PSERC

# Detection, Prevention and Mitigation of Cascading Events

## *Part III of Final Project Report*

**Power Systems Engineering Research Center**

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**Power Systems Engineering Research Center**

# Detection, Prevention and Mitigation of Cascading Events

## Final Project Report

## Part III


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**Power Systems Engineering Research Center**

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# Executive Summary

Power systems are under increasing stress as market policies introduce new economic objectives for operation. To achieve those economic objectives, power systems are being operated closer to their limits. As a result, any one of a large number of factors (such as weak connections, unexpected events, hidden failures in protection system, and human errors) may cause a system to lose stability, possibly leading to catastrophic failure. Therefore, there is a need for a systematic study and design of a comprehensive system control strategy to mitigate the possibility of such catastrophic failures.

Among the control strategies, controlled system islanding is the final resort to save a system from a blackout. In the literature, many approaches have been proposed to undertake controlled islanding. Some approaches only take static power flow into consideration; others require a great deal of computational effort. Following large disturbances, groups of generators tend to swing together. Research has focused on control strategies to maintain stability of inter-area oscillations between groups of machines. The slow coherency-based generator grouping is one potential method for capturing the movement of generators between groups under disturbance. The research issue is how to take advantage of the information from the slow coherency generator grouping method to island the system in a controlled way by tripping an identified set of transmission lines.

In this third part of the final project report, a comprehensive approach is proposed for controlled islanding. The approach uses slow coherency based generator grouping to initiate controlled power system islanding based on the minimal cutset technique from graph theory by calculating the net flow through the cutset. The proposed approach has been demonstrated on a 29-generator, 179-bus model of the WECC system. The approach has been implemented using Matlab. The code is available on request.

The results show that the controlled islanding approach, with an adaptive load-shedding scheme, has the advantage of shedding fewer loads than that from islanding based on practical experience. Furthermore, with the new islanding scheme, the system experiences less frequency oscillation than islanding based on practical experience. This approach could be implemented as a critical EMS function for use in extreme power system conditions requiring islanding to stop a cascading event. The next steps in developing this application include application to a large realistic power system, implementation of the islanding scheme in an EMS setting, use of PMU measurements to facilitate controlled islanding, and development of an efficient protection scheme which included transfer tripping to create the desired islands.

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# 1. Introduction

## 1.1 Problem Description

With the advent of deregulation and restructuring, power systems have come under increasing stress as deregulation has introduced several new economic objectives for operation. Since systems are being operated close to their limits, weak connections, unexpected events, hidden failures in protection system, human errors, and a host of other factors may cause a system to lose stability and even lead to catastrophic failure. Therefore, the need for a systematic study and design of a comprehensive system control strategy is gaining more and more attention.

The work described in this research is one component of the project “Detection, Prevention and Mitigation of Cascading Events” granted by the Power Systems Engineering Research Center (PSERC). This work is directed at enhancing reliability of interconnected power systems and preventing cascading outages: “When a power system is subjected to large disturbances, and vulnerability analysis indicates that the system is approaching a potential catastrophic failure, control actions need to be taken to steer the system away from severe consequences, and to limit the extent of the disturbance[1]. In this project three steps are proposed to address this problem:

1. Detect Major Disturbances and Protective Relay Operations Leading to Cascading Events.
2. Utilize Wide Area Measurement-Based Remedial Action.
3. Initialize Controlled System Islanding with Selective Under Frequency Load Shedding.

As a part of this project, Controlled System Islanding acts as the final resort to save the system from a blackout. This report covers in detail the work conducted under Step 3.

## 1.2 Power System Reliability

The North American Electric Reliability Council (NERC) Planning Standards define two components of reliability, a) adequacy of supply and b) transmission security:

**Adequacy** is the ability of electric systems to supply the aggregate electrical demand and energy requirements of customers at all times, taking into account scheduled and reasonably-expected unscheduled outage of system elements.

**Security** is the ability of electric systems to withstand sudden disturbances such as electrical short circuits or unanticipated loss of system elements. Corrective control strategies contribute to the security problem in many such ways, as circuit overload, voltage problems, and transient problems.

### 1.3 Power System Operating States

The bulk power grid is the largest and most complex interconnected network ever devised by man, which makes its control an extremely difficult task. Generally, the ability of a power system to survive a given disturbance depends on its operating condition at the time of occurrence, and any adaptive control scheme needs to be designed in such way that it will only be activated when the system is in an appropriate operating condition.

In order to facilitate investigation of power system security and design of appropriate control strategies, power systems can be conceptually classified into five operational states: Normal, Alert, Emergency, In Extremis, and Restorative [2]. Various preventive and corrective control strategies for coping with power systems in the different operational states have been studied. Figure 1-1 illustrates these operating states and the transitions which can take place between states. Figure 1-1 also shows the relative corrective control strategies according to different system operating conditions. [3]

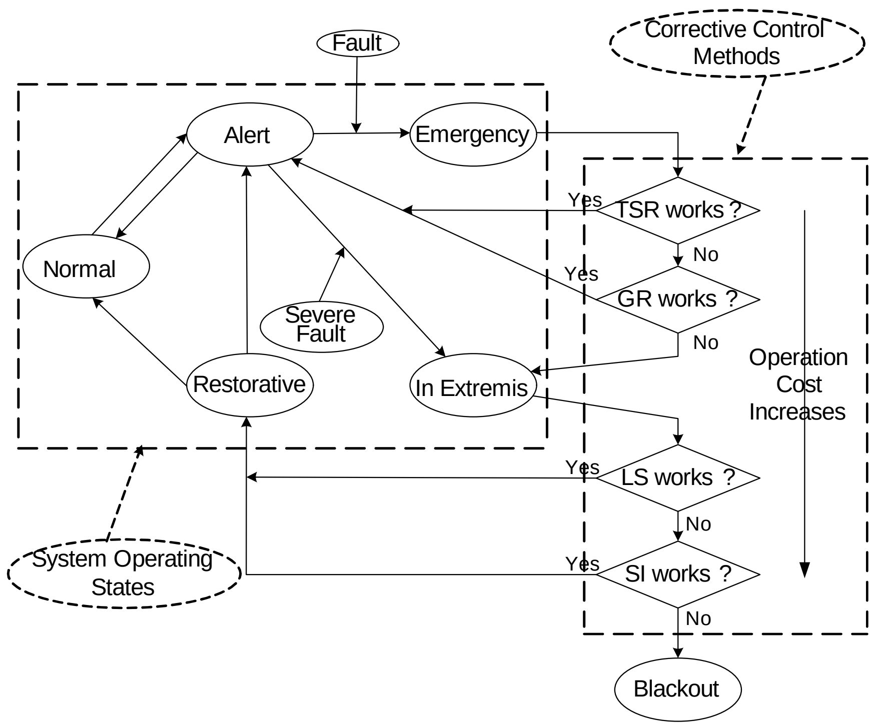


**Figure 1-1 Power system operating states and relative corrective control strategies**

Normal State

In this state, all the system variables are in the normal range and no system component is being overloaded. The system operates in a secure manner and is able to withstand a contingency without violating any constraints.

## Alert State

The system enters the alert state when the system condition is degraded. In this state, all the system variables are still within the acceptable range and no constraints have been violated. However, the system components may overload when an N-1 contingency occurs and leads the system into an emergency state. The system may also directly transit from the alert state to the in extremis state if the disturbance is severe enough.

## Emergency State

When the system is in the alert state, a sufficiently large contingency event may bring the system to the emergency state, where system voltages at many buses go below the normal range and the one or more system components may experience overloading. In this state, the system is in operation and may be restored back to the alert state by initiating corrective control strategies such as transmission system reconfiguration (TSR), generation rescheduling (GR), and load shedding (LS), etc.

## In Extremis

The system enters the in extremis state if the relative corrective controls are not applied or are ineffective when the system is in the emergency state. Corrective control strategies in this state include Load shedding (LS) and controlled system islanding (CSI). These controls are intended to prevent total system blackout and preserve as much of the system as possible.

## Restorative State

This state depicts a condition where control strategies are being deployed to reconnect all system components and to restore system load. Depending on the system condition, the system may transfer to the alert state, or directly transit back to the normal state.

In summary, when a severe fault occurs in a power system, the system may enter the emergency state or even the in extremis state, where the system may encounter an overloading condition, voltage violations, cascading failures, or even loss of stability requiring that system operators take appropriate corrective control actions. It is well known that transmission system reconfiguration (TSR) (including line switching and bus-bar switching) and controlled system islanding (CSI) are two effective corrective control strategies for various system operational states. When the system is in the emergency state, TSR may change the power flow distribution and voltage profiles and consequently solve the problems of overloads and voltage violations caused by system faults. However, when the system is being operated close to its limits, TSR may not successfully relieve all the overloads and voltage violations for some severe faults, and consequently the system may lose stability or even suffer catastrophic failure. More aggressive corrective control strategies, such as LS and/or CSI must be used to prevent catastrophic failure.

### 1.4 Problem Statement

On one hand, power system islanding is usually considered such a rare or improbable event that it seems not to merit special consideration. On the other hand, the significant impact of unintentional islanding on power system and electricity customers leads many individuals to have great concern about this situation. On November 9th, 1965, the largest power system blackout in history occurred. The northeast power system broke up 4 seconds after an initial disturbance, and 30 million people were without electricity for as long as 13 hours. On August 14th, 2003, widespread power blackouts occurred in the Northeastern United States and in Southeastern Canada, affecting eight states and two provinces with combined population of approximately 50 million people [4]. In reality, most intentional islanding schemes are based on engineering experience, and lack either theoretical analysis or reality validation. Therefore, it is our intention to develop an adaptive islanding scheme by taking into account not only system dynamic characteristics, but also the topology of the power network. This adaptive islanding approach breaks the system up into smaller islands at slightly reduced capacity, with an added advantage that the system can be restored very quickly.

Requirements and considerations in forming islands:

1) Frequency deviation considerations: Active power imbalance between generation and load induces a frequency deviation from the nominal value. Low-frequency deviations especially cause many more problems in power systems than high-frequency deviations. Therefore, in this report, approaches have been proposed to only deal with islands with excess load.
2) Voltage stability considerations in reactive power balance: Under certain circumstances, the system within the island could collapse due to cascading events initiated by voltage instability. Therefore, it is important to consider the reactive power balance within the island.
3) Restoration considerations with regard to black-start capability or remote cranking power capability: All power systems require contingency arrangements to enable a restart in the unexpected event that all or part of the system is out of service. The process of restoring the power system is commonly referred to as "Black Start". It entails "islanded" power subsystems being started individually and then gradually being reconnected in order to restore system integrity.
4) Flexibility: automation as our goal for power system islanding. The islanding approach should be designed in such a way that it can provide the system operator a reasonable islanding solution without a great deal of human interaction. However, it should also be able to acquire and utilize information from human evaluation and prediction to improve performance.

Slow coherency has been widely used in industry power system dynamics studies to reduce system scale while not affecting accuracy. In this research, it has been a practice to group generators with similar behavior, referred to as coherent generators [5]. Slow coherency has very nice features, such as: 1) The coherent groups of generators are almost independent of the size of the disturbance. 2) The coherent groups are independent of the level of detail used in modeling the generating units. The first feature provides a theoretical background with which to design an islanding approach independent of disturbance, which would make it possible to design a controlled islanding scheme prior to the disturbance. The second feature simply states that classical generator model can be used in grouping analysis, which may save the computation effort dramatically.

However, various studies indicate that generator coherency may change with respect to the change in the system operating point, and therefore grouping information at one particular operating point may not be suitable to use in another operating point. Since we believe that system dynamic characteristics should be considered in system islanding, and slow coherency is one of most widely-used approaches to group generators for processing system islanding in such a way that generators in one group shall be included in one island, further study may be needed to investigate the relationship between the generator coherency and system operating points.

## 1.5 Report Organization

A brief introduction has been presented in section 1. The rest of this report is organized as follows.

Section 2 details the concept of slow-coherency-based generator grouping and the motivation for introducing minimal cutsets into power system islanding. Starting with a review of relevant literature, slow coherency and some graph theory terminology are introduced in this chapter. A system islanding scheme using these concepts is also presented in this chapter. The proposed approach answers the following questions in detail: where is islanding initiated? How does it work? What are the advantages and disadvantages of the proposed method? Results obtained from the WECC 29-Generator and 179-Bus system are also given in this section.

Conclusions, future work, and contributions are presented in section 3.

# 2. Slow-Coherency-Grouping-Based Islanding Using Minimal Cutsets

## 2.1 Relevant Literature Review

Following large disturbances, groups of generators tend to swing together. Attention has thus been drawn to the stability of inter-area oscillations between groups of machines. These oscillations are lower in frequency than local oscillations between electrically-close machines. As a result, there is a separation in time scale between these two phenomena. Additionally, several comprehensive software packages for computing such low frequencies in large power systems are available with which to analyze the participation of the machines in these oscillations.

In References [6], [7], [8], [9] and [10], a slow-coherency approach based on the two-time-scale model has been successfully applied to the partitioning of a power system network into groups of coherent generators.

In the literature, there are some other approaches for the detection of islanding. In Reference [11], a spectral method for identifying groups of strongly connected sub-networks in a large-scale interconnected power system grid is presented as an alternative to long-standing singular perturbation-based coherency techniques. Reference [12] introduces an algorithm based on the breadth-first-search (BFS) algorithm from graph theory for island detection and isolation. In Reference [13], an interesting method based on the occurrence of singularity in Newton power flow is illustrated. Reference [14] gives an active technique based on the voltage-magnitude variation method of a distributed generation unit for detecting islanding. In [15], the authors present an interesting method for system splitting by using the OBDD technique. In the case of splitting a system into two islands, each load bus belongs either to one island or the other. This relationship can be captured by a Boolean variable. A software package called ‘BuDDY’ [16] has been utilized to determine the value of these Boolean variables in order to cap the generation and load imbalance within limits in the island. However, for better system islanding, the dynamic characteristics of system, in particular dynamics of generators and loads, should be taken into consideration. The slow-coherency approach of generator grouping, which has been widely studied in the literature, provides a potential for capturing the movement of generators between groups under disturbance. It has the ability to capture both system dynamics and network topology. Therefore, in this approach, we use slow coherency as our grouping technique.

Based on slow coherency, the generators in the system may be divided into several groups. For two interconnected generator groups, reference [5] presents an islanding method for constructing a small sub-network using the center bus which is one of the buses in the group boundary. This sub-network is referred to as the interface network. A brute force search is then conducted on the interface network to determine the cutsets where the islands are formed. For each island candidate, the total load and generation are calculated, and the island with minimum load-generation imbalance is picked up as the optimal cutset if no other criteria have been considered. This approach converts the objective of finding the optimal cutset from that of searching the whole network into that of searching the interface network, making the searching space much smaller. However, this approach still involves considerable computational effort particularly that of the brute force search applied. Furthermore, it is system-dependent since for some specific system, it may return fairly good results, while it may not for others. In this report, a new slow-coherency-grouping based approach using minimal cutsets is presented to solve this type of problem.

Minimal cutsets have been previously investigated in communication, network topology, and network (particularly, power system) reliability analysis (maximum flow and connectivity) [17], [18], [19], [20]. As shown in this approach, it also has the potential for determining where to actually island the system.

In the remainder of this chapter, an introduction to the basic concept of slow coherency is provided.

## 2.2 Slow Coherency

In the controlled-islanding self-healing approach, it is critical to determine the optimum set of islands for a given operating condition. An elegant and flexible approach to islanding can result in significant benefit to the post-fault corrective control actions that follow the islanding, including a load-shedding procedure and a load-restoration procedure. Generally, islanding is system-dependent. Reference [21] indicates that the choice of islands is almost disturbance-independent, which makes it easy to implement a fairly general corrective control scheme for a given system.

Slow coherency was originally used in the development of dynamic equivalents for transient-stability studies. Several methods have been used to identify coherent groups of generators [7], [22]. In all these methods, there are two common assumptions:

The coherent groups of generators are almost independent of the size of the disturbance.

The coherent groups are independent of the level of detail used in modeling the generating unit.

The first assumption is based on the observation that the coherency behavior of a generator is not significantly changed as the clearing time of a specific fault is increased. Although the amount of detail of the generator model can affect the simulated swing curve, it does not radically change the basic network characteristics such as inter-area modes. This forms the basis of the second assumption. In the following section, a brief introduction of slow coherency is given.

