Regulating Hazardous Materials Transportation by Dual Toll Pricing

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Abstract

We investigate toll setting as a policy tool to mitigate the risk of hazardous material (hazmat) shipment in transportation networks. We formulate the problem as a bi-level program wherein the first level aims at minimizing the risk via dual toll setting, and the second level explores the user equilibrium decision of the regular vehicles and hazmat carriers given the toll. We decompose the formulation into first-stage and second-stage, and suggest separate methods to solve each stage. Our two-stage solution methodology guarantees that we can always obtain nonnegative valid dual tolls regardless of the solution to the first-stage problem. We present a summary of our computational experiments on various problem instances to support the proposed methods. A general dual-toll setting problem where the regulator rather wishes to minimize a combination of risk and the paid tolls is also considered. To solve this true bilevel model, we provide a heuristic algorithm which decomposes the problem into subproblems each being solved using a line search routine. Numerical experiments have been carried out to verify the efficiency of the heuristic. Finally, we implement the two-stage procedure and the decomposition heuristic on a problem instance from the Sioux Falls network to illustrate the insights that can be obtained through the corresponding dual-toll policies.

Keywords: hazardous material transportation; toll setting; non-convex optimization; bi-level programming

1 Introduction and Literature Review

While the phrase “hazardous material” (hazmat) almost always carries a negative connotation they are an integral part of our daily lives and industrial development which necessitates large volumes of

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shipments. Despite the extensive use of hazmats from fueling our vehicles and heating our homes to farming, medical and manufacturing purposes, they are potentially disastrous to the people and the environment. Common examples are explosives, gases, flammable liquids and poisonous substances shipped by trucks, trains, vessels, and planes containing undesirable consequences in the event that they release or explode due to an accident. According to the U.S. Department of Transportation Pipeline and Hazardous Materials Agency, among the close to 1 million daily shipments of hazmats crisscrossing the United States, during the year 2013, 16,769 hazmat incidents have been recorded. Nevertheless, these incidents caused a total of 12 fatalities, 23 major and 123 minor injuries and damages of over $114 million. A recent example is the November 2013 gasoline truck collision in Guyton, Georgia which resulted in gasoline spillage, fire and explosion leading to 3 fatalities and over $250 thousand damage and cleanup costs. Another major incident is the July 2013 train accident in Canada where a runaway 72-car train slammed into the center of the Quebec town of Lac-Megantic in the early morning. Tank cars full of oil exploded and burned in the heart of the commercial district. Other than the $7.8 million preliminary environmental costs, the remains of 42 people were recovered, and five people are reported missing (Pipeline and Hazardous Materials Administration (PHMSA), 2013). These incidents underscore the importance of overseeing safe, reliable and environmentally sound hazmat transportation. This study specifically focuses on regulating hazmat transportation in road networks motivated by the fact that in 85% of the hazmat incidents truck has been used as the mode of transportation.

In the literature of hazmat routing, risk measurement typically relies on two link attributes: 
\textit{accident probability} and \textit{accident consequence}. The accident consequence can be the population, which may be measured using the $\lambda$-neighborhood concept (Batta and Chiu, 1988), or it can also include damages to individuals who are directly involved in an incident as well as damage to the environment and properties (Abkowitz and Cheng, 1988). Various measures of hazmat accident risk can be defined by considering different distributions for these two link attributes. Following this scheme, popular hazmat risk measures include incident probability (Saccomanno and Chan, 1985), population exposure (ReVelle et al., 1991), perceived risk (Abkowitz et al., 1992), conditional probability (Sivakumar et al., 1993), and expected risk (Alp, 1995; Jin and Batta, 1997). More recently, Kang et al. (2011) and Toumazis et al. (2013) proposed quantile-based methods such as value-at-risk and conditional value-at-risk.

To mitigate hazmat transport risk, authorities usually attempt to separate hazmat flow from regular traffic flow while directing hazmat trucks to less populated areas. For doing so two groups of policies are available to the governments: \textit{network design} and \textit{toll setting}. In the former approach, the authority restricts hazmat carriers from using certain road segments by imposing (permanent or time-based) curfews. Although much work has been done in applying this policy for hazmat transport risk management (e.g. Kara and Verter, 2004), it does not consider driver’s preferences in route selection. In contrast, toll setting is a relatively unexplored risk mitigation policy proposed by Marcotte et al. (2009) which diverts hazmat carriers to less risky areas based on their economical and time preferences by assigning tolls to road segments. Other than the generated revenue to improve
road infrastructure, toll setting can yield a win-win scenario for both carriers and transportation agencies by providing more flexible solutions to drivers.

Toll pricing is most often used to reduce traffic congestion. This is achieved by assigning tolls to certain links to encourage drivers to choose less-congested routes (i.e. congestion pricing). The congestion pricing problem can be usually categorized as first and second best pricing problems. The first-best policy or marginal social cost pricing (Arnott and Small, 1994) refers to a problem where network regulators assign tolls that are equal to marginal external costs on each individual link such that the carriers’ route choices optimize the collective use of the network. In this context, Hearn and Ramana (1998) obtain a valid toll vector such that the resulting tolled user equilibrium flow equates the system optimum flow pattern. For a given toll set, various objectives (e.g., minimizing the total tolls collected, minimizing the number of toll booths) can be optimized with respect to the tolls. In contrast, in the second-best policy as defined by Johansson-Stenman and Sterner (1998) not every individual link of the network can charge tolls. Such kind of tolls are referred to as second-best tolls.

In the field of hazmat routing, toll setting (TS) was first proposed by Marcotte et al. (2009) as an effective means to mitigate hazmat transport risk by deviating the hazmat carriers from using certain road segments due to their own considerations. Their findings indicate that in contrast with a network design (ND) policy (e.g. Kara and Verter, 2004; Erkut and Gzara, 2008) which does not take the drivers’ route selection into account, TS has significant potential as a policy tool because it is more flexible and effective. To spread the risk in an equitable way while minimizing the network total risk, Bianco et al. (2012) expand the work of Marcotte et al. (2009) by introducing the idea that the toll paid by a carrier on a network link depends on the usage of that link by all carriers. Hence, to deter the carriers from using links with high concentration of total risk and achieve equity, they suggest minimizing the maximum link total risk.

An overview of the existing literature on hazmat risk mitigation policies reveals that the majority of them focus either on the so-called restrictive network design policy, or single toll pricing which excludes the inevitable impact of regular vehicles on hazmat risk. Even though Wang et al. (2012) proposed a dual toll pricing problem to regulate both types of traffic, their model assumes a linear travel delay function (for analytical tractability) which does not capture the regular traffic congestion properly. A nonlinear travel delay function, used by the U.S. Bureau of Public Roads and Sheffi (1985), better explains the network congestion due to regular traffic flow, and consequently provides more efficient pricing strategies to reduce the hazmat risk. Our dual-toll pricing model uses a nonlinear travel delay function and a duration-population-frequency risk measure to model the dependence of hazmat risk on congestion induced by regular traffic.

