An Ontology of Preference-Based Multiobjective Evolutionary Algorithms

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Abstract—User preference integration is of great importance in multiobjective optimization, in particular in many objective optimization. Preferences have long been considered in traditional multicriteria decision making (MCDM) which is based on mathematical programming and recently it is integrated in evolutionary multiobjective optimization (EMO), resulting in focus on preferred parts of the Pareto front instead of the whole Pareto front. The number of publications and results on preference-based multiobjective evolutionary algorithms (PMOEAs) has increased rapidly over the past decade. There already exists a large variety of preference handling methods and EMO methods, which have been combined in various ways. This article proposes to use the Web Ontology Language (OWL) to model and systematize the results developed in this field. An extensive review of the existing work is provided, based on which an ontology is built and instantiated with state of the art results. The OWL ontology is made public and open to future extension. Moreover, the usage of the ontology is exemplified for different use-cases, including training new researchers into this knowledge domain, querying for methods that match an application problem in engineering optimization, checking existence of combinations of preference models and EMO techniques, and discovering opportunities for new research and open research questions.

Index Terms—Preference, Evolutionary Multiobjective Optimization (EMO), Multicriteria decision making (MCDM), OWL Ontology, Prot´eg´e.

I. INTRODUCTION

MOST real-world optimization problems involve multiple objectives (or criteria) to be considered simultaneously. The objectives are usually conflicting, which means improvement of one objective cannot be achieved without deteriorating other objective(s). This kind of problem is regarded as multiobjective optimization problems (MOPs). Unlike single objective optimization resulting in a single optimum, the result of MOPs consists of multiple trade-off solutions called Pareto optimal solutions, and the image in the objective space is referred to as Pareto front (PF).

As [1] indicates, conventional multiple criteria decision making (MCDM) [2, 3] and evolutionary multiobjective optimization (EMO) [4, 5] are two main research fields dealing with MOPs. Here, conventional MCDM refers to methods focusing on mathematical programming, aiming at finding one solution that best fits the preference of a decision maker (DM). In contrast, Multiobjective Evolutionary Algorithms (MOEAs), which solve EMO problems, are population-based algorithms whose purpose is finding a set of solutions that best approximate the whole PF, without consideration of DM preferences. Once the set is generated, it will be presented to the DM for selection of a single solution at the second step.

The two methodologies have both pros and cons as stated in [6]. Much synergy can be gained from the collaboration of MCDM and EMO research areas. MCDM can help shrink the set of alternative solutions obtained by EMO, it is also good remedy for many objective optimization (objectives greater than or equal to four) when selection based on Pareto optimal relation is not sufficient. EMO can assist MCDM with difficult problems which mathematical programming cannot handle, such as nondifferentiable or discontinuous functions optimization, nonconvexity conditions, etc. These motivations are the origin of preference-based multiobjective evolutionary algorithms (PMOEAs) research.

A typical categorization of MCDM depends on when the preference information is asked from the DM: a-priori, a-posteriori or interactively, which refer to the preference obtaining before, after, or during the optimization process respectively. This is also a typical categorization of PMOEAs [7]. It is believed that the first PMOEA was proposed in 1993 [8] by Fonseca et al. The 2004 and 2006 Dagstuhl seminar witnessed an initiation of many results in PMOEAs [9] when researchers in EMO and MCDM fields gathered together to know each other and stimulate cooperation. Since then, large numbers of methods and algorithms were proposed and published. Major EMO conferences now have an MCDM track and vice versa [10], which will also be the case for EMO 2017 in Münster, Germany. Good review papers exist along with the development of PMOEAs [11–15]. Due to the variety of approaches and contexts where preference modelling is combined, to obtain a systematical view of PMOEAs becomes increasingly complex. In addition to the interaction moment with the DM, PMOEAs can also be classified by type of preference information used (implicit preference or explicit preference [16]), by ways
preferences are integrated in (change of objectives or change of MOEAs), by type of the internal preference model, etc. Understanding PMOEA requires background knowledge of both MCDM and EMO fields. Overviews help researchers to figure out what has already been done, what are the relationships between different methods and what can be done in the future. Ontologies are currently the most suited way to formally describe knowledge in a standard way, by means of comprehensive notations and graphical representations, to help understand concepts and relationships in complex knowledge domains [17]. They have been widely investigated in knowledge management, information system integration, semantic web, and electronic commerce, but underexplored for EMO and MCDM fields knowledge representation.

The ontology describes concepts, relationships, classes, individuals, formal logics axioms and objects of a particular domain [18]. It allows for a shared and common understanding of the structure of information among people or software agents, it also enables reuse of domain knowledge. With the help of a PMOEA domain ontology, researchers can easily understand, learn and assess existing methods, discover potential combinations and seek an appropriate method for a specific problem or application. To the best of our knowledge, formal logics-based ontologies have not been used in the PMOEA knowledge domain yet. However, in [19] and [20] OWL ontologies were presented for diversity-oriented optimization and Evolutionary Computation, respectively, revealing the suitability and relevance of this type of knowledge representation for the optimization algorithms research domain.

In this paper we propose a PMOEA ontology built with Protégé [21], a state-of-the-art ontology editor and framework. New contributions of this paper can be summarized as follows:

1. A thorough review of the PMOEA field is given and the main characteristics of each algorithm are analyzed.
2. An OWL ontology of PMOEsas is built with Protégé and made public for the research community to comment, annotate, (re)use, extend and complement.
3. Use cases of the PMOEA ontology are provided for e-learning support, research results assessment, knowledge base querying and optimization problem matching.
4. The PMOEA Ontology provides an overview and new perspective on this scientific area. Other ontologies can be built in related scientific areas that need a systematical view and shared conceptualization, stimulated by this initial step on PMOEA. The strength of this approach relies on its generalization and knowledge inference ability, its formal logics basis and ontology automatic processing by computational means, which is essential for knowledge management of large scale and complex knowledge domains.

The rest of this paper is organized as follows. In section II a review of state-of-the-art results in the PMOEA field is given as well as the scientific background of the semantic web and ontology design areas. In section III the method and procedure of building the PMOEA ontology is described in detail. Use cases of the ontology are exemplified in section IV Conclusions are drawn in section V.

II. PRELIMINARIES AND RELATED WORK

A. MOEAs

A minimization MOP is defined as follows [4] without loss of generality:

\[
\begin{align*}
\min f(x) &= [f_1(x), f_2(x), \ldots, f_M(x)]^T \\
g_j(x) &\geq 0 \quad j = 1, 2, \ldots, P \\
h_k(x) &= 0 \quad k = 1, 2, \ldots, Q \\
x_i^L &\leq x_i \leq x_i^U \quad i = 1, 2, \ldots, n
\end{align*}
\]

in which solution \(x\) is an \(n\)-dimensional decision vector \(x = (x_1, x_2, \ldots, x_n) \in \mathbb{R}^n\). Each variable \(x_i\) is bounded between lower bound \(x_i^L\) and upper bound \(x_i^U\). \(f_i\) is the \(i\)-th objective function and there are \(M\) objectives in total. \(P\) and \(Q\) specify the numbers of inequality and equality constraints, respectively. Alternatively, one might also consider combinatorial MOP in which \(\mathbb{R}^n\) is replaced by \(\mathbb{Z}^n\) or some other discrete search space.

The resolution of a MOP is a set of trade-off solutions, called non-dominated solutions or Pareto optimal solutions. The related definitions are as follows [2].

A vector \(u = (u_1, \ldots, u_M)^T\) is said to dominate another vector \(v = (v_1, \ldots, v_M)^T\), denoted as \(u \prec v\), if \(\forall i \in \{1, \ldots, M\}\), \(u_i \leq v_i\) and \(u \neq v\).

A feasible solution \(x^* \in \Omega\), where \(\Omega\) is the decision space of problem (1), is called a Pareto optimal solution, iff \(\exists y \in \Omega\) such that \(F(y) \prec F(x^*)\). The set of all Pareto optimal solutions is called Pareto set (PS) and the image of the PS in the objective space is called the Pareto front (PF).