### 2.2.1 Modes and Time Scales in Power Systems

A power system can be modeled as a set of nonlinear differential equations and algebraic equations. Small-signal stability analysis can be used to investigate system behavior under small disturbances. In this context, the system can be linearized for the purpose of analysis.

Suppose an unforced dynamic system is defined as the following:

$$\dot{x}(t) = Ax(t); \qquad x(0) = \zeta \tag{2.1}$$

The solution will be

$$x(t) = \exp(At)\zeta = \sum_{i=1}^{n} \exp(\lambda_i t) v_i [w_i{}'\zeta] \tag{2.2}$$

where $\lambda_i$ is the $i^{th}$ eigenvalue of matrix $A$, while $v_i$ and $w_i$ are its right and left eigenvectors respectively.

The definition of the mode in this context is as follows: the $i^{th}$ mode is $\exp(\lambda_i t)v_i$, which is defined by the direction of right eigenvector $v_i$ and the time-domain characteristic of associated eigenvalue $\lambda_i$.

It can be seen that the dynamic behavior of state $x$ is actually a linear combination of the dynamic behavior of modes in the linear system. The elements in the right eigenvector $v_i$ quantifies the contribution of mode $i$ on the particular state.

This concept is important for the understanding of the grouping algorithm of slow-coherency theory. Slow-coherency analysis shows that partitioning according to the $r$ slowest modes will produce the weakest connection between areas. After the $r$ slowest modes are selected, the corresponding columns of the modal matrix will determine the effect of the selected modes on the state variables. If two rows of the eigenvector matrix have the same entries corresponding to the $r$ modes, the corresponding machines will be coherent with each other with respect to the selected modes.

Models of large-scale systems involve interacting dynamic phenomena of widely-differing speeds. To analyze the various stability problems, power system dynamics are usually modeled into the following four time scales:

- Long-term dynamics (several minutes and slower): Boiler dynamics, daily load cycles, etc.
- Mid-term dynamics (1-5 min): Load Tap Changers (LTC), Automatic Generation Control (AGC), thermostat-controlled loads, generator over-excitation limiters, etc.
- Transient dynamics (seconds): Generators, Automatic Voltage Regulators (AVR), governors, induction motors, HVDC controllers, etc.
- Practically instantaneous (less than a msec): Electromagnetic and network transients, various electronically controlled loads, etc.

In models of large-scale interconnected systems, dynamics of different speeds are frequently observed. With appropriate partitioning of a power system into areas, the motion of the center of angle associated with each area is much slower than the "synchronizing" oscillations between any two machines in the same area. A physical interpretation of this phenomenon is that the connections between the machines within an area are strong while those between the areas are weak. Therefore, the machines within the same areas interact on a short-term basis. On a long- term basis, when these fast dynamics have decayed, the machines in the same area move together, that is, they are "coherent" with respect to the slow modes. These slow dynamics, which are represented by the area centers of angle, are due to the interaction between groups of machines through the weak connections which may become important in the long term.

### 2.2.2 The Explicit Singular Perturbation Form

In the slow-coherency approach, singular perturbation techniques can be used to separate larger power systems into slow and fast dynamic sections. The low-frequency oscillations between coherent groups of stiffly-connected machines are referred to as the more relevant slow dynamics and the less significant fast dynamics are the higher frequency oscillations between machines within the coherent groups. [23]

Assume that the state variables of an $n^{th}$ order system can be divided into $r$ "slow" state $y$ and $n$-$r$ "fast" state $z$, that is

$$\begin{aligned} dy/dt &= f(y,z,t), \qquad y(t_0) = y_0 \\ dz/dt &= G(y,z,t), \qquad z(t_0) = z_0 \end{aligned} \tag{2.3}$$

The quasi-steady state approach assumes that the only states used for long-term studies are $y$, while the differential equations for $z$ are reduced to algebraic or transcendental equations by setting $dz/dt = 0$. The quasi-steady state model is thus

$$\begin{aligned} dy_s/dt &= f(y_s,z_s,t), \qquad y_s(t_0) = y_0 \\ 0 &= G(y_s,z_s,t). \end{aligned} \tag{2.4}$$

An inconsistency of this approach is that the requirement that $z_s$ must be constant due to the assumption made above, is violated by equation (2.3) which defines $z_s$ as a time-variant variable. A rigorous approach is to treat the situation as a two-time scale singular perturbation problem.

A new time variable τ is introduced to express the fast phenomena, defined by

$$\tau = (t - t_0)/\varepsilon$$

where $t_0$ is the initial value of $t$ and $\varepsilon$ is a ratio of time scale.

By rescaling $G$ as $g=\varepsilon G$, we get the **explicit** singular perturbation form.

$$
\begin{aligned}
dy/dt &= f(y,z,t), \qquad y(t_0)=y_0 \\
\varepsilon dz/dt &= g(y,z,t), \qquad z(t_0)=z_0
\end{aligned}
\tag{2.5}
$$

To investigate quasi-steady state models, it is assumed that

$$dy_f/dt = 0, dz_f/dt = 0$$

It is known that $y_f(t)$ can be any value, and here we assume that $y_f(t)$ is 0 and $y_s(t_0)=y_0$. In the limit as $\varepsilon$ is approaching 0, this model defines the quasi-steady states $y_s(t)$, $z_s(t)$ as

$$
\begin{aligned}
dy_s/dt &= f(y_s,z_s,t), \qquad y_s(t_0)=y_0 \\
0 &= g(y_s,z_s,t).
\end{aligned}
\tag{2.6}
$$

The value of $z_s(t_0)$ can be obtained from the above equations if $\partial g(y_s)/\partial z_s$ .is non-singular.

To obtain the fast parts of $y$ and $z$, equation (2.5) is rewritten in terms of the fast time-scale $\tau$:

$$
\begin{aligned}
dy/d\tau &= \varepsilon f(y,z,t_0+\varepsilon\tau) \\
dz/d\tau &= g(y,z,t_0+\varepsilon\tau)
\end{aligned}
\tag{2.7}
$$

which leads to $dy/d\tau = 0$ as $\varepsilon$ approaches 0, which means that the slow variable $y$ is constant in the fast time scale. Also, it is assumed that $dz_s/d\tau = 0$, which yields the following dynamics model in fast time scale:

$$
\begin{aligned}
&dy/d\tau = 0 \\
&dz_f/d\tau = g(y_0, z_f(\tau)+z_s(t_0), t_0), \qquad z_f(0) = z_0 - z_s(t_0)
\end{aligned}
\tag{2.8}
$$

The separated lower-order models are in error because they assume $\varepsilon$ is approaching 0, instead of the actual positive $\varepsilon$. This parameter perturbation is called singular, since the dependence of the solutions of (explicit form) on $\varepsilon$ is not continuous.

However, in power systems, it is expected that slow state $y$ will be continuous in $\varepsilon$ and the discontinuity in fast state $z$ can be corrected by $z_f$, if we assume that with well-damped fast modes, the state $z$ rapidly reaches its quasi-steady state $z_s$.

When the state $z$ exhibits high-frequency oscillations, the state $y$ is still approximated by $y_s(t)$ due to the "averaging" or filtering effect.

An $0(\varepsilon)$ perturbation form of y, z is therefore given by the following based on the slow model in equation (2.6) and the fast model in equation (2.8).

$$\begin{aligned} y(t) &= y_s(t) + 0(\varepsilon), \\ z(t) &= z_s(t) + z_f(\tau) + 0(\varepsilon) \end{aligned} \tag{2.9}$$

### 2.2.3 Equilibrium and Conservation Properties in LTI systems

When the model of a two time scale system is expressed in terms of physical variables such as those in power systems, it is often not in an explicit form, which requires that $\partial g(y_s)/\partial z_s$ be nonsingular along $y_s(t)$ and $z_s(t)$. When this condition is violated, the explicit form of the two-time scale model cannot be obtained.

Consider the $n$-dimensional system

$$\varepsilon dx/dt = dx/d\tau = A(\varepsilon)x = (A_0 + \varepsilon A_1(\varepsilon))x \tag{2.10}$$

If $A_0$ is nonsingular, $x \rightarrow 0$ as $\varepsilon \rightarrow 0$, no slow phenomenon would exist and the system would not have two-time scales. If $A_0$ is singular with rank $p$, by letting $\varepsilon \rightarrow 0$, the following equation is obtained:

$$dx/d\tau = A_0 x \tag{2.11}$$

It is observed that $A_0$ has a $v$-dimensional equilibrium subspace or manifold, as follows:

$$S = \{x : A_0 x = 0\} \tag{2.12}$$

where $v$ is the rank of null space of $A_0$ and $v+p=n$.

Equation (2.12) indicates that model (2.11) has the **equilibrium property**.

If the rows of a $p \times n$ matrix $Q$ span the row space of $A_0$, then $S$ can also be denoted as $S = \{x : Qx = 0\}$.

To investigate the conservation property of (2.11), a $v \times n$ matrix P is defined such that it spans the left null space of $A_0$, that is, $PA_0 = 0$. Therefore,

$$Pdx/d\tau = d(Px)/d\tau = PA_0x = 0$$

which induces that

$$Px(\tau) = Px(0)\text{, for all } x(0) \text{ in } R^n. \tag{2.13}$$

This means that for each value of *x(0),* the trajectory of (2.11) is confined to a translation of a *v*-dimensional subspace, defined in (2.13). Therefore, the system has the conservation property. This *v*-dimensional subspace is orthogonal to the rows of *P* and contains the initial point *x(0)*, defined as follows:

$$F_{x(0)} = \{x : Px = Px(0)\} \tag{2.14}$$

Based on these two properties, time scales in nonexplicit models can be examined to make them explicit by defining a set of coordinates. In the fast time scale, slow motions of a two time scale system remain constant (interpreted as an equilibrium property) while fast motions are restricted to a linear manifold (interpreted as a conservation property).

### 2.2.4 Time Scale Separation in Non-Explicit Models

For the models shown in (2.10), we can define a transformation matrix

$$T = \begin{bmatrix} P \\ Q \end{bmatrix}\text{, and its inverse, } T^{-1} = \begin{bmatrix} V & W \end{bmatrix}.$$

and define new states *y* and *z*, such that

$$\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} P \\ Q \end{bmatrix} x\,. \tag{2.15}$$

Therefore, equation (2.10) has been transformed into

$$\begin{aligned} Tdx/dt &= \begin{bmatrix} dy/dt \\ dz/dt \end{bmatrix} \\ &= T(A_0/\varepsilon + A_1(\varepsilon))x \\ &= T(A_0/\varepsilon + A_1(\varepsilon))T^{-1}\begin{bmatrix} y \\ z \end{bmatrix} \\ &= \begin{bmatrix} PA_1(\varepsilon)V & PA_1(\varepsilon)W \\ QA_1(\varepsilon)V & QA_0W/\varepsilon + QA_1(\varepsilon)W \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} \end{aligned} \tag{2.16}$$

that is,

$$
\begin{aligned}
dy/dt &= A_s(\varepsilon)y + A_{sf}(\varepsilon)z \\
\varepsilon dz/dt &= \varepsilon A_{fs}(\varepsilon)y + A_f(\varepsilon)z
\end{aligned}
\qquad (2.17)
$$

where

$$
\begin{aligned}
&A_s(\varepsilon) = PA_1(\varepsilon)V, \qquad A_{sf}(\varepsilon) = PA_1(\varepsilon)W \\
&A_{fs}(\varepsilon) = QA_1(\varepsilon)V, \quad A_f(\varepsilon) = QA_0W + \varepsilon QA_1(\varepsilon)W
\end{aligned}
$$

Equation (2.17) shows an **explicit** form for model (2.10), because $A_f(0)=QA_0W$ is non-singular.

It is of the interest to mention that the concept of equilibrium and conservation can be extended to a non-linear system to induce the explicit form. More detailed information may be obtained in [23].

### 2.2.5 Coherency and Grouping Algorithms

As mentioned in previous sections, it has been observed that in multi-machine transients after a disturbance some synchronous machines have the tendency to "swing together". Such coherent machines can be grouped into "coherent areas". A coherency- based grouping approach requires the states to be coherent with respect to a selected set of modes $\sigma_\alpha$ of the system. This approach allows coherency to be examined in terms of the rows of an eigenvector matrix $V$ which can be used to find coherent groups of states.

Most grouping criteria result in coherency states that are disturbance-dependent because they simultaneously treat the following two tasks:

Select the modes which are excited by a given disturbance or a set of disturbances, and
Find the states with the same content of disturbed modes.

The slow-coherency-based approach only addresses the second task, that is, how to find coherency states for a given set of the $r$ slowest modes. The selection of the slowest modes results in slow coherent groups such that the areas of the system are partitioned along the weakest boundaries. Detailed information may be obtained in [23].
In this approach, disturbances are modeled as initial conditions. Therefore, a linear system may be modeled as the following form:

$$
\dot{x} = Ax, \qquad x(0) = x_0 \qquad (2.18)
$$

where the state $x$ is an $n$-vector.

Suppose $\sigma_\alpha = \{\lambda_1, \lambda_2, ..., \lambda_r\}$, where $\lambda_i$ is an eigenvalue of $A$ associated with a dominant mode. The definition of **Coherency** is that the states $x_i$ and $x_j$ are coherent with respect to

$\sigma_\alpha$ if and only if the $\sigma_\alpha$-modes are unobservable from $z_k$, where $z_k$ is defined as $x_j$-$x_i$.

This definition implies that coherent states have the same impact as dominant modes on dynamics, which means the relative rows of *V* are identical. Modes with high frequency and high damping are neglected in long-term studies. By concentrating only on the $\sigma_\alpha$-modes the coherency study will be independent of the location of disturbance.

For an *n*-machine power system, the classical model is defined as the following:

$$\begin{aligned} \dot{\delta}_i &= (\omega_i - \omega_0)\omega_R \\ 2H_i\dot{\omega}_i &= -D_i(\omega_i - \omega_0) + (P_{mi} - P_{ei}) \end{aligned} \tag{2.19}$$

where

$\delta_i$ Rotor angle of machine *i* in radians

$\omega_i$ Speed of machine i, in per unit (pu)

$\omega_0$ Reference speed, in per unit (pu). Here $\omega_0 = 1$.

$P_{mi}$ Mechanical input power of machine i, in pu

$P_{ei}$ Electrical output power of machine i, in pu

$H_i$ Inertia constant of machine i, in seconds

$D_i$ Damping constant of machine i, in pu

$\omega_R$ Base frequency, in radians per second. (376.99 rad / s).

In this model, the mechanical input power $P_{mi}$ is assumed to constant. The electrical output power is

$$P_{ei} = \sum_{j=1, j\neq i}^{n} V_i V_j [B_{ij}\sin(\delta_i - \delta_j) + G_{ij}\cos(\delta_i - \delta_j)] + V_i G_{ii}^2$$
$$i = 1, \ldots, n$$

where $V_i$ is behind transient reactance the machine per unit voltage, which is assumed to be constant. Loads are modeled by constant impedance, such that load buses may be eliminated from the $Y_{bus}$ matrix. $G_{ij}$ and $B_{ij}$ are the real and image entries of $Y_{bus}$. Linearizing the model about the an equilibrium operating point:

$$\begin{aligned} \Delta\dot{\delta}_i &= \omega_R \Delta\omega_i \\ 2H_i\dot{\omega}_i &= -D_i\Delta\omega_i + \sum_{j=1}^{n} \overline{k}_{ij}\Delta\delta_j \end{aligned} \tag{2.20}$$

where

$$\overline{k}_{ij} = V_i V_j [B_{ij}\cos(\delta_i - \delta_j) - G_{ij}\sin(\delta_i - \delta_j)], \quad \text{j} \neq \text{i}$$

$$\overline{k}_{ii} = -\sum_{j=1, j\neq i}^{n} \overline{k}_{ij}$$

Neglecting the damping constants which do not significantly change the mode shape and the line conductance which are relatively small compared with the line reactance, a second order dynamic model can be obtained as

$$\ddot{X} = M^{-1}KX = AX, \quad X(0) = X_0 \qquad (2.21)$$

where

$$x_i = \Delta\delta_i$$
$$m_i = 2H_i / \omega_R$$
$$M = diag(m_1, m_2, ..., m_n)$$
$$k_{ij} = V_i V_j B_{ij}\cos(\delta_i - \delta_j), \quad \text{j} \neq \text{i}$$
$$k_{ii} = -\sum_{j=1, j\neq i}^{n} k_{ij}$$

It has been observed that matrix $K$ has a zero eigenvalue with eigenvector $u$ where $u = [1\ 1\ \ldots\ 1]^{\mathrm{T}}$. Furthermore, $K$ is symmetric if $B$ is symmetric which is true for transmission networks without phase shifters. In general, $B_{ij}$ are positive and $\delta_i - \delta_j$ are small, which implies that $K$ is a negative semi-definite matrix and the eigenvalues of $A$ are non-positive.