From the standpoint of methodology, our solution procedure expands the work of Marcotte et al. (2009). They suggest a single toll pricing problem to minimize hazmat risk, and develop a bilevel program. Further, they propose a 2-stage solution method and claim that to ensure obtaining a valid toll in the second stage, the solution to the first stage problem must be optimal. In our work we consider nonlinearity of the travel delay function, which makes the objective function of the first
stage problem non-convex; hence an exact optimal solution is difficult to find. Nevertheless, our solution methodology guarantees obtaining valid dual tolls while having an approximately optimal solution to the first stage problem.

The rest of this paper is organized as follows. The next section reviews technical contributions related to our research. Section 3 develops the dual toll pricing model, encompassing the explicit definition of the problem, the mathematical formulation of the general model, and discusses the obstacles toward solving the model. Section 4 is devoted to our solution methodology including the solution procedures to the first stage and details of the method we introduce for the second stage problem. Computational experiments based upon different example problems are also provided in this section. Section 5 explains an alternative heuristic method to solve the dual toll pricing as a single level problem. We present an application of our proposed algorithm on the realistic road network of Sioux Falls in Section 6. Finally, Section 7 concludes the paper and suggests some future research directions.

2 Review of Related Technical Contributions

We review technical contributions in past research efforts for both regular and hazmat traffic control that use toll pricing. We build upon these techniques in developing our solution methodology.

2.1 Regular Traffic Control

In regular traffic control, the primary objective of the network administrator is to minimize a certain system total cost function such as the total travel time or the total emissions—denoted by system optimum. Consider a directed network \( \mathbb{G}(N,A) \), with a set of nodes \( N \), and a set of arcs \( A \). Let \( f_{ij}(v_{ij}) \), a function of arc traffic volume \( v_{ij} \), denote such a performance measure in each arc \( (i,j) \in N \) that the network administrator wants to minimize. We assume there is travel demand \( d^w \) for each O-D pair \( w \) in the set of O-D pairs \( W \). The administrator’s problem can be written as follows:

\[
\min_{v \in V} \sum_{(i,j) \in A} f_{ij}(v_{ij})
\]

where

\[
V := \left\{ v : v_{ij} = \sum_{w \in W} y^w_{ij} \quad \forall (i,j) \in A, \right. \\
\left. \sum_{j: (i,j) \in A} y^w_{ij} - \sum_{j: (j,i) \in A} y^w_{ji} = b^w_i, \quad \forall i \in N, \forall w \in W, \\ y^w_{ij} \geq 0, \quad \forall (i,j) \in A, \forall w \in W \right\},
\]

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$y^w_{ij}$ is the traffic flow in arc $(i, j)$ for O-D pair $w$, $b^w_i = d^w$ if node $i$ is the origin of O-D pair $w$, $b^w_i = -d^w$ if node $i$ is the destination of O-D pair $w$, and $b^w_i = 0$ for all other intermediate nodes.

We let a solution to the administrator’s problem (1) to be $\bar{v}$.

While $\bar{v}$ is the desired traffic flow of the administrator, network users will not necessarily follow the desire. Instead, they are often interested in their own benefits; hence a game-theoretic model is necessary. Given a tolling scheme from the administrator, according to the Wardrop’s first principle (e.g. Florian and Hearn, 1995), the users, in an attempt to minimize their individual travel cost, will take the User Equilibrium flow pattern. When arc traveling time/cost is $c_{ij}(v_{ij})$, the user equilibrium of network users can be modeled as a variational inequality (e.g. Dafermos, 1980). Let us introduce toll prices $\tau_{ij}$ for each arc $(i, j) \in \mathcal{A}$. Therefore, the objective of the toll pricing problem is to make the desired flow pattern $\bar{v}$ the tolled-user equilibrium flow that satisfies the following variational inequality:

$$\sum_{(i,j) \in \mathcal{A}} (c_{ij}(\bar{v}_{ij}) + \tau_{ij})^T (v_{ij} - \bar{v}_{ij}) \geq 0, \quad \forall v \in \mathcal{V}. \quad (3)$$

Any toll vector $\tau$ that makes (3) hold is called a valid toll (Hearn and Ramana, 1998). Bergendorff et al. (1997) develop and prove the following property:

**Lemma 1.** For $\tau$ to be a valid toll vector for $\bar{v} \in \mathcal{V}$, there should exist a vector $\lambda$ such that

$$c_{ij}(\bar{v}_{ij}) + \tau_{ij} \geq \lambda^w_i - \lambda^w_j, \quad \forall (i, j) \in \mathcal{A}, w \in \mathcal{W}, \quad (4)$$

$$\sum_{(i,j) \in \mathcal{A}} [c_{ij}(\bar{v}_{ij}) + \tau_{ij}]\bar{v}_{ij} = \sum_{i \in \mathcal{N}} \sum_{w \in \mathcal{W}} \lambda^w_i b^w_i. \quad (5)$$

We notice that to obtain constraints (4) and (5), one can construct the Lagrangian of variational inequality (3), and derive the KKT conditions.

Usually, there are multiple valid tolls and one may choose a toll based on another criteria. A popular choice is to minimize the total toll collected. Hence, for a given $\bar{v}$, we solve:

$$\text{RTP}(\bar{v}) : \min_{\tau \geq 0} \sum_{(i,j) \in \mathcal{A}} \bar{v}_{ij}\tau_{ij}, \quad (6)$$

subject to (4) and (5).

The regular-toll problem (RTP) is an instance of the Inverse Optimization (IO) approach (e.g. Ahuja and Orlin, 2001) to find toll prices. Mathematically, arc tolls can be positive or negative (subsidy). However, it is not logical to consider subsidies when trying to regulate traffic. The existence of a nonnegative valid toll vector $\tau \geq 0$ is assured by the following theorem due to Yin and Lawphongpanich (2006):

**Theorem 1.** If $f_{ij}()$ is a monotonically increasing function for all $(i, j) \in \mathcal{A}$, there exists a nonnegative valid toll vector $\tau \geq 0$ for the traffic pattern $\bar{v}$. (i.e. RTP($\bar{v}$) is feasible).
2.2 Hazmat Traffic Regulation

To guide hazmat traffic to a certain desired pattern, we can follow an analogous process as introduced by Marcotte et al. (2009). On the directed network $G(N, A)$, we let $S$ denote the set of shipments (similar to the set of O-D pairs for regular traffic case), $n^s$ the number of trucks for each shipment $s \in S$, and $x^s_{ij}$ a binary routing variable corresponding to shipment $s$ on arc $(i, j) \in A$. For a certain performance measure $g_{ij}(\cdot)$ for each arc $(i, j) \in A$, the hazmat network administrator’s problem can be written as follows:

$$\min_{x \in X} \sum_{s \in S} \sum_{(i,j) \in A} g_{ij}(x^s_{ij}),$$

(7)

where

$$X := \left\{ x : \sum_{j:(i,j) \in A} x^s_{ij} - \sum_{j:(j,i) \in A} x^s_{ji} = e^s_i \quad \forall i \in N, s \in S, \quad x^s_{ij} \in \{0,1\} \quad \forall (i,j) \in A, s \in S \right\},$$

(8)

e^s_i = 1 \text{ if node } i \text{ is the origin of shipment } s, e^s_i = -1 \text{ if node } i \text{ is the destination of shipment } s, \text{ and } e^s_i = 0 \text{ for all other intermediate nodes } i. \text{ We let } \bar{x} \text{ denote a solution to problem (7)}.