Over the last 20 years a variety of MOEAs have been proposed and demonstrated a great success in approximating the PF. There are several survey papers on MOEAs from different aspects [22, 23]. Here we focus on classification of MOEA and for each category, we identify the characteristic elements of the algorithms that can be changed to embrace preferences.

1) Diversity VS Convergence based: Maintaining diverse solutions and getting close to the PF are the two main goals of MOEAs. Traditional MOEAs were designed to improve both convergence and diversity, such as the famous Non-dominated Sorting Genetic Algorithm-II (NSGA-II) [24]. Strength Pareto Evolutionary Algorithm 2 (SPEA2) [25], Pareto Archive Evolutionary Strategy (PAES) [26], Pareto Enveloped-based Selection Algorithm-II (PESA-II) [27], etc.

Fitness assignment and diversity methods are the key elements of algorithms in this category, where preference information can be considered and integrated. For example, Pareto front sorting and crowding distance calculation in NSGA-II, fitness assignment and clustering method in SPEA2, grid size in PAES and hyperbox size in PESA-II.

2) Indicator based: Set quality indicator is an important metric to evaluate MOEAs. Indicator-based MOEAs use an indicator to guide the search, resulting in a set of solutions that maximize the indicator. Algorithms in this category include IBEA [28], SMS-EMOA [29], HypE [30] and R2-EMOA [31]. Since quality indicator and its computation are essential to this category of algorithms, characteristic elements specific for each indicator can be used to embed preferences of the
DM. For instance, reference point selection and Hypervolume calculation can be modified to incorporate DM’s preferences in IBEA, SMS-EMOA and HypE. Calculation of R2 indicator can be adjusted to take into account DM preferences in R2-EMOA.

3) Decomposition-based: The decomposition of a multiobjective problem is based on a conventional method that transforms a multiobjective problem into several single objective optimization problems. MOEA/D [32] and NSGA-III [33] are two representative algorithms in this category. MOEA/D and its variants decompose the problem into multiple aggregation functions, while NSGA-III decomposes the objective space into subregions without using any aggregation functions.

Weight vectors and reference points are core elements for MOEA/D and NSGA-III respectively. Preference information can be incorporated via weights and reference points.

4) PSO-based: Particle Swarm Optimization (PSO) is a population-based stochastic optimization approach inspired by the behavior of bird flocking. Valuable research has been produced in PSO-based MOEAs during the last years, being MOPSO [34] among the most representative contributions in this area. The key idea in PSO-based MOEAs is how to select the global best and local best particles, which are also the characteristic elements to incorporate DM’s preferences.

5) Memetic-based: Memetic MOEAs are a kind of hybrid MOEAs that combine global search with local search. After the traditional variation operators, usually a local search method is applied, such as in Pareto memetic algorithm (PMA) [35]. A weighted scalarization function is defined by a weight vector and a reference point. At first, two solutions being good on the current scalarization function are selected probabilistically and recombined to get offsprings, then a local heuristic is applied to improve the quality of the offspring solutions. Weight vector and reference point are the key elements to import preferences in memetic algorithms.

6) Probabilistic-based: Probabilistic-based MOEAs build a probability distribution model of promising solutions by extracting global statistical information from previous search. New solutions are generated based on the model instead of classical crossover or variation. Representative algorithms in this category include crowding population-based Ant Colony Optimization (ACO) [36], multiobjective quantum-inspired genetic algorithm (QGA) [37], and Voronoi-based estimation of distribution algorithm (EDA) [38]. Preference information can be combined with pheromone matrices in ACO-based algorithms, Q-bit individual in QGA-based algorithms and variable linkage information in EDA-based algorithms.

7) Coevolution-based: Coevolution is an approach of evolving multiple subpopulations simultaneously to tackle a complicated problem. It can also be interpreted as decomposing a problem into a set of subproblems and evolving a subpopulation for each of them. Components from different subpopulations are combined to form a complete solution. Preference Inspired Coevolutionary Algorithm (PICEA) [39] proposed a new method to co-evolve a population of solutions and a population of preference vectors, which is so far the only preference integration in coevolutionary algorithms.

Apart from the above elements according to the specialty of each algorithm, there are also common elements that can be modified to incorporate preferences. For example, changing dominance relation is a most widely used method in PMOEA, regardless of MOEA classification. Initialization, fitness evaluation and termination criterion have also been used in PMOEA to combine preference information.

B. PMOEAs

The ultimate goal of multiobjective optimization is to help the DM choose the most preferred solution. Hence, the integration of MCDM approaches, which facilitate such choice, becomes indispensable. PMOEAs aim at finding the preferred part of the PF according to the preference information given by the DM. Embracement of the DM’s preferences into MOEAs have attracted wide attention due to the following advantages:

1) It is difficult for EMO to handle many objective problems (when there are more than three objectives). Since almost all of the solutions tend to be non-dominated, selection pressure will be lost when many solutions belong to the PF. Having the preference information of the DM as a selection criterion is a good way to address this problem [40–42].

2) Inspecting and choosing solutions from the whole PF is not a trivial task for the DM. The visualization of high-dimensional space further aggravates the difficulty. If the DM gives some (even vague) information about his/her preferences, then preferred parts of the PF will be emphasized, relieving the selection burden of the DM.

3) Taking into account preference information, MOEAs can only focus on searching regions favored by the DM, so that solutions the DM prefers will be obtained more quickly. In other words, computational efforts for finding unwanted solutions will be avoided.

Due to these advantages, a large number of publications have focused on integrating preferences in EMO. There are several criteria that can be used to classify PMOEAs, such as interaction moment in [7] and preference information in [14]. Here, we adopt the taxonomy of [14] with slight changes. The categories are Reference point-based approaches, Reference direction-based approaches, Preference region-based approaches, Trade-off-based approaches, Objective comparison-based approaches, Solution comparison-based approaches, Outranking-based approaches, Knee point-based approaches.

1) Reference point-based approaches: Compared to other articulation of preferences, reference point first proposed by Wierzbicki [44] in MCDM, is now the most widely used method in PMOEAs due to its natural and intuitive meaning. In general, a reference point is a user-defined point in objective space that represents the DM’s aspiration level for each objective. The optimization search can be stopped once a point that dominates or equal to the reference point is found. When the reference point is infeasible, the closer a solution is to the reference point, the more it is preferred.

1It is also called aspiration level or goal vector, or preference point in [43]. Not to be confused with reference point in Hypervolume
a) Pareto Dominance: The earliest PMOEA was regarded to be proposed by Fonseca and Fleming in multiobjective genetic algorithm (MOGA) [8], where a goal vector (or reference point) was used to articulate preferences. The main idea is to define a new dominance relation that gives higher priority to objectives that do not satisfy the predefined goals. This method was extended by Tan et al. [45], who proposed a new goal-sequence dominance relation incorporating goal and hard/soft priority information. It is also capable of dealing with multiple reference points by introducing logical "OR" and "AND" operations.

Utilizing the reference point to change dominance relation was also adopted in g-dominance [46] and r-dominance [47]. In g-dominance, solutions that satisfy all the aspiration levels are preferred over solutions that satisfy some of the aspiration levels. In r-dominance, solution \( x \) is referred to r-dominate solution \( y \) when \( x \) Pareto dominates \( y \), or the weighted Euclidean distance between \( x \) and the reference point is smaller than the distance between \( y \) and the reference point to a predefined extent. g-dominance can be combined with any metaheuristic algorithm, so can r-dominance because they only change the dominance relation. However, g-dominance does not comply with the Pareto dominance, which means a dominated solution may be preferred to the solution that dominates it. Fortunately, r-dominance preserves the Pareto dominance. Later, r-NSGA-II was developed in the context of group decision making, in which a Weighted Negotiation Support System for Group Preference Aggregation (W-NSS-GPA) was proposed [48]. The reference point of each DM should be provided as input of the system and the DMs’ importances are modelled as weights. It yields a single Social Reference Point (SRP) as output which is considered as an aggregation of all the DMs’ preferences.

