Algorithm for Image Processing Using Improved Median Filter and Comparison of Mean, Median and Improved Median Filter

Gajanand Gupta

Abstract—An improved median filter algorithm is implemented for the de-noising of highly corrupted images and edge preservation. Mean, Median and improved mean filter is used for the noise detection. Fundamental of image processing, image degradation and restoration processes are illustrated. The pictures are corrupted with different noise density and reconstructed. The noise is Gaussian and impulse (salt-and pepper) noise. An algorithm is designed to calculate the PSNR and MSE. The result is discussed for Mean, Median and improved Median filter with different noise density.

Index Terms — FILTERS, MATLAB, MSE, PSNR.

I. INTRODUCTION

Digital image processing is a subfield of digital signal processing. Digital image processing has many advantages over analog image processing; it allows a much wider range of algorithms to be applied to the input data and can avoid problems such as the build-up of noise and signal distortion during processing.

An image May be defined as a two-dimensional function f(x, y). Where x and y are spatial (plane) coordinates, and the amplitude of f at any pair of coordinates (x, y) is called the intensity or gray level of the image at that point. When x, y and the amplitude values of f are all finite discrete quantities, we call the image as digital image. The field of digital image processing refers to processing digital images by means of a digital computer. A digital image is composed of a finite number of elements, each of which has a particular location and value. These elements are referred to as picture elements, image elements, pels, and pixels. Pixel is the term most widely used to denote the elements of a digital image [1].

Vision is the most advanced of our senses, so it is not surprising that the images play the single most important role in human perception. However, unlike humans who are limited to the visual band of the electromagnetic (EM) spectrum imaging machines cover almost the entire EM spectrum ranging from gamma to radio waves. They can operate on the images generated by sources that humans are not accustomed to associating with images. Thus digital image processing encompasses a wide and varied field of applications.

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Gajanand Gupta, department of electronics and communication, Jaipur Natoanal University, Jaipur, India, +919649726814, (e-mail: gajanand2007@gmail.com).

II. STEPS IN DIGITAL IMAGE PROCESSING

In Image representation one is concerned with the characterization of the quantity that each picture element represents. An image could represent luminance of objects in a scene, the absorption characteristics of the body tissue, the radar cross section of the target, the temperature profile of the region or the gravitational field in an area. In general, any two dimensional function that bears information can be considered an image.

Figure 1: fundamental steps in digital image processing Image representation and modeling

An important consideration in image representation is the fidelity or intelligibility criteria for measuring the quality of an image or the Performance of processing technique.
Specification of such measures requires models of perception of contrast, spatial frequencies, and colors and so on. The fundamental requirement of digital processing is that images be sampled and quantized. The sampling rate has to be large enough to preserve the useful information in an image. It is determined by the bandwidth of the image.

A. Image enhancement

In image enhancement, the goal is to accentuate certain image features for subsequent analysis or for image display. Examples include contrast and edge enhancement, pseudo coloring, noise filtering, sharpening and magnifying. Image enhancement is useful in feature extraction, image analysis and visual information display. The enhancement process itself does not increase the inherent information display in the data. It simply emphasizes certain specified image characteristics. Enhancement algorithms are generally interactive and application dependent.

Image enhancement techniques such as contrast stretching, map each grey level into another grey level by a pre determined transformation. An example is the histogram-equalization method, where the input levels are matched so that the output grey level distribution is uniform. This has been found to be powerful method of enhancement of low contrast images. Other enhancement techniques perform local neighborhood operations as in convolution; transform operations as the discrete Fourier transforms.

B. Image restoration

It refers to removal or minimization of known degradations in an image. This includes de-blurring of images degraded by the limitations of a sensor or its environment, noise filtering and correction of geometric distortion or non-linearities due to sensors.

A fundamental result in filtering theory used commonly for image restoration is called Weiner filter. This filter gives the best linear mean square estimate of the object from the observation. It can be implemented in frequency domain via the fast unitary transform, in spatial domain by two dimensional recursive techniques similar to Kalman filtering or by FIR non-recursive filters. It can also be implemented as a semi-recursive that employs a unitary transformation is one of the dimensions and a recursive filter in the other. Several other image restoration methods such as least squares, constraint least squares and spline interpolation methods can be shown to belong to the class of Weiner filtering algorithms. Other methods such as maximum likelihood, minimum entropy are non-linear techniques that require iterative solutions.

