Abstract— The orthogonality between sub-carriers in OFDM systems is disrupted by offsets between the carrier frequencies in the transmitter and receiver as well as by the so-called IQ mismatch in the mixers. When treated separately, effective algorithms exist for estimation and compensation of frequency offset as well as IQ imbalance. However, with both effects present, such algorithms do not lead to useful estimates of the related parameters. In this paper, we propose a novel technique for the estimation of the frequency offset in the presence of IQ imbalance. The proposed algorithm is tested on an OFDM communication system designed based on the specifications of IEEE 802.11a for wireless local area networks. Simulation results show that the frequency offset estimation algorithm works well even with large values of gain and phase imbalances in the receiver mixer. When an estimate of the frequency offset is obtained, and the complex baseband samples are corrected accordingly, any effective technique for the compensation of IQ imbalance can be applied.

Keywords: Wireless Communication OFDM, Direct Conversion, Frequency Offset, IQ imbalance

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is an effective and spectrally efficient signaling technique for communication over frequency selective fading channels. While in OFDM the spectrum of sub-carriers are allowed to overlap, sub-carriers are made orthogonal by choosing their frequencies to be multiples of the inverse of the symbols period. If the orthogonality between the sub-carriers is disrupted by any means, OFDM transmission will be rendered largely ineffective.

Various sources could affect the orthogonality of the OFDM sub-carriers. These include Doppler shifts, offsets between the carrier frequencies in the transmitter and receiver, as well as the so-called IQ-mismatches in the mixers [1-2].

When the transmitter and receiver carriers have a frequency offset, the complex baseband signal at the receiver will exhibit a time-dependent phase rotation which is also a function of the amount of frequency difference. In addition to affecting orthogonality between sub-carriers, frequency offset causes inter-carrier interference (ICI) to occur [1-4].

IQ mismatch refers to phase and gain imbalance between in-phase (I) and quadrature (Q) paths [5-6]. Specifically, phase mismatch occurs when the phase difference between the local oscillator signals for I and Q channels is not exactly 90 degrees. Gain imbalance refers to a gain mismatch in the path of the I and Q signals. The occurrence of IQ mismatch is especially pronounced in direct conversion receiver architectures where the radio frequency (RF) signal is converted directly to baseband in one mixing operation [7-8]. Such architectures have been under active investigation in recent years, since they offer potentials for low cost transceivers with low power consumptions, which are suitable for mobile terminals.

Various techniques exist for compensation of the frequency offset as well as calibration of the IQ imbalance in OFDM receivers when each effect is treated separately [2-6]. The estimation of the frequency offset, for example, can be performed by repeating a block of transmit data and examining the phases of the received samples in time or frequency domains in successive blocks [3]. However, as we will show later, the above procedure is not effective when there is gain or phase mismatch in mixers of the transmitter or receiver, since the phase difference between successive repeated samples are no longer solely due to the difference in the carrier frequencies.

As for the IQ mismatch problem, several methods have been proposed for estimation and compensation of such effects. In [5], an adaptive algorithm is proposed which is suitable for an OFDM system when mismatches exist in one or both mixers. Non-adaptive calibration techniques for OFDM transceivers are discussed in [6]. For this algorithms to perform effectively, however, the received baseband samples need to be first corrected for the frequency offset.

In this paper, we propose a novel technique for the estimation of frequency offset in presence of IQ mismatch in the receiver. In the next section, basic principles behind OFDM modulation are described. Next, we briefly review algorithms for frequency offset estimation and non-adaptive IQ mismatch estimation when each problem is addressed separately and investigate their performances when both effects are present. In Section IV, a new technique for estimation of frequency offset with IQ mismatch in the receiver is presented. Section V contains the simulation results of the proposed method.

II. OFDM SIGNALLING

In OFDM, each symbol of a block of size N is modulated onto a sub-carrier whose frequency is $f_k = k / T$, where $k$ is the sub-carrier index and $T$ is the duration of sub-carrier waveforms. By setting the frequency spacing to $1/T$, sub-carriers are made orthogonal. The continuous-time OFDM
signal $x(t)$, is obtained by adding the sub-carrier modulated waveforms together, such that:

$$x(t) = \frac{1}{N} \sum_{k=K}^{K+N-1} X[k] e^{j2\pi f_k t}, \quad t \in [0, T]$$  \hspace{1cm} (2.1)

where $X[k]$ is the $k^{th}$ data symbol, and we have considered that only sub-carrier in the range $[-K, K]$ where $N \geq (2K+1)$, contain non-zero symbols. Equation (2.1) shows that samples of $x(t)$ can be obtained by performing an inverse discrete Fourier transform (IDFT) on the $N$-symbol block. A complete OFDM symbol is formed by adding a guard time from the end of each block to the beginning, which helps to eliminate inter symbol interference as well as inter channel interference.