Another dominance relation changed by reference point is the Chebyshev preference relation [49] [50]. A Region Of Interest (ROI) is defined with reference point and a threshold, solutions in this ROI are compared by the classical Pareto dominance relation, while solutions outside of the ROI are compared by their Chebyshev achievement function value [2].

b) Crowding Distance: Apart from dominance relation, reference point can also be used to change crowding distance, as it is done in R-NSGA-II [51]. Here, the original crowding distance is replaced by a preference distance, which prefers points close to the reference point concerning Euclidean distance. R-NSGA-II was later extended by Siegmund et al. [52] to speed up the search under a limited number of evaluations by changing selection and diversity mechanisms, and was also extended by Filatovas et al. [53] to consider several scalarizing functions simultaneously. In [54], preference was articulated by a positive reference point and a negative reference point, they are also integrated in crowding distance to form a new metric called similarity. The aim is to find Pareto optimal solutions that are close to the positive reference point and far away from the negative reference point.

c) Achievement Scalarizing Function: Another reference point-based PMOEA is weighting achievement scalarizing function genetic algorithm (WASF-GA) [55]. It applies an achievement scalarizing function (ASF) and classifies the individuals into several fronts based on ASF values. The main purpose of this method is to generate a well-distributed set of non-dominated solutions approximating the ROI defined by a reference point. The authors also published the interactive version of WASF-GA [56]. WASF-GA is similar to Reference Direction Based NSGA-II (RD-NSGA-II) [57] with regard to ASF and classification of the individuals. The difference between those two PMOEAs is that WASF-GA employs a set of weight vectors in the ASF and fixes the reference point, while RD-NSGA-II fixes the reference direction (or weight vector) and applies a set of reference points along the direction.

d) Indicator: Reference points have also been applied to indicator-based MOEAs. In PBEA [58], \( \varepsilon \)-indicator has been modified to incorporate ASF values, it can be used in both ways: a-priori and interactively. The method was extended in Parallel Multiple Reference Point Evolutionary Algorithm (PMRPEA) [59] to explore areas guided by multiple reference points. PMRPEA’s main idea is to solve different PBEAs in parallel and focus on multiobjective optimization problems of discrete and combinatorial nature. Another method considering a set of reference points is Aspiration Set EMOA [60], which aims at approximating multiple reference points in a single run using averaged Hausdorff distance as quality indicator.

e) Others: Reference point can also be integrated in the constraints, such as the goal-constraint approach proposed in [61]. Swarm-based MOEAs is another category that incorporates reference point, such as Reference point-based PSO using a Steady-State approach (RPSO-SS) [62]. It proposes to use distance to reference points as criterion in selection strategy, for the sake of finding a set of solutions near the reference points provided by the DM. Wickramasinghe et al. use Multiobjective Differential Evolution and PSO (MDEPSO) algorithm to hybridize two MCDM methods: reference point and light beam search [63]. A distance metric is used to measure the closeness of each particle to the reference points. Particles will update their velocities and positions based to this metric and move towards the reference points.

f) Performance Measure: How to compare different PMOEAs is also an important issue. Traditional EMO metrics are not suitable because they measure convergence and diversity with respect to the whole PF, without consideration of the preference information. Some contributions have been made in this topic. A new performance metric for reference point-based MOEAs was proposed in [64], where a reference set (or composite front) and a preferred region were defined based on reference point offered by the DM. Existing EMO performance metrics can be applied according to this preferred region. A testing framework to compare different reference point-based interactive methods was introduced in [65]. An artificial DM consisting of the steady part and the current context was constructed to mimic actions of a human DM, which makes comparison of interactive methods in a controlled environment possible.

2) Reference direction-based approaches: Reference direction method [66] and light beam search [67], which can be considered as extensions of reference point method, have also been investigated in the PMOEA field. Light Beam Search-
based EMO \[68\] and Reference Direction-Based NSGA-II (RD-NSGA-II) \[57\] are the representatives of reference direction-based approaches. Light beam search has also been considered in Multiobjective Differential Evolution and PSO (MDEPSO) in \[63\], as mentioned in the previous section.

Branke and Deb proposed a biased crowding distance approach \[69\] to find a biased distribution on the PF. A direction vector should be provided by the DM to define an iso-utility function, the crowding distance in NSGA-II is modified to focus solutions parallel to this iso-utility function. This approach was regarded as an objective scaling method in \[13\].

3) Preference region-based approaches: Instead of using a single point to represent preferences, a preference region in the objective space favored by the DM is also popular. Usually, it is difficult for the DM to give a precise reference point. Hence, a vague range of candidate values for such point could be used as a reasonable alternative. Moreover, functions that reflect the objective values and DM’s degree of satisfaction are included in this category, two representatives are Desirability Function and density function in the objective space\[1\] as discussed next.

Desirability functions (DFs) \[70\], \[71\] are widely investigated in the current PMOEA field for its simple and intuitive meaning. DFs nonlinearly transform the objective values in a desired region into the desirability domain \([0, 1]\). 0 stands for unacceptable and 1 represents fully satisfied. Improving objective quality is reflected by an increasing of desirability. By changing the values of objectives corresponding to 0 and 1, the DF can focus the search on different regions of the PF. DFs have already been successfully introduced in combination with NSGA-II \[72\], MOPSO \[73\] and SMS-EMOA \[74\] on both benchmark problems and practical problems. In order to alleviate the computational burden of SMS-EMOA, Trautmann et al. proposed to use the desirability index (DI), which is a DF-based scalarization, as the second-level selection criterion in the non-dominated sorting \[75\].

Another popular method to integrate preference is a density function in the objective space. In \[43\], \[76\], \[77\], weighted hypervolume is used to guide the search towards ROI by utilizing a variety of density functions in the objective space, including stressing one objective, one reference point, a preference region. The preference-specific hypervolume indicator is introduced into the set-based evolutionary algorithm called W-hyP-E, which is based on Monte Carlo sampling and capable of handling problems with an arbitrary number of objectives. This kind of preference can also be integrated with NSGA-II and SPEA2, as demonstrated in \[78\]. It can yield similar results compared to the hypervolume approach and requires less computational effort. The above two methods (Desirability and density function) can be integrated. Emmerich et al. use Desirability Function to define a density in the objective space and provide a probabilistic interpretation for it \[79\].

Karahan and Köksal devised a territory defining steady-state elitist evolutionary algorithm (TDEA) \[80\] which defines a territory around each individual solution to prevent crowding. They also proposed a preference-based approach, called pTDEA, to assign different sizes of territories for preferred regions and non-preferred regions. Preferred regions have smaller territories so that a denser coverage can be achieved around them. An interactive version of this method has also been proposed in \[81\].

In \[82\], an interactive decision making approach is embedded in preference-inspired co-evolutionary algorithm (PICEA-g). The DM can only brush his/her preferred region in the objective space without specifying any parameters. Goal vectors are generated according to this region and co-evolved with solution vectors, in order to achieve solutions in the ROI brushed by the DM.

A hyperplane in the objective space is constructed to articulate preference in \[83\]. The DM can visually specify his/her preference by center and spread vectors of Gaussian functions on the hyperplane. This preference model is embedded into the framework of NSGA-II and applies to many-objective knapsack problems \[84\].

Recently, Yang et al. proposed a method for effectively approximating a preferred part of PF based on multiobjective efficient global optimization (EGO) \[85\]. It uses truncated expected hypervolume improvement (TEHVI) as infill criterion, making it possible to set a ROI on the Pareto front and focus the search effectively on this preferred region. In this method for the first time preferences are integrated in surrogate-assisted optimization.

4) Trade-off-based approaches: Trade-offs have been widely investigated in MCDM literature. According to Miettinen \[86\], trade-offs can be objective or subjective. On the one hand, a trade-off that depends on the structure of the problem (i.e. the change in one objective in respect to change of another one, when moving from a feasible solution to another), is objective trade-off. On the other hand, a trade-off that represents how much the DM is ready to sacrifice in the value of some objectives in order to improve another objective(s). This is so-called subjective trade-off. PMOEA\(
eds deal with subjective trade-offs.

In the guided MOEA proposed by Branke et al. \[87\], subjective trade-offs are given by the DM in the form of statement: “one unit improvement in objective i is worth at most a\(_{ij}\) units degradation in objective j”. The basic idea of this approach is to modify the dominance relation: a solution \(x\) is preferred to a non-dominated solution \(y\) if it does not violate the specified subjective trade-offs, or maximally acceptable trade-offs.