As opposed to the user equilibrium problem in the regular traffic case wherein an O-D pair flow can split in many routes, we assume hazmat carriers use the shortest path for all trucks of each shipment. Therefore, with $c_{ij}$ a nonnegative travel time for each arc $(i, j) \in A$, and a hazmat toll vector $t$, hazmat carriers determine their routes by solving the following multi-commodity shortest path problem:

$$\min_{x \in X} \sum_{s \in S} \sum_{(i,j) \in A} n^s(c_{ij} + t_{ij}) x^s_{ij}. \tag{9}$$

A valid hazmat toll vector $t$ makes the desired hazmat traffic pattern $\bar{x}$ an optimal solution to (9). Referring to Marcotte et al. (2009), Section 4, we can characterize a valid hazmat toll:

**Lemma 2.** For $t$ to be a valid toll vector for $\bar{x} \in X$, there should exist a vector $\pi$ such that

$$\pi^s_i - \pi^s_j \leq n^s(c_{ij} + t_{ij}) \quad \forall (i,j) \in A, s \in S, \tag{10}$$

$$\bar{x}^s_{ij}(\pi^s_i - \pi^s_j - n^s(c_{ij} + t_{ij})) = 0 \quad \forall (i,j) \in A, s \in S. \tag{11}$$

Similar to the regular traffic case, multiple valid toll vectors may exist and one can choose a toll to minimize the total toll collected:

$$\text{HTP}(\bar{x}) : \min_{t \geq 0} \sum_{s \in S} \sum_{(i,j) \in A} n^s \bar{x}^s_{ij} t_{ij}, \quad \text{subject to (10) and (11).} \tag{12}$$
The following theorem guarantees the existence of a nonnegative valid toll vector $t \geq 0$:

**Theorem 2.** If $g_{ij}(\cdot)$ is a monotonically increasing function for all $(i,j) \in A$, there exists a nonnegative valid toll vector $t \geq 0$ for the traffic pattern $\bar{x}$.

**Proof.** We can revise the proof of Marcotte et al. (2009, Proposition 1). Although the original proof is written for a linear program with nonnegative cost coefficients, the same argument applies to any increasing function $g_{ij}(\cdot)$. □

3 Hazardous-Network Dual Toll Pricing Problem

In this section, we introduce the dual toll pricing problem to minimize the risk of hazardous material transportation.

3.1 Problem Description

In our network, there are two distinct groups of decision makers: the local government, and the network users including regular vehicles’ drivers and hazmat carriers. We note that although the tolls on the road network are determined by the government, it is actually the routing decisions of the users on the tolled network that determine not only their own travel cost, but also the associated transport risk. We assume that the government has a leader position aiming at minimizing population exposure as the risk measure, whereas users follow regulation while minimizing their travel cost. To capture this leader-follower relationship, we use a bilevel framework to represent our model with the government in level 1 and the users in level 2 as depicted in Figure 1.

Once the government decides on the toll charges in level 1, two different sets of users (i.e., regular traffic and hazmat carriers) will minimize their total travel cost in level 2. Specifically, the regular traffic will take the tolled user equilibrium flow, and apart from them, the hazmat carriers will choose the shortest paths between every origin and destination pair considering both time and economical preferences. As drivers’ decisions do not guarantee the desired least-risk solution, one would expect the government to account for the risk implications of users’ route choices. In
response to such behavior, the government can indirectly influence the users’ decisions through toll pricing. We seek a dual toll policy that produces a mutually acceptable solution.

### 3.2 A Bilevel Model

Now we turn to the mathematical formulation of the bilevel structure we propose for the hazardous-network dual toll pricing problem.

We represent the existing road system by the directed network $G(N, A)$, with a set of nodes $N$, and a set of arcs $A$. Consider a regular traffic demand $d^w$ for each O-D pair $w$ in regular O-D pair set $W$. In addition, there is a set $S$ of O-D shipments for hazmat traffic with each shipment $s \in S$ representing a specific hazmat type with a certain level of risk exposure. Let $n^s$ be the number of trucks needed to complete every shipment $s \in S$. Finally, let $\rho_{ij}$ denote a measure of hazmat transportation risk in arc $(i, j) \in A$, for example, population along each arc. Table 1 provides a summary of the notation used in the formulations.

We make two simplifying assumptions for analytical tractability of our model. First, we assume that network users have perfect information of the current status of the network. Second, there is no stochasticity in the travel time and the behavior of users implying that our model is deterministic.

 Armed with the above notation and assumptions, we are now ready to introduce our mathematical model. By imposing separate toll policies $\tau$ and $t$, we guide regular and hazmat traffic to
a solution of network administrator’s problem of the following form:

$$\min_{v \in V, x \in X} z(v, x) = \sum_{s \in S} \sum_{(i,j) \in A} n^s c_{ij}(v_{ij}) \rho_{ij} x^s_{ij},$$  \hspace{1cm} (13)$$

with \(V\) and \(X\) being as constraint sets (2) and (8), respectively.

The objective function \(z(v, x)\) is called a duration-population-frequency measure of hazmat exposure in arc \((i, j)\) (Wang et al., 2012) as it involves the product of \(c_{ij}, \rho_{ij}\) and \(n^s\). To determine the travel time on each link given the flow, we use the US Bureau of Public Roads (BPR) function. Algebraically, the function is of the form \(c_{ij}(v_{ij}) = c^0_{ij}(1 + 0.15(v_{ij}/l_{ij})^4)\), where \(v_{ij}\) is the amount of the flow on link \((i, j)\). In a road network, the number of hazmat trucks is relatively small when compared to the flow of regular traffic. Therefore, in calculating the link travel time, congestion induced by the traffic flow of hazmat trucks can be ignored. Consequently, we assume the link travel delay function \(c_{ij}\) to be only a function of regular traffic flow \(v_{ij}\). The remaining parameters, \(c^0_{ij}\) and \(l_{ij}\), represent the free-flow travel time and capacity for link \((i, j)\), respectively.

Once the administrator makes a decision on toll charges for regular and hazmat traffic (upper level), the network users’ routing problem (lower level) boils down to two separate decision making problems. The regular vehicles’ drivers follow the tolled-user equilibrium flow pattern that satisfies the variational inequality of the form (3) discussed in Section 2.1. Parallel with the regular traffic, hazmat carriers also try to optimize their individual utility assuming that all trucks associated to the same shipment take the same path. Therefore, similar to Section 2.2, hazmat carriers take the shortest path between each specified O-D pair by solving equation (9).