C. Image analysis

It is concerned with making quantitative measurements from an image to produce a description of it. In simplest form, this task is reading a label on a grocery item, sorting different parts on an assembly line or measuring the size and orientation of blood cells in a medical image. More advanced image analysis systems measure quantitative information and use it to make a sophisticated decision such as controlling an arm of a robot to move an object after identifying it or navigating an aircraft with the aid of images acquired along its trajectory.

Image analysis techniques require extraction of certain features that aid in the identification of an object. Segmentation techniques are used to isolate the desired object. Quantitative measurements of object features allow classification and description of the image.

D. Image data compression

The amount of data associated with visual information is so large that its storage would require enormous storage capability. Although the capacities of several storage media are substantial, their access speeds are usually inversely proportional to their capacities. Typical television images data rates exceeding ten million bytes per second. There are other image sources that generate even higher data rates. Storage and transmission of such data requires large capacity or bandwidth could be very expensive. Image data compression techniques are concerned with reduction of the number if bits required to store or transmit images without any appreciable loss of information [2].

Image transmission applications are in broadcast television; remote sensing via satellite, aircraft, radar, sonar, teleconferencing, computer communications and facsimile transmission. Image storage is required most commonly for educational and business documents, medical images used in patient monitoring system. Because of their wide applications, data compression is of great importance in digital image processing.

III. Mathematical Model

As Figure 2 shows, the degradation process is modeled as a degradation function that, together with an additive noise term, operates on an input image f(x, y) to produce a degraded image g(x, y). Given g(x, y) some knowledge about the degradation function H, and some knowledge about the additive noise term n(x, y) the objective of restoration is to maintain an estimate f*(x, y) of the original image. We want the estimate to be as close as possible to the original image and in general the more we know about H and n the closer f*(x, y) will be to f(x, y).

If H is a linear, position invariant process, then the degraded image is given in the spatial domain by

\[ g(x, y) = h(x, y) * f(x, y) + n(x, y) \]  

(1)

Figure 2: A model of the image degradation/restoration process

Where h(x, y) is the spatial representation of the degradation function and, the symbol “*” indicates spatial convolution. Convolution in spatial domain is equal to multiplication in the frequency domain, so we may write the model in Equation (1) in an equivalent frequency domain representation:
Bipolar impulse noise is also called salt-and-pepper noise randomly distributed over the image. For this reason equal impulse noise values will resemble salt-and-pepper noise. If neither probability is zero and especially if they are approximately equal impulse noise values will resemble salt-and-pepper noise granules randomly distributed in the image. For this reason bipolar impulse noise is also called salt-and-pepper noise. Shot and spike noise terms are also used to refer to this type of noise.

Noise impulses can be negative or positive. Scaling usually is part of the image digitizing process. Because impulse corruption usually is large compared with the strength of the signal image, impulse noise generally is digitized as extreme (pure white or black) values in an image. Thus the assumption usually is that a and b are “saturated” values in the sense that they are equal to the minimum and maximum allowed values in the digitized image. As a result, negative impulses appear as black (pepper) points in an image. For the same reason, positive impulses appear white (salt) noise. For an 8-bit image this means that a = 0 (black) and b = 255 (white).

### A. Gaussian Noise

The PDF of a Gaussian random variable, z is given by

\[
p(z) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(z-\mu)^2}{2\sigma^2}}
\]

Where z represents gray level, \(\mu\) is the mean of average value of z, and \(\sigma\) is its standard deviation. The standard deviation squared, \(\sigma^2\) is called the variance of z.

Because of its mathematical tractability in both the spatial and frequency domains, Gaussian (also called normal) noise models are frequently used in practice. In fact, this tractability is so convenient that it often results in Gaussian models being used in situations in which they are marginally applicable at best.

### B. Impulse (salt-and-pepper) noise

The PDF of impulse noise is given by

\[
p(z) = \begin{cases} 
  P_a & \text{for } z = a \\
  P_b & \text{for } z = b \\
  0 & \text{otherwise}
\end{cases}
\]

If \(b > a\), gray-level \(b\) will appear as a light dot in the image. Conversely a leva will appear like a dark dot. If either \(P_a\) or \(P_b\) is zero, the impulse noise is called uni-polar. If neither probability is zero and especially if they are approximately equal impulse noise values will resemble salt-and-pepper noise granules randomly distributed over the image. For this reason bipolar impulse noise is also called salt-and-pepper noise. Shot and spike noise terms are also used to refer to this type of noise.