At the receiver, the complex baseband signal due to an OFDM symbol is given by

$$y(t) = \frac{1}{N} \sum_{k=K}^{K+N-1} X[k] H[k] e^{j2\pi f_k t/N} + w(t)$$  \hspace{1cm} (2.2)

where $H[k]$ represents the combined effect of the channel and all continuous-time blocks in the path of the signal and $w(t)$ represents the additive Gaussian noise. Sampling $y(t)$ at instants $t = nT/2N$ yields:

$$y[n] = \frac{1}{N} \sum_{k=K}^{K+N-1} X[k] H[k] e^{j2\pi f_k n/N} + w[n]$$  \hspace{1cm} (2.3)

where $0 : n : N - 1$. If a discrete Fourier Transform (DFT) is performed on the block of $N$ samples, the demodulated OFDM sequence becomes:

$$Y[k] = X[k] H[k] + W[k]$$  \hspace{1cm} (2.4)

For each sub-channel, $Y[k]$ is multiplied by a one-tap complex equalizer, so that the effect of $H[k]$ is removed. The data samples are recovered after demodulation of the equalized samples.

### III. FREQUENCY OFFSET AND IQ IMBALANCE

Fig. 1 shows a generic diagram of the front-end of an OFDM receiver with a direct conversion mixer which shows the effects of frequency offset and IQ imbalance in the receiver. Here, the received RF signal is denoted by $y_{RF}(t)$ and $\hat{d}[n]$ is the demodulated bit sequence. The difference between the carrier frequencies in the transmitter and receiver mixers is shown by the parameter $\Delta \alpha = 2\pi f_c$. The gain and phase imbalance parameters are denoted by $\theta$ and $\epsilon$, respectively.

Let us first consider the frequency offset problem in absence of any IQ mismatch (i.e., with $\theta = \epsilon = 0$). With a frequency offset $\Delta \alpha$, the complex envelope of the received sequence will have a multiplicative factor of the form $\exp(j \Delta \alpha_n T_s)$, where $n$ is the sample index and $T_s$ is the sample period. If $\beta$ denotes the ratio of the frequency offset to the sub-carrier separation, Equation (2.3) will become

$$\hat{y}[n] = \frac{1}{N} \sum_{k=K}^{K+N-1} X[k] H[k] e^{j2\pi (k+\beta) N/n} + w[n]$$  \hspace{1cm} (3.1)

The presence of the factor, $\exp(j 2\pi n \beta / N)$, disturbs the orthogonality between sub-carriers and results in ICI. It also causes degradation of signal-to-noise ratio (SNR) in each sub-carrier.

![Fig. 1: OFDM receiver with IQ imbalance and frequency offset](image.png)

An effective approach for frequency offset estimation is to consider the phase shift between samples in two subsequent sub-channels [3]. For example, suppose an OFDM symbol of length $N$ is transmitted twice in a row and the samples corresponding to the two received block of samples are denoted as $\hat{Y}_{1}[k]$ and $\hat{Y}_{2}[k]$ with $0 \leq k \leq N-1$. In absence of any frequency offset (and noise), the product $\hat{Y}_{1}[k] \hat{Y}_{2}^{*}[k]$ is expected to be real. A non-zero phase value for this product is attributed to the carrier frequency offset. It is shown in [3], that with $(2K+1)$ non-zero symbols in each OFDM symbol, a maximum likelihood estimate of $\beta$ is given by

$$\hat{\beta} = \frac{1}{2\pi} \arg \left\{ \sum_{k=K}^{K+N-1} \hat{Y}_{1}[k] \hat{Y}_{2}^{*}[k] \right\}$$  \hspace{1cm} (3.2)

where the channel noise is assumed to be additive white Gaussian (AWG).