The hazardous-network dual toll pricing problem lends itself to a bilevel programming formulation of the following form:

$$\begin{align*}
\text{(TS)} & \min_{v \in V, x \in X, \tau \geq 0, t \geq 0} z(v, x) = \sum_{s \in S} \sum_{(i,j) \in A} n^s c_{ij}(v_{ij}) \rho_{ij} x^s_{ij}, \\
\text{(R)} & \sum_{(i,j) \in A} (c_{ij}(v_{ij}) + \sigma_R \tau_{ij})^\top (v'_{ij} - v_{ij}) \geq 0, \quad \forall v' \in V, \\
\text{(H)} & \min_{x \in X} \sum_{s \in S} \sum_{(i,j) \in A} n^s (c_{ij}(v_{ij}) + \sigma_H t_{ij}) x^s_{ij},
\end{align*}$$

where \(v_{ij}\) solves the following tolled-user equilibrium problem:

$$\sum_{(i,j) \in A} (c_{ij}(v_{ij}) + \sigma_R \tau_{ij})^\top (v'_{ij} - v_{ij}) \geq 0, \quad \forall v' \in V,$$  \hspace{1cm} (15)$$

with \(V\) being as set (2), and simultaneously \(x^s_{ij}\) solves the shortest path problem:

$$\min_{x \in X} \sum_{s \in S} \sum_{(i,j) \in A} n^s (c_{ij}(v_{ij}) + \sigma_H t_{ij}) x^s_{ij},$$  \hspace{1cm} (16)$$

with \(X\) being as set (8). Overall, equations (14)–(16) represent the bilevel model denoted by problem (TS). It is understood that in (TS), equation (14) corresponds to the upper-level (or administrator) problem, while the lower-level problem consists of two distinct problems (R) and (H) which are represented by equations (15)–(16). We note that the main decision variables of
the upper-level problem, i.e., $\tau_{ij}$ and $t_{ij}$, constitute parameters for the lower-level problems (R) and (H), respectively. We also note that problem (H) contains the decision variables of the regular vehicles’ problem, i.e., $v_{ij}$, which implies that hazmat carriers’ route choice is influenced by regular traffic flow through the link travel time function $c_{ij}(v_{ij})$.

Bilevel programming has a rich body of literature in transportation network regulation problems. For a recent review of applications we refer the reader to Yang and Bell (1998, 2001) and Colson et al. (2005). In the context of solution methodology, a comprehensive overview of the state of the art bilevel optimization algorithms is provided by Bard (1998). For linear bilevel problems, the most common approach is based on substitution of the lower-level problem with its equivalent KKT conditions. However, when bilevel problem involves binary or integer variables, branch and bound has been the dominant solution approach. Kara and Verter (2004) were the first to propose a bilevel formulation for the hazmat ND problem. They reformulate the problem as a single-level MIP and use CPLEX to solve the problem. However, their method cannot solve large-scale problems in a reasonable amount of computation time. A different approach to solve the network design bilevel formulation can be found in Erkut and Gzara (2008). Instead of solving the problem as a single level linear program, they suggest a heuristic algorithm that iterates between the upper-level and the lower-level problems. Although their solution method improves the computational performance, the solutions are sub-optimal.

Referring to problem (TS), we can infer that other than the equilibrium constraint (15), the hazmat shortest path problem (16) can be also seen as an equilibrium equation due to each shipment tends to route all its trucks through the shortest path, i.e., Wardrop’s first principle. Therefore, the bilevel problem (TS) is a special case of Mathematical Programs with Equilibrium Constraints (MPEC). For an overview of these problems and their solution algorithms, we refer the interested reader to Luo et al. (1996). Although MPEC are in general very difficult to solve, an easy solution methodology can be applied upon existence of the appropriate structural properties. In Sections 4 and 5, we propose our solution methodologies to the bilevel problem (TS) which take advantage of its analytical properties.

4 Solution Methodology

We exploit specific structural properties to demonstrate that problem (TS) as a MPEC is equivalent to a two-stage problem which can be solved efficiently. We propose methods to solve each stage of the problem.

In the context of transportation network regulation, a relevant study in terms of solution methodology is Hearn and Ramana (1998). They model the congestion toll pricing problem as MPEC, and apply a two-stage process analogous to the one introduced in Section 2.1 to obtain valid tolls. Similarly, Wang et al. (2012) show the equivalence of their dual toll pricing problem modeled as MPEC to a two-stage problem which can be solved easily. From the analytical point of view, the two-stage process is implementable when the administrator focuses on minimizing hazmat
transport risk but not travel costs (including paid tolls). In our case, there is no toll decision variable in the administrator’s objective function (14), thus an equivalent two-stage problem exists for problem (TS). In the following, we will further show that it is always possible for the administrator to find a nonnegative dual toll policy that induces minimum hazmat risk.

The first stage problem corresponds to determining the minimum-risk flow pattern, i.e., the administrator’s ideal solution. The minimum-risk problem (MR) is comprised of the objective function of problem (TS), i.e., equation (14), and the flow conservation constraints (2) and (8), without considering toll vectors. That is, we solve the following problem to obtain regular and hazmat traffic flow vectors inducing the minimum level of risk to the network:

\[
(MR) \quad \min_{v \in V, x \in X} z(v, x) = \sum_{s \in S} \sum_{(i,j) \in A} n^s c_{ij}(v_{ij}) \rho_{ij} x^s_{ij},
\]  

(17)

We do not enforce the resulting traffic pattern to be a user equilibrium. (MR) is a non-convex problem. Wang et al. (2012) considered linear delay functions for $c_{ij}(\cdot)$ and used a null-space active set method for solving the resulting non-convex quadratic programming problem. In contrast we use a nonlinear BPR function as described in Section 3.2. Under this function we can demonstrate that problem (17) is not even pseudo-convex or quasi-convex (can be proven by a counter-example). Thus, finding an optimal solution to (MR) is challenging.

An important issue surrounding the existence of nonnegative valid toll vector pairs, i.e., $\tau$ and $t$, is that our numerical solution to (17) may be sub-optimal; hence the existence results may not be guaranteed. For the existence of a nonnegative valid regular toll vector $\tau$, one can refer to Yin and Lawphongpanich (2006, Theorems 2 and 3). The existence of a nonnegative valid hazmat toll vector $t$ is assured by Marcotte et al. (2009, Proposition 1). We propose a simple post-iteration to the numerical solution to (MR) which guarantees the existence of a nonnegative valid toll vector pair.