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### Restoration in the presence of Noise Only-Spatial Filtering

When the only degradation present in an image is noise, Equation (1) and (2) become

\[
g(x,y) = f(x,y) + \sigma(x,y) \tag{3}
\]

and

\[
G(u,v) = F(u,v) + N(u,v) \tag{4}
\]

The noise terms are unknown, so subtracting them from \(g(x,y)\) or \(G(u,v)\) is not a realistic option. In the case of periodic noise, it usually is possible to estimate \(N(u,v)\) from the spectrum of \(G(u,v)\). In this case \(N(u,v)\) can be subtracted from \(G(u,v)\) to obtain an estimate of the original image [4]. Spatial filtering is the method of choice in situations when only additive noise is present.

### Filters

#### A. Arithmetic mean filters

This is the simplest of the mean filters. Let \(S_{xy}\) represents the set of coordinates in a rectangular sub image window of size \(m \times n\) centered at point \((x,y)\). The arithmetic mean filtering process computes the average value of the corrupted image \(g(x,y)\) in the area defined by \(S_{xy}\). The value of the restored image \(f\) at any point \((x,y)\) is simply the arithmetic mean computed using the pixels in the region defined by \(S_{xy}\). In other words,

\[
\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t) \in S_{xy}} g(s,t)
\]

This operation can be implemented using a convolution mask in which all coefficients have value \(1/mn\). A mean filter simply smooths local variations in an image. Noise is reduced as a result of blurring [5].

#### B. Median filter

The best known order-statistics filter is the median filter, which replaces the value of a pixel by the median of the gray levels in the neighborhood of that pixel:

\[
\hat{f}(x,y) = \text{median} \{ g(s,t) \}_{(s,t) \in S_{xy}}
\]

The original value of the pixel is included in the computation of the median. Median filters are quite popular because, for certain types of random noise they provide excellent noise reduction capabilities, with considerably less blurring than linear smoothing filters of similar size [6][7].

### IV. ALGORITHM

#### Algorithm for the improved median filter

To remove salt and pepper noise from the corrupted image the below described algorithm is used.
Algorithm for Image Processing Using Improved Median Filter and Comparison of Mean, Median and Improved Median Filter

Step 1: A two dimensional window (denoted by $3\times3$ W) of size $3\times3$ is selected and centered around the processed pixel $p(x, y)$ in the corrupted image.

Step 2: Sort the pixels in the selected window according to the ascending order and find the median pixel value denoted by $P_{\text{med}}$, maximum pixel value ($P_{\text{max}}$) and minimum pixel value ($P_{\text{min}}$) of the sorted vector $V_0$. Now the first and last elements of the vector $V_0$ is the $P_{\text{min}}$ and $P_{\text{max}}$ respectively and the middle element of the vector is the $P_{\text{med}}$.

Step 3: If the processed pixel is within the range $P_{\text{min}} < p(x, y) < P_{\text{max}}$, then the pixel and it is left unchanged. Otherwise $p(x, y)$ is classified as corrupted pixel.

Step 4: If $p(x, y)$ is corrupted pixel, then we have the following two cases:

Case 1: If $P_{\text{min}} < P_{\text{med}} < P_{\text{max}}$ and $0 < P_{\text{med}} < 255$, replace the corrupted pixel $p(x, y)$ with $P_{\text{med}}$

Case 2: If the condition in case 1 is not satisfied then $P_{\text{med}}$ is a noisy pixel. In this case compute the difference between each pair of adjacent pixel across the sorted vector $V_0$ and obtain the difference vector $V_D$. Then find the maximum difference in the $V_D$ and mark its corresponding pixel in the $V_0$ to the processed pixel.

Step 5: Step 1 to step 4 are repeated until the processing is completed for the entire image [8].