When the gain and phase mismatch values are not zero in the mixer at the receiver, the above relation between phases of the samples of $\hat{Y}_{1}[k]$ and $\hat{Y}_{2}[k]$ no longer holds. To illustrate this, the algorithm above is tested on an OFDM communication system whose parameters are set based on the IEEE 802.11a standard for wireless local area networks (WLAN) [9]. Here, a block of 64 symbols is formed at the IDFT input in the transmitter, 48 of which are filled with data symbols. Other sub-carriers are filled with known symbols to be used as pilots or with zeros. The sub-carrier spacing is 312.5 kHz. In this work, the data sequence is modulated using 16-QAM. SNR per bit (denoted as $E_b/N_0$) is set at a relatively large value.
Fig. 3 shows a plot of the average of frequency offset estimate (in points-per-million (ppm)) obtained by applying Equation (3.2) vs. the corresponding true value in absence of any IQ imbalance. (A nominal carrier frequency of 5GHz is considered.) The offset ranges from 0 to 24 ppm, corresponding to $\beta$ ranging from 0 to 0.39. It is observed that the algorithm produces unbiased estimate of the unknown parameter. The figure also shows a plot of the average of the estimate when gain and phase mismatch values are set at 2 dB and 10 degrees, respectively. The estimate obtained under these conditions is now biased where the amount of bias changes with the value of the frequency offset.

In addition, Fig. 4 shows the standard deviation of the estimates obtained in presence and absence of IQ imbalance. It is observed that the standard deviation of the estimate vs. the actual frequency offset is relatively constant for the selected channel SNR when no mismatch exists. With IQ imbalance, however, the variance changes as frequency offset is varied. Notice that even when the estimate has a small bias for certain values of frequency offset, the variance is relatively large. Such a poor performance was similarly observed when other frequency offset estimation algorithms were employed.

Next, with zero frequency offset and with non-zero values of gain and phase mismatches, it can be shown that [6]:

$$\hat{Y}[k] = \gamma Y[k] + \lambda Y'[\neg k]$$

(3.3)

where

$$\gamma = 0.5[1 + (1 + \varepsilon)(\cos(\theta) + j \sin(\theta))]$$

(3.4)

$$\lambda = 0.5[1 - (1 + \varepsilon)(\cos(\theta) + j \sin(\theta))]$$

(3.5)

Equation (3.3) shows that the sample at the $k$th sub-carrier will be multiplied by a factor $\gamma$ and added to an interference term related to the symbol at the $\neg k$th sub-carrier. In [6], the above relation is exploited to design a calibration technique which is based on transmitting specific OFDM symbols such that selected sub-channels, such as $k'$ are filled with zero, and a corresponding sub-channel ($\neg k'$) contains a known symbol. The algorithm is based on monitoring the output of the DFT operation at sub-carriers which were filled with zeros. Notice that based on Equation (3.3) and in absence of noise, any non-zero values at these bins are due to the gain and phase mismatches.

While the algorithm above performs well for even large values of gain and phase mismatch in absence of frequency offset [6], it is not effective if frequency offset is non-zero and the related phase rotation of the complex samples are not corrected. In fact, if samples exhibit small frequency offset or have not been corrected perfectly, the performance of the algorithm is adversely affected.

In the next section, we describe a new technique for frequency offset estimation, which is effective even in the presence of severe IQ mismatch. Once the estimate is obtained, the complex samples can be corrected and then any effective technique for IQ imbalance estimation and compensation can be applied.
where $I[k]$ denotes the effect of ICI which is given by

$$I[k] = \sum_{i=k}^{K} Y[i] \frac{\sin(\pi \epsilon)}{N \sin(\pi (l-k+\epsilon)/N)} e^{j\pi N^{-1} N \epsilon} e^{j\pi N^{-1} k}$$

and $W[k]$ shows the effect of additive noise. The reduction in signal amplitude and the phase rotation of the samples as well as the effect of ICI caused by the frequency offset is evident in above relations.

The relation between $\tilde{Y}$ and $\hat{Y}$ will again be similar to Equation (3.3) so that

$$\hat{Y}[k] = \gamma \tilde{Y}[k] + \lambda \hat{Y}[-k]$$

where $\gamma$ and $\lambda$ are defined in Equations (3.4) and (3.5).

Let us suppose that a specific OFDM symbol, $X_p[k]$, is formed such that only one of the sub-carriers denoted by $k_p$, contains a non-zero value, i.e.,

$$X_p[k] = \begin{cases} P & \text{if } k = k_p \\ 0 & \text{if } k \neq k_p \end{cases}$$

where $P$ denotes the symbol stored in sub-carrier $k_p$. When $X_p[k]$ is transmitted through the channel, using Equations (2.4) and (3.1) to (3.3), $\tilde{Y}[k]$ at sub-carrier $k_p$ becomes:

$$\tilde{Y}_p[k] = P \times H[k] \frac{\sin(\pi \epsilon)}{N \sin(\pi \epsilon)} e^{j\pi N^{-1} N \epsilon} + W[k]$$

where a subscript $p$ is added to $\tilde{Y}[k]$ to indicate that this signal is due to symbol $X_p[k]$. At all other sub-carriers, we have

$$\tilde{Y}_p[k] = P \times H[k] \frac{\sin(\pi \epsilon)}{N \sin(\pi (k_p-k+\epsilon)/N)} e^{j\pi N^{-1} N \epsilon} e^{j\pi N^{-1} k} + W[k], \quad (k \neq k_p)$$

The above relation shows that, even in absence of noise, all other sub-carriers (besides $k_p$) will contain non-zero terms due to the ICI effects. These terms are further affected by IQ mismatch according to Equation (3.3).