Apart from dominance relation, trade-offs can also be employed to sort additional front in the best non-dominated set, just as pNSGA-II \[88\] did. In this method a set \(F_0\) is found which satisfies the acceptable trade-offs from the current best front \(F_1\). Solutions in this front are assigned with rank 0, which is better than the rank of \(F_1\). The remaining steps of pNSGA-II are the same as of NSGA-II \[24\].

Trade-offs can also be integrated with set quality indicators. A concept of cone-based hypervolume indicators (CHI) is proposed and theoretically investigated in \[89\], \[90\]. The idea is to use a family of polyhedral cones with scalable opening angle \(\gamma\) to express preferences in the sense of trade-

\[1\] We call it density function here, although originally it is referred to as weight function. As a weight we would rather consider an integral over the density function.
off constraints. Furthermore, the authors present two searching algorithms to obtain solutions that are compatible with the given preference model, i.e., to find a subset of at most \( \mu \) points that maximize the CHI.

5) Objective comparison-based approaches: Objective comparison is statements of preference on objectives, such as “prefer \( f_1 \) to \( f_2 \)” or classify objectives qualitatively as “most important, important, less important”, or describe their importance quantitatively as weights. Many researches use weights at early stages, such as in [91] and [92]. Since it is difficult for the DM to specify weights accurately, vague values, or fuzzy preference became popular. Jin and Sendhoff [93] transformed fuzzy preferences on objectives into weight intervals, which is similar to the idea of Cvetkovic and Parmee in [94]. The difference is that a dynamic weighted aggregation approach was adopted in [93], which transformed the multiobjective optimization problem into a single objective optimization with imprecise weights, or changing weights dynamically during the optimization process.

Rachmawati and Srinivasan introduced relative importance of objectives, including strict preference, equality of importance, and incomparability between pair of objectives [95]. An elicitation algorithm was also proposed to assist a human DM to construct a coherent overall preference model, which is then combined with NSGA-II to obtain a subset of PF.

Brockhoff et al. utilized two preference articulation approaches in the interactive W-HypE [96]. One is to let the DM choose the most preferred solution from a set of alternative solutions (this belongs to “Solution comparison-based approaches”, see the next category), the other approach uses comparative preference statements, such as “prefer \( f_1 \) < 0.5 to \( f_2 \) < 0.3”. This preference information is then transformed to parameters in the objective space density function used by W-HypE [77].

A preference-based solution selection method has been proposed by consideration of fuzzy measure and fuzzy integral. Preference information is provided by an objective pairwise comparison matrix and has been combined with multiobjective particle swarm optimization [97] and multiobjective quantum-inspired evolutionary algorithm [41]. Application of the method on path following footstep optimization for humanoid robots is also investigated in [98].

6) Solution comparison-based approaches: Solution comparison is often used in interactive PMOEAs where the DM does pairwise comparison, rank or grade a sample of representative solutions, or choose the best (and worst) solution(s) in a sample set. The earliest interactive PMOEA may be the method proposed by Phelps and Köksalan [99]. They assumed a linear value function and used the DM’s pairwise comparison to determine the most discriminating function compatible with the preferences. The method is tested on multiobjective combinatorial optimization problems. Similarly, Fowler et al. adopted a more general quasi-concave utility function to represent the DM preferences [100]. The best and worst solutions in the sample set are selected by the DM and used to create convex preference cones in the objective space, which are utilized to identify inferior solutions. A similar interactive genetic algorithm and experimental framework to [99] were adopted here.

The Necessary-preference-enhanced Evolutionary Multiobjective Optimizer (NEMO-I) [101] presented by Branke et al. is a combination of interactive evolutionary multiobjective optimization with robust ordinal regression (ROR). At regular interactions, the DM is asked to compare some pairs of solutions in the current population. The whole set of additive value functions compatible with this preference information is utilized and integrated in a modified version of NSGA-II, in order to find solutions satisfying the preferences. To reduce the computational complexity, which is a main shortcoming of NEMO-I, the authors proposed NEMO-II [102], which compares each solution to all other solutions as a set, instead of pairwise comparison. While NEMO-I and NEMO-II both consider whole sets of value functions compatible with the preference information, the authors also presented NEMO-0 [103] using only the most representative value function. The DM is also asked to do pairwise comparison and the internal value function, which is used in subsequent generations to rank solutions incomparable based on dominance relation, will update with this information.

Dominance-based Rough Set Approach (DRSA) is an MCDM approach based on “if...,then...” rules deduction [104]. It has been integrated in interactive EMO in [105], where two schemes were proposed to collect user preference obtained through interaction with the DM. Then, a set of decision rules is induced and used for ranking solutions in the current population. One scheme is to make the DM sort some solutions into “relatively good” and “others”, the other scheme requires pairwise comparison among representative solutions by the DM.

Deb et al. suggested a progressively interactive EMO using value functions (PI-EMO-VF) [106]. The authors derived a most discriminating polynomial value function from DM’s ranking of sample solutions during the interaction phase (or DM call, which is the terminology in the literature). Once a most discriminating value function has been identified, it is combined in a new domination principle as well as a preference-based termination criterion. Sinha et al. proposed another progressively interactive EMO algorithm based on polyhedral cone (PI-EMO-PC) [107]. This polyhedral cone is constructed according to DM’s best selection in a set of alternative solutions, and it is used to eliminate a part of the search space for a more focused search. The domination principle is modified, so is the termination criterion. Sinha et al. also investigated PI-EMO-VF and PI-EMO-PC under limited number of DM calls [108]. The construction and application of the preference polyhedral cone were extended for interval MOPs in [109].

Battiti and Passerini proposed to use support vector machine (SVM) to learn an arbitrary value function in the Brain-Computer EMOA [110]. In the interaction phase, the DM is presented with a set of sample solutions and asked to rank them (at least partially). The rank result is then used to train the SVM to get an approximate value function, which replaces the crowding distance in NSGA-II by their learned value function values.

Cruz-Reyes et al. introduced the Hybrid-MultiCriteria Sort-
ing Genetic Algorithm (H-MCSGA) [111], in which the selective pressure based on dominance is strengthened by assigning solutions into ordered categories. A reference set of solutions belonging to different categories is kept and updated to capture the preference. The goal of this method is to find non-dominated solutions belonging to the best category.

Solution comparison can also be incorporated with other MOEAs, such as Pareto memetic algorithm [112], MOEA/D [113] and territory defining evolutionary algorithm [114] (where Neural Network is utilized to learn the preferences).

7) Outranking-based approaches: An outranking relation is a binary relation $S$ defined on the set of potential solutions (also called actions) $A$ such that $a S b$ if there are enough arguments to decide that $a$ is at least as good as $b$, whereas there is no essential argument to refuse that statement [115]. Outranking relation is widely investigated in the MCDM field.

Fernandez et al. proposed a Non-outranking Sorting Genetic Algorithm (NOSGA) [116] to consider a binary fuzzy preference relation that expresses the degree of truth of predicate " $x$ is at least as good as $y$ ". Pareto dominance is replaced by outranking relation and it searches for the non-strictly outranked frontier which is a subset of the PF. The method is extended to increase the selective pressure towards the best compromise solution later in NOSGA-II [117].

8) Knee point-based approaches: Knee points of PF are characterized by the fact that a small improvement in one objective will arouse a large deterioration in other objectives. It is similar to trade-off-based approaches, but in trade-off-based approaches the trade-off constraint is explicitly provided by the DM, while in knee point-based approaches, no explicit preference is given, knee point (point with the maximal trade-off) is regarded as most preferred by the DM.

Bechikh et al. proposed KR-NSGA-II [118] based on R-NSGA-II [51], in which mobile reference points are used to modify crowding distance in the traditional NSGA-II. In each generation, knee points in the current population are found and serve as reference points, the method can be used both a-priori and interactively. TKR-NSGA-II [119] is an enhanced version of KR-NSGA-II [118], where the approach to find knee points has been improved. A similar idea has been used in knee point-driven MOEA [42], where knee points of the nondominated fronts in the current population are used as a secondary selection criterion after the dominance relation. The authors also show that a preference over knee points can be considered as an approximation of the preference over a larger hypervolume. It should be noticed that SMS-EMOA [29] automatically focuses on knee points in that solutions are denser around knee points which is due to SMS-EMOA internally maximizing hypervolume.

