The Capacity of Wireless Networks

Lijun Chen

08/14/2006

Hard Limits

- Lots of hard limits: thermodynamic laws, Bode integral, Shannon's channel capacity, …
- New dimension in a network setting--scalability
  - The scaling of the network resource (such as network capacity) that is determined by the constraints resulting from the physical interactions among individual systems
  - The scaling of the usage of network resource such as communication overhead that is necessary for the operation of the network functionality
- May have important implications for network architecture and protocols
  - E.g., one reason for reactive routing in ad hoc networks is the poor scaling of the network capacity and high communication overhead of proactive routing
- Just begin to look at the related issues…

Outline

❑ Introduction to ad hoc wireless networks

❑ The capacity of the static networks

    ❑ Based on P. Gupta and P. R. Kumar, Trans. Inform. Theo., 2000

❑ Discussions

Ad Hoc Wireless Networks
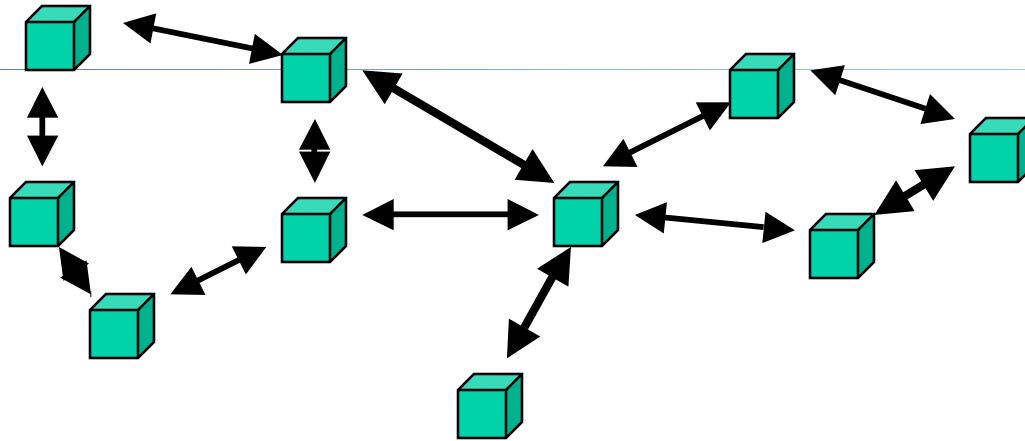
- ❑ Nodes can send, receive, and relay for each other
- ❑ No backbone infrastructure, no centralized control
- ❑ Interference limited: transmitting nodes create interference for other nodes
- ❑ Questions:
  - ❑ How does the network capacity scale with the number of nodes?
  - ❑ Any ways to improve the capacity?

Capacity of the Static Network

□ Network capacity: the achievable rate of end-to-end transmissions

  ❑ The rate of the point-to-point transmission is given
  ❑ Depends on routing and scheduling
  ❑ Not in a strict information theoretic sense
  ❑ Throughput capacity: (end-to-end rate) X (SD distance)
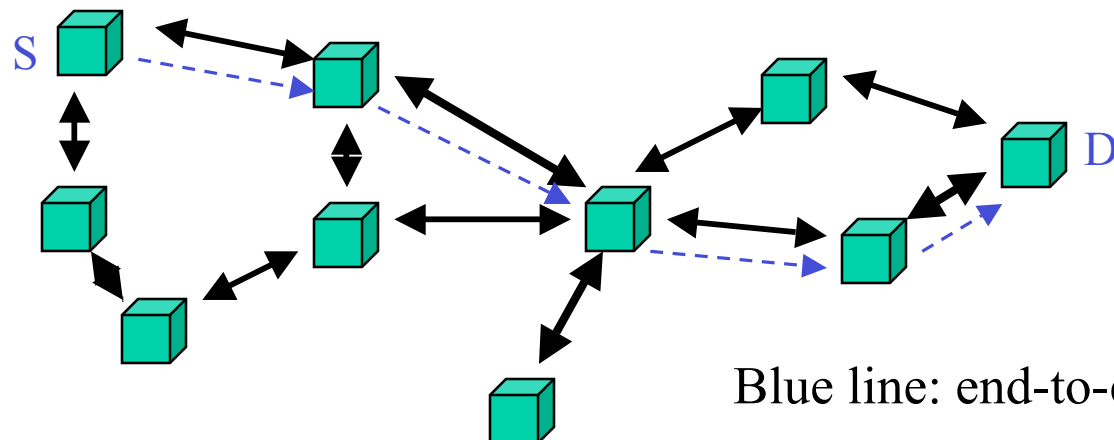