We let $\tilde{v} \in V$ and $\tilde{x} \in X$ denote an approximate solution to (17) obtained using an algorithm. Then the post-iteration is that we solve two following optimization problems:

\[
\tilde{v}^{l+1} = \arg \min_{v \in V} z(v, \tilde{x}^l)
\]

(18)

\[
\tilde{x}^{l+1} = \arg \min_{x \in X} z(\tilde{v}^{l+1}, x),
\]

(19)

until convergence, starting with an initial solution $\tilde{x}^0 = \tilde{x}$ when $l = 0$. Note that (18) is an instance of convex traffic assignment problem which can be solved efficiently using, e.g., the Frank-Wolfe algorithm (Frank and Wolfe, 1956) and its variants, and (19) is a pure network flow problem (one shortest-path problem per carrier). Finally, let $(\tilde{v}, \tilde{x})$ represent a solution to the post-iteration phase.

Despite the simplicity, the post-iteration has two major outcomes. First, although the pair $(\tilde{v}, \tilde{x})$ may not be an optimal solution to (17), the risk value corresponding to $(\tilde{v}, \tilde{x})$ is at least as
Obtain an approximate solution to problem (MR), i.e. \((\tilde{v}, \tilde{x})\)

Perform post-iteration phase on \((\tilde{v}, \tilde{x})\)
Iteratively apply (18) and (19) to obtain \((\tilde{v}, \tilde{x})\)

Perform Inverse Optimization phase on \((\tilde{v}, \tilde{x})\)
Solve RTP\((\tilde{v})\) to obtain \(\tau \geq 0\), and HTP\((\tilde{x})\) to obtain \(t \geq 0\)

Figure 2: The Two-Stage Process Flowchart for Problem (TS)

good as the one induced by \((\tilde{v}, \tilde{x})\). That is:

\[
z(\tilde{v}, \tilde{x}) \geq \min_{v \in V} z(v, \tilde{x}) = z(\tilde{v}, \tilde{x}) \geq \min_{x \in X} z(\tilde{v}, x) = z(\tilde{v}, \tilde{x}),
\]

implying that post-iteration improves the quality of the solution to (MR). Second, after the post-iteration (18) and (19), we obtain the following existence results:

**Theorem 3.** For any approximate solution \(\tilde{v} \in V\) and \(\tilde{x} \in X\) to (MR), there exists a nonnegative valid dual-toll pair \((\tau, t) \geq 0\) for the traffic pattern \((\tilde{v}, \tilde{x})\) as obtained by (18) and (19).

**Proof.** Since \(z(\cdot, \tilde{x})\) is an increasing function, there exists a nonnegative valid regular toll \(\tau\) by Theorem 1. Similarly the existence of a nonnegative valid hazmat toll \(t\) is assured by the monotonicity of \(z(\tilde{v}, \cdot)\) and Theorem 2.

After obtaining \(\tilde{v}\) and \(\tilde{x}\), in the second stage we use the inverse optimization problems RTP\((\tilde{v})\) as equation (6), and HTP\((\tilde{x})\) as equation (12) to determine toll vectors \(\tau\) and \(t\), whose existence is guaranteed by Theorem 3. We note that the inverse optimization enforces achievement of dual-toll vector \((\tau, t)\) such that \(\tilde{v}\) and \(\tilde{x}\) are a tolled-user equilibrium and shortest-paths, respectively. The flowchart of the whole two-stage process is illustrated in Figure 2 for easy reference. Therefore, the original problem (TS) is analytically equivalent to this two-stage problem. In what follows, we focus on the computational methods to separately solve each stage of the two-stage problem.

### 4.1 The First Stage Problem

Note that (MR) is a non-convex optimization problem with two disjoint linear constraint sets \(V\) and \(X\). We further observe that for any given \(x\), \(\min_{v \in V} z(v, x)\) is an easy convex minimization with linear constraints, and for any given \(v\), \(\min_{x \in X} z(v, x)\) is also an easy convex minimization with linear constraints. However, a cyclic minimization approach will not guarantee optimality. On the other hand, other methods such as branch-and-bound which guarantee the optimality are likely to be computationally expensive.
For solving the first stage problem, we propose the Frank-Wolfe algorithm, which is devised for convex optimization problems. Although there is no guarantee on the optimality for the non-convex problem (MR), it is computationally efficient. We will show that an approximate solution is enough for the dual-toll pricing purpose, and there always exists a nonnegative dual-toll pair if a simple post-iteration is applied at the end of the Frank-Wolfe algorithm. For doing so, we apply the Modified Frank-Wolfe algorithm proposed by Fukushima (1984) which can be implemented almost as easily as the original one. Taking advantage of the information obtained at some previous iterations in determining the actual search direction, the modified version yields a considerably better performance than the ordinary Frank-Wolfe algorithm. Since implementation of the Modified Frank-Wolfe algorithm to problem (MR) is relatively straightforward, the details are described in a supplementary document (Appendix A, Part I).

It is evident that this algorithm is a heuristic approach for the non-convex problem (MR) and does not guarantee the global optimality. To study the quality of the solution provided by Modified Frank-Wolfe algorithm, we use two linearization approaches whose solution possesses the desirable property of global optimality. We then use those solutions to justify our results from the heuristic method. Note that throughout Section 4.1, we use the terms “Modified Frank-Wolfe” and “heuristic” interchangeably.

The linearization idea is rooted in the work of Wang and Lo (2010) who transform the network design problem with equilibrium constraints into a mixed-integer linear program that assures global optimality. We incorporate a similar logic to our model to linearize the travel time function. Since the non-convexity of (MR) stems from the nonlinear travel time function, we devise a scheme to approximate the BPR function $c_{ij}(v_{ij})$ with a set of piecewise-linear functions. For doing so, the key is to discretize the travel time function into small regions and approximate each region with a linear function. Below we provide our main logic towards the establishment of the two linearization approaches briefly. However, the comprehensive descriptions are delegated to the supplementary document (Appendix A, Parts II and III) since the details are somewhat tedious.

In the first linearization approach, referred to as piecewise-linear model 1, transformation of the nonlinear objective function (17) involves two parts. The first is to linearize the travel time function and the second is to linearize the resultant bilinear terms. For doing so, we partition the feasible domain of $v_{ij}$ into $N$ segments each denoted as $[n]$. We note that the larger the number of partitions, the finer the discretization adopted, resulting in a more accurate global optimal solution. Then we modify the original idea of Wang and Lo (2010) for the single variable function $c_{ij}(v_{ij})$, and specify a linear function to approximate $c_{ij}(v_{ij})$ within each region $[n]$. However, after replacing the linear transformation of the travel time in the objective function (17), the result is still nonlinear due to the bilinear terms. Every bilinear term is denoted by the product of a nonnegative continuous and a binary variable, namely $v_{ij}$ and $x_{sj}$. Therefore, we adapt the approach proposed by Gupte et al. (2013) to replace the bilinear term with an equivalent continuous variable. Eventually, by combining all the constraints required for transformations, we can convert nonlinear problem (MR) to a mixed-integer linear problem referred to as (MILP-1).
The second linearization approach in transformation of the objective function (17) to a linear problem considers the entire nonlinear term in problem (MR) as a single function with two variables, namely $v_{ij}$ and $x_{ij}^s$. Therefore, the linearization relies solely on the piecewise technique introduced by Wang and Lo (2010). In essence, in piecewise-linear model 2, we incorporate their idea of linearizing a two-variable nonlinear function while taking advantage of the binary nature of some variables. Consequently, another equivalent MILP model is obtained for problem (MR) which we refer to as (MILP-2).