C. Ontology

Ontologies are content theories (i.e. theories that explain the specific factors that motivate behavior) about the sorts of objects, properties of objects, and relations between objects that are possible in a specified domain of knowledge [120]. First addressed by the Knowledge Acquisition Community and aiming at "knowledge modeling", formal logics ontology design and engineering is now an important research and application topic in many domains, such as the Semantic Web [121], knowledge management, e-Learning, e-Commerce, etc. The most widely accepted definition of ontology in this context is given by Gruber [122]: “An ontology is a formal explicit specification of a shared conceptualization for a domain of interest.” It is formal logics-based, allowing for mathematical treatment and computational processing. It has explicit specification, because concepts, properties, functions and axioms are explicitly defined. It is shared with standard notation, aiming at the representation of consensual knowledge. In essence, it is conceptualization, or abstract model of some phenomena in the world.

There have been many contributions in the last years concerning so called knowledge technologies in general and knowledge technology languages in particular. In February 2004, the World Wide Web Consortium (W3C) announced the final approval of two key Semantic Web technologies. They are the Resource Description Framework (RDF) [123] and the Web Ontology Language (OWL) [124]. Both standards make use of the eXtensible Markup Language (XML) for the definition of text-based documents syntax/structure, by means of tags that can be added to parts of the text documents, promoting interoperability between applications that exchange machine comprehensible information.

The role of OWL is to set a common agreed vocabulary to describe a subject domain. It may be categorized into three sub-languages, i.e. OWL-Lite, OWL-DL and OWL-Full. OWL-Lite is the least expressive, the syntactically simplest sub-language, intended to situations where only a simple class hierarchy and simple constraints are needed. OWL-DL (Description Logics), is much more expressive than OWL-Lite and is based on Description Logics (DL), a decidable fragment of First Order Logic, which makes automated reasoning possible. It can compute the classification hierarchy automatically and check for inconsistencies in an ontology. OWL-Full is the most expressive, intended to situations of high expressiveness but it is incapable of automated reasoning.

One of the key features of OWL-DL ontologies is that they can be processed by a reasoner. Several reasoners are freely available for the research community to use (e.g. Pellet, FaCT++, HermiT), which are also integrated into other widely adopted ontology design and engineering tools (e.g. Protégé ontology editor). An OWL-DL ontology was built for the PMOEA knowledge domain and will be presented in the next sections of this paper.

The current amount and diversity of knowledge sources, representations and vocabularies in most of the research domains, motivated the appearance of more systematic knowledge management processes, which include knowledge identification, capturing, structuring, preservation, dissemination, usage and assessment. Simultaneously, software tools to provide support for these activities have been developed in the last years and are widely used by the research community. Protégé is a free, open-source platform, which serves a growing number of uses with a suite of tools to construct domain models and knowledge-based applications with ontologies [21]. It has WebProtégé and Protégé Desktop and it supports the (OWL 2)
Web Ontology Language. It was developed and maintained by Stanford Center for Biomedical Informatics Research (BMIR) and now has more than 300 thousand registered users. People from different background can publish and import OWL ontologies for research freely.

According to Noy [125], the reasons to develop an ontology are given below:

1. To share common understanding of the structure of information among people or software agents. It is of high value and interest in a knowledge domain to share and use the same underlying ontology. Additionally, computer agents can extract and aggregate information from different sources, answer more complex and complete user (or other computer systems) queries, as well as providing or using as input each others knowledge basis. As PMOEA connects MCDM and EMO communities, there are plenty of relevant concepts and relations between concepts. Hence, ontology can help to define a machine-interpretable vocabulary in this domain, based on which reasoning and further analysis can be done.

2. To enable reuse of domain knowledge. This is one of the main benefits of developing an ontology. If one group of researchers develops an ontology, others can reuse, compose and extend it for their domains. For example, PMOEA domain includes concepts such as preference information (including reference points, reference direction, etc.), multiobjective optimization problem and MOEAs. Preference information can be reused in MCDM community and detailed concepts in MOEAs can also be of interest in (single objective) evolutionary algorithms contexts.

3. To make domain assumptions explicit. Explicit specifications of domain knowledge are useful for new users who must learn the concepts and relations in the domain. In the scientific literature there are often ambiguities of how a certain term is interpreted. For instance in PMOEA one might debate whether a swarm-based or immune-based algorithm is still a MOEA. An ontology does not resolve the question, but could help to make a viewpoint explicit and thereby avoid confusion.

4. To separate domain knowledge from the operational knowledge. In some sense developing an ontology is like defining a set of data and their structure for other users or softwares to use. Different types of users, domain-independent applications and software agents use ontologies as input data. For instance the PMOEA ontology might serve a automatic text-mining or literature retrieval system to understand the meaning of certain concepts or words (e.g., to identify synonyms or subsumption of keywords). The ontology itself is not doing the operation of searching, but it provides important information for it.

5. To analyze domain knowledge. Usually a domain ontology is not an ultimate goal in itself. More useful information can be gained by analyzing the ontology. By building the PMOEA ontology we can easily analyze what kind of preference has been integrated in what kind of MOEA, through what kind of integration. We can also query for methods that can deal with a specific kind of problem, or find potential combination of MCDM and EMO for future reasearch. One possible use is to extract building blocks which can be used as operators within different kinds of MOEAs and design new PMOEAs.

There are various ontology applications in different fields. For example, ontologies are used in Knowledge Management [126] to support knowledge visualization, search, retrieval and personalization, serve as the basis for information gathering and integration. Ontologies have also been used in Recommender Systems, e-Learning, e-Commerce, Semantic Interoperability, Bioinformatics and so on.

Since PMOEA is a research area built on intersection between EMO and MCDM, there is a requirement to build the PMOEA ontology for researchers in both fields. As far as we know, such ontology has not been reported so far, but Evolutionary Computation (EC) ontology [20] and diversity-oriented optimization ontology [19] have appeared recently, which can help to build the PMOEA ontology.

III. BUILDING THE PMOEA ONTOLOGY

We build our PMOEA ontology with Protégé Desktop. An OWL ontology consists of classes, properties and individuals. A class describes a group of concepts with the same properties in the domain. Classes may also be described by the necessary and sufficient conditions an individual must verify to belong to that class. For example, PMOEA is the class of preference-based multiobjective evolutionary algorithms. A class can have subclasses that represent concepts more specific than the superclass. The hierarchy of classes, organized in a tree structure that relates classes by isa relation, defines the taxonomy adopted in the ontology. For example, MOP has Academic_Problem and Realworld_Problem as subclasses. Properties include object properties and data properties. An object property is a binary relation to relate classes or individuals. For instance, canSolve is an object property that can relate PMOEA and MOP, which indicates the capability of one specific PMOEA with regard to solving one specific problem. A data property relates classes or individuals with a designed primitive data-type (e.g. integer, boolean). For example, hasDevelopingYear is a data property of PMOEA with datatype "integer". Individuals represent objects (class instances) in the domain of interest, for instance R-NSGA-II [51] is an individual of PMOEA.

As suggested by Noy [125], there is no one "correct" way or methodology for developing ontologies. However, the guidelines provided in [125] represent a generalized set of steps, widely accepted in this area as best practices for ontology design and engineering, which are presented in detail next and are followed in our work.

A. Determine the domain and scope of the ontology

The ontology we built addresses preference-based multiobjective evolutionary algorithms, or PMOEAs. Generally speaking, it is a combination of EMO and MCDM research areas. PMOEAs utilize the preference information provided by the DM, a-priori or interactively, to guide the search towards the interesting parts of the PF instead of searching for the whole set (as most MOEAs do).