Blue line: end-to-end transmission

Capacity of the Static Network

❐ Consider two types of networks

    ❑ arbitrary network: nodes are arbitrarily located

    ❑ random network: nodes are placed randomly

Two Transmission Models

□ Protocol model

  ❑ transmission successful if there are no other senders within a distance $(1+\Delta)r$ of the receiver, where r is the distance from the sender to the receiver
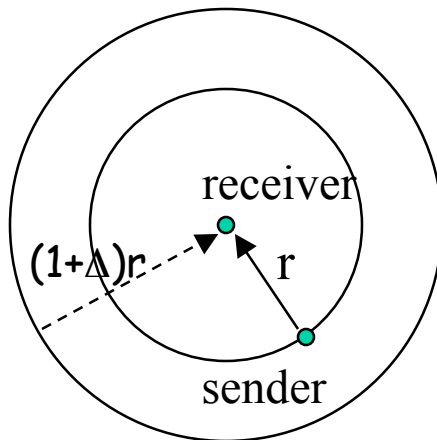
□ Physical (interference) model
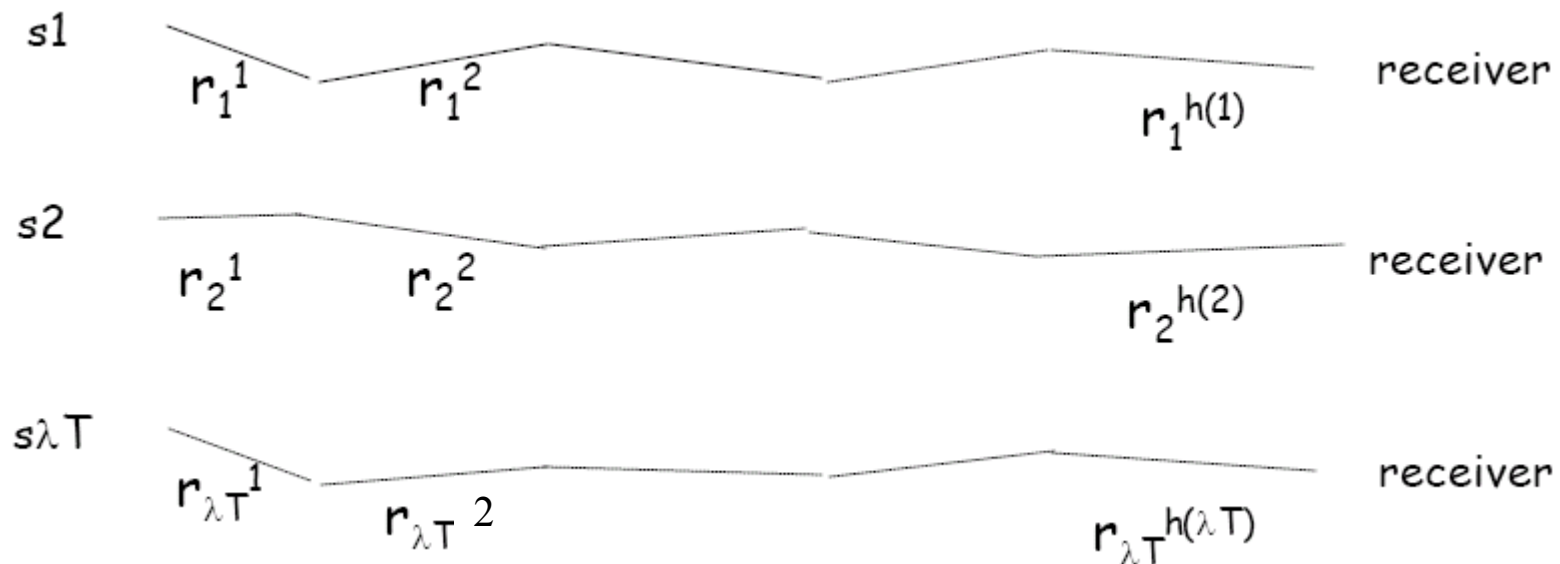
  ❑ Transmission successful if

$$SNIR = \frac{P_i r_i^{-\alpha}}{N + \sum_{j \neq i} P_j r_j^{-\alpha}} \geq \beta$$
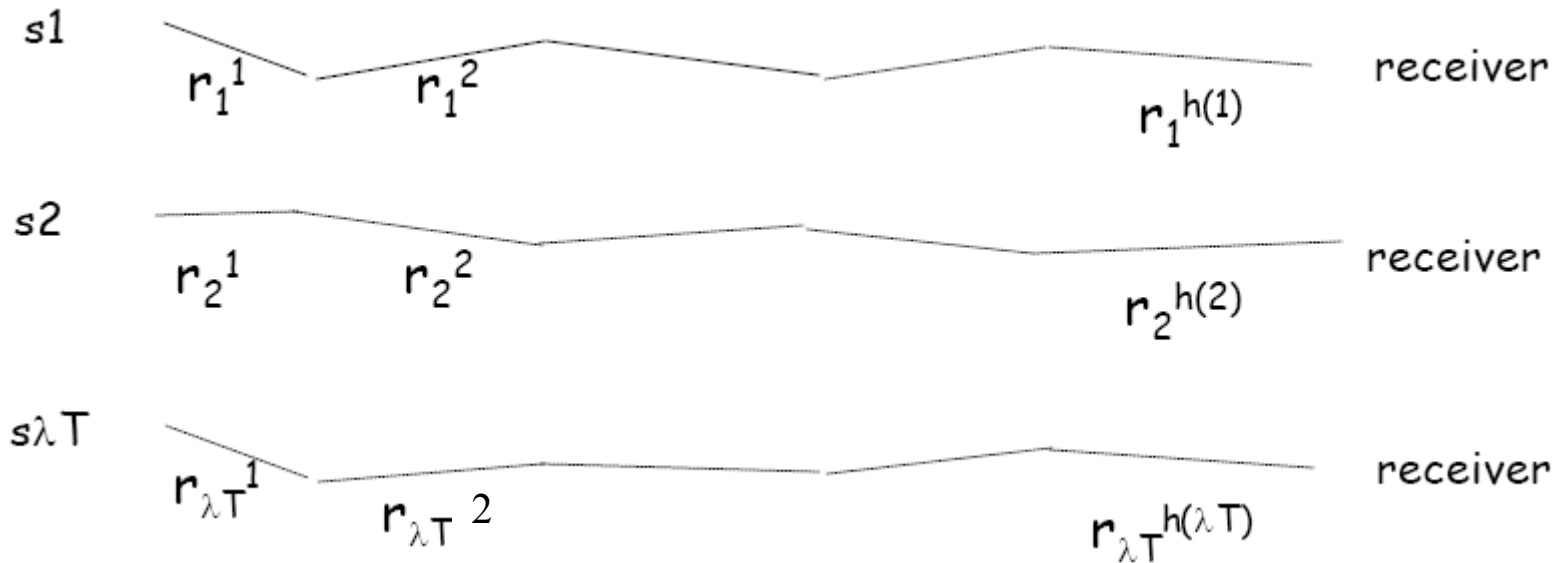
Arbitrary Networks

- domain is a disk of unit area
- there are n nodes in the domain
- each node can transmit at W bits/sec
- a node can not transmit and receive at the same

Capacity: Upper Bound

- Assume the network sends a total of λnT end-to-end bits in T seconds
- Chop time into a total of WT slots in T seconds; each slot is the time to transmit one bit and decision can be made for each bit
- Assume the b-th bit makes a total of h(b) hops from the sender to the receiver
- Let r_b^h denote the hop-length of the h-th hop of the b-th bit

