Computational Morphology of Curves

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Summary

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- **Minimum Spanning Trees**
- **Reconstructing Curves from Clean Data**
- **Reconstruction from Noisy Data**
- **Probabilistic Parameter Estimation**
- **Results**
- **Conclusions**

Introduction

- ***Problem statement:*** *"given a set of 2D points, find a sub-set which we can sort in an order compatible with the natural trace of the curve, as perceived by humans."*
- ***Motivation:*** *"Reproducing human perception of shapes from dot patterns is a classical problem in low-level computer vision, pattern recognition, and cluster analysis."*

Minimum Spanning Trees

- ***Minimum Spanning Tree – MST***
  - A MST for a weighted graph is a spanning tree for which the sum of edges’ weights is minimal (for us the edge weight is the euclidean point distance).

Minimum Spanning Trees

- ***Bridge Theorem:*** *Let* **A∪B** *be a partition of the set of vertices of a connected weighted graph* ***G****. Then any shortest edge in* ***G*** *connecting a vertex of* ***A*** *and a vertex from* ***B*** *is an edge of a* ***MST(G)****.*

Curves from Clean Data

- *A **Chord** in a path is an edge joining nonconsecutive vertices*
- *A **Short Chord** is a chord that is shorter them some edge of sub-path connecting its extremes.*
- ***Theorem 2:** A spanning path is a **MST** if and only if it has not short chords.*

Curves from Clean Data

- *A **Tubular Neighborhood** for a curve **C** is a set **T** containing **C** such that every point of **T** belongs to exactly one line segment totally contained in **T** and normal to **C**.*
- *A **Sample** of a curve **C** is **Dense** if there is a real number $\varepsilon>0$ such that no two consecutive samples points are more than ε apart, and the closed disks of radius ε, centered at the sample points, form a **Tubular Neighborhood** of C.*

Curves from Clean Data

- ***Theorem 3:****Let* ***p*** *and* ***q*** *be points on a arc* ***C,*** *such that q is inside a tubular disk* ***D*** *centered at* ***p.*** *Then the subarc of* ***C*** *between* ***p*** *and* ***q*** *is completely inside the disk with diameter* ***pq****.*
- ***Theorem 4:****Let* ***S*** *be a dense sample on an arc* ***C*** *containing the extremes of* ***C****. Then the euclidean* ***MST*** *of* ***S*** *coincides with the polygonal path induced by* ***C*** *on* ***S****, and is therefore a geometric graph model for* ***C****.*

Curves from Clean Data

- ***Deleting Bridges*** *(Detecting Connected Components)*
  - *Every edge greater than a given threshold is deleted*
- ***Closing Loops***
  - *Every edge incident to degree 1 vertices and with weight lower than a given threshold is added*

Curves from Noisy Data

- ***Topological length*** *of a path is the number of its edges*
- ***Euclidean length*** *of a path is the sum of weights (euclidean distances) of its edges*
- *A* ***diameter path*** *is a path with maximal length (in the topological or euclidean sense)*
- *To filter out the noise from an* ***MSF*** *(minimum spanning forest), a diameter path is computed for each connected component*

Parameter Estimation

- ***Chebychev's Inequalities***
  - *Let X a random variable, μ and σ^2 its mean and variance (respectively). Then holds:*

$$P(|X - \mu| > \epsilon) \leqslant (\mu / \epsilon)$$
$$\leqslant (\sigma^2 / \epsilon^2)$$

Parameter Estimation

- *From Chebychev's inequalities, we can estimate thresholds which ensure an* ***upper bound*** *(0<p<1) on the probability of existing bridges (or loops not closed) in the output reconstruction*

$$p \geqslant (\sigma^2/\epsilon^2) \geqslant \mathrm{P}(|\mathrm{X} - \mu| > \epsilon) \Leftrightarrow \epsilon \geqslant \sigma/p^{1/2}$$

$$\epsilon = \lceil \sigma/p^{1/2} \rceil \Rightarrow \mathrm{P}(|\mathrm{X} - \mu| > \epsilon) \leqslant p$$

- *If the given probability is too low ($p \approx 0$), edges other than bridges can be discarded!!!* **[Trade-off]**

Results

http://www.impa.br/~ijamj/courses/computational_geometry/project/results/

Conclusions

- ***As proved in* [de Figueiredo and Gomes]*, it is possible to construct a correct polygonal approximation for the curve underlying a point cloud.***
- ***Contrary to our feeling, we had bad results when the sample set is sparse.***

Conclusions

- ***As noted in* [de Figueiredo and Gomes], *the reconstruction algorithm is very sensible to the distance of the noisy samples from the ground truth.***
  - ***As well as to the ratio of noisy and on-curve samples (which must be kept as small as possible).***

References

- Computational morphology of curves
  Luiz Henrique de Figueiredo, Jonas Gomes.
  The Visual Computer. *11(2)*, pp. 105-112, 1995.

Thank you !!!

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