At least two possible ways of integrating EMO and MCDM can be identified according to [127]: "evolutionary algorithm
in MCDM” and “MCDM in EMO” approaches. On the one hand, in “MCDM in EMO” methods, preference information originated from MCDM, such as reference points, trade-offs, Desirability Function, is incorporated into the procedure of EMO, leading to a biased distribution or partial region of the PF. On the other hand, ”evolutionary algorithm in MCDM” follows the main procedure of MCDM method and utilizes evolutionary algorithms to get intermediate solutions (such as PIE [127], RD-NSGA-II [57]).

In addition to concepts and knowledge representation from MCDM and EMO, PMOEA ontology also intends to provide support for questions like "What methods can deal with a specific type of problem?", "What is the implementation language of that method?", etc. Therefore, knowledge related to MOP, optimization software frameworks are also considered in the ontology.

B. Consider using the existing ontologies

As far as we know, Evolutionary Computation (EC) Ontology [20], Diversity-Oriented Optimization Ontology [19] and Semantic MCDM (SeMCDM) [128] are related to the PMOEA field to some extent. Because they were designed with strong focus on specific domain knowledge operationalization, we can not reuse them directly, but some common concepts and vocabularies can be adopted.

While the EC ontology was designed to include simple and general EC concepts to support e-Learning applications, the SeMCDM architecture proposed a three-layered knowledge representation, addressing agents negotiation for self-organization application scenarios (e.g. autonomous adaptive devices). Here an ontology for the categorization of MCDM methods was introduced to enable the selection of the adequate methods for the SeMCDM architecture runtime decision making processes.

In contrast to the contributions and motivations of the above mentioned ontologies, the PMOEA ontology intends to be a complete and detailed formal description of the PMOEA knowledge domain, for knowledge harmonization and sharing within the PMOEA research community. Since the PMOEA, EC, MCDM and Diversity-Oriented Optimization Ontology define and use a subset of common concepts, this overlapping knowledge was taken into account in the PMOEA ontology design in two ways. The first way is to adopt the same naming for identical concepts, such as hasAuthor, hasDevelopingYear, canSolve, UtilityFunction. The other way is to specify the relations between them using OWL relations: owl:sameAs, owl:equivalentProperty and owl:equivalentClass, to map identical individuals, properties and classes between ontologies respectively. For example, SetQualityIndicator of the PMOEA ontology is owl:equivalentClass to Indicator of Diversity-Oriented Optimization Ontology.

C. Enumerate important terms in the ontology

It is important to make a comprehensive list of terms related to the PMOEA domain, without thinking of the overlap between some concepts. They are modelled as classes and properties in the next three sections.

D. Define the classes and the class hierarchy

A graph view of main class hierarchy is shown in Fig[1]. The class and class hierarchy (taxonomy) of our ontology include the following, all of them are extendable if needed:

PMOEA is the class of preference-based multiobjective evolutionary algorithms. This is the core class in our ontology, and each algorithm in this field is represented by an individual of this class. PMOEA is a subclass of MOEA.

MOP indicates multiobjective optimization problem. Subclasses of MOP include Academic_Problem and Real-world_Problem. Academic_Problem has subclasses DTLZ, Knapsack, WFG, ZDT.

InteractionTime indicates the moment when the DM interacts with the optimization process. a-priori, a-posteriori and progressive are subclasses.

PreferenceInformationFromDM refers to the information provided by the DM to express his/her preference. Subclasses include BudgetofDMcalls, SolutionComparison (PairwiseComparison, SampleRanks and SampleSorts are subclasses), DesirabilityFunction, ReferencePoint, PreferenceRegion, ReferenceDirection, Trade-off, ObjectiveComparison, OutrankingParameters, Indicators.

MetaHeuristic indicates the searching method used by the PMOEA, MOEA (multiobjective evolutionary algorithms) and SOEA (single-objective evolutionary algorithms) are subclasses of MetaHeuristic class, MOEA contains DiversityVSConvergence_based, Indicator_based, Decomposition_based, Swarm_based, Memetic, Coevolution_based subclasses.

PreferenceIntegration defines how the preference information is integrated in the search method, i.e. what is modified in the optimization process to embrace preference. Subclasses of PreferenceIntegration are the following: CrowdingDistance, DominanceRelation, Objectives, SetQualityIndicator, Constraints, TerminationCriterion, SelectionCriterion, Fitness, FrontSorting, Initialiation, ParticleUpdate, ASF (achievement scalarizing function), TerritorySize.

PreferenceModel specifies the preference model used in the PMOEA. It is strongly related to PreferenceInformationFromDM, but it focuses on the internal model utilized by the algorithm, which DM does not care or know. PreferenceModel subclasses include AchievementScalarizingFunction, FuzzyLogic, DecisionRules, OutrankingRelation, PolyhedralConeBased, UtilityFunction (which has Linear, AdditivePiecewiseLinear, GeneralAdditive, ChoquetIntegral, Polynomial, DesirabilityFunction as subclasses), PreferenceRegion, LightBeamSearch, PreferencePoint, KneePoint.

LearningMethod is used to indicate the learning method used by some PMOEAs (usually interactive PMOEAs) to mimic the DM’s preferences, when a real DM is not available. LearningMethod subclasses include OrdinalRegression, LinearProgramming, QuadraticProgramming, SupportVector-Machine, NeuralNetwork.

ResultInfluence defines the type of the result, which can be classified as OneSolution and SetOfSolutions. BiasedDistribution and PartialRange are subclasses of SetOfSolutions.

Researcher specifies the authors of the papers.
**ImplementationLibrary** indicates the library or framework used by metaheuristics, **ProgrammingLanguage** specifies the language used for implementation.

### E. Define the relations between classes (Object Properties)

Object properties are binary relations on individuals, they may be functional, transitive, symmetric and reflexive. Object properties may have a domain and a range specified. For example, **R-NSGA-II canSolve ZDT3**, **canSolve** is an object property whose domain is **MetaHeuristic** and range is **MOP**. The main object properties in our ontology are listed in **TABLE I**. Note that one individual can be related to several individuals with the same object property, such as **R-NSGA-II hasPreferenceInformationFromDM ReferencePoint** and **R-NSGA-II hasPreferenceInformationFromDM weights** hold at the same time.

**OntoGraf** is a (Protégé plugin) visualization tool that allows visual, interactive navigation of the relationships in OWL.

In our ontology there are 78 PMOEA individuals currently. Fig 2 presents R-NSGA-II [51] as an example of a PMOEA individual.

Individuals of MOP, another important class in the proposed PMOEA ontology, were created by describing them as theoretical or real-world, continuous/discrete/mixed integer, many objective or multimodal and if they deal with expensive evaluation functions or not.

Table III

<table>
<thead>
<tr>
<th>Data Property</th>
<th>Domain</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>isContinuousProblem</td>
<td>MOP</td>
<td>boolean</td>
</tr>
<tr>
<td>isDiscreteProblem</td>
<td>MOP</td>
<td>boolean</td>
</tr>
<tr>
<td>isMixedIntegerProblem</td>
<td>MOP</td>
<td>boolean</td>
</tr>
<tr>
<td>isManyObjectiveProblem</td>
<td>MOP</td>
<td>boolean</td>
</tr>
<tr>
<td>isMultimodalProblem</td>
<td>MOP</td>
<td>boolean</td>
</tr>
<tr>
<td>isNoisyProblem</td>
<td>MOP</td>
<td>boolean</td>
</tr>
<tr>
<td>hasExpenseEvaluation</td>
<td>MOP</td>
<td>boolean</td>
</tr>
<tr>
<td>hasNumberOfObjectives</td>
<td>MOP</td>
<td>boolean</td>
</tr>
<tr>
<td>hasDevelopingYear</td>
<td>MetaHeuristic, MOP</td>
<td>string</td>
</tr>
<tr>
<td>hasReference</td>
<td>PMOEA</td>
<td>boolean</td>
</tr>
<tr>
<td>hasMultipleRegionOfInterest</td>
<td>PMOEA</td>
<td>boolean</td>
</tr>
<tr>
<td>hasSpreadControl</td>
<td>PMOEA</td>
<td>boolean</td>
</tr>
<tr>
<td>preservesParetoDominance</td>
<td>PMOEA</td>
<td>boolean</td>
</tr>
</tbody>
</table>