s1 $\qquad$ receiver
r_1^1 $\qquad$ r_1^2 $\qquad$ $r_1^{h(1)}$

s2 $\qquad$ receiver
r_2^1 $\qquad$ r_2^2 $\qquad$ $r_2^{h(2)}$

$s\lambda T$ $\qquad$ receiver
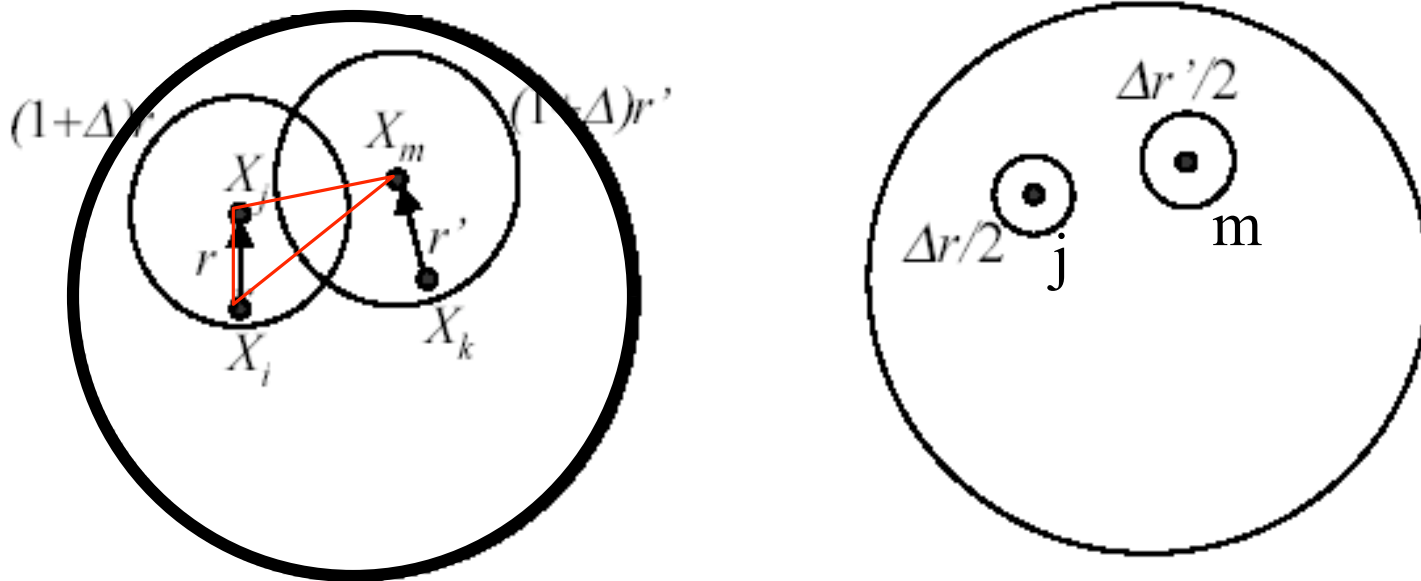$r_{\lambda T}^1$ $\qquad$ $r_{\lambda T}^2$ $\qquad$ $r_{\lambda T}^{h(\lambda T)}$

Hop-Count Constraint

Since there are a total of WT slots, and during each slot at most n/2 nodes can transmit, we have

$$\sum_{b=1}^{\lambda n T} h(b) \leq \frac{WTn}{2}$$
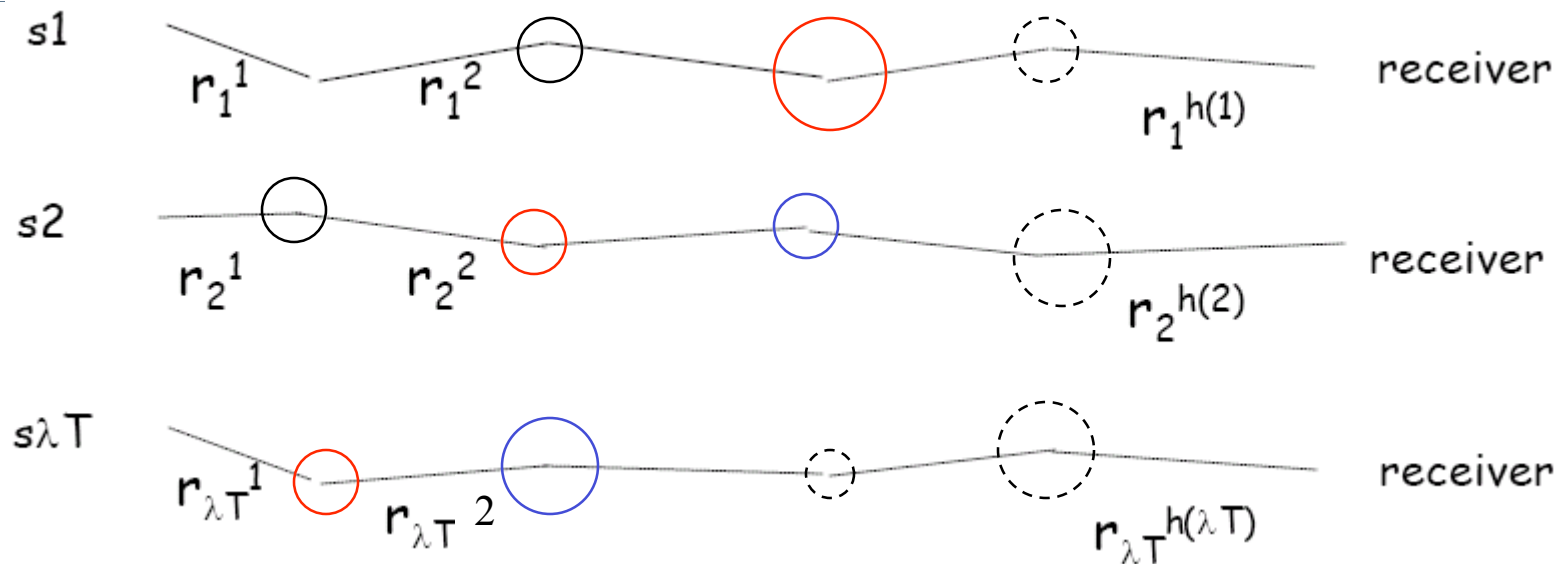
Area Constraint: Two Nodes

Triangle inequality: $D_{jm} + r >= D_{im} >= (1+\Delta)r'$
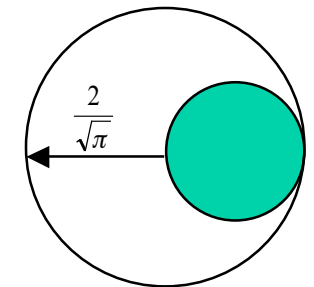
$D_{jm} + r' >= D_{jk} >= (1+\Delta)r$

$\Rightarrow$

$D_{jm} >= (r+r') \Delta / 2$

Area Constraint: Global Picture

s1 $r_1^{\,1}$ $r_1^{\,2}$ receiver $r_1^{\,h(1)}$

s2 $r_2^{\,1}$ $r_2^{\,2}$ receiver $r_2^{\,h(2)}$

$s\lambda T$ $r_{\lambda T}^{\,1}$ $r_{\lambda T}^{\,2}$ receiver $r_{\lambda T}^{\,h(\lambda T)}$

since each circle has at least ¼ of its area in the unit disk,

$$\frac{2}{\sqrt{\pi}}$$

$$\sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)} \frac{1}{4}\pi\left(\frac{\Delta r_b^h}{2}\right)^2 \le WT \;\rightarrow\; \sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)} (r_b^h)^2 \le \frac{16WT}{\pi\Delta^2}$$