Similar to the first order dynamic system, same implication is applicable in the second order dynamic system.

Starting with (2.21), assuming

$$x_1 = x, x_2 = \dot{x}$$

Equation (2.21) may be rewritten as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & I_n \\ A & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad (2.22)$$

Assume $V$ to be a $\sigma_\alpha$-eigenbasis matrix of $A$, and $\Lambda = diag(\lambda_1, \lambda_2, ..., \lambda_r)$. Based on $AV = V\Lambda$, it is easy to obtain

$$\begin{bmatrix} 0 & I_n \\ A & 0 \end{bmatrix} \begin{bmatrix} V & 0 \\ 0 & V \end{bmatrix} = \begin{bmatrix} 0 & V \\ AV & 0 \end{bmatrix} = \begin{bmatrix} V & 0 \\ 0 & V \end{bmatrix} \begin{bmatrix} 0 & I_r \\ \Lambda & 0 \end{bmatrix} \tag{2.23}$$

which means that

$$\begin{bmatrix} V & 0 \\ 0 & V \end{bmatrix}$$

is a $\sigma_\alpha$-eigenbasis matrix of

$$\begin{bmatrix} 0 & I_n \\ A & 0 \end{bmatrix}$$

From the definition, $x_i$ and $x_j$ are coherent if and only if the $i^{th}$ and $j^{th}$ rows of $V$ are identical. This implies that to examine the coherency of the second order system such as that of (2.21), only the $\sigma_\alpha$-eigenbasis matrix of $A$ is required.

Usually in the real dynamic network of a real system, the coherency definition may not be exactly satisfied. Thus, if this definition is applied to a real system, there will be, in general, more coherency groups than the number of modes in $\sigma_\alpha$, which means that there are too many groups to be used in islanding. As a result, an approach to finding near-coherent groups will be presented such that the total number of near-coherent groups is equal to the number of modes in $\sigma_\alpha$. The areas formed by these near-coherent groups are still coherent with small perturbation.

The coherency based grouping algorithm has been summarized as follows: [23]

Choose the number of groups and the set of the slowest modes $\sigma_\alpha$.
Compute a basis matrix $V$ of the σα-eigenspace for a given ordering of the x variables containing slow modes.
Apply Gaussian elimination with complete pivoting to $V$ and obtain the set of reference machines. Each group will then have one and only one reference machine. $V_1$ is the matrix composed of the rows of the matrix $V$ related to the reference machines.
Compute $L = VV_1^{-1}$ for the set of reference machines chosen in step 3).
Determine the group that each generator belongs to from the matrix $L$ by comparing the row of each generator with the row of the reference machines.

A 3-machine system will be chosen to illustrate this coherency based grouping algorithm. Suppose two slowest modes have been chosen and the $\sigma_\alpha$-eigenspace matrix $V$ has the following form:

$$\begin{array}{ccc} & \lambda_1 & \lambda_2 \\ x_1 & 0.577 & -0.287 \\ x_2 & 0.577 & 0.827 \\ x_3 & 0.577 & 0.483 \end{array} \tag{2.24}$$

The procedure of Gaussian elimination with complete pivoting will be shown in the following steps.

The largest number in (2.24) is 0.827. Therefore, the first and second rows and the first and second column can be exchanged to obtain

$$\begin{array}{ccc} & \lambda_2 & \lambda_1 \\ x_2 & 0.827 & 0.577 \\ x_1 & -0.287 & 0.577 \\ x_3 & 0.483 & 0.577 \end{array}$$

Then, the number 0.827 can be used as a pivot to eliminate the remainder of the first column. The result is shown as follows:

$$\begin{array}{ccc} & \lambda_2 & \lambda_1 \\ x_2 & 0.827 & 0.577 \\ x_1 & 0 & 0.831 \\ x_3 & 0 & 0.239 \end{array}$$

Excluding the first row and first column of the matrix $V$, the largest number is 0.831, and the procedure terminates because all the pivots have been found and the reference states are $x_2$ and $x_1$, shown as follows:

$$V_1 = \begin{bmatrix} 0.577 & 0.827 \\ 0.577 & -0.287 \end{bmatrix}$$

Therefore,

$$L = VV_1^{-1} = \begin{bmatrix} 0.577 & -0.287 \\ 0.577 & 0.827 \\ 0.577 & 0.483 \end{bmatrix} \begin{bmatrix} 0.577 & 0.827 \\ 0.577 & -0.287 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 0.577 & -0.287 \\ 0.577 & 0.827 \\ 0.577 & 0.483 \end{bmatrix} \begin{bmatrix} 0.4465 & 1.2866 \\ 0.8977 & -0.8977 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0.6912 & 0.3088 \end{bmatrix}$$

It can be concluded that machine 2 and machine 3 are with the same coherent group and machine 1 itself is another coherent group since the number 0.6912 is closer to 1 than the number 0.3088.

In summary, slow coherency assumes that the state variables of an $n^{th}$ order system are divided into $r$ slow states $Y$, and $(n-r)$ fast states $Z$, in which the $r$ slowest states represent $r$ groups with the slow coherency.

Slow coherency solves the problem of identifying the theoretically weakest connection in a complex power system network. Previous work shows that groups of generators with slow coherency may be determined using Gaussian elimination with complete pivoting on the eigen subspace matrix after selection of $r$ slowest modes $\sigma_a$. In [7], it has been proven through linear analysis that with selection of the $r$ slowest modes, the aggregated system will have the weakest connection between groups of generators.

The weak connection form best states the reason for islanding based on slow coherency grouping. That is, when the disturbance occurs, the slow dynamics in the transient time scale must be separated, which could propagate the disturbance very quickly, by islanding on the weak connections. The slow dynamics will mostly remain constant or change slowly on the tie lines between the areas.

Slow coherency is actually a physical manifestation of a weak connection, which is a network characteristic. In many large-scale practical systems, there always exist groups of strongly interacting units with weak connections between groups. However, weak connections can become strong connections with significant interactions after a long time interval. When a large disturbance happens, it is imperative to disconnect the weak connections before the slow interaction becomes significant.

## 2.3 Graph Theoretic Terminology

Graph theory has developed as a branch of mathematics during the second half of 19th century, and has boomed since 1930. The Swiss mathematician Leonard Euler (1707-1783) is undoubtedly the father of graph theory. His famous problem of the Bridge of Königsberg, has been viewed as the first problem in graph theory. Graph theory is basically the mathematical study of the properties of formal mathematical structures called graphs. Although mathematicians are responsible for much of its development and growth, sociologists and engineers alike are looking enthusiastically toward graph theory to solve problems in their fields [24], [25].

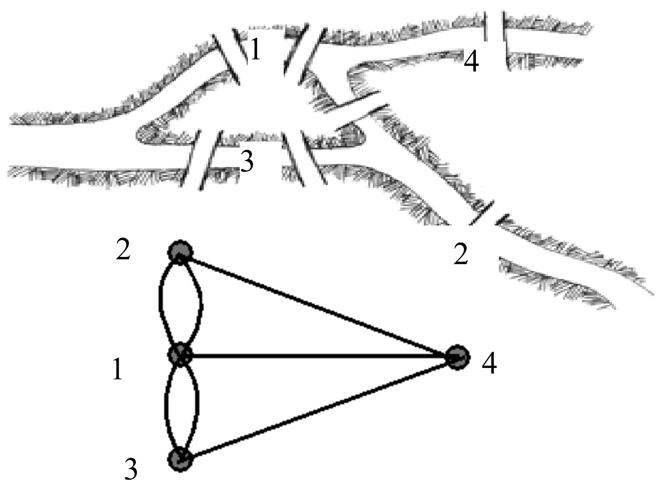


**Figure 2-1 Konigsberg Bridge and its graph representation**

**Definition 2.1** ***Graph:*** A **graph** is a pair of sets ($V$, $E$) where $V$ is the vertex-set and $E$ the edge-set is a family of pairs (possibly directed) of $V$. It is usually denoted as $G \equiv G(V, E)$. Graph are simple abstractions of reality. In this sense, graphs are diagrammatical models of systems. However, not every system can be represented in the form of a graph. As a general rule, any system involving **binary relationships** can be represented in the form of a graph.

**Definition 2.2** ***Connected Graph:*** A graph is connected if there is a path connecting every pair of vertices. A graph that is not connected is said to be disconnected. Its vertices $V$ can be divided into two nonempty subsets $V_1$ and $V_2$ such that vertices of $V_1$ are not adjacent to those of $V_2$. A subgraph $G_1(V_1,E_1)$ of a graph $G(V, E)$ is a graph with vertices $V_1$ and edges $E_1$ such that $V_1 \subset V$ and $E_1 \subset E$. A maximal connected subgraph is called a **component** of a graph. Clearly a graph is connected if it has only one component. The subgraph inside the dashed circle shown in Figure 2-2 one of the components of a graph $E$.

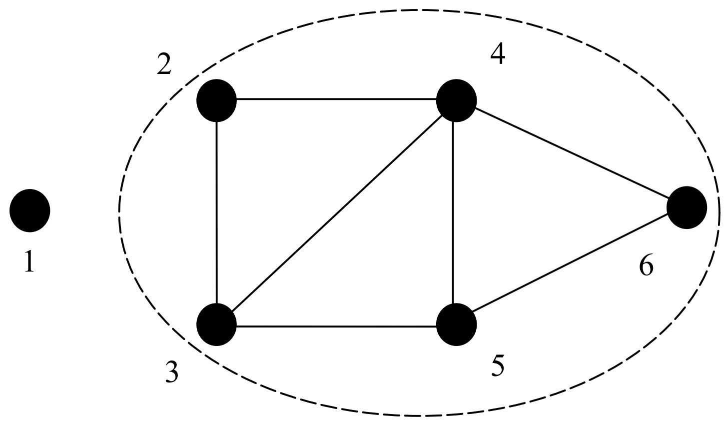


**Figure 2-2 Illustration a component of a graph**

**Definition 2.3** ***Minimal Cutsets (MC)*****:** A **disconnecting set** of a connected graph $G(V, E)$ is a set of edges $E_1 \in E$ such that after the removal of $E_1$ the residual graph $G_1(V_1, E-E_1)$ is no longer connected. This set of edges is called **cutset.** For a given graph $G=(V, E)$, a subset of edges $E_1 \subset E$ is a minimal cutset if and only if deleting all edges in $C$ would divide $G$ into two connected components. It is also called **proper cutset**.

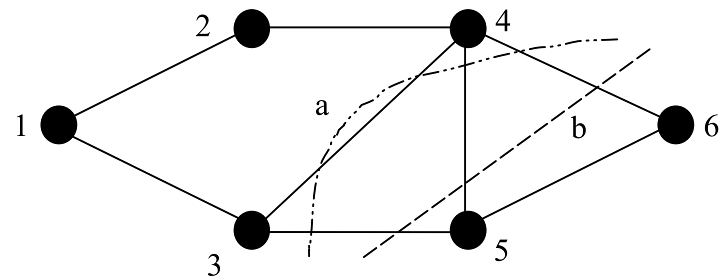


**Figure 2-3 Illustration of: a) a cutset and b) a minimal cutset**

**Definition 2.4** ***Vertices Contraction (VC)*****:** Given a graph $G$ and one adjacent vertices pair $\{x, y\} \in V$, we define $G/\{x, y\}$, the vertices contraction of pair $\{x, y\}$, by deleting $x$ and replacing each edge of the form $\{w, x\}$ by an edge $\{w, y\}$. If this process creates parallel edges, only one edge will remain in the graph. Any self-loops are also eliminated.

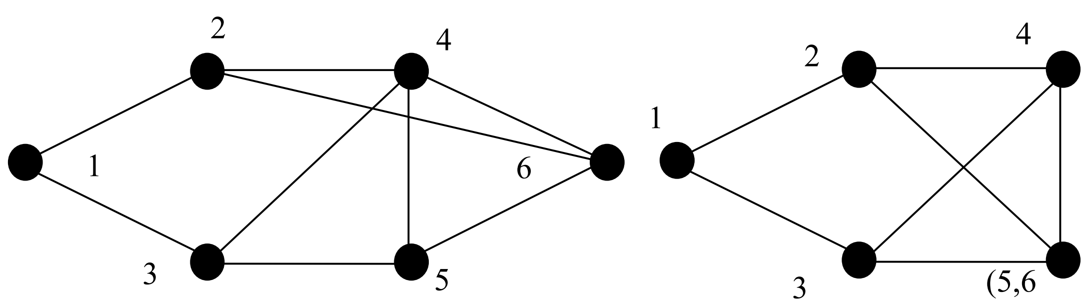


**Figure 2-4 Illustration of vertices contraction on vertex 5 and 6**

**Definition 2.5** ***Depth First Search (DFS)*****:** *Depth first search* is a graph search algorithm which extends the current path as far as possible before backtracking to the last choice point and trying the next alternative path. Extremes are searched first. DFS tends to require less memory, as only nodes on the "current" path need to be stored. However, DFS may fail to find a solution if it enters a cycle in the graph. This can be avoided if we never extend a path to a node which it already contains. DFS can be easily implemented with a *recursion* process.

**Definition 2.6** ***Breadth First Search (BFS):*** *Breadth first search* is a graph search algorithm which tries all one-step extensions of current paths before trying larger extensions. This requires all current paths to be kept in memory simultaneously or at least their end points. Extremes are searched last. Compared to Depth first search, breadth first search does not have a cycling problem. Usually, BFS can be realized with a queue. Modified Breadth first search tree (BST), which is a tree constructed through BFS with specific properties.

**Definition 2.7** ***Minimum Spanning Tree (MST)*****:** A connected, undirected acyclic graph is called a *tree*. *Spanning Trees* are trees that are subgraphs of $G$ and contain every vertex of $G$. In a weighted connected graph $G = (V, E)$, it is often of interest to determine a spanning tree with minimum total edge weight – that is, such that the sum of the weights of all edges is minimum. Such a tree is called *Minimum Spanning Tree*.

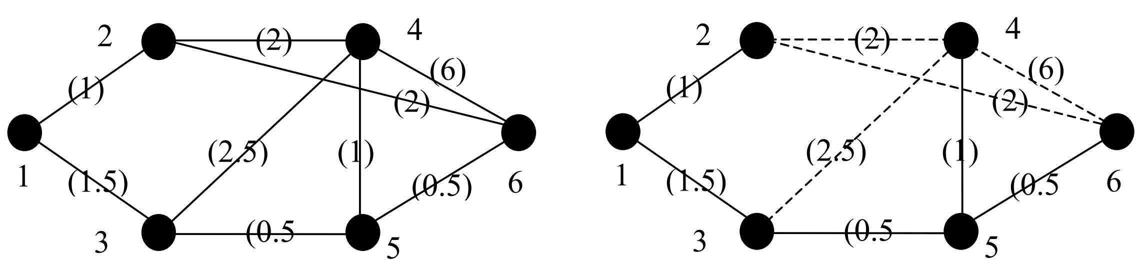


**Figure 2-5 Illustration of the minimum spanning tree**

**Definition 2.8** ***Steiner Tree***: A minimum-weight tree connects a designated set of vertices, called terminals, in a weighted graph or points in a space. The tree may include non-terminals, which are called *Steiner vertices* or *Steiner points*. The Steiner tree problem is distinguished from the minimum spanning tree problem in that we are permitted to select intermediate connection points to reduce the cost of the tree.