G. Create Individuals

Individuals of PMOEA are algorithms named after the original paper or abbreviations of the method. We create every individual by answering the following questions, the answers are used for property in the parentheses:

1. What type of preference information should the DM provide? (hasPreferenceInformationFromDM)
2. When should s/he provide the preference information? (hasInteractionTime)
3. What is the preference model of this algorithm? (hasPreferenceModel)
4. What is the searching algorithm (metaheuristic) used in this method? (hasSearchAlgorithm)
5. How is the preference information integrated in the searching algorithm? (hasPreferenceIntegration)
6. What is the learning method in this algorithm, if the DM is not available? (hasLearningMethod)
7. What type of result is obtained by the algorithm: one solution or part of the PF? (hasResultInfluence)
8. Who introduced this algorithm and when? (hasAuthor, hasDevelopingYear)
9. What problems are tested in the simulation experiments by this algorithm? (canSolve)
10. Can the algorithm deal with multiple ROI in one run? (hasMultipleRegionOfInterest)
11. Does the algorithm have spread control methodology if its result is part of the PF? (hasSpreadControl)
12. Is it compatible with Pareto dominance relation? (preservesParetoDominance)
13. What other PMOEAs are compared in this paper? (hasComparison)
14. Does the algorithm have an extension or interactive version (if it is of a-priori type)? (hasExtension, hasInteractiveVersion)
15. What library and programming language are used to implement this algorithm? (useLibrary, useLanguage)

F. Define the properties of classes (Data Properties)

Data properties link an individual to an XML Schema Datatype value or an RDF literal. In other words, they describe relationships between an individual and data values. For example, hasDevelopingYear is a data property of R-NSGA-II with datatype "integer", which indicates the year when R-NSGA-II was proposed. The main data properties defined in our ontology are listed in Table II.

<table>
<thead>
<tr>
<th>Data Property</th>
<th>Domain</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>preservesParetoDominance</td>
<td>PMOEA</td>
<td>boolean</td>
</tr>
<tr>
<td>hasInteractionTime</td>
<td>MetaHeuristic</td>
<td>integer</td>
</tr>
<tr>
<td>hasSearchAlgorithm</td>
<td>MetaHeuristic</td>
<td>integer</td>
</tr>
<tr>
<td>hasComparison</td>
<td>MetaHeuristic</td>
<td>integer</td>
</tr>
<tr>
<td>hasExtension</td>
<td>MetaHeuristic</td>
<td>integer</td>
</tr>
<tr>
<td>useLibrary</td>
<td>MetaHeuristic</td>
<td>integer</td>
</tr>
<tr>
<td>useLanguage</td>
<td>MetaHeuristic</td>
<td>integer</td>
</tr>
<tr>
<td>hasMultipleRegionOfInterest</td>
<td>PMOEA</td>
<td>boolean</td>
</tr>
<tr>
<td>hasInteractiveVersion</td>
<td>PMOEA</td>
<td>boolean</td>
</tr>
<tr>
<td>hasInteractiveVersionOf</td>
<td>PMOEA</td>
<td>boolean</td>
</tr>
<tr>
<td>hasExtension, transitive</td>
<td>PMOEA</td>
<td>boolean</td>
</tr>
</tbody>
</table>

There are three data properties that describe the characteristics of PMOEA: hasMultipleRegionOfInterest indicates whether the method offers DM the ability to obtain more than one ROI in one run. hasSpreadControl describes whether the method allows the DM to control the spread of the obtained ROI, preservesParetoDominance shows whether the method preserves the order induced by Pareto dominance. These are important properties of PMOEA also examined by Bechikh [14].

E. Define the properties of classes (Object Properties)

We use these properties to describe relationships between individuals and other properties. For example, hasSearchAlgorithm is an object property of the class MetaHeuristic, which indicates the algorithm used in this method. Similarly, hasExtension describes whether the algorithm has an extension or interactive version. These properties are important for understanding the characteristics of the algorithms.

<table>
<thead>
<tr>
<th>Object Property</th>
<th>Domain</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>hasResultInfluence</td>
<td>MetaHeuristic</td>
<td>PreferenceInfluence</td>
</tr>
<tr>
<td>hasPreferenceModel</td>
<td>MOP</td>
<td>PreferenceModel</td>
</tr>
<tr>
<td>canSolve</td>
<td>MOP</td>
<td>boolean</td>
</tr>
<tr>
<td>hasSearchAlgorithm</td>
<td>MOP</td>
<td>MetaHeuristic</td>
</tr>
<tr>
<td>hasInteractionTime</td>
<td>MOP</td>
<td>InteractionTime</td>
</tr>
<tr>
<td>hasAuthor</td>
<td>MOP</td>
<td>Researcher</td>
</tr>
<tr>
<td>hasPreferenceInformationFromDM</td>
<td>PreferenceInformationFromDM</td>
<td>PreferenceIntegration</td>
</tr>
<tr>
<td>hasInteractiveVersion</td>
<td>PreferenceIntegration</td>
<td>LearningMethod</td>
</tr>
<tr>
<td>isInteractiveVersionOf</td>
<td>PreferenceIntegration</td>
<td>boolean</td>
</tr>
<tr>
<td>hasComparison</td>
<td>MetaHeuristic</td>
<td>MetaHeuristic</td>
</tr>
<tr>
<td>isExtensionOf</td>
<td>MetaHeuristic</td>
<td>MetaHeuristic</td>
</tr>
<tr>
<td>hasExtension</td>
<td>MetaHeuristic</td>
<td>MetaHeuristic</td>
</tr>
<tr>
<td>useLibrary</td>
<td>MetaHeuristic</td>
<td>ImplementationLibrary</td>
</tr>
<tr>
<td>useLanguage</td>
<td>MetaHeuristic</td>
<td>ProgrammingLanguage</td>
</tr>
</tbody>
</table>

Ontologies. As Fig 2 shows, all the classes that are related to PMOEA are rectangles and relations between them (object properties) are represented with lines.

F. Define the properties of classes (Data Properties)

Data properties link an individual to an XML Schema Datatype value or an RDF literal. In other words, they describe relationships between an individual and data values. For example, hasDevelopingYear is a data property of R-NSGA-II with datatype "integer", which indicates the year when R-NSGA-II was proposed. The main data properties defined in our ontology are listed in Table II.

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4. What is the searching algorithm (metaheuristic) used in this method? (hasSearchAlgorithm)
5. How is the preference information integrated in the searching algorithm? (hasPreferenceIntegration)
6. What is the learning method in this algorithm, if the DM is not available? (hasLearningMethod)
7. What type of result is obtained by the algorithm: one solution or part of the PF? (hasResultInfluence)
8. Who introduced this algorithm and when? (hasAuthor, hasDevelopingYear)
9. What problems are tested in the simulation experiments by this algorithm? (canSolve)
10. Can the algorithm deal with multiple ROI in one run? (hasMultipleRegionOfInterest)
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14. Does the algorithm have an extension or interactive version (if it is of a-priori type)? (hasExtension, hasInteractiveVersion)
15. What library and programming language are used to implement this algorithm? (useLibrary, useLanguage)

In our ontology there are 78 PMOEA individuals currently. Fig 2 presents R-NSGA-II [51] as an example of a PMOEA individual.

Individuals of MOP, another important class in the proposed PMOEA ontology, were created by describing them as theoretical or real-world, continuous/discrete/mixed integer, many objective or multimodal and if they deal with expensive evaluation functions or not.
H. Publish and reuse

We have uploaded our ontology into the WebProtégé [129], which is an ontology development environment for the Web that makes it easy to create, upload, modify and share ontologies for collaborative viewing and editing. Our PMOEA ontology is published here. You can view, comment, edit (after permission) and download the ontology for reuse and research.

IV. USING THE PMOEA ONTOLOGY

Since an ontology has been created, a lot of knowledge management tasks can be performed using the ontology together with other tools such as DL-based inference engines or reasoners, query languages and visualization tools, which are available in a diversity of free ontology management software (e.g. Protégé).

