB with a resolution of 1 dB as depicted in Fig. 3. This figure shows that the system has the capability to acquire the ST for CFO values as high as 0.5. In the tracking mode, \(\Delta = \epsilon = 0\), the proposed synchronizer have demonstrated a very high accuracy even at SNR values less than 4 dB.

Similarly, we assessed the performance of the CFO estimator in the acquisition and tracking modes using the same OFDM system considered earlier. As demonstrated by Fig. 4, the system can not produce reliable estimates in the presence of large CFO offsets. This implies that the synchronization process should start by estimating the symbol timing, then the CFO can be estimated accurately. In the tracking mode, the proposed synchronizer was able to achieve substantially small MSE values for SNR smaller than 3 dB.

To performance of the proposed estimator is compared to the performance of the maximum likelihood synchronization (MLS) algorithm presented in [22] which is based on the correlation of CP samples in the received OFDM signal. The MSE of both estimators is evaluated for a range of SNR from -5 dB to 5 dB with a resolution of 1 dB as depicted in Figure 5. In this figure, we initially estimated the ST using both techniques, the estimated ST values are compensated, then the CFO was estimated correspondingly. Hence, the CFO is estimated in the presence of some ST uncertainty. It can be seen from this figure that the proposed joint estimator considerably outperforms the MLS estimator and it is considerably more robust to ST errors. Although the results are not presented in this work, simulations results have confirmed that the proposed system is robust and accurate even in frequency selective fading channels.

V. Conclusion and Future Work

We presented a new technique for blind and joint estimation of symbol timing and carrier frequency offset in OFDM systems. The joint estimator exploited the interference introduced at the fast Fourier transform output when a carrier frequency or timing offsets are present. The synchronization parameters were selected such that the interference is minimized. It was confirmed by using simulation results that the proposed joint estimator is highly efficient and robust even at low SNRs.

As it was observed from the presented results, the searching process was the major part of the system which determines the system complexity. Moreover, the accuracy of the CFO estimates were limited to the search step. Our future work includes the developing low complexity alternatives for the
linear search processes. In addition, the system performance will be evaluated in frequency selective fading channels for various system parameters.

REFERENCES