Notice that for a noiseless channel, if the frequency offset is zero, or if the samples are corrected for any non-zero offset, $\hat{Y}_p[k]$ will only contain non-zero terms in sub-carrier $k_p$. With IQ imbalance, there will be non-zero terms at sub-carriers $k_p$ and $-k_p$.

The basic principle behind the estimation algorithm is to find a value for the frequency offset estimate such that the total power in all sub-carriers except $k_p$ and $-k_p$ is minimized. Toward that end, we define the following cost function:

$$E_f = \sum_{k=-K}^{K} \left| \hat{Y}_p[k] \right|^2$$

For $|\beta| < 1$, i.e., for frequency offsets less than the sub-carrier spacing, plots of $E_f$ shows a unique minimum with respect to $\beta$. The algorithm, therefore, works as follows:

**Step 1**: Transmit a tone at the frequency $\omega_c + 2\pi k_p \Delta f$.

**Step 2**: Select an initial value for $\beta$.

**Step 3**: Correct the received samples for frequency offset, $\beta$.

**Step 4**: Compute $E_f$ and choose a new estimate for $\beta$ with a reduced cost function by moving in a direction opposite to the gradient of $E_f$ with respect to $\beta$.

**Step 5**: If the algorithm has not converged with sufficient desired accuracy, go to Step 3.

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The algorithm above can be modified in a number of ways to allow for faster convergence. For example, the number of sub-channels in the transmitted OFDM symbol $X_p[k]$ which contain non-zero terms, could be increased. This, in turn, will result in larger ICI terms when frequency offset is non-zero. In addition, in defining the cost function, a weight factor can be attributed to the magnitude for each sub-carrier such that

$$E_f = \sum_{k=-K}^{K} C[k] \left| \hat{Y}_p[k] \right|^2$$

where $C[k]$ is a positive real term. The weight function above can be specified so that only sub-carriers which are closer to $k_p$, and hence, are expected to have more significant ICI values are would have larger values.
V. SIMULATION RESULTS

To evaluate the proposed algorithm, an OFDM system designed based on IEEE 802.11a standard for WLAN’s was again considered. The transmit OFDM symbol was designed to contain two pilot tones. A weight function was designed so that the weight of a channel was lower the further it was from the pilots. Given the shape of the cost function with a unique minimum in the range of interest, a variable step-size search mechanism was utilized. Here, if the value of $E_f$ was higher for a new value of $\beta$, the step size was lowered and $\beta$ was moved in a direction opposite to the previous change. On the other hand, if $E_f$ was lower, the step size remained the same and $\beta$ was moved in the same direction. This technique proved to result in fast convergence of the algorithm.

Fig. 5 shows the average of the estimate of the frequency offset vs. the actual value when the parameter $\beta$ is changed from –0.15 to 0.15 (corresponding to -9.4 ppm to 9.4 ppm offset with a 5GHz nominal carrier frequency) for an AWGN channel at a relatively high SNR. The plots show the results with and without IQ mismatch in the receiver mixer. With IQ imbalance present, the phase and gain imbalance parameters were again set at 10° and 2 dB, respectively. The estimated value was obtained when the iterative algorithm had converged. The plots show that the algorithm produces unbiased estimate of the frequency offset in both cases. The standard deviation of the estimate is also plotted in Fig. 6, which clearly does not vary as the frequency offset changes. As mentioned previously, once an estimate of the frequency offset is obtained and the samples are corrected accordingly, an IQ imbalance compensation algorithm can be applied.

VI. CONCLUSIONS

In this paper, the problem of frequency offset estimation in presence of IQ imbalance in the receiver mixers was investigated for OFDM communication systems. Generally speaking, frequency offset estimation and IQ imbalance correction algorithms do not work effectively when both phenomena are present. We proposed an iterative technique which used specially designed OFDM symbols to yield an estimate of the frequency offset. Simulation results show that the algorithm performs well even in presence of severe IQ imbalance. After the complex baseband samples are corrected for the effect of frequency offset, the IQ imbalance can be corrected using any of the available techniques.

REFERENCES