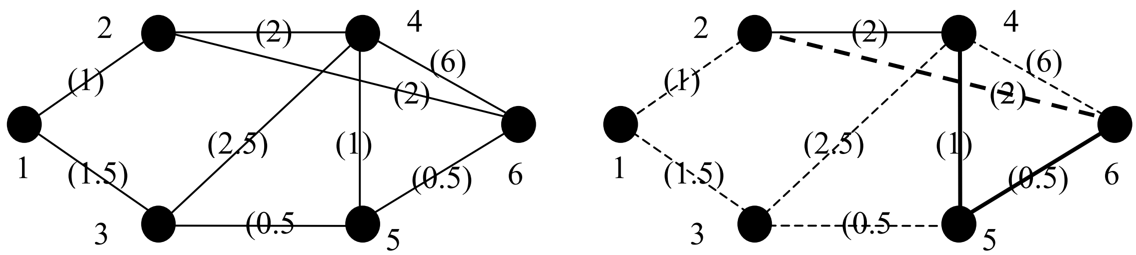


**Figure 2-6 Illustration of the difference between a Steiner tree and a minimum spanning tree**

To find a path connecting vertices 2, 4 and 6 with a minimum cost, a Steiner tree will consist of vertices 2, 4, 5 and 6, where vertex 5 is the Steiner point. However, a minimum spanning tree will consist of vertices 4, 2 and 6.

## 2.4 Realization in Power System

### 2.4.1 Motivation

Power systems are composed of buses and transmission lines connecting the buses. There are both generator buses and load buses with various capacities. Electrical power flows among those transmission lines in certain directions. Therefore, it is very convenient to consider a power system network as a directed graph with different weights at its vertices.

Figure 2-7 illustrates the diagram of one typical 3-generator 5-bus system and its graph representation. It can be seen that the graph is only the representation of the binary relationship between the pairs of buses in the system.

One of the most important requirements for islanding is to minimize the real power imbalance within the islands to benefit restoration. After an island is formed, the imbalance between the real power supply and load demand is usually calculated by computing all the generator vertices and load vertices, which requires a great deal of computation [5]. One may ask the question: What if we consider the branches connecting this island with other islands instead of browsing all vertices within this island? This intuitively makes sense, because most of the time, the number of tripping lines is limited in order to form an island.

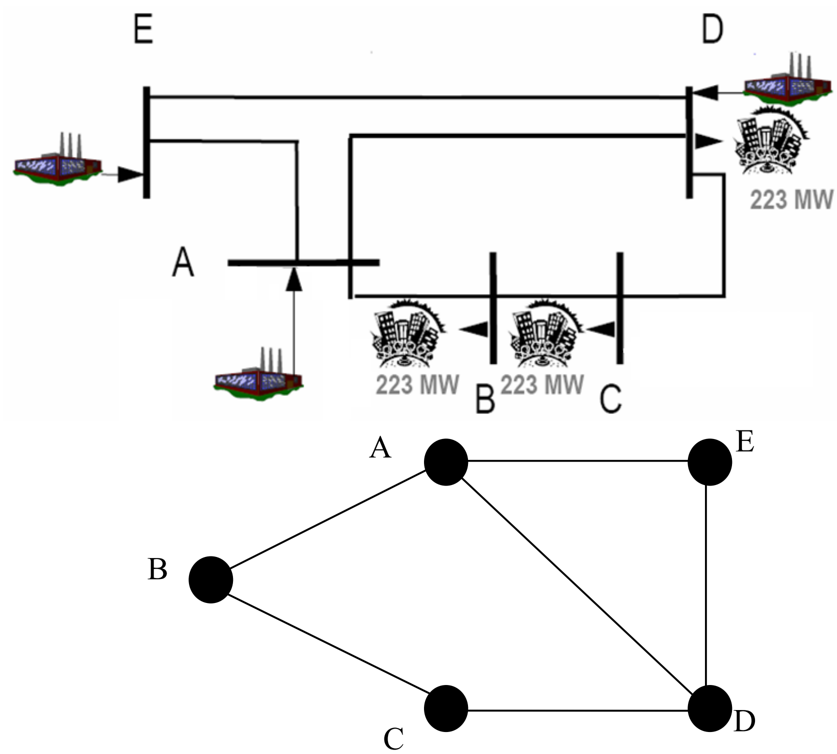


**Figure 2-7 Illustration of a typical 3 generator 5 bus system and its graph representation**

The power flows in the transmission line also contain information about the distribution of generators throughout the system. Once the island is formed, the net flow in the tripping lines indicates the exact difference between the real generation and the load within the island (we assume that the losses can be ignored without the loss of generality).

Therefore, the problem can be converted into one of searching the minimal cutsets (MCs) to construct the island with the minimal net flow. We can decompose the islanding problem into two stages:

1. Find Minimal Cutsets candidates
2. Obtain Optimal Minimal Cutset by various criteria

Generally, the edge-searching approach may result in inefficiency in computation since generally there are more edges than vertices in the network. However, most power systems, at the transmission system level, are sparse, which results in little difference between vertices and edges in terms of numbers.

The advantage of this method is that we can decompose the islanding problem into two stages: In the first stage, we find the cutsets disconnecting the sets of generators; in the second stage, we check the net flow on each cutset to obtain the optimal cutset. Another advantage is that, in the second stage, we can apply any additional criteria to formulate the optimization function under different conditions, such as the requirements for system restoration, while the first stage remains unchanged.

Other advantages of this method are that, besides the general criteria mentioned previously, other user-specified requirements can also be included during islanding, such as:

1. Specification as to which lines may not be disconnected. This is simply done by blocking such a line from the cutsets' candidates.
2. Specification as to which area will remain untouched. This can be done by aggregating such an area into one bus.

### 2.4.2 Software Structure

In order to demonstrate the applicability of this idea, an automatic power system islanding program has been developed to automatically determine where to create an island using minimal cutsets and a breadth first searching (BFS) flag-based depth-first searching (DFS) technique based on Graph Theory. Figure 2-8 illustrates the software structure of this approach.

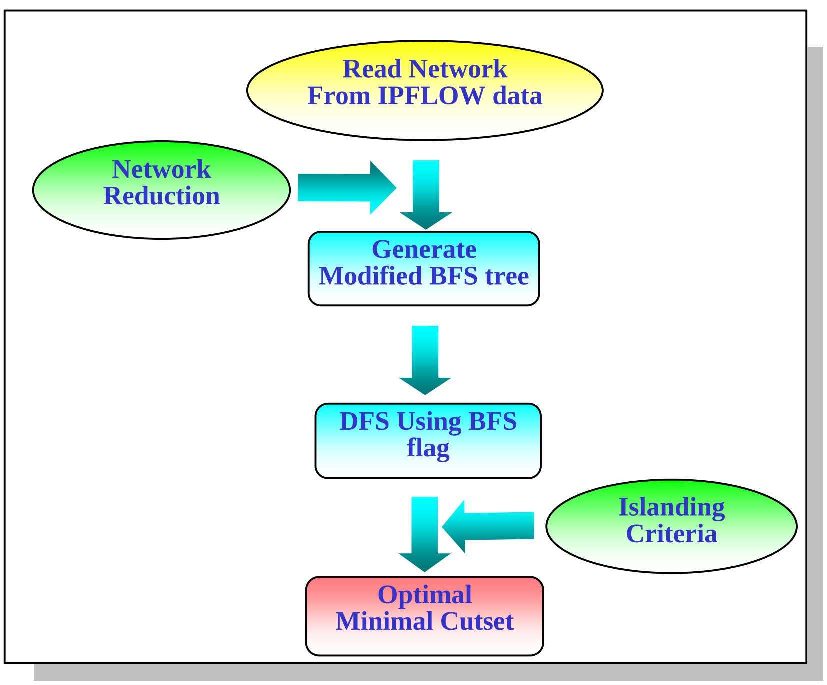


**Figure 2-8 Software structure**

The four main components are:

1. Network reduction
2. Generation of a modified BFS tree with no offspring in sink vertex
3. Conduct of a DFS search with a BFS flag to enumerate all possible MCs
4. Application of islanding to select the optimal MC.

**Network reduction**

As one of the main components in this approach, network reduction plays a significant role in islanding performance and optimality as the network scale increases. Performance and Optimality are two goals that must be dealt with. Here, optimality means the minimum value for our objective function, and performance indicates the computational effort (how long it is going to take to reach the optimality). It is our goal to achieve global optimality as closely as possible while maintaining high performance. However, there is sometimes a tradeoff between these two factors. As we keep reducing the network by a given criteria, we may also deviate from the globally optimal solution since the searching space has been reduced. A heuristic scheme using a *Contraction Factor* has been provided to make it possible to adjust both the performance and optimality level. The layered representation for Network Reduction is shown in Figure 2-9, which is composed of 3 layers, the original network, pre-reduction, and network reduction composed of first stage reduction and second stage reduction.

*Original Network*: In the stage, bus numbers are directly read from PSAPAC ipflow file.

*Pre-Reduction*: Bus numbers are re-ordered beginning with the number 1, since the index of the MATLAB vector variable begins with 1. The variable *bus* is used to record the actual bus numbers.

*First Stage Reduction*: Buses with degree one are reduced from the network. The network scale is reduced. Two functions ***fs2pr*** and ***pr2fs*** are used to convert the bus numbers from the first stage reduction index to pre-reduction

*Second Stage Reduction*: Vertices contraction is applied in this stage, in which the network will be greatly reduced to a reasonable scale. Vertices categorization basically investigates each system vertex and decides which generator group it belongs to. This is also referred to as the ***Generator Bus Extension Process***, after which the uncategorized vertices construct the area from which the minimal cutset will be obtained. The procedure can be illustrated as follows:

1. Find buses to connect generator buses with each other. This can be done by searching using the Steiner tree technique.
2. For each remaining bus, find the shortest path to the generator buses in other areas. If the path crosses the extensive generator buses, it means that this bus is located inside the boundary Π formed by the extensive generator buses. Update the extensive generator buses boundary by adding part of the path.

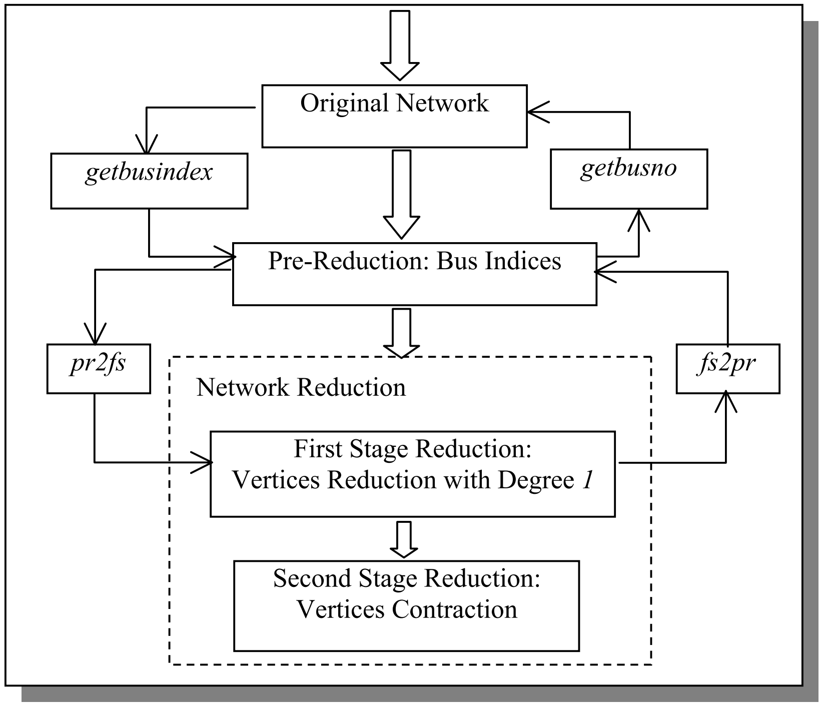


**Figure 2-9 Layered representation of Reduction Component**

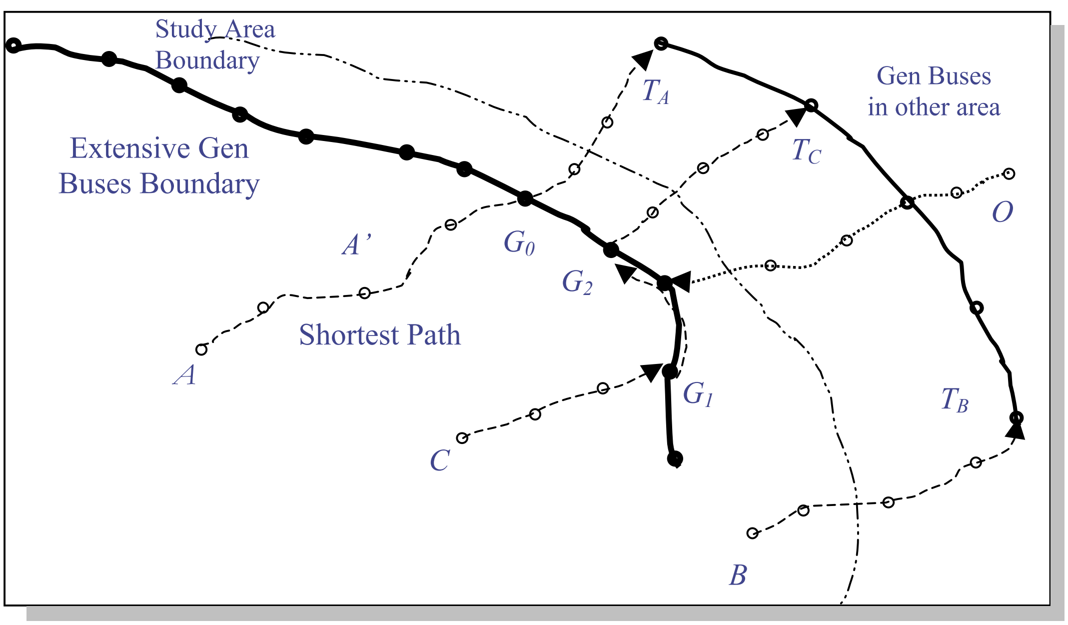


**Figure 2-10 Generator Bus Extension Process**

**The Modified BFS tree based DFS approach**

A modified BFS tree is generated such that there is no offspring for the sink vertex. It is used as a flag to ensure that there is no reexamination of recurring subsets when a DFS is conducted to traverse the reduced graph generated from network reduction. This is because when the graph contains a cycle graph $C_k$, DFS will visit some vertices several times. Each vertex in the BFS tree has an ordered number. The inclusion-exclusion principle is used to organize the vertices according to their position in the BFS tree of the graph. To do this, all vertices in the outline that have a lower order BFS are omitted.

The following figure shows pseudo code of the BFS tree flag-based DFS searching algorithm.

```
/* FINDALLCUTSET (adjmatrix, v, t, F) */
/* adjmatrix is the adjacency matrix of the graph.
/* v and F are initialized as the source vertex, t is initialized as
/* the sink vertex.
If F has not been recorded
   Record F into cutsets set;
Find the outline of F excluding t;
Remove the lower order vertices than v which has been already taken from the
   outline;
For each vertex in the remaining outline of v
   Add this vertex into F;
   Update v to the vertex in F with the lowest order;
   FINDALLCUTSET (origmatrix, v, t, F);
```

**Figure 2-11 Pseudocode of the BFS tree flag based DFS searching algorithm**

## 2.5 Two Comprehensive Approaches for System Wide Islanding

As addressed above, by using the proposed approach a feasible solution to the islanding problem can be found. Without loss of generality, consider the islands formed in Figure 2-12. $H_1$, $H_2$, and $H_3$ are the total inertia of the load-rich islands; $H_4$ is the total inertia of a generation rich island.

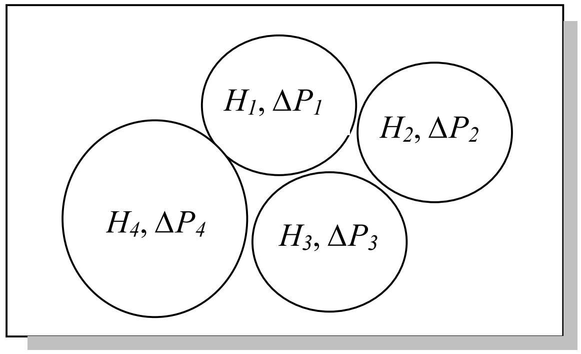


**Figure 2-12 Islands with feasible cutsets**

From the load-generation balance point of view, the optimal solution is to minimize the net flow of each of the islands $H_1$, $H_2$, and $H_3$, while maintaining $\Delta P_i/H_i$ constant among the islands $H_1$, $H_2$ and $H_3$. This means that the average real power imbalance per inertia should kept the same as nearly as possible among those load-rich islands. Here reactive power requirements and other restoration criteria have not been taken into consideration. Two applicable approaches to deal with this optimization are presented in the following:

### 1. Tuning Trial-Error Iterations

Generator rotor speed deviation is captured by the swing equation, shown in (2.19), if the damping effect is ignored. It is observed that the change of generator rotor speed deviation is determined by the ratio of the difference between mechanical input and electrical output over the machine inertia, denoted as $\Delta P_{m,i} / H_{m,i}$. Furthermore, it is our intention to maintain the frequency decline almost the same across all the islands.