12

Upper Bound

Assume L is the average (direct-line) distance for all source-destination pairs.

$$\lambda nTL \leq \sum_{b=1}^{\lambda nT} \sum_{h=1}^{h(b)} r_b^h$$

$\longrightarrow$

$$\lambda nTL \leq \sum_{b=1}^{\lambda nT} \sum_{h=1}^{h(b)} r_b^h \leq \sqrt{\sum_{b=1}^{\lambda nT} h(b)} \sqrt{\sum_{b=1}^{\lambda nT} \sum_{h=1}^{h(b)} (r_b^h)^2}$$

$\longrightarrow$

$$\lambda nTL \leq \sqrt{\frac{WTn}{2}} \sqrt{\frac{16WT}{\pi\Delta^2}} = \sqrt{\frac{8}{\pi}} \frac{WT}{\Delta} \sqrt{n}$$

$\longrightarrow$

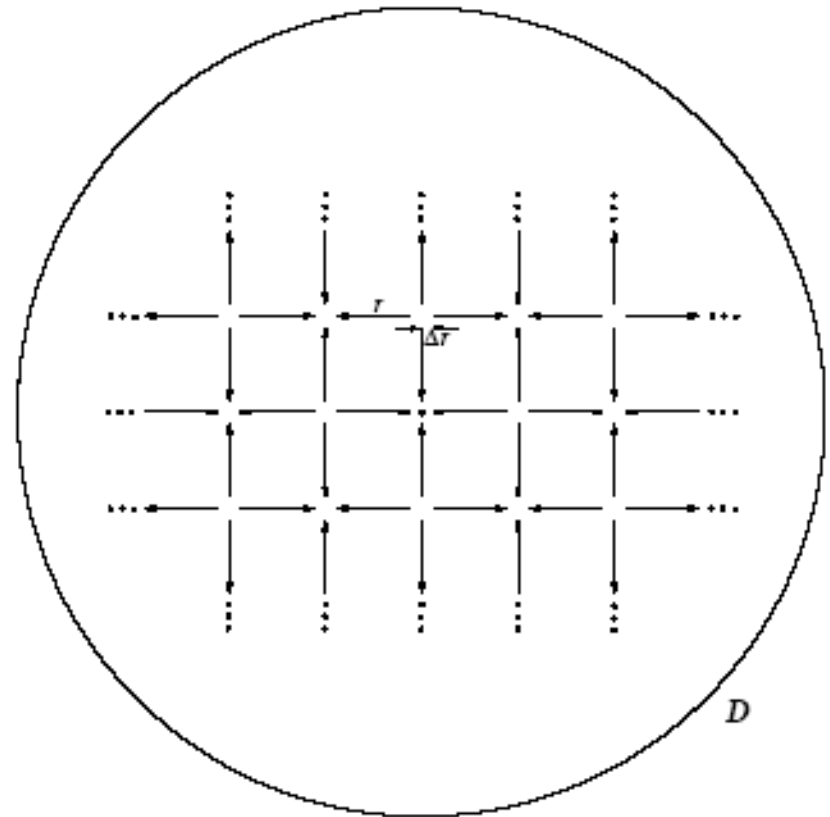$$\lambda nL \leq \sqrt{\frac{8}{\pi}} \frac{W}{\Delta} \sqrt{n}$$

A Constructive Lower Bound

□ n/2 senders and receivers:

$$r = \frac{1}{1+2\Delta} \frac{1}{\sqrt{\frac{n}{4}} + \sqrt{2\pi}}$$

□ Total bit-meters:

$$\frac{W}{1+2\Delta} \frac{n}{\sqrt{n} + \sqrt{8\pi}}$$

Capacity of Arbitrary Network

$$\frac{w}{1+2\Delta}\frac{n}{\sqrt{n}+\sqrt{8\pi}} \leq \lambda nL \leq \sqrt{\frac{8}{\pi}}\frac{w}{\Delta}\sqrt{n}$$

$\Rightarrow$ the average transport capacity for each node

$$\lambda L \propto \frac{1}{\sqrt{n}}$$

Random Networks

□ Uniform distribution of nodes and sources/destinations

□ Equal throughput $\lambda(n)$ bits/sec for all SD pairs

□ Each node chooses same power level, and hence equal transmitting radius r(n)

Required bit transmissions

- Assume mean length of each SD pair is L
- Average number of hops for each end-to-end bit:
  - **L/r(n)**

- Total required bit transmissions per second to support $\lambda(n)$:
  - **Lnλ(n)/r(n)**

Offered Bit Transmissions

□ What is the maximum number of bit trans-missions in one second?

  ❑ space used per transmission:
  - at least $\pi \Delta^2 r^2(n)/16$

  ❑ number of simultaneous transmissions at most:
  - $1 / (\pi \Delta^2 r^2(n)/16) = 16 / \pi \Delta^2 r^2(n)$