Reasoners can check the consistency of an ontology, perform subsumption checking (check if a concept is a subset of another concept) and infer relations that are not explicitly given by the user. The reasoner can automatically build a new inferred hierarchy of classes, by inferring new class/subclass relations and new individual membership relations to a class, based on necessary and sufficient condition axioms defined by the user. The “manually constructed” class hierarchy is called the asserted hierarchy and the class hierarchy automatically computed by the reasoner is called the inferred hierarchy. Being able to use a reasoner to automatically compute consistency and the inferred class hierarchy is one of the major benefits of building an ontology using the OWL-DL sub-language. Without a reasoner it is very difficult to keep large ontologies in a maintainable and logically correct state.

For example, ReferencePoint_based and ObjectiveSpaceTransformation_based are two inferred classes with conditions "hasPreferenceInformationFromDM value ReferencePoint" and "hasPreferenceIntegration value ObjectiveTransformation" respectively. They are shown in a different color in Fig.1.

An example of inferred object property is shown in Fig.4. The explanation to draw this property assertion is also given. We can see because isExtensionOf is transitive, "W_HypE isExtensionOf W_IBEA" and "W_HypE' isExtensionOf W_HypE" are explicitly defined by the user, "W_HypE' isExtensionOf W_IBEA" can be inferred by the reasoner.

DL Query is another important feature of Protégé, which can help to access, analyze and explore the knowledge domain described in the ontology. One simple query and results are shown below.

Query: Which PMOEAs can solve many objective problems? DL Query expression of this query is: canSolve some (isManyObjectiveProblem value true).

Query results show that 42 PMOEAs can solve many objective problems (Fig.5). Note that DL inference ability is much beyond simple relational database queries. If an algorithm A solves problem B and problem B is many objective, then algorithm A can solve many objective problems.
In fact, any property or combination of properties can be queried, next we give a more complex example.

Query: Which PMOEAs use preference region as preference information, integrate it with set quality indicator and are proposed after 2011?

In Fig. 5 query results show that $W_{HypE}$ and TEHVI meet all the conditions. It also shows the explanation of the result for the $W_{HypE}$ case. Note that because the OWL ontology design and DL tools are based on first order logics, we are able not only to query and formulate hypotheses to be checked for the knowledge domain, but also able to get the (formal logics) explanation that lead to the given results/conclusions. Following this approach we build an ontology-based expert system.

Based on the features presented above, we can use the PMOEA ontology for different purposes, typical use cases are given next.

A. Learning

PMOEA ontology is an essential resource to get acquainted with the field, and understand concepts and relations between concepts. Explicit specifications of domain knowledge can be accessed, inferred knowledge and corresponding explanations are provided with support of visualization tools, which are useful for new users who must learn what terms and relations in the domain mean.
Fig. 6. Complex Query and Explanation

B. Reviewing and Research Assessment

With the help of DL Query, researchers can easily find what they want to know. For example, they can find out if similar research has been done before, what papers compared one specified method (e.g., who used R-NSGA-II [51] as comparison algorithm with their proposed method?), who used one specific benchmark problem (e.g., ZDT1) if they want to compare results on the same problem. They can also query for relations of algorithms (e.g., extension and interactive version) or search for algorithms that use the same preference articulation (e.g., reference point).

C. Classification and Analysis

There are various criteria to classify the PMOEAs, e.g., interaction time, preference information, search algorithm, to name a few. Protégé can add the results of a query to new classes, thus making classification easy and flexible. This type of decisions must be done according to the knowledge domain expert and knowledge engineering perspectives. For example, we can query for PMOEAs that use reference point as preference information from the DM and add the results to class ReferencePoint-based PMOEA, which is a subclass of PMOEA. Any PMOEA to be added in the future that uses reference point will be automatically sorted into this subclass. TABLE III shows one classification of PMOEAs based on used preference information and type of MetaHeuristics. MetaHeuristics are generally grouped into MOEAs and single objective EAs, while MOEAs are grouped into Convergence&Diversity-based (such as NSGA-II [24], SPEA2 [25], etc.), Indicator-based (such as IBEA [28], SMS-EMOA [29]), other MOEAs (including MOPSO [34], MOEA/D [32], etc.). Of course, PMOEAs can also be classified by some other criteria if needed.

D. Finding new research topics

Apart from blank cells in Table III researchers can easily query combination of preferences and MOEAs or integration method of MOEAs to see whether such combinations have already been included in some PMOEAs. If all the MCDM methods are included in the ontology, we can find out that not all of them are considered in PMOEAs. Classification of objective functions, for example, is one category of interactive MCDM methods according to Miettinen et al. [86]. At each interaction phase, the DM is shown the current Pareto optimal solution and asked to classify the objectives into several categories, i.e. whose values should be improved (till some desired aspiration level), whose values can be impaired (till some upper bound), whose values are temporarily allowed to change freely. This kind of MCDM method has not been integrated in EMO, which can be a topic to consider in future research.

E. Finding the right method for an application

Application to real world problems is the ultimate goal of development of new PMOEAs. Different problems have diverse characteristics such as type of the decision variables used, e.g., continuous/discrete; or number of objectives, e.g., multi- or many-objectives; or type of functions, e.g., multi-modal; or shape of PF, e.g., convex, concave, disconnected. PMOEA ontology can help to find methods that suit a specific problem.

For example, a satellite task scheduling problem requires to make optimal plan of tasks for the satellite to execute in order to achieve maximal profit, in accordance with all the operational constraints. It can be modeled as a multiobjective combinatorial optimization problem similar to the well-known knapsack problem. The objectives to consider are maximizing...
the total profit and minimizing the total cost. If the DM chooses pairwise comparison as preference articulation, then we can search the PMOEA ontology for a feasible algorithm for this problem. Fig.7 shows that iPMA [112] and IEM-CO [99] has pairwise comparison as preference articulation method and use knapsack problems or discrete problems in the experiment, which are reasonable suggestions for our application problem. Note that we do not assume that the indicated approach is the only one for solving the satellite scheduling problem, but we find formulating it as a discrete knapsack problem as easy to apply.

F. Reuse and Extension

The proposed PMOEA ontology offers a framework for all the algorithms which integrate MCDM and EMO. So far it includes 78 PMOEA individuals. The PMOEA ontology is open for new methods to be added with the development of MCDM and EMO fields. With the help of WebProtégé users can easily edit (e.g. revise properties, add comments), create (new individuals/classes/properties), delete (wrong/redundant individuals/classes/properties) and download (the OWL file). The value of the proposed ontology will increase with the progressive accumulation of more information. EMO researchers can help and develop the MOEA classes, MCDM researchers can assist with the preference information classes, MOP class can also be extended by adding more practical and engineering applications. In a nutshell, PMOEA ontology helps to bring together researchers of MCDM and EMO fields as well as practitioners who use the PMOEA ontology. It provides a knowledge sharing platform for both academic research and real-world application.

V. Conclusion

Preference-based multiobjective evolutionary algorithms, which combine EMO and MCDM fields, are addressed in this

<table>
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<tr>
<th>MetaHeuristic Preference</th>
<th>Convergence&amp;Diversity based</th>
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<th>other MOEAs</th>
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TABLE III
CLASSIFICATION OF PMOEA
paper. We propose a novel method to systematize and manage the current knowledge and research results in the PMOEA field by means of ontology design and engineering. For the first time, an ontology is designed and used in this field, showing high benefits and advantages for knowledge management in the EMO, MCDM and PMOEA domains. After providing an overview of PMOEAs, we presented the process followed to build the PMOEA ontology using Protégé, introduced typical use cases for the built ontology, including learning support, querying, discovering new research opportunities and application-method matching.

It is believed that, the more attention and information PMOEA ontology attracts, the higher its value will be, for both the research community of PMOEA and practitioners who are using the PMOEAs. Therefore, reuse and extension of the proposed PMOEA ontology are welcome to help its growth. In the future, on the one hand new appearing MCDM, MOEAs and PMOEAs will be taken into consideration as a part of the proposed PMOEA ontology. On the other hand, benchmarks of practical applications based on papers dealing with practical problems will be collected and support for PMOEA implementation codes management will be introduced.

REFERENCES


M. Gong, F. Liu, W. Zhang, L. Jiao, and Q. Zhang, “Interactive MOEA/D for multi-objective decision making,” in Proceedings of the...