Therefore, by extending this concept from the generator to the island, a Tuning Index *TI* is first defined to indicate the degree to which each island needs to be tuned:

$$TI = \frac{\Delta P}{H} \tag{2.25}$$

Obviously, islands with higher values of $\Delta P/H$ have a higher potential to be tuned if real power imbalance is of concern. These values are expressed as a vector $[\Delta P_i/H_i]$ denoted as the *TI* vector.

The algorithm will then expand the islands having the smallest *TI* among those which have intersections with the islands having the largest *TI*. The aim is to reduce the largest *TI*, which increases the smallest *TI*.

An island can be expanded by including its outline. However, one should keep in mind that the expansion should exclude the generators in other islands. Minimum spanning tree (MST) or Steiner tree techniques can be used to keep the generator buses from being included. This would also give maximal space for neighboring islands to expand.

For the example considered in Figure 2-12, suppose $H_1$ has the largest *TI*, and $H_2$ has the smallest *TI* among those islands which intersect with $H_1$. $H_2$ will be expanded by including its outline.

In general this approach will not reach the optimal solution in a single tuning procedure. Several iterations are needed until the error (as computed by equation 2.26), is less than a specified tolerance.

$$\varepsilon = \sqrt{\sum_i \frac{\left(\frac{\Delta P_i}{H_i} - \overline{\frac{\Delta P}{H}}\right)^2}{n}} \tag{2.26}$$

where $\overline{\frac{\Delta P}{H}} = \frac{1}{n}\sum_i \frac{\Delta P_i}{H_i}$

**2. Aggregated Island Approach**

An alternative for finding the optimal cutset for all islands will be addressed below:

1) Based on the Tuning Indices, find the reasonable cutsets for all the generator groups.
2) Determine the load-rich islands.
3) Consider all those generators in interconnected load-rich islands as one group, and determine the minimal cutsets for this aggregated group with minimal net flow, which corresponds to the aggregated islands.
4) Assume that once the minimal cutset for the aggregated group is acquired the optimal cutset for these individual groups can always be found.
5) Calculate the load-generation imbalance within the aggregated islands. If only the load-generation imbalance is considered, index $\Delta P_i/H_i$ among those individual islands should be maintained to be equal. By applying this principle, the load-generation imbalances within each individual island can be calculated.
6) Taking other criteria associated with restoration into account; and based on appropriate priority indices, the islanding procedure can be re-run again with an estimation of the load-generation imbalance within each island.

If some load-rich islands are interconnected, the minimal cutsets for the aggregated island is the combination of the minimal cutsets. Here only one aggregated island is taken into consideration. For a system which is comprised of multiple aggregated islands, method A should be used. First, the number of islands existing in the system should be determined. Second, by using method (B), connected islands are considered as one island, and only isolated islands are taken into account. Next, using method (A), tune each isolated to reach the condition where equation (1) in [26] holds. The procedure is shown schematically in Figure 2-13.

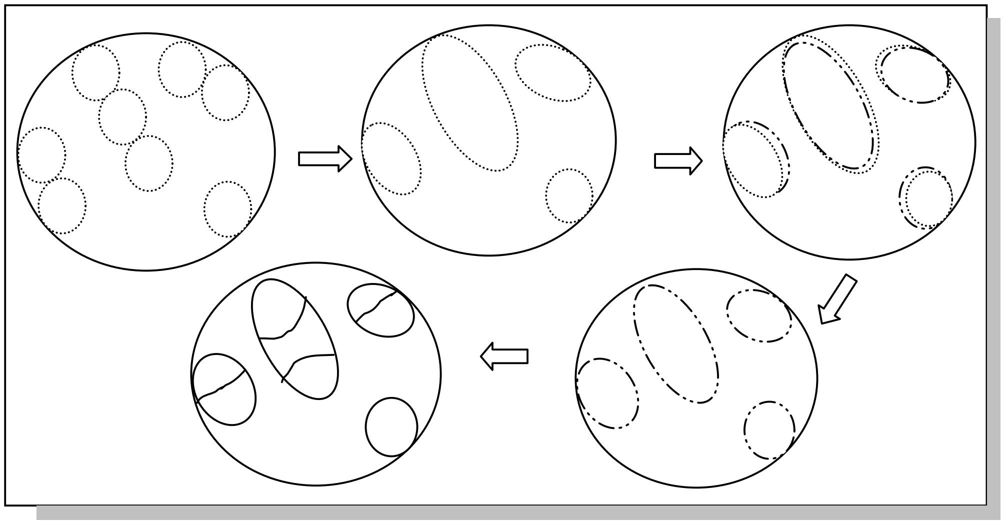

**Figure 2-13 Final approach to system islanding**

For an aggregated island containing less than two individual islands, separation is much simpler However, in the case where there are more than two islands within an aggregated island, each separate island needs to be calculated and identified; consequently. We need to first specify the source vertices *S* which represent the generators to be identified within the island, and sink vertices *T*, which will be generators in the remainder of the aggregated island, as shown in step 1, Figure 2-14. This procedure will continue as shown in step 1, Figure 2-14, until no more islands need to be identified.

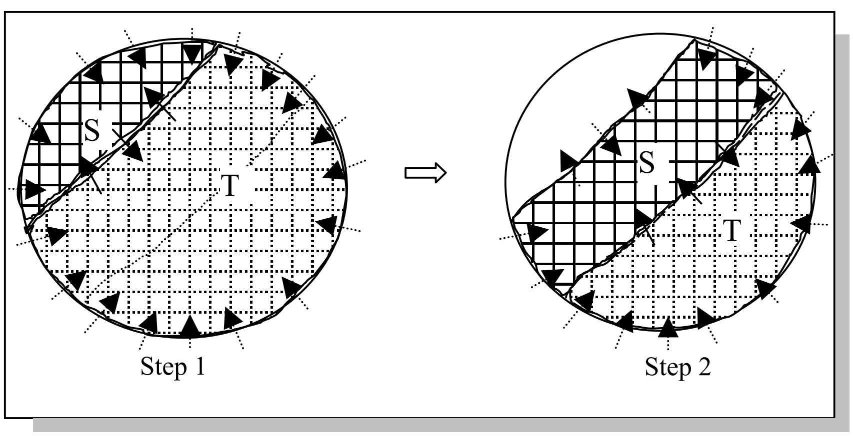


**Figure 2-14 Separate individual islands in one aggregated island**

## 2.6 New Governor Model

So far, a novel approach has been presented for performing power system islanding. This method is independent of the models of generators, exciters and governors. However, more accurate models are needed to represent a real system when time-domain simulation is conducted. In these simulations, frequency response needs to be identified and a load shedding scheme will also be designed.

Studies have demonstrated that representing base loading of generators and generator load controllers has a dramatic positive effect on simulation results, not only in frequency deviation studies (reserve, under frequency load shedding, etc.), but also on the results of many system stability studies, such as those used to set transfer limits, remedial action, etc. Simulations of real-time events, including staged and random generator trips in the WECC system [27], have indicated that there is a wide difference in the frequency response values produced by simulations and those recorded by disturbance-monitoring equipment. Differences of the order of 50% to 60% have been noted in both transient peaks and "settling" frequencies. Governor and load-modeling issues were highlighted in previous work during 2000 by the Task Force of the WECC's Modeling & Validation Work Group (M&VWG) for further investigation.

In [27],[28], the WECC Modeling & Validation Work Group has proposed a new governor model for the WECC system to resolve wide differences in the frequency response values produced by simulations and those recorded by disturbance-monitoring equipment.

### 2.6.1 Model Description

Two new models have been developed for use in WECC studies. The *ggov1* model referenced in [28] is a generic thermal governor/turbine model that incorporates base loading and a load controller, as shown in Figure 2-15. The model, *lcfb1* [28], is identical in structure to the load controller portion of *ggov1*, and can be used in tandem with any governor model currently defined in any power system transient stability program, as shown in Figure 2-16.

Thermal plants not currently modeled with a governor in the WECC database should be added using the *ggov1* model. All gas turbine units should use the *ggov1* model. Hydro units that operate under load control should use the *lcfb1* model in addition to the appropriate hydro governor model.

Existing *ieeeg1* models may be used with the addition of the *lcfb1* load-controller model if it applies. Alternatively, the new *ggov1* model may be used for such units with appropriate data supplied for it.

Upon initialization, base-loaded units and load-controllers are assigned values equal to the generator dispatched value specified in the power flow data in the *ggov1* and *lcfb1* models. If the effects of a load (or any set point other than frequency) controller are to be included, the output of the unit will be reset to the value of $P_{MWSET}$. The speed at which the resetting takes place is controlled by the value of $K_{IMW}$ ($K_I$ in model *lcfb1*).

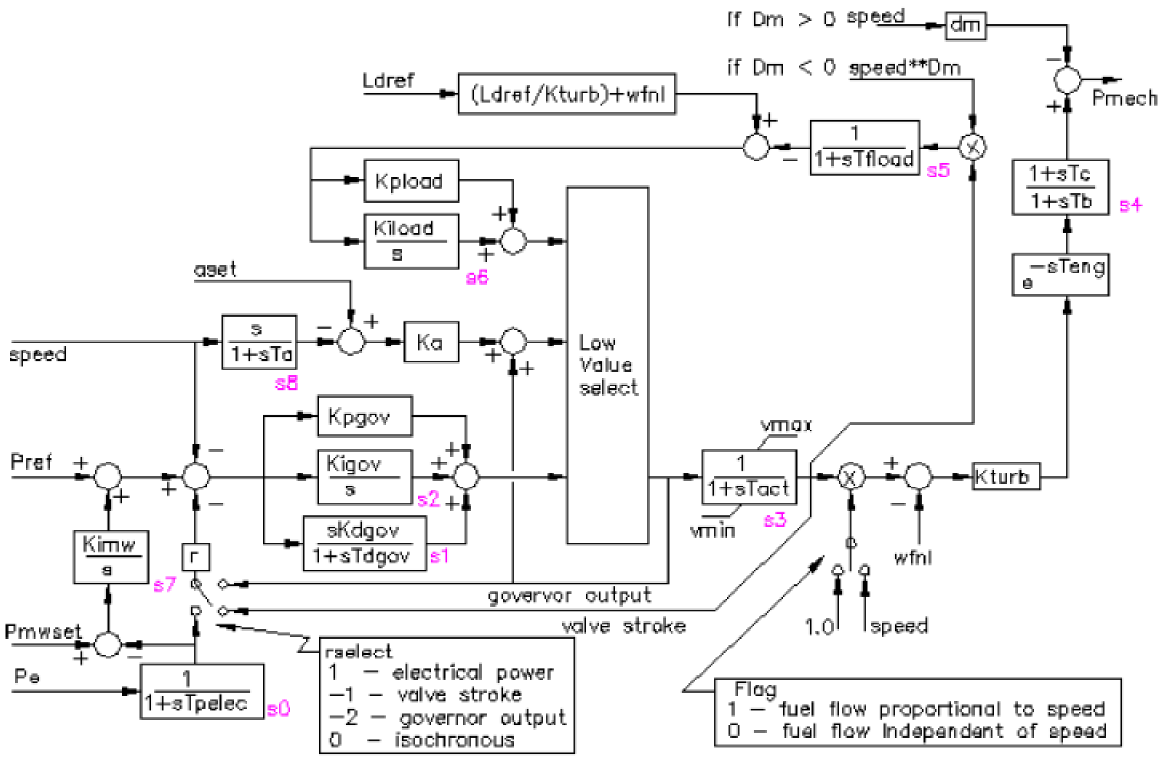


**Figure 2-15 Generic thermal governor/turbine model**

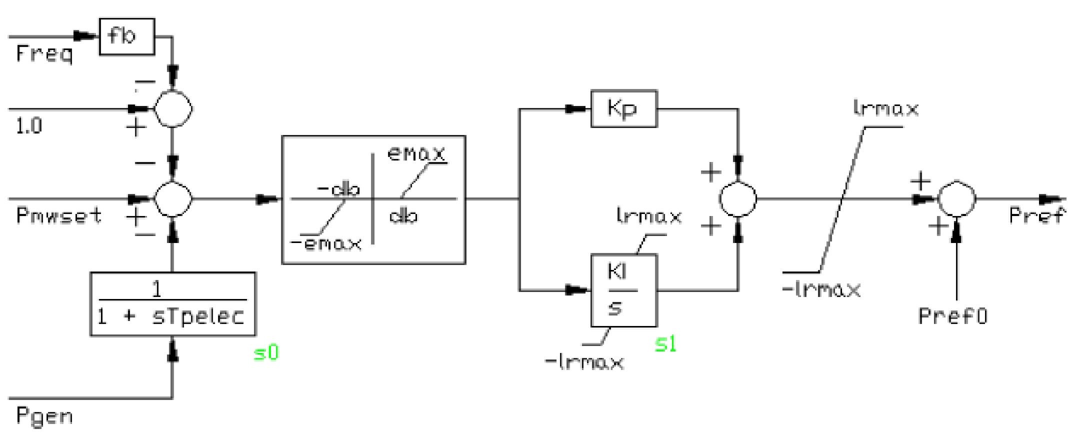


**Figure 2-16 Load controller potion of governor model**

## 2.7 Adaptive Load Shedding

An adaptive load shedding scheme is required to help preserve the security of generation and interconnected transmission systems during major system frequency declining events. Such a program is essential to minimize the risk of total system collapse; to protect generating equipment and transmission facilities against damage; to provide for equitable load shedding (interruption of electric supply to customers), and to help ensure the overall reliability of the interconnected systems.

Load shedding resulting from a system under-frequency event should be controlled so as to balance generation and customer demand (load), to permit rapid restoration of electric service to customer demand that has been interrupted, and, when necessary, to re-establish transmission interconnection ties.

In our approach, controlled system islanding divides the power system into islands. Some of these islands are load-rich and others may be generation-rich. Generally, in a load-rich island, the situation is more severe. The system frequency will drop because of the generation shortage. If the frequency falls below a certain set point (e.g., 57.5 Hz), the generation protection system will begin operation and trip the generator, further reducing the generation in the island and making the system frequency decline even more. In the worst case, the entire island will experience blackout.

### 2.7.1 Definition of Load Shedding

According to the North American Electric Reliability Council (NERC) definition, load shedding is the process of deliberately removing (either manually or automatically) pre-selected customer demand from a power system in response to an abnormal condition to maintain the integrity of the system and minimize overall customer outages [29].

### 2.7.2 General Requirements of the Automatic Under-Frequency Load Shedding

As already pointed out, controlled system islanding is the last resort to prevent a system from total collapse. A load-shedding scheme is the ultimate strategy to prevent total blackout in load-rich islands. This is due to the facts that a continuous generation shortage leads to persistent low frequency, which may activate the unit's protection scheme to trip units out of the system and further decrease the frequency. This may reach to the point that all the units trip out and the system shuts down.

Based on the characteristic and nature of under-frequency load shedding (UFLS), the following aspects should be the major considerations for designing a load shedding scheme: amount of load to be shed at each step, frequency threshold, step size and number of steps, time delay, and priorities.

The literature describes two types of load-shedding schemes: load shedding based on frequency decline and load shedding based on rate of frequency decline. Load-shedding schemes used before the 1980s were almost all based on frequency decline (UFLS). This conventional load shedding scheme has the following disadvantages: 1) longer low-frequency system operation caused by slower UFLS action; 2) possible excess of load shed and associated frequency overshooting.

An adaptive load-shedding scheme which takes the rate of frequency decline into consideration has been proposed.