  ❑ total bits per second
  - $16\,W / \pi \Delta^2 r^2(n)$

Capacity for Random Networks

Required ~ offered

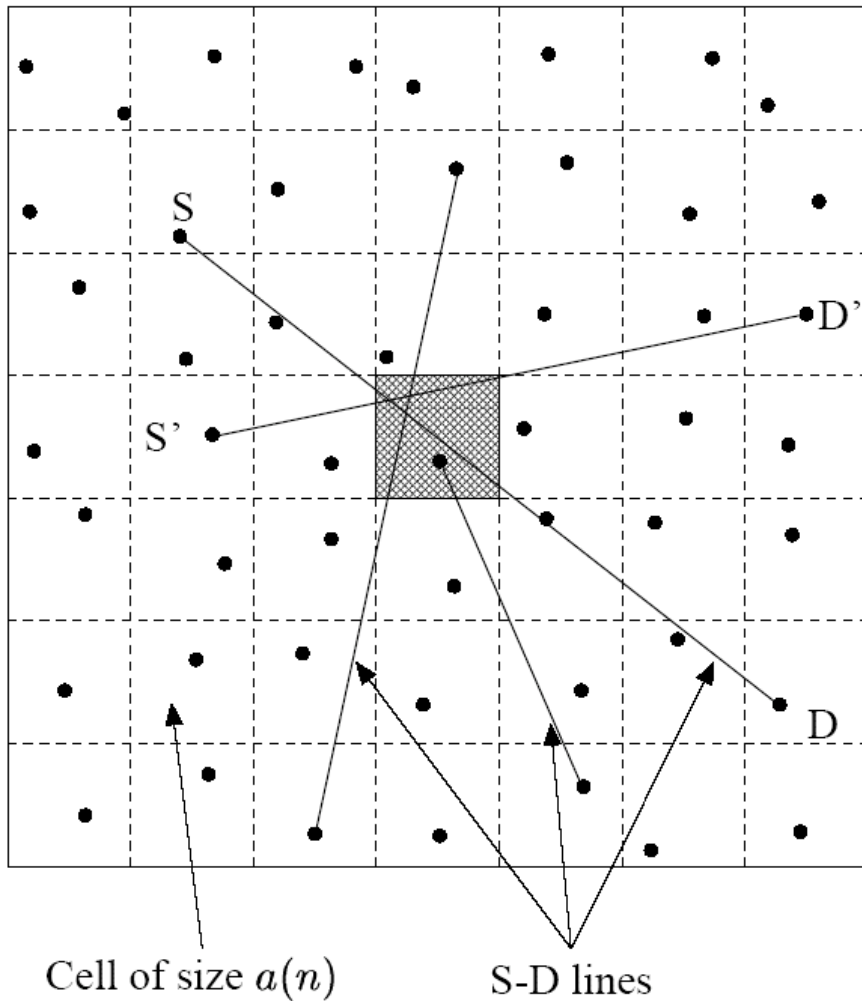**L n λ (n) / r(n)** ~ 16 W / $\pi\Delta^2 r^2(n)$

$\Rightarrow$

$\lambda(n)$ ~ 16W/($\pi\Delta$2Lnr(n))

To maintain connectivity, requires r(n) on the order of $\sqrt{\frac{\log n}{n}}$

$\Rightarrow$

$$\lambda(n) \propto \frac{1}{\sqrt{n \log n}}$$
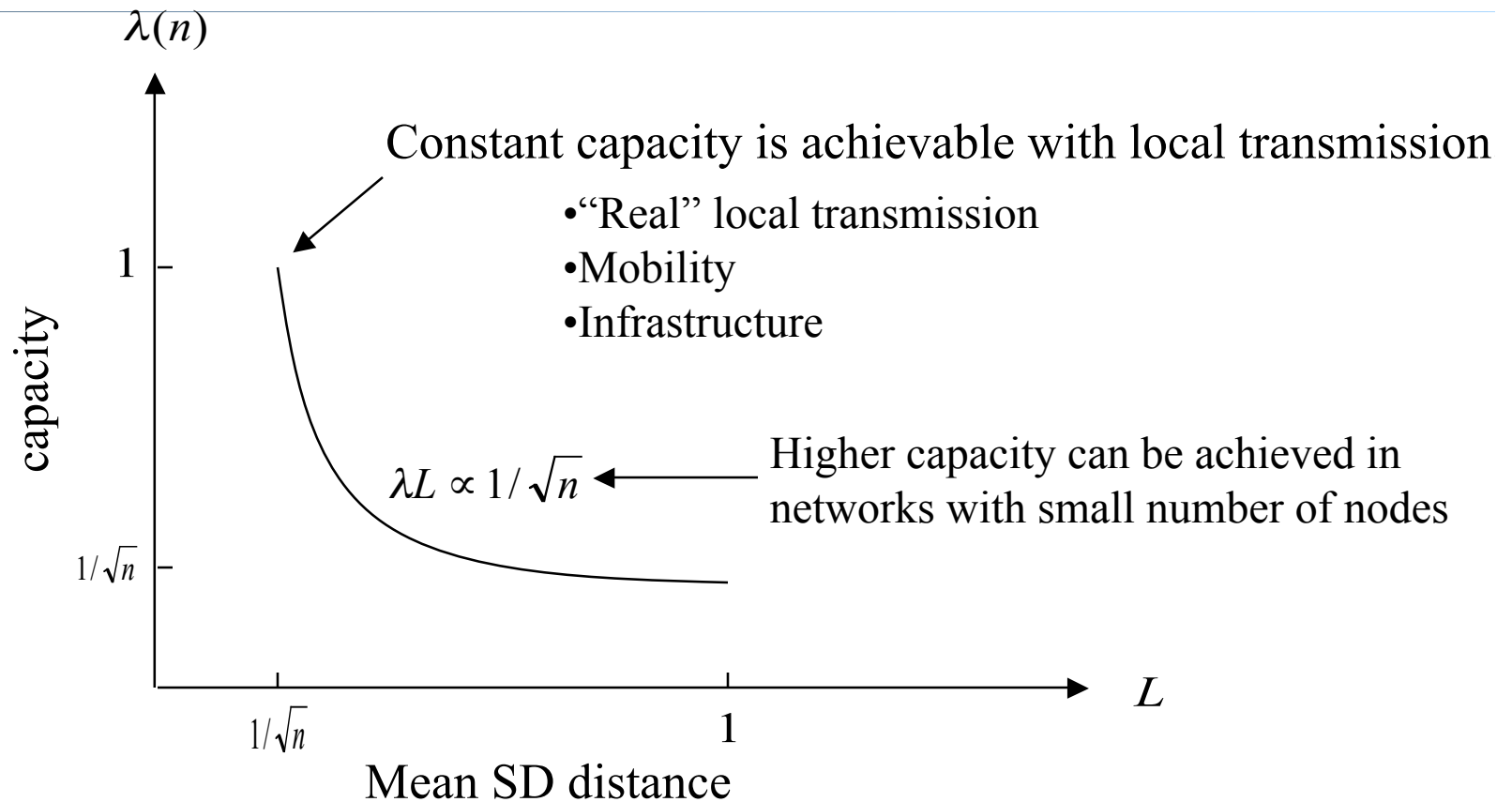
A Little Bit More Details
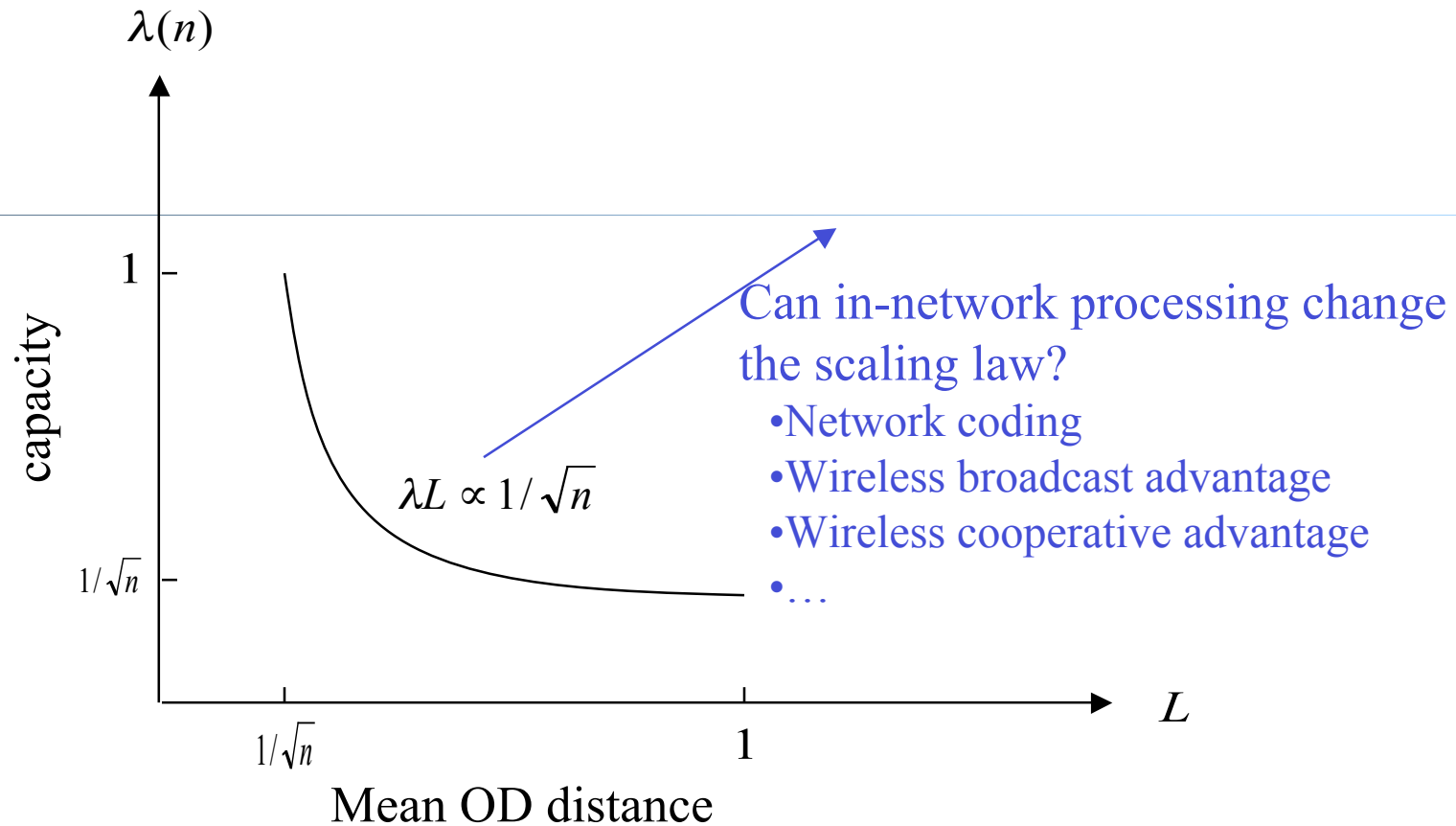