The remainder of this section is mainly devoted to study the performance of the Modified Frank-Wolfe algorithm when applied to problem (MR). That is to verify that the solutions calculated by the heuristic are valid approximations when compared with the solutions obtained by (MILP-1/2).

For this purpose, we consider three networks for our test problem generation, having 8, 10, and 15 nodes, respectively. For each instance, the initial data includes start nodes, end nodes and arc free flow travel time. In the trip table, i.e., Origin-Destination (OD) pairs and OD demands, we consider all possible paired permutations of nodes for the regular traffic case while for the hazmat we randomly generate a few shipments due to the low volume of this type of traffic. In addition, we use Census site information to better estimate the arc population whereas the arc capacity and regular OD demand are sampled from the established test problems data available on the Transportation Network Test problems website (Bar-Gera, 2013) to effectively account for the network congestion. Table 2 provides a summary of these under-study networks. However, additional information on each network topology and the parameter values can be found in the supplementary document (Appendix B). We used Java as the programming language. When required, for solving LP or MILP, we use CPLEX 12.5.1 integrated with Java platform. All experiments were performed on an Intel(R) Processor 2.99 GHz computer. The supplementary document (Appendix A, Part IV) entails the complete version of our extensive experiments on the efficiency of the heuristic algorithm and the two exact models. Here, we underscore our most significant observations as well as the final comparative analysis.

<table>
<thead>
<tr>
<th>Test Problem</th>
<th>No. of Nodes</th>
<th>No. of Links</th>
<th>No. of OD pairs</th>
<th>Regular</th>
<th>Hazmat</th>
</tr>
</thead>
<tbody>
<tr>
<td>8-node network</td>
<td>8</td>
<td>13</td>
<td>27</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>10-node network</td>
<td>10</td>
<td>20</td>
<td>39</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>15-node network</td>
<td>15</td>
<td>28</td>
<td>97</td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Test networks

Our numerical tests on the Modified Frank-Wolfe algorithm show that starting from a large number of initial feasible solutions does not affect the quality of the (MR) objective value significantly. This establishes the robustness of the heuristic, and guarantees that generating a few initial solutions is sufficient to obtain a good approximation. Moreover, although we noticed that both congestion and network size increase the computational effort required by the algorithm, the (MR) problem is still solved very quickly even for the biggest test problem. For instance, the average
The computation time observed to solve problem (MR) for the 15-node network is less than 23 seconds.

To demonstrate the convergence of the Modified Frank-Wolfe to optimality, for the test problems illustrated in Table 2, we solve (MILP-1) and (MILP-2) using CPLEX 12.5.1 to obtain the corresponding optimal solutions. We note that generally in both (MILP-1/2) finer partition schemes yield more accurate solutions, at the expense of longer calculation times.

We now investigate how the two MILPs are different in obtaining the optimal solution. For doing so, either a time limit or an optimality gap is defined as the stopping criteria for (MILP-1/2). Our observations reveal that piecewise-linear model 1 always takes longer computational times to reach to the specified optimality gap when compared to model 2. Similarly, when the allotted time for running the MILPs is limited to a certain value, better optimality gaps are achieved under (MILP-2).

Let us summarize the results of our comparative analysis on the performance of the Piecewise Linear Models 1 and 2. First, a salient feature of both proposed linearization methods is that for every resolution scheme adopted we can state definitively that the solution found is globally optimal, with finer resolutions resulting to better approximations of the MR objective value. Moreover, the MILP-1 typically produces higher quality solutions but it is computationally more expensive to implement comparing with MILP-2. In fact the constraint set added to MILP-1 to remove the bilinear terms increases the computational complexity of the resultant MILP. However, transformation of bilinear terms is an exact process which along with approximating a single-variable BPR function generates more accurate solutions than that of MILP-2 approximating a two-variable function.

We recognize that by CPLEX parameter fine tuning, bounding the big-M constants, and devising an optimal discretization scheme, one can improve the computational efficiency of the linearization methods while achieving lower optimality gaps. However, we would like to emphasize that our focus in applying the piecewise-linear models is on obtaining a qualified solution and justification of heuristic results rather than solution efficiency.

To verify the solution quality of the heuristic we present a summary of our computational results for the three algorithms, namely Modified Frank-Wolfe, Piecewise Linear Model 1 and 2. We compare the quality of the solutions provided by each of the algorithms as well as the time required to obtain such solutions. Notice that here by solution we denote the MR objective value found by each method which for the two piecewise-linear cases it is determined by calculating equation (17) for the flow solutions obtained by (MILP-1/2) to remove the linearization error. Specifically, for the Frank-Wolfe algorithm, starting from a few initial solutions, we choose the least Min-Risk objective value and the corresponding total computational time. Similarly, for piecewise-linear models 1 and 2, the best found MR objective value is adopted for comparison purposes. As the required computation time, we report the instant of observing the solution by MILPs. We also present the percentage gap between the solution of the heuristic with the one provided by either of the two linearization approaches. The results prior to the post-iteration step are shown in Table 3.

From Table 3 it is observed that the computation time required to obtain the solution of
Table 3: Solution performance of the Modified Frank-Wolfe vs. MILP-1 and MILP-2. Run times are in seconds.

(MR) problem is significantly reduced under the Modified Frank-Wolfe algorithm for all problem instances. Although the piecewise-linear models produce a better exact MR objective value in all cases, the gaps between the heuristic solution and the ones provided by linearization methods are very small. Owing to the global optimality of the solution obtained from the linearization methods, we conclude that the Modified Frank-Wolfe also converges to the optimal solution of (MR) problem.

The Modified Frank-Wolfe can be applied to the first-stage problem (17) to determine a regular flow pattern  \( \tilde{v} \in V \) and a hazmat flow pattern  \( \tilde{x} \in X \) inducing a sufficiently low level of risk to the network. In the next section, we will further show that given such an approximate minimum-risk flow pattern, it is always guaranteed to obtain a nonnegative valid toll pair.