A threshold value ($M_0$) is defined in each island, such that, if the rate of frequency decline after islanding at one load exceeds $M_0$, a new load-shedding scheme will be deployed. Otherwise, a conventional load shedding scheme will be deployed. [30]

$$M_0 = \frac{60 \times P_{L\Delta}}{2\sum H_i}, \quad where \;\; P_{L\Delta} = 0.3 \times P_{sys} \tag{2.27}$$

where $P_{L\Delta}$ is the minimum load deficit that could drive the system frequency down to 57*Hz*, which is the minimal operational frequency.

## 2.8 Results for the Test System

### 2.8.1 Grouping Results from DYNRED

In this section we will demonstrate the efficacy of slow-coherency-based grouping and automatic islanding by applying minimal cutsets on the WECC 29-Generator 179-Bus test system. The system has a total generation of 61,410MW and 12,325Mvar. It has a total load of 60,785MW and 15,351Mvar. The Dynamic Reduction Program (DYNRED) from EPRI's Power System Analysis Package (PSAPAC) [31] was chosen to form groups of coherent generators based on an improved slow-coherency method developed by GE [32] to deal with large systems and achieve more precise results. The user can specify the tolerance value, the number of slow modes, and the number of eigenvalues being calculated. Then, with the help of the automatic islanding program, the optimal minimal cutset of the island may be determined, taking into account the least generation load imbalance and topological requirements.

The DYNRED program has been employed to find groups of generators with slow coherency on the 179-Bus system as a base case. The 29 generators are divided into 4 groups by the slow-coherency program as shown by the dotted lines in Figure 2-17. The dashed lines indicate these four groups. The detailed grouping information has been shown in **TABLE 2-1**. Fast dynamics are propagated through the weak connections determined by the boundary between groups of generators. To develop a better understanding of the proposed approach, the minimal cutsets between the south island and the rest of the system are first determined. Once the minimal cutset of the south island is found, we can, if necessary, continue to find other islands by removing the south island from the network and treating the rest of network as the whole network.

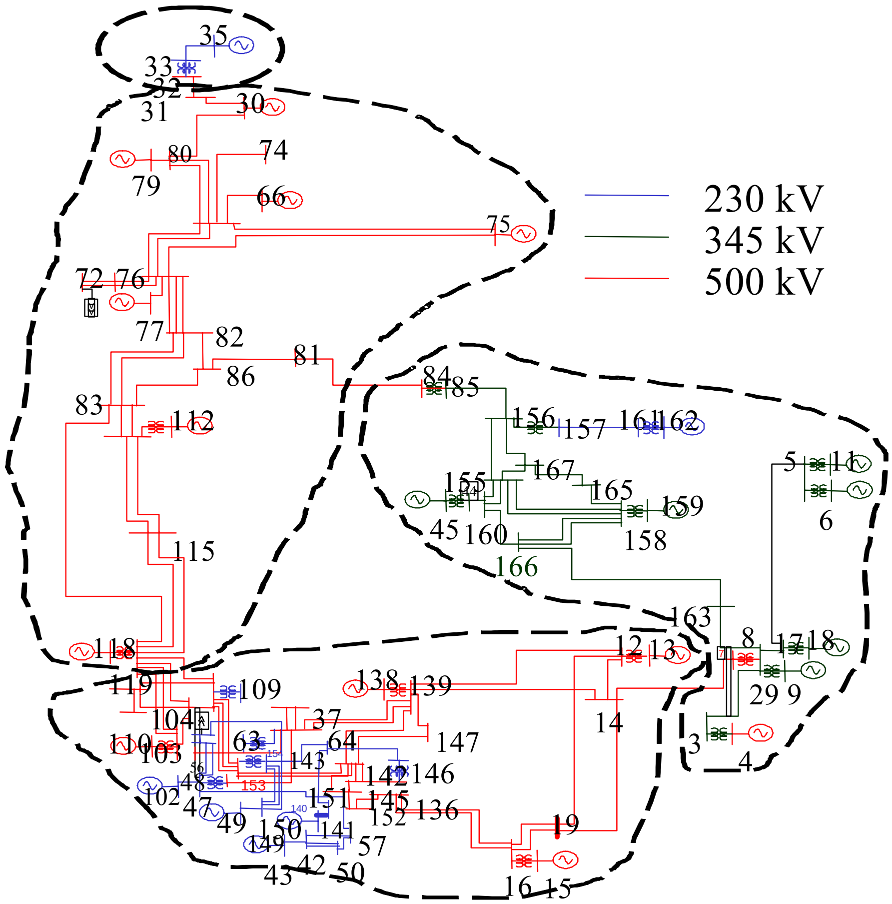


**Figure 2-17 Generator groups formed by slow coherency**

**TABLE 2-1 GENERATOR GROUPING RESULTS FROM DYNRED**

| Group No. | Generator No. |
|---|---|
| 1 | 140, 40, 103, 138, 43, 144, 148, 13, 47, 15, 149 |
| 2 | 11, 36, 4, 6, 159, 9, 45, 162, 18 |
| 3 | 35 |
| 4 | 79, 30, 70, 77, 65, 112, 116, 118 |

### 2.8.2 Graph Representation

Figure 2-18 denotes the graph representation of the WECC 29-generator, 179-bus system, where the largest font designates the generator buses in the south island and the middle-sized font designates the generator buses in other islands. It can seen that each double circuit is considered as one edge in the graph, because the controlled system islanding action always disconnects both lines in the double circuit rather than just a single line. This is due to the fact that, other than transmission system reconfiguration, controlled system islanding is mainly intended to change the system topology rather than system flow. Therefore, this simplification will not affect the final result.

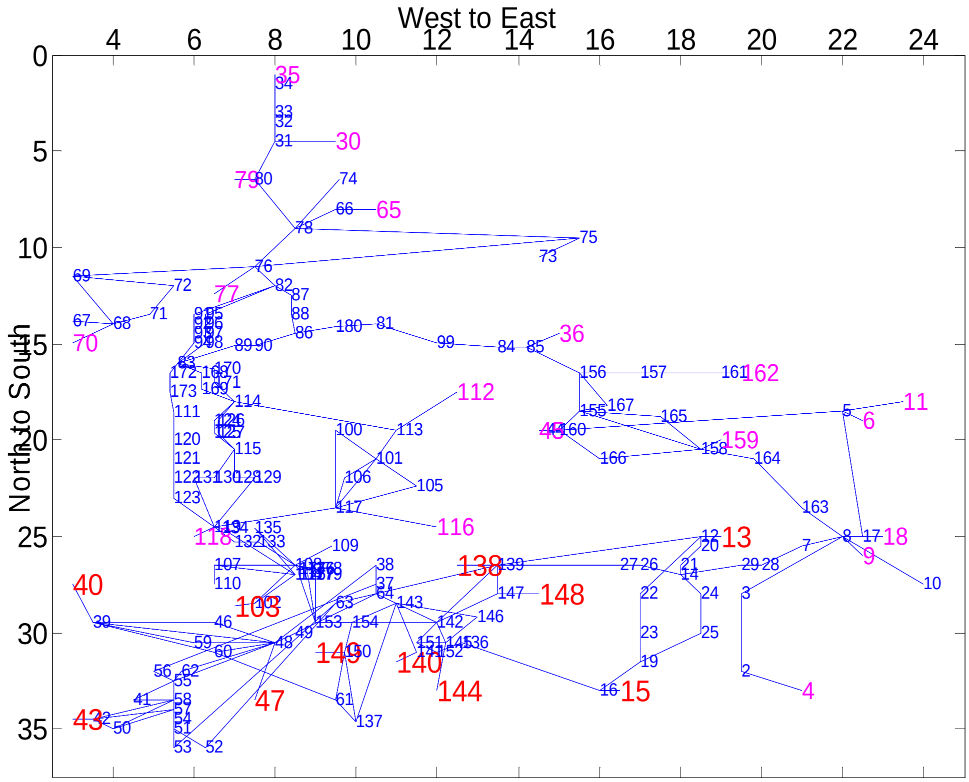


**Figure 2-18 Graph of WECC 29-179 System**

To demonstrate that controlled system islanding is almost independent of disturbance locations, two scenarios with different disturbances have been examined to illustrate the procedure of this approach.

**Scenario 1**

As indicated in Figure 2-8 and Figure 2-9, our approach starts with network reduction, which can be divided into first-stage reduction and second-stage reduction.

After first-stage reduction, the WECC system has been reduced from 189 vertices and 222 edges to 132 vertices and 175 edges.

Based on the assumptions given earlier, the set of source vertices S and the set of sink vertices T should both be connected. To achieve this, other buses are included with the minimum spanning tree technique to make the set of generator buses in the south island and the set of generator buses in the rest of the area both connected. Then the network is reduced to a 21-vertex graph after applying vertices contraction, shown below.

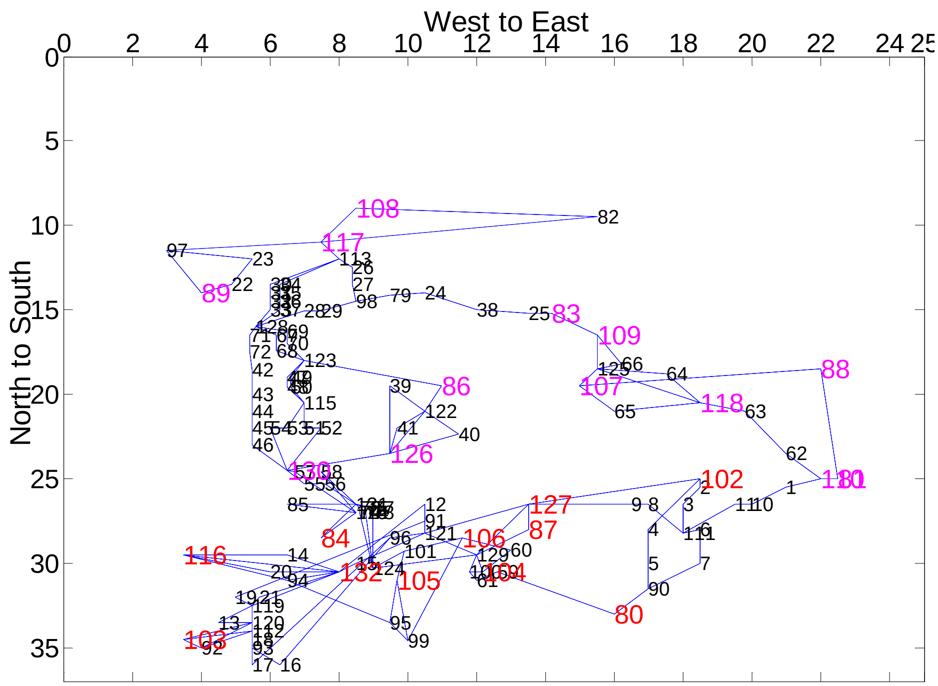


**Figure 2-19 Graph of WECC 29-179 System after first stage reduction**

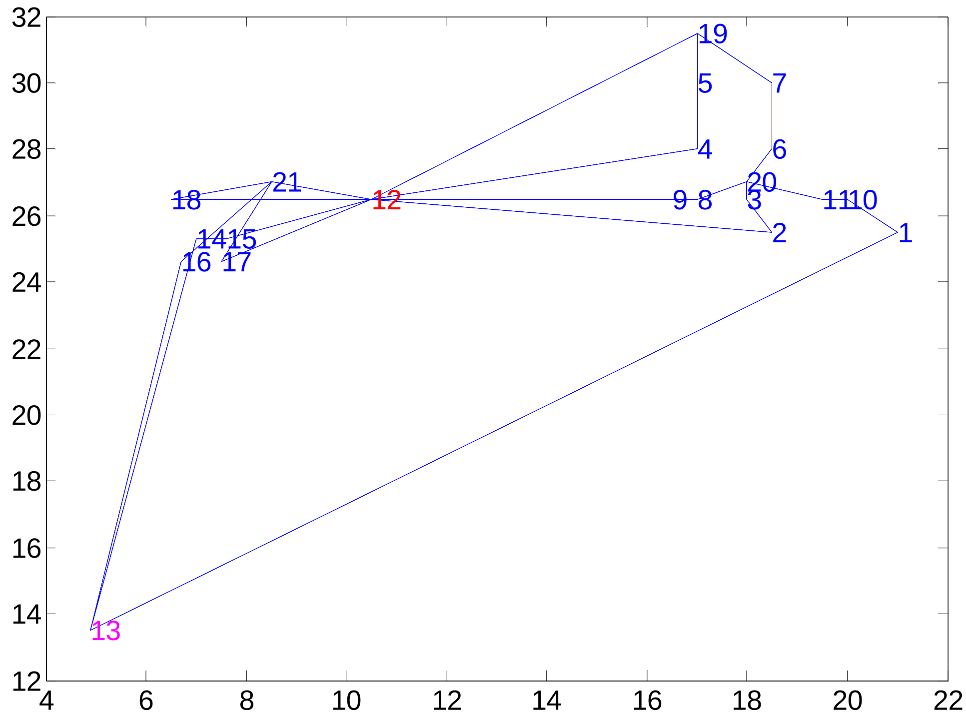


**Figure 2-20 Network representation after vertices contraction**

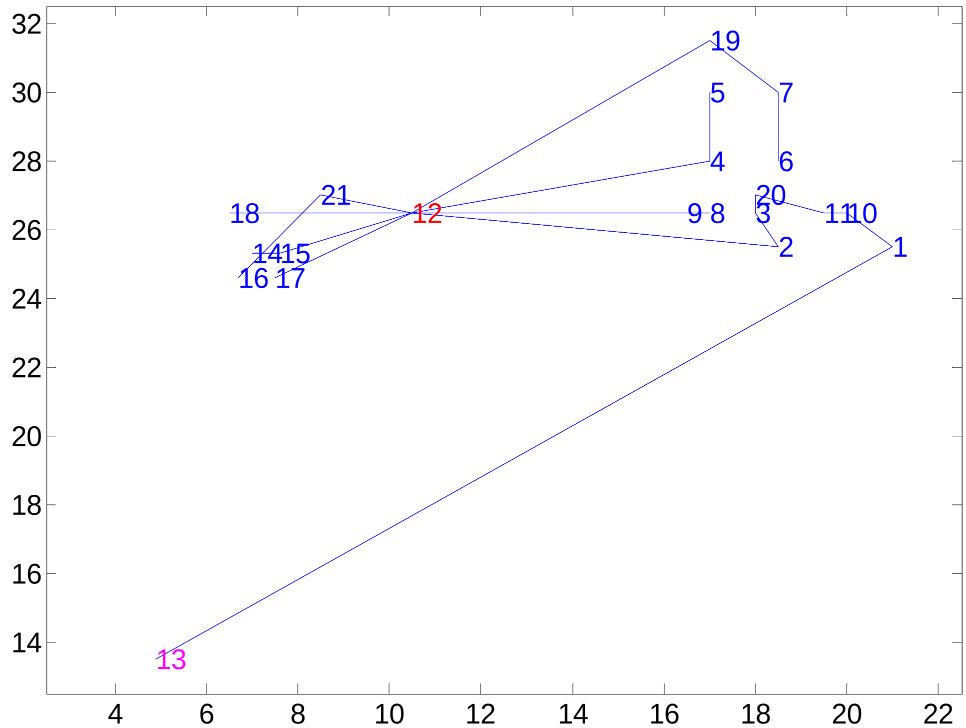


**Figure 2-21 Modified BFS tree**

In Figure 2-20, vertex 12 is the source vertex, which is the aggregated vertex of the extensive generator buses in the south island, and vertex 13 is the sink vertex, which is the aggregated vertex of the extensive generator buses in the rest of the network. During the vertices contraction, other buses are included to make the set of generator buses in the south island and the set of generator buses in the rest of the network both connected.

Starting with the source vertex 12, the modified BFS tree is obtained as shown in Figure 2-21.

A recursive function with BFS tree flag-based DFS searching technique returns the following choices of 24 minimal cutsets with 3 lines, 210 cutsets with 5 lines, 162 cutsets with 6 lines, 324 cutsets with 7 lines, and 324 cutsets with 8 lines. TABLE 2-2 summarizes the minimal cutsets with different numbers of lines and minimal load-generation imbalance.