Cell of size $a(n)$ S-D lines

Need to guarantee
- Successful transmission along each S-D line
- Each cell can be active for a constant fraction of time
- Load balance: the number of S-D lines passing each cell is of the same order

Discussions

Constant capacity is achievable with local transmission
- "Real" local transmission
- Mobility
- Infrastructure

$\lambda L \propto 1/\sqrt{n}$

Higher capacity can be achieved in networks with small number of nodes

Using infrastructure and mobility do not really change the scaling law. They are equivalent to reduce the effective SD distance or the number of hops taken by each bit.

The graph shows $\lambda(n)$ on the vertical axis (labeled "capacity") versus L on the horizontal axis (labeled "Mean OD distance"). A decreasing curve labeled $\lambda L \propto 1/\sqrt{n}$ is shown. Vertical axis marks at 1 and $1/\sqrt{n}$; horizontal axis marks at $1/\sqrt{n}$ and 1.

Can in-network processing change the scaling law?
- Network coding
- Wireless broadcast advantage
- Wireless cooperative advantage
- …

Network layer solution seems not to help much. We need to go deep into physical layer to exploit all possibilities.

Caution: this may 'blur' the interfaces between Phys, MAC and Networking layers, and replace them with a new 'giant' layer.

References

☐ [1] P. Gupta and P. R. Kumar, *The capacity of wireless networks*, IEEE Trans. Inform. Theo., 2000

☐ [2] M. Grossglauser and D. Tse, *Mobility increases the capacity of ad-hoc wireless networks*, INFOCOM 2001

☐ [3] E. Gamal, J. Mammen, B. Prabhakar and D. Shah, *Throughput-Delay Trade-off in Wireless Networks*, INFOCOM 2004

☐ [4] G. Sharma, R. R. Mazumdar and N. B. Shroff, *Delay and Capacity Trade-offs in Mobile Ad Hoc Networks: A Global Perspective*, INFOCOM 2006

☐ [5] R. Yang, Lecture notes.

Thanks

Backup Slides

Mobility Increases the Capacity

- The reason for low capacity is that the number of relay nodes a packet goes through is $n^{1/2}$
- To exploit mobility can decrease hops and thus increase capacity
- The location $X_i(t)$ is stationary and ergodic with stationary uniform distribution
- Trajectory between user is iid
- Use physical model

Direct transmission

□ Only allow transmission between S-D pair when they are close to each other

❏ A hard constraint

$$\sum_{i \in s(t)} \left| X_i(t) - X_{j(i)}(t) \right|^\alpha \leq B \quad B = 2^\alpha \pi^{-\alpha/2} \frac{\beta + L}{\beta}$$

□ An impossibility result:

$$\Pr\{\lambda(n) = cn^{-\frac{1}{1+\alpha/2}} R \text{ is feasible }\} = 0$$

□ Intuition: interference-limited v.s. distance-limited

Transmission with Relaying

- Need relay to increase transmission probability
- How many relays are enough?
  - One relay is enough!
  - The probability for transmission is ~ O(1)
- Each packet is transmitted either in one step or two step

Scheduling Policy

□ Two phase scheduling
- ❑ Transmission from Source nodes to relaying nodes or destination
- ❑ Transmission from Source nodes or relaying nodes to destination

□ In each phase
- ❑ Randomly choose θn nodes (0<θ<1) as sender
- ❑ Transmit at unit power
- ❑ Successful transmission Nt