V. SIMULATION

The 8-bit images of dimensions $M_1 \times M_2 (= 512 \times 512)$ pixels is used for simulations. The pixels $s(i, j)$ for $1 \leq i \leq M_1$ and $1 \leq j \leq M_2$, of the image is corrupted by adding impulse noise, with noise density ranging from 0.1 to 0.8. In all the simulations, square windows of dimensions $N \times N$ pixels and with different values of width $N (= 3, 5, 7)$ are used. The Peak signal to noise ratio (PSNR) is used to compare the relative filtering performance of various filters. The PSNR between the filtered output image $y(i, j)$ and the original image $s(i, j)$ of dimensions $M_1 \times M_2$ pixels is defined as:

$$\text{PSNR} = 20 \cdot \log_{10}\left(\frac{\text{MAX}_i}{\text{MSE}}\right)$$

Where $\text{MAX}_i$ is max pixel value of the image and MSE is defined as

$$\text{MSE} = \frac{\sum_{i=1}^{M_1} \sum_{j=1}^{M_2} [y(i, j) - s(i, j)]^2}{M_1 \times M_2}$$

It can be seen that Peak signal to noise ratio (PSNR) is closely related to mean square error (MSE) [9].

VI. RESULT ANALYSIS

PSNR values for different filters on ‘Lena’ image

<table>
<thead>
<tr>
<th>Filter</th>
<th>Noise Density</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10%</td>
<td>20%</td>
<td>30%</td>
<td>40%</td>
<td>50%</td>
<td>60%</td>
<td>70%</td>
<td>80%</td>
</tr>
<tr>
<td>Median</td>
<td>30.84</td>
<td>29.34</td>
<td>27.71</td>
<td>26.07</td>
<td>24.08</td>
<td>20.54</td>
<td>17.04</td>
<td>12.91</td>
</tr>
<tr>
<td>Improved Median</td>
<td>28.23</td>
<td>27.12</td>
<td>26.92</td>
<td>24.53</td>
<td>23.77</td>
<td>21.76</td>
<td>18.94</td>
<td>15.78</td>
</tr>
</tbody>
</table>

Table I: PSNR at different noise density for ‘lena’ image
Figure 3: Plot for PSNR values of lena image

Figure 4: (a) original image (b) corrupted with 60% noise (c) output from mean filter (d) output from median filter (e) output from improved median filter
PSNR values for different filters on ‘House’ image

Table II: PSNR at different noise density for house image

<table>
<thead>
<tr>
<th>Filter</th>
<th>10%</th>
<th>20%</th>
<th>30%</th>
<th>40%</th>
<th>50%</th>
<th>60%</th>
<th>70%</th>
<th>80%</th>
<th>90%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>23.08</td>
<td>19.98</td>
<td>17.73</td>
<td>15.99</td>
<td>14.77</td>
<td>13.60</td>
<td>12.55</td>
<td>11.65</td>
<td>10.78</td>
</tr>
<tr>
<td>Median</td>
<td>30.65</td>
<td>29.13</td>
<td>27.05</td>
<td>25.04</td>
<td>22.88</td>
<td>19.33</td>
<td>15.82</td>
<td>11.93</td>
<td>8.24</td>
</tr>
<tr>
<td>Improved Median</td>
<td>24.66</td>
<td>24.35</td>
<td>23.99</td>
<td>23.57</td>
<td>22.12</td>
<td>18.97</td>
<td>15.29</td>
<td>11.35</td>
<td>8.04</td>
</tr>
</tbody>
</table>

Figure 5: Plot for PSNR values of House image
VII. CONCLUSION AND FUTURE SCOPE

A novel method based on efficient noise detection algorithm is studied here for effectively de-noising extremely corrupted images and better edges preservation. The studied filter is based on order-statistic filtering and uses an impulse noise detector.

The good performance of the standard median filter is severely impaired in extremely impulse noise. The innovative aspects of the studied method are simple and effective algorithm for noise cancellation across a wide range of noise densities from 10% to 98%, while preserving high quality of restored image. The key success of such performance delivery is mainly due to highly accurate noise detection accomplished.
by this algorithm. In addition, the studied method uses simple fixed length window, and hence, it requires significantly lower processing time compared with other methods. The simulation results show that the studied method can be applied to different types of image and provide very satisfying results. It has significant improvement over the existing methods. In the future, various techniques can be considered to incorporate in this scheme to further improve the performance and preserve more edges in both highly and lowly corrupted images.

REFERENCES


GAJANAND GUPTA has received his B.E degree in electronics and communication from Anna University, Chennai in 2008. He has received his M. Tech from Jaipur National University, Jaipur in 2011. He is presently working with JNU, Jaipur as a Lecturer. He has published various papers in national conferences. His research interest is software defined radio and signal processing.