**TABLE 2-2 MINIMAL CUTSETS WITH DIFFERENT NUMBER OF LINES REMOVED**

| No. of lines removed | 3 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|
| Cutsets number | 24 | 210 | 162 | 324 | 324 |
| Minimal Cutset with Minimal active power imbalance | 14 29<br>104 134<br>108 133 | 102 104<br>14 29<br>108 133<br>108 135<br>108 107 | 16 19<br>12 20<br>12 22<br>104 134<br>139 27<br>108 133 | 102 104<br>19 25<br>12 20<br>139 27<br>108 133<br>108 135<br>108 107 | 16 19<br>102 104<br>12 20<br>12 22<br>139 27<br>108 133<br>108 135<br>108 107 |
| Net Flow (MW) | -2076.35 | -1464.98 | -1434. 17 | -1442. 28 | -822.80 |

Figure 2-22 shows the relationship between the number of lines removed and load generation imbalance within the island. It is very clear that there is a trade-off; with more lines removed, there is less imbalance.

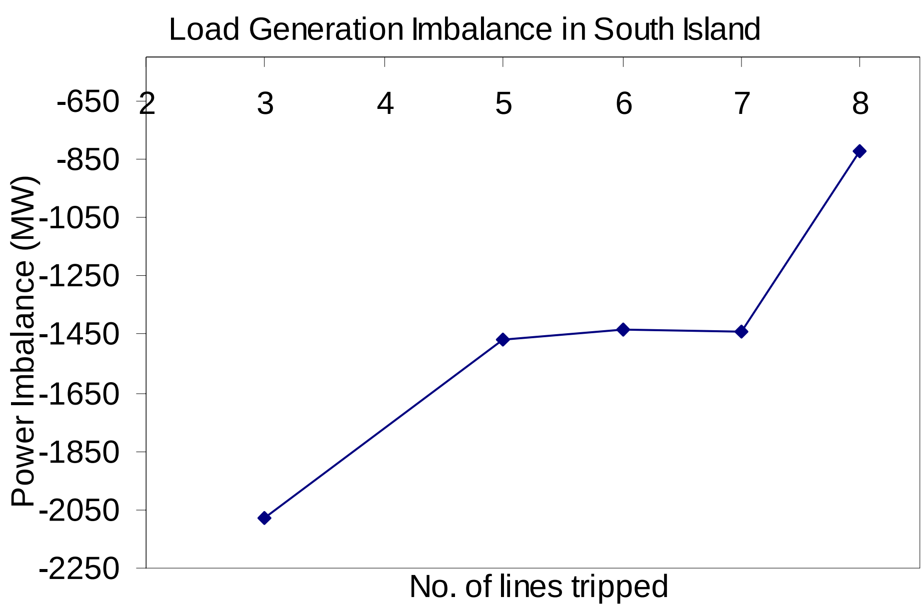


**Figure 2-22 Relationship between number of line removed and active power imbalance**

The situation is a bit more complicated when a major contingency is taken into account. A large contingency has been applied to the WECC system with characteristics such that transmission lines 83-168, 83-170, 83-172 are disconnected at the same time. This actually cuts the WECC system in the East. According to the method described in Section 2.5, in order to handle a system with more than two islands, either a Tuning Trial-Error or an Aggregated Island approach may be used to form the island in a systematic manner. In this case, the Aggregated Island approach is applied to island the system into two subsystems (one load-rich, the other generation-rich), along with the contingency. Once this is done, a Trial-Error approach is conducted in the aggregated load-rich island to form two islands.

TABLE 2-3 provides detailed information about the load-rich island after the Aggregated Island approach has been applied.

**TABLE 2-3 AGGREGATED LOAD RICH ISLAND**

| | Generator (Bus No.) | Cutset (Bus No.) | Inertia (S) | Net Flow (MW) | TI (MW/S) |
|---|---|---|---|---|---|
| Aggregated Load Rich Island | 15, 103, 148, 13, 43, 144, 149,140, 40, 138, 47, 112, 116, 118 | 168 83<br>170 83<br>172 83<br>14 29 | 1310.05 | -4106.71 | -3.1348 |

Two islands have been created by applying the Tuning Trial-Error approach to the aggregated island. TABLE 2-4 illustrates the detailed information of these two islands. The last column intuitively gives the idea of how fast the average rotor angle of generators in this island will move once the island is actually formed. It is ideally expected that the TI values for island 1 and 2 will be the same. However, depending on the topology of the real situation, these values are most likely not the same, although they are as close as possible. Figure 2-23 shows the final minimal cutset used to island the system.

**TABLE 2-4 DETAILED INFORMATION FOR THE TWO SOUTH ISLANDS IN SCENARIO I**

| | Generator (Bus No.) | Cutset (Bus No.) | Inertia (S) | Net Flow (MW) | TI (MW/S) |
|---|---|---|---|---|---|
| Island 1 | 15, 103, 148, 13, 43, 144, 149,140, 40, 138, 47 | **132 119**<br>**134 119**<br>14 29 | 966.66 | -2084.46 | -2.1563 |
| Island 2 | 112, 116, 118 | 168 83<br>170 83<br>172 83<br>**119 132**<br>**119 134** | 343.39 | -2022.24 | -5.8891 |

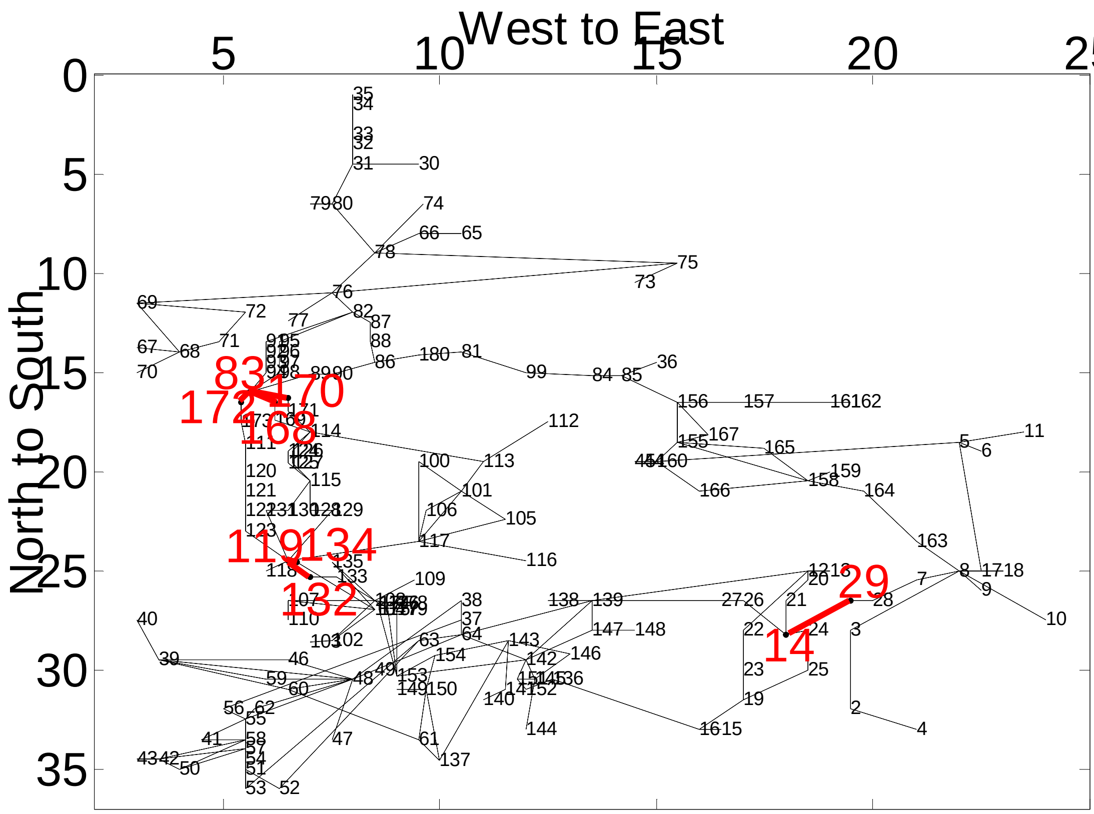


**Figure 2-23 Final minimal cutset to island the system in Scenario I**

**Scenario 2**

In this scenario, a contingency has been applied such that lines 139-27, 139-12, and 136-16 (dct) have been disconnected. This leads to the disconnection of the southern area from the east. The contingency is severe enough to make the system unstable if no self-healing strategies have been initiated.

The recursive function with BFS tree flag based DFS searching technique returns very similar cutest outcomes as shown in scenario 1. However, since the nominal south island has been split into two parts due to the contingency, the optimal cutest shown in TABLE 2-4 in scenario 1 may not be applicable any longer.

As stated in scenario I, all cutsets candidates have been obtained from the recursive function before applying the contingency. With 3 lines tripped, the minimal cutset with minimum net flow will be 14-29, 104-134, and 108-133 as shown in **TABLE 2-4**. However, after the contingency has occurred, the final cutset is line 132-119, 134-119, 136-16, 139-12, and 139-27 as shown in TABLE 2-5. Figure 2-24 shows the final cutset to island the system. One may find that the final cutset in scenario II is very similar to that in scenario I. This is due to the special network topology of the southern WECC system. This part of the system connects the rest of the system only through two independent paths: east path starting with bus 24, and west path near bus 108. Therefore, it becomes totally independent in searching minimal cutsets in east path and west path. That is the reason that part of the final cutset: line 132-119 and line 119-134 does not change in both scenarios.

**TABLE 2-5 DETAILED INFORMATION FOR THE SOUTH ISLAND IN SCENARIO II**

| | Generator (Bus No.) | Cutset (Bus No.) | Inertia (S) | Net Flow (MW) | TI (MW/S) |
|---|---|---|---|---|---|
| Island | 103 148 43<br>144 149 140<br>40 138 47 | **132 119**<br>**134 119**<br>136 16<br>139 12<br>139 27 | 720.158 | -4632.34 | -6.4324 |

One may notice from Figure 2-24 that no line is tripped to form an island with generator 13 and 15; instead, these two generators connect to the rest of the system in spite of the fact that a weak connection exists, the connection to other generators, such as generator 11, is still relative tight. Also, the proposed island with generator 13 and 15 is in reality too small in scale to form an island.

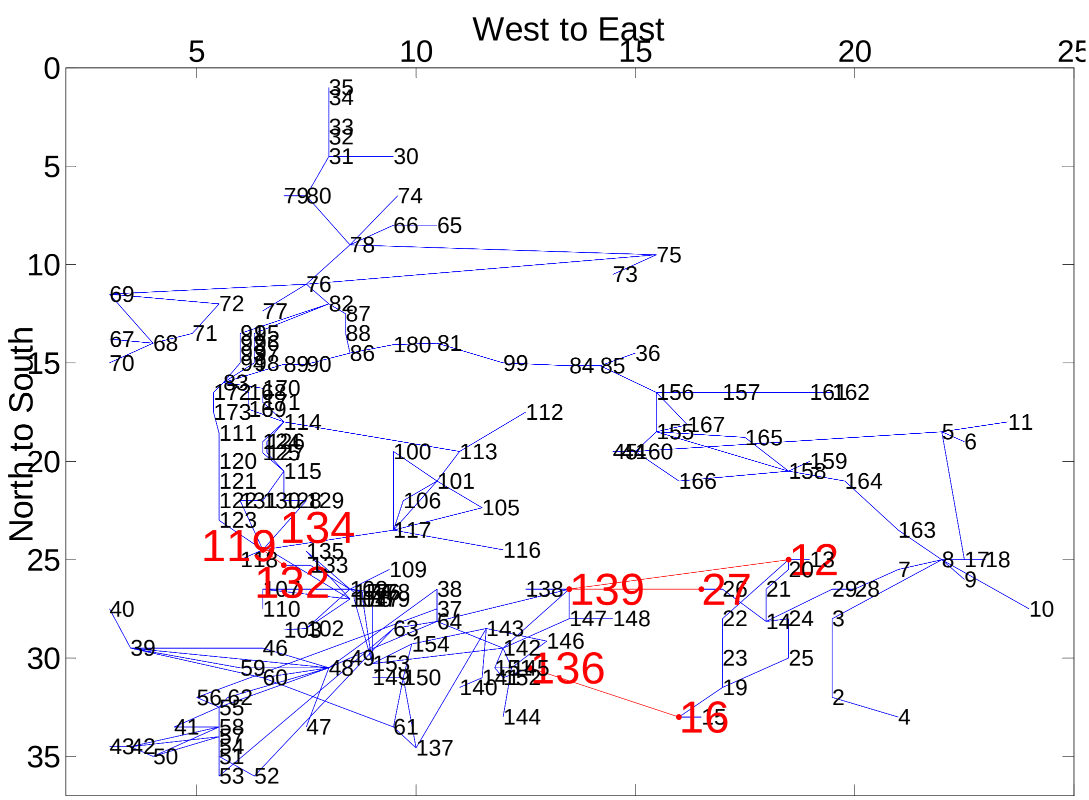


**Figure 2-24 Final minimal cutset to island the system in Scenario II**

## 2.9 Transient Simulation

To verify the advantages of the new islanding approach, it is necessary to conduct time-domain simulation and to investigate the system's transient performance after islanding. Two scenarios for accomplishing this task have been provided by to the grouping results in section 2.8.

**Scenario 1**

As a result of a severe fault, three 500KV transmission lines (83-168, 83-170, 83-172) are tripped at time 0 s, and the path from north to south along the east thus has been disconnected.

Four cases have been studied:

1. No self-healing strategy;
2. At time 0.087 s, form islands by tripping line 132-119, 134-119, and 14-29, without any load-shedding scheme installed;
3. At time 0.087 s, form islands by tripping line 132-119, 134-119, and 14-29 with conventional load-shedding scheme installed; and
4. At time 0.087 s, form islands by tripping line 132-119, 134-119, and 14-29 with the new adaptive load-shedding scheme installed.

In cases 2, 3, and 4, two islands have been formed to prevent the cascading event addressed in section 0.

In Island 1, there are 12 generators: 104, 149, 44, 145, 150, 141, 41, 139, 48, 113, 117 and 119. The total system inertia is 966.66s and total real power generation is 15477.7 MW. Therefore, the value of $M_0$ can be obtained as follows:

$$M_{0,1} = 60 \times \frac{0.3P_{sys}}{2 \times \sum H} = 60 \times \frac{0.3 \times 154.477}{2 \times 966.66} = -1.441(Hz/s)$$

Similarly, in Island 2, there are 3 generators: 112, 116 and 118. The total system inertia is 343.39s and total real power generation is 5118 MW. Therefore, the value of $M_0$ can be obtained as follows

$$M_{0,2} = 60 \times \frac{0.3P_{sys}}{2 \times \sum H} = 60 \times \frac{0.3 \times 51.18}{2 \times 343.39} = -1.341(Hz/s)$$

The new load-shedding scheme is developed as shown in TABLE 2-6. When the fault occurs, the rate of frequency decline at each bus is calculated and compared with the value from (2.27). If the rate of frequency decline at each bus is increased, the new load shedding scheme will be activated, shown as the second row in TABLE 2-6, in which 25 percent of the total load is shed with zero cycle delay in the first step. The character C in the table denotes cycle. Otherwise, the conventional load-shedding scheme will be activated, as shown in the last row. The result from system transient simulation after applying contingency and islanding techniques indicates that the rate of frequency decline in the south island does not exceed the threshold value $M_0$. Therefore, the conventional load-shedding scheme has been applied at each bus in the south island. However, a large load deficit in the central island results in the application of the new load-shedding scheme.

**TABLE 2-6 STEP SIZE OF THE NEW LOAD SHEDDING SCHEME**

| | 59.5Hz | 59.3Hz | 58.8Hz | 58.6Hz | 58.3Hz |
|---|---|---|---|---|---|
| $M_i$>$M_0$ | 25% (0C) | 5% (6C) | 5% (6C) | 4% (12C) | 4% (18C) |
| $M_i$<$M_0$ | | 15% (28C) | 25% (18C) | | |

For the purpose of comparison with our new islanding scheme, islanding based on practical experience has been also studied, such that, after a fault at time 0 s, four lines are tripped to form the islands as follows: 139-12, 139-27, 136-16(dct). Simulation shows that the new islanding using both the conventional and the new load shedding scheme has the advantage of shedding fewer loads than that from islanding based on practical experience. Furthermore, there is less frequency oscillation detected at Generator 118 when new islanding is applied, compared to islanding based on practical experience as shown in Figure 2-25 and Figure 2-26.

**TABLE 2-7** shows that new islanding method with new load-shedding scheme has the advantage of shedding fewer loads when compared with conventional load-shedding scheme, which indicates new load-shedding scheme indeed captures the frequency drop and sheds the loads ahead of the time based on the rate of the change of the frequency decline.