Capacity Result

❑ Successful transmission probability is ~O(n)
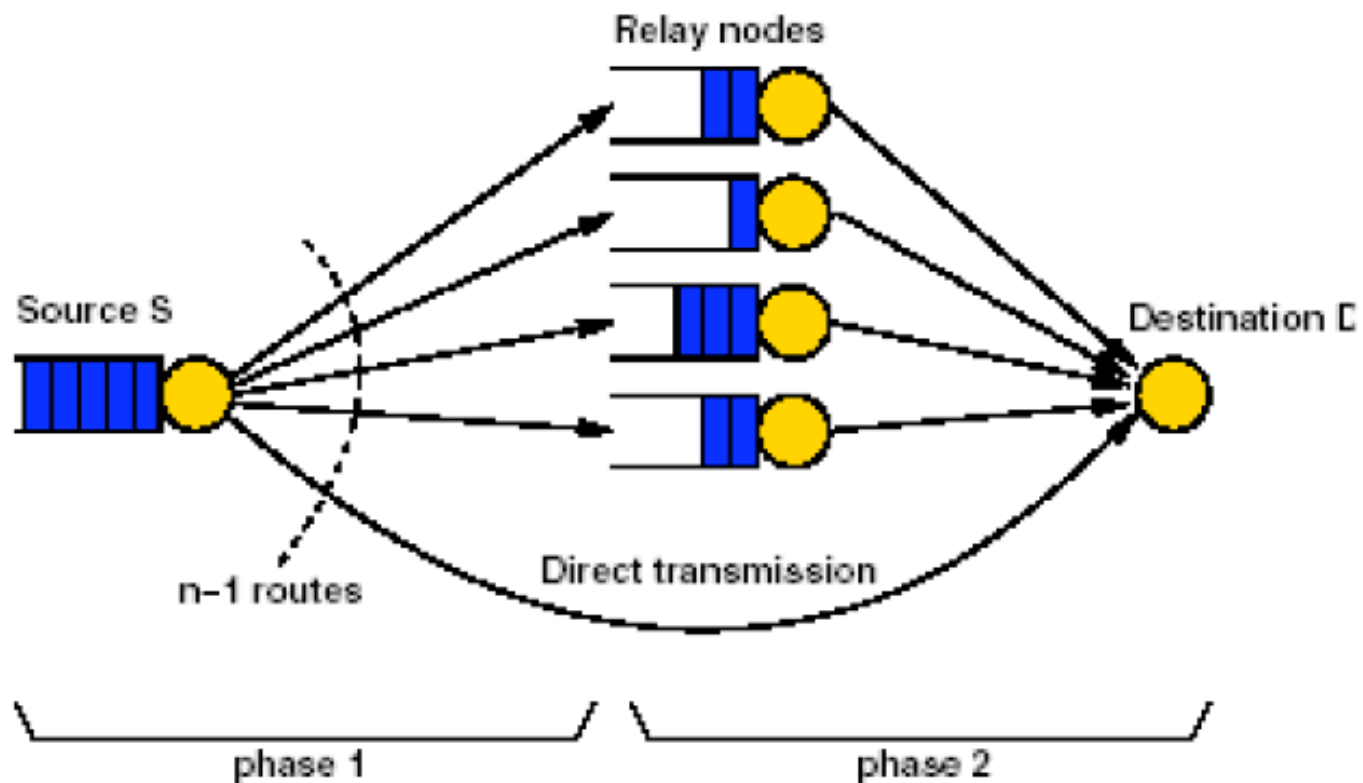
$$\lim_{n \to \infty} \frac{E(N_t)}{n} = \phi > 0$$

❑ The two phase algorithm achieves a throughput per S-D pair of O(1), i.e. we can find c>0 that

$$\Pr\{\lambda(n) = cR \text{ is feasible }\} = 1$$

❑ Note that c is at most φ/2

Intuition

❑ Multi-user diversity

Messages So Far

- ❏ Constant capacity can be achieved
- ❏ How about delay? Scaling behavior?
- ❏ What's the capacity-delay tradeoffs

Delay for the Static Networks

❑ Consider random networks

❑ Delay is proportional to the hop counts

   ❑ Constant S-D distance on average

   ❑ Hop length $r(n) \sim (\log n/n)^{1/2}$

   ❑ Delay $\sim 1/r(n) = (n/\log n)^{1/2}$

❑ Capacity-delay tradeoff $C \sim D/n$

❑ Can prove it is optimal

# More Details

- Divide the unit torus using a square grid into square cells, each of area a(n)~logn/n.

- A cellular time-division multi-access (TDMA) transmission scheme is used, in which, each cell becomes *active*, i.e., its nodes can transmit successfully to nodes in the cell or in neighboring cells, at regularly scheduled *cell timeslots*.

- Let the straight line connecting a source S to its destination D be denoted as an S-D line. A source S transmits data to its destination D by hops along the adjacent cells lying on its S-D line.

- When a cell becomes active, it transmits a single packet for each of the S-D lines passing through it. This is again performed using a TDMA scheme that slots each cell timeslot into *packet time-slots*.

Cell of size $a(n)$ S-D lines

Cell time-slots

Packet time-slots
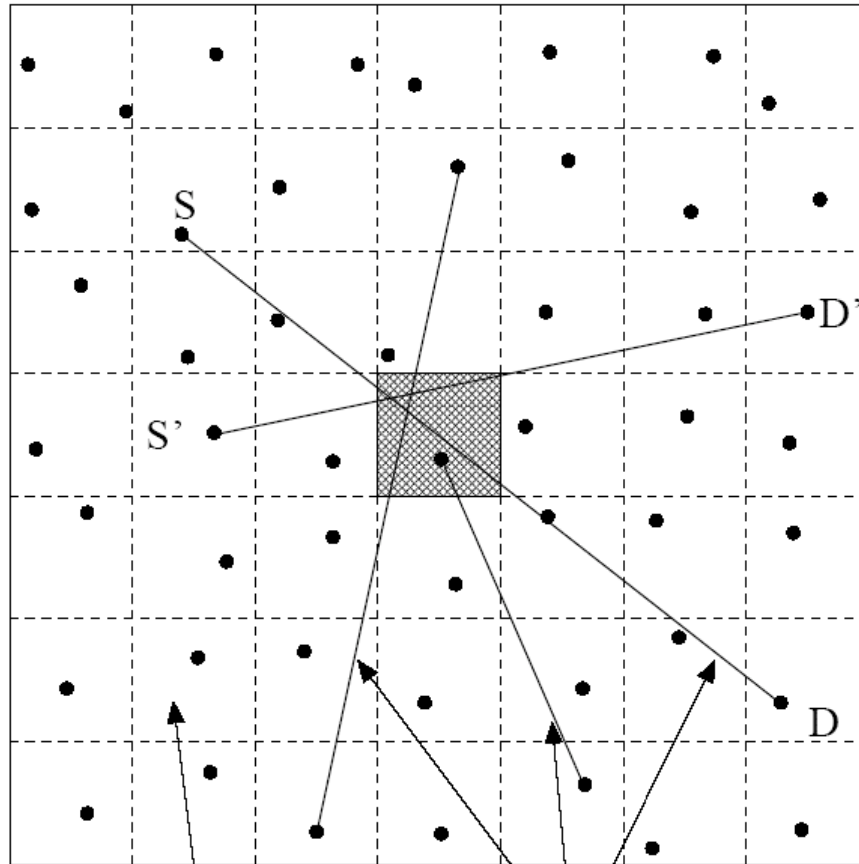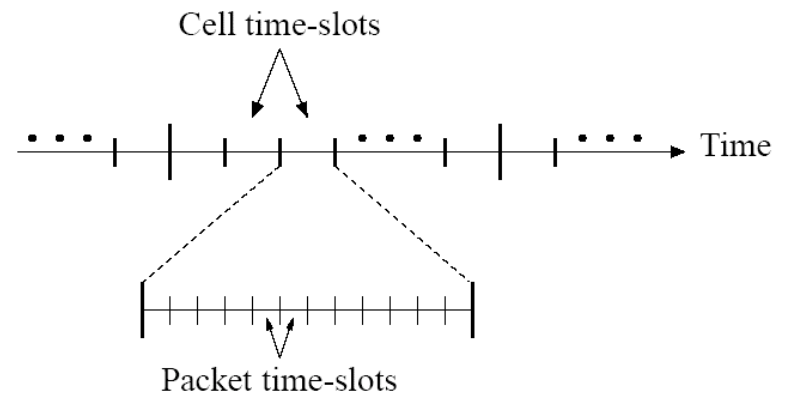
Time

Capacity-delay Tradeoff

□ Lemma 1: Each cell will have at least one node whp, thus guaranteeing successful transmission along each S-D line.

□ Lemma 2: Each cell can be active for a constant fraction of time, independent of n.

□ Lemma 3: The number of S-D lines passing through any cell is $\sim n(a(n))^{1/2}$

□ Capacity, delay and tradeoff: $C \sim 1/n(a(n))^{1/2}$, $D \sim 1/(a(n))^{1/2}$, $C \sim D/n$.

Delay for the Mobile Network

□ Divide the unit torus into n square cells, each of area 1/n

□ each node moves according to an independent Brownian motion

□ Each packet is relayed at most once.

□ Each cell becomes active once in every $1+c_1$ cell time-slots

□ Transmission is always between two nodes in the same cell

- Delay will be the order of the time for two node to meet in the same cell
- Each node randomly walk on a $n^{1/2}$ X $n^{1/2}$ discrete torus
- The relative position between two node is another random walk, and the meeting time corresponds to the hitting time of the state $(0, 0)$, which is ~nlogn
- Delay ~nlogn

More Results

❑ The capacity-delay tradeoff is determined by the routing and scheduling policy and mobility model

❑ For random walk model of mobility (reference [3])
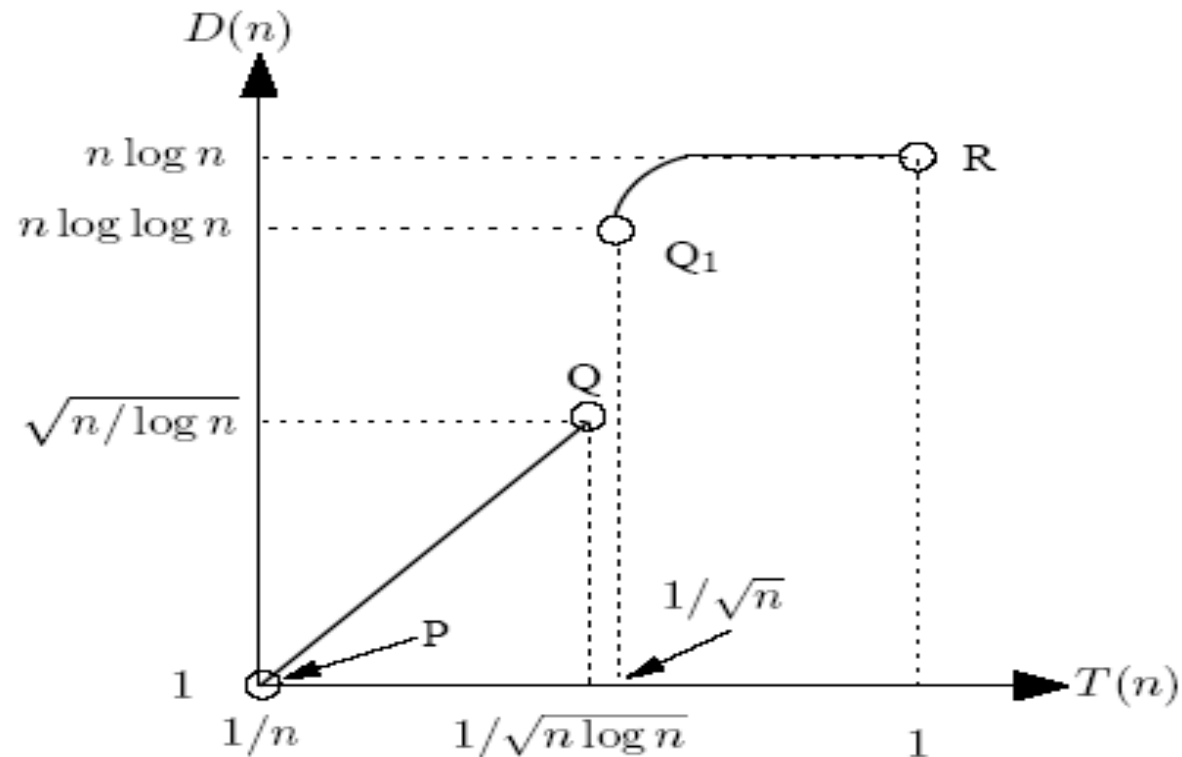
More Results (con'ed)

❑ There exists other results for other mobility models and scheduling/routing policies

❑ Question: to achieve a better throughput than a static network, what's the minimum delay that must be tolerant? (reference [4])