**TABLE 2-7 COMPARISON OF NEW ISLANDING WITH TWO LOAD SHEDDING SCHEME**

| | Generation Load Imbalance (MW) | Inertia (S) | New Islanding with Load Shedding Scheme (MW) | |
|---|---|---|---|---|
| | | | Conventional | New |
| Island 1 | **Generation:** 15477.70 **Load:** 17373.60 | 966.66 | 2220.84 (12.8%) | 2220.84 (12.8%) |
| Island 2 | **Generation:** 5118 **Load:** 7005.9 | 343.39 | 2439.51 (34.8%) | 2081.19 (29.7%) |

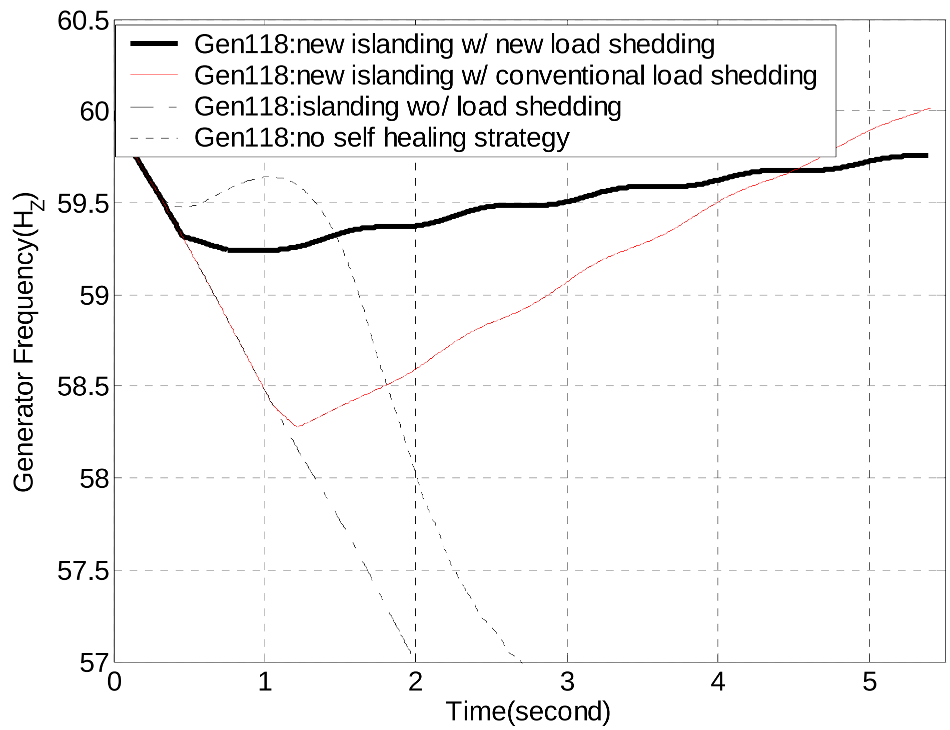


**Figure 2-25 Generator frequency under different scenarios at Generator 118**

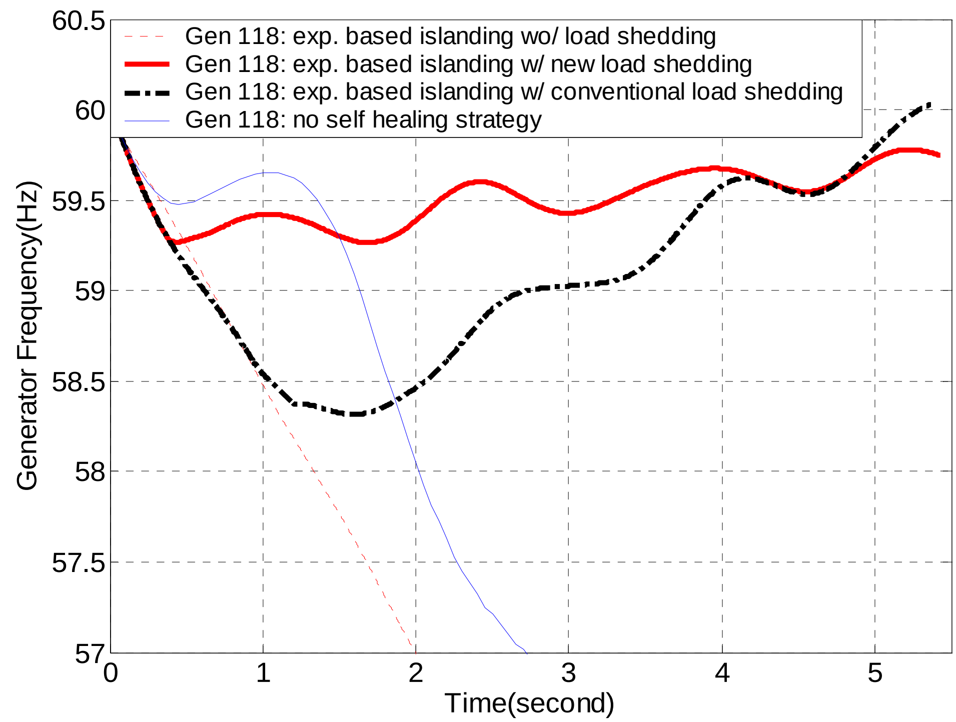


**Figure 2-26 Generator frequency under different scenarios at Generator 118**

**Scenario 2**

In this scenario, three 500KV transmission lines (12-139, 27-139, 16-136 dct) are tripped at time 0 s, and the path from north to south along east thus has been disconnected.

Four scenarios have been studied:

1. No self healing strategy;
2. At time 0.2 s, form the islands but without any load-shedding scheme installed;
3. At time 0.2 s, form the island with the conventional load-shedding scheme installed;
4. At time 0.2 s, form the island with the new adaptive load-shedding scheme installed

The final island in southern California has a total load of 15673.2 MW, and total generation of 11147.7MW. Therefore, the real power imbalance is 4624.23MW. The total inertia is 720.158 s. The value of M0 can be obtained as follows:

$$M_0 = 60 \times \frac{0.3P_{sys}}{2 \times \sum H} = 60 \times \frac{0.3 \times 111.477}{2 \times 720.158} = 1.3932(Hz/s)$$

**TABLE 2-8 STEP SIZE OF THE NEW LOAD-SHEDDING SCHEME**

| | 59.5Hz | 59.3Hz | 58.8Hz | 58.6Hz | 58.3Hz |
|---|---|---|---|---|---|
| $M_i > M_0$ | 24% (0C) | 5% (6C) | 5% (6C) | 4% (12C) | 4% (18C) |
| $M_i < M_0$ | | 5% (28C) | 24% (18C) | | |

**TABLE 2-9 COMPARISON OF NEW ISLANDING WITH TWO LOAD-SHEDDING SCHEME**

| | Generation Load Imbalance (MW) | Inertia (S) | New Islanding with Load Shedding Scheme (MW) | |
|---|---|---|---|---|
| | | | Conventional | New |
| South Island | **Generation:** 11147.7 **Load:** 15673.2 | 720.158 | 5572.22 (35.6%) | 4792.92 (30.6%) |

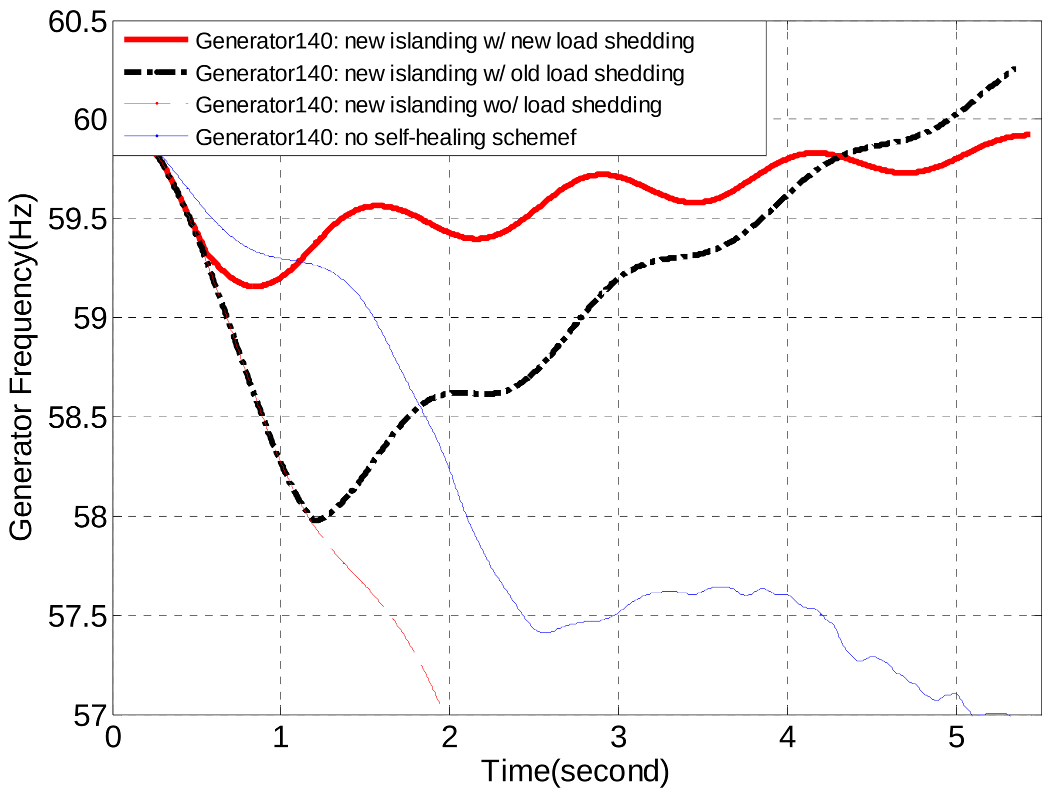


**Figure 2-27 Generator frequency under different scenarios at Generator 140**

## 2.10 Summary

In this section, an automatic power system islanding program has been described, and detailed descriptions of its motivation, functionality, and drawbacks have been addressed. It can be seen that this approach relies heavily on system reduction; since this BFS flag-based DFS searching technique requires significant computational effort for a large-scale system. It is therefore important to reduce the system scale by utilizing system topology information and system dynamic characteristics. To reduce system scale prior to using the minimal cutsets approach, one possible approach would be to eliminate each bus which is neither a generator bus nor a load bus, because those buses will not come into consideration under every situation. The system can thus be reduced to an equivalent system but with lower scale. The minimal-cutset based islanding approach can then be applied to this reduced system, and the optimal cutset can be found. We may then map this optimal cutset back into the original system.

The final result of islanding may be affected by contingencies, especially when a 3-phase short-circuit fault occurs in the system. Sometimes such a fault shorts main transmission lines, which leads in turn to relay or breaker action and system isolation may not be avoidable. Some kinds of faults will also cut off the generator groups originally produced by the slow-coherency method. In either case, more islands will be produced as a result.

PSAPAC/DYNRED can be used to decide generator grouping based on the slow-coherency technique. Without consideration of contingencies, the number of groups may be specified by users. After the islanding program is executed, the same number of islands is generated, with each island including one generator group. However, when contingencies are taken into account, more islands will most likely make the original islanding less optimal. Therefore, the islanding program should be re-run.

The location of the contingency is very important for islanding. It may be located in one island, or at the boundary between two islands, or it may break one island into two.

Another issue requiring attention is the determination of line trips that may cause island separation. It is possible that only a subset of lines tripped may contribute to island separation (whether the contingency cut is minimal). Also, the contingency may influence more than two aggregated islands. For simplicity, the location of contingency is assumed to be limited within one island in our approach.

Various studies indicate that slow coherency may be affected by a change in system topology, which could also be due to the contingency. Slow coherency does not promise consistency if system topology is changed. If the change in system topology happens at a weak connection, however, slow coherency will not be affected. If the topology change occurs at a strong connection, slow coherency will be changed, and grouping may need to be re-run. A more critical question would be: Does the coherency between generators vary with different load conditions? References [21],[22] present a new algorithm to compute the coherency index by using a matrix of eigenvectors corresponding to the small eigenvalues (slow modes) and row vectors associated with the generator rotor angles. Those slow modes change their time constants and contribution to system state with respect to the change of the system operating point. It would not be surprising if the coherency index indeed changes.

# 3. Conclusions and Future Work

## 3.1 Conclusions

Power system islanding is considered as such a rare or improbable event that it may seem that it does not deserve a great deal more attention. However, the result of unintentional islanding on power systems and electricity customers leads individuals and the public to have great concern. It is critically important to bring to the table the question of how to conduct controlled system islanding as a last resort when large disturbances occur in the system, especially under the circumstance of a deregulated power market in which power systems are being operated close to their limits.

In this research, the following technical issues has been discussed regarding automatic power system islanding, taking both system network topology and component dynamic characteristics into consideration. The primary issue is how to find the paths that will propagate cascading events once a large disturbance is initialized in the system.

This includes two important components.

Generator Grouping based on Slow Coherency

The slow-coherency technique has mainly been used to conduct power system network reduction. However, it has also shown great applicability in power system generator grouping to investigate the strong connections among coherent generators and the weak connections among general groups of generators.

Based on slow coherency theory, it is the weak connection between the groups of the generators that will most likely have the greatest impact on the system and propagate cascading events. Two assumptions have been made: 1) the coherent groups of generators are almost independent of the size of the disturbance so that the linearized model can be used to determine coherency; and 2) the coherent groups are independent of the level of detail used in modeling the generating unit so that a classical model may be used to model the generator.

Once the grouping information is obtained, the power system network may be reduced in scale in such a way that generators in the same group can be represented as only one single bus in the reduced network.

Minimal Cutset Based Islanding

A minimal cutset technique which originates from Graph Theory has been applied to search the system to find the boundary of each island. In the literature, most islanding schemes have focused on vertices (buses) other than edges (lines), since it is very straightforward to enumerate all the buses to obtain the imbalance of real power within the islands. However, the transition from vertices to lines makes it possible to obtain the same information by computing only the power flowing through the lines connecting to other islands.

The advantage by doing this is that the number of those lines is limited, one of the requirements of islanding. Therefore, the problem has been simplified into searching the minimal cutsets (MCs) to construct the island with the minimal net flow. We can decompose the islanding problem into two stages: a) find Minimal Cutset candidates; b) obtain the Optimal Minimal Cutset by various criteria. An automatic power system islanding program using minimal cutsets and breadth-first searching (BFS) flag-based depth-first searching (DFS) technique of Graph Theory has been developed to automatically determine where to create the island.

From the optimal-cutset and time-domain simulation for the WECC 29 generator and 179 bus system, it has been shown that the controlled islanding approach with an adaptive load-shedding scheme has the advantage of shedding fewer loads than that from islanding based on practical experience. Furthermore, it has also been shown that with the new islanding scheme, the system experiences less frequency oscillation than with islanding based on practical experience.

## 3.2 Future Work

Future work will focus on how to improve the performance of this approach and apply it to much larger scale systems. There is significant necessary work to be done since academic research work almost always involves small-scale systems with several hundred buses or less. In order to handle large-scale systems, the program needs to be modified such that it can provide a better mechanism to support large data structures and to manipulate large-volume data efficiently.

## 3.3 Contributions

For modern power systems, catastrophic cascading events can cause huge losses to the economy and society. By using the minimal cutest technique, a controlled system islanding program has been developed in this report, based on slow coherency grouping information. The most significant contributions may be summarized as follows:

1. A comprehensive approach: Different from other approaches in the literature, this approach takes both system dynamic characteristics and power system network topology into consideration.
2. Two tier islanding scheme: this approach makes it possible to decompose the islanding scheme into two stages: 1) consider the system dynamics and find out the weak connection among generators; 2) find out the minimal cutset space based on the system topology information and obtain the optimal cutset by computing the net real flow on each cutset. Another advantage is that, in the second stage, we can also apply any additional criteria to formulate the optimization objective function under different conditions, such as the requirements for system restoration, while the first stage remains unchanged.

3. Both slow-coherency theory and the minimal cutsets method have been widely used in different applications. However, this is the first time they have been introduced together to solve the system-islanding problem. This may be of great interest to the power industry after the recent blackout in the United States and Canada [4] because the proposed method provides a completely new strategy for corrective action following large disturbances in power system.

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