4.2 The Second Stage Problem

Let \((\tilde{v}, \tilde{x})\) denote an approximate minimum-risk flow obtained by solving the first-stage problem (MR). To assure the existence of a nonnegative valid dual-toll vector \((\tau, t)\), the post-iteration step is carried out. That is the optimization problems (18) and (19) are solved using the Frank-Wolfe algorithm (Frank and Wolfe, 1956) and a linear programming solver, respectively, to obtain \((\tilde{v}, \tilde{x})\). Now according to Theorem 3, we can determine a nonnegative regular toll vector \(\tau\) by solving

\[
\text{RTP}(\tilde{v}) : \quad \min_{\tau \geq 0} \sum_{(i,j) \in A} \tilde{v}_{ij}\tau_{ij},
\]

subject to

\[
c_{ij}(\tilde{v}_{ij}) + \sigma_R\tau_{ij} \geq \lambda^w_i - \lambda^w_j, \quad \forall (i, j) \in A, w \in W,
\]

\[
\sum_{(i,j) \in A} [c_{ij}(\tilde{v}_{ij}) + \sigma_R\tau_{ij}]\tilde{v}_{ij} = \sum_{i \in N} \sum_{w \in W} \lambda^w_i b^w_i.
\]
and a nonnegative hazmat toll vector \( t \) using

\[
\text{HTP}(\tilde{x}) : \min_{t \geq 0} \sum_{s \in S} \sum_{(i,j) \in A} n^s \tilde{x}_{ij}^s t_{ij}, \tag{24}
\]

subject to

\[
\pi_i^s - \pi_j^s \leq n^s (c_{ij} + \sigma_H t_{ij}) \quad \forall (i, j) \in A, s \in S, \tag{25}
\]

\[
\tilde{x}_{ij}^s (\pi_i^s - \pi_j^s - n^s (c_{ij} + \sigma_H t_{ij})) = 0 \quad \forall (i, j) \in A, s \in S. \tag{26}
\]

Both inverse optimization problems \( \text{RTP}(\tilde{v}) \) and \( \text{HTP}(\tilde{x}) \) are linear programs which can be solved efficiently.

To computationally test the validity of our claim, we also implemented the second stage on the problem instances described in Section 4.1. Our numerical experiments reveal that there always exists a nonnegative valid dual-toll vector \((\tau, t)\) resulting in a tolled-user equilibrium flow pattern for regular vehicles and shortest-path flows for hazmat trucks for which the corresponding risk is sufficiently low. That is, our results are consistent with Theorem 3.

Further, in Section 6, we will explain the impact of charging nonnegative dual-tolls, with the nonlinear travel delay function incorporated in problem (TS), on risk mitigation using a case study.

\section{5 A Single-Level Approach}

So far, in problem (TS) the sole objective of the administrator is to minimize the population exposure. However, when the administrator is interested in minimizing a combination of hazmat transport risk and paid tolls, or when there are constraints on toll values, the dual-toll pricing problem becomes truly bilevel, and the two-stage solution methodology is no longer a feasible approach. Therefore, in this section, we assume that in problem (TS), the objective function (14) is replaced with

\[
(GTS) \min_{v \in V, x \in X, \tau \geq 0, t \geq 0} z(v, x, \tau, t) = \sum_{s \in S} \sum_{(i,j) \in A} n^s c_{ij} (v_{ij}^s) \rho_{ij} x_{ij}^s \\
+ \varphi \left( \sum_{(i,j) \in A} \tau_{ij} v_{ij} + \sum_{s \in S} \sum_{(i,j) \in A} n^s t_{ij} x_{ij}^s \right), \tag{27}
\]

where problems (R) and (H) are kept same as in Section 3.2, and \( \varphi \) is a parameter converting users’ cost into population exposure units. We call this general toll setting problem as (GTS), wherein the administrator’s goal of charging toll is to route network users to minimize population exposure while keeping the users’ cost as low as possible. A common methodology to solve problem (GTS) is to convert it to a single-level mixed-integer linear program. Although single-level MIP formulations are applied in solving the general hazmat toll problem by Marcotte et al. (2009), in our case, due to nonconvexity of the population exposure term as well as nonlinearity of the carrier cost terms, the equivalent MILP reformulation is too computationally intensive. For this reason, in
the current section, our main purpose is to suggest a heuristic for finding an approximate solution to problem \( (GTS) \) which is markedly more efficient than the MILP. Our algorithm is a variation to the Equilibrium-Decomposed Optimization (EDO) heuristic first introduced for the continuous equilibrium network design problem by Suwansirikul et al. (1987). In the following subsections, we briefly discuss the obstacles toward solving bi-level problem \( (GTS) \) with the conventional EDO heuristic, and present a formal statement of the altered EDO, namely 2-Step EDO, along with numerical tests.

5.1 The 2-Step EDO Heuristic

The major difficulty in solving problem \( (GTS) \) is due to the unknown functional forms. In particular, with \( \tau \) and \( t \) being the two vectors of decision variables determined by the administrator, we note that \( v(\tau) \) is a UE flow pattern wherein \( v_{ij}, \forall (i,j) \in A \), is an implicit function of regular toll \( \tau \), i.e., \( v_{ij} = v_{ij}(\tau) \). Similarly, \( x^s_{ij} = x^s_{ij}(t), \forall (i,j) \in A, \forall s \in S \), denotes the hazmat shortest path flow pattern \( x \) as an implicit function of the hazmat toll \( t \). To avoid these unknown functional forms, when the arc improvement vector constitutes the single decision variable of the administrator, the conventional EDO of Suwansirikul et al. (1987) suggests decomposition of the problem into subproblems corresponding to each arc. At each iteration, a new vector of arc improvements is generated by solving a user optimized equilibrium problem and simultaneously minimizing the subproblems with a one-dimensional search routine. For the two decision variable case, although one may perform a dual search for both \( \tau \) and \( t \), we choose to limit the one-dimensional search routines to search only for \( \tau \) in order to reduce the approximation error and increase the computational efficiency. We now describe a modification of the Suwansirikul et al. (1987)’s decomposition scheme tailored to problem \( (GTS) \).

An appropriate approach for \( (GTS) \) is to decompose it into interacting subproblems with regard to regular toll, \( \tau \), one for each arc considered to be tolled. Each subproblem has the following form

**subproblem of arc \( (i,j) \):**

\[
\min_{\tau} z_{ij}(\tau) = \sum_{s \in S} n^s c_{ij}(v_{ij}(\tau)) \rho_{ij} x^s_{ij}(t) + \varphi \left( \tau_{ij} v_{ij}(\tau) + \sum_{s \in S} n^s t_{ij} x^s_{ij}(t) \right).
\]  

(28)

In fact, in equation (28), assuming that \( x_{ij}(t) \) and \( t \) are available from another routine, we transformed the bivariate problem into univariate unconstrained nonlinear subproblems with respect to \( \tau \). The interaction among the subproblems stems from the fact that the value of \( v_{ij} \) depends on the vector \( \tau \) for all arc tolls. Referred to as Modified EDO heuristic, our algorithm is similar to the conventional EDO in its strategy of applying simultaneous line searches along each arc, as well as solving only one user equilibrium at each iteration. Nevertheless, it is different in that equation (27) is generally non differentiable due to binarity of \( x \) which prohibits the use of derivative-based line search techniques such as Bolzano search. Accordingly, Modified EDO can only employ derivative-free line search methods such as Golden Section or Fibonacci Search methods which rely on the
value of the function at a candidate point. The Modified EDO algorithm is as follows:

**Modified EDO**

**Step 0. Initialization**

- For the links considered for regular toll charge, let $L^0 = (\ldots, L^0_{ij}, \ldots)$ and $U^0 = (\ldots, U^0_{ij}, \ldots)$ represent the initial lower and upper bounds on $\tau$, respectively.
- Solve the UE problem (R), using Frank-Wolfe algorithm, once for $\tau = L^0$ to obtain the regular flow vector $v(L^0)$, and once for $\tau = U^0$ to obtain $v(U^0)$.
- Obtain hazmat shortest path flow $x(t)|_{L^0}$ and hazmat toll $t|_{L^0}$ corresponding to $v(L^0)$. Similarly, obtain $x(t)|_{U^0}$ and hazmat toll $t|_{U^0}$ corresponding to $v(U^0)$. For doing so, solve the following bi-level model for hazmat toll setting given a regular flow vector $\tilde{v}$:

$$\begin{align*}
\text{(HTS)} \quad & \min_{x \in \{0,1\}, t \geq 0} \sum_{s \in S} \sum_{(i,j) \in A} n^s c_{ij}(\tilde{v}_{ij}) \rho_{ij} x^s_{ij} + \varphi \sum_{s \in S} \sum_{(i,j) \in A} n^s t_{ij} x^s_{ij} \\
\text{subject to} \quad & \sum_{(i,j) \in A} x^s_{ij} - \sum_{(j,i) \in A} x^s_{ji} = e^s_i \quad \forall i \in N \quad \forall s \in S.
\end{align*}$$

We note that upon replacing the lower-level problem (H) by its primal-dual optimality conditions and linearizing the bilinear terms as described in Gupte et al. (2013), problem (HTS) is transformed into a single-level linear model which can be solved with a powerful linear programming software such as Cplex.
- For each arc $(i, j)$, evaluate $z_{ij}(L^0)$ and $z_{ij}(U^0)$. Set $k = 1$ and go to step $k$.

**Step k. Iterative search for $\tau$**

- For each $(i, j)$, specify a new point $\tau^k_{ij}$ in the interval $[L^k_{ij} - 1, U^k_{ij} - 1]$ using a line search procedure such as Golden Section.
- Solve the UE problem (R) for given $\tau^k$ and get $v(\tau^k)$.
- Solve problem (HTS) for given $v(\tau^k)$ and obtain $x(t)|_{\tau^k}$ and $t|_{\tau^k}$.
- For each arc $(i, j)$, evaluate $z_{ij}(\tau^k)$.
- Calculate new intervals $[L^k_{ij}, U^k_{ij}]$, for each arc $(i, j)$ by Golden Section.
- For all $(i, j)$, if $U^k_{ij} - L^k_{ij} < \Delta$, stop and set $\tau^*_{ij} = (L^k_{ij} + U^k_{ij})/2$; otherwise set $k = k + 1$.

Since the line search procedure is based on function evaluations, the quality of the solution provided by Modified EDO is very sensitive to the magnitudes of the risk and revenue terms. In particular, our numerical observations on the test problems of Table 2 reveal that for some cases, in the first
iterations, large values of regular toll yield revenue terms of higher magnitudes. Hence, initially the algorithm favors the revenue term without making any significant improvement to the risk. Upon obtaining a revenue term with the same order of magnitude as risk, the algorithm starts minimizing both terms almost equally. At this stage, the intervals of $\tau$ are no longer wide enough to substantially minimize the risk, and consequently we can get low quality solutions. This motivates us to suggest a 2-Step EDO which resolves the shortcomings of the Modified EDO.

The 2-Step EDO uses Modified EDO algorithm in two consecutive steps. Ignoring the revenue terms in problem (GTS), the first step applies Modified EDO to obtain a solution $(\tau^I, v^I, t^I, x^I)$ which only minimizes the risk. From the underlying information in problem (GTS), one can notice that for any link $(i, j)$, regardless of the amount of regular flow (i.e., $v^I_{ij}$), without having a hazmat truck passing through the link, the corresponding risk value is zero implying that charging toll not only is neutral to risk reduction but also increases the revenue. Therefore, we incorporate the following modification to the current solution:

Modification to the current regular toll vector, $\tau^I$:

- $\forall s \in S, \forall (i, j) \in A$: if $x_{ij}^s = 0$, set $\tau^I_{ij} = 0$;
- Otherwise, set $L_{ij}^{\text{new}} = L_{ij}^0$ and $U_{ij}^{\text{new}} = \tau^I_{ij}$.

Starting from the new toll bounds, $L^{\text{new}}$ and $U^{\text{new}}$, the second step applies Modified EDO to problem (GTS) to minimize risk and revenue simultaneously. In fact, modifying the first step solution helps the second step search procedure to start from the minimum risk solution and gradually redirect the exploration towards minimizing the revenue at the expense of a higher risk. This allows the algorithm to efficiently search in the direction of risk and revenue and lead to a qualified solution which favors both objective terms. Figure 3 illustrates the procedure undertaken by the 2-Step EDO algorithm.

5.2 Numerical Tests

To evaluate the performance of the Modified EDO and 2-Step EDO heuristics in terms of computational effort and solution efficiency, we present results of our experiments on test problems of Table 2 as well as Test network 1 in Suwansirikul et al. (1987, Figure 1). Also, we report the results of the LINGO approach as an exact method that serves to verify that the solutions calculated by the two heuristics are in fact valid approximations to problem (GTS). LINGO, produced by LINDO Systems Inc., includes a set of built-in solvers to tackle a wide variety of linear, nonlinear, and integer problems. For nonlinear models, it applies various techniques to obtain locally or globally optimal solutions. In our experiments, we used Extended LINGO 11.0 which can handle unlimited number of constraints and variables. Table 4 depicts a summary of our comparative analysis results. As a companion, Table 5 describes the details of the network for each case.

For each test network, we consider multiple levels of regular and hazmat OD pair to capture different regular traffic congestion and hazmat travel demand. Increasing the number of OD pairs generates more constraints and consequently increases the computational effort needed to arrive at
Step 2: Minimize Risk, Regular Revenue, and Hazmat Revenue

Consider the new regular toll bounds, \( L_{\text{new}}, U_{\text{new}} \).

Step 1: Minimize the Risk

Consider sub problem of link \((i, j)\) as
\[
z_{ij}(\tau) = \sum_{s \in S} n^s c_{ij}(v_{ij}(\tau)) p_{ij} x_{ij}^s(\tau).
\]
Apply modified EDO to obtain the 1st step solution \( \tau^I, v^I, t^I, x^I \).

Modify the 1st step regular toll solution i.e., \( \tau^I \)

Use hazmat shortest path information, i.e., \( x^I \),

Obtain new regular toll bounds \( L_{\text{new}}, U_{\text{new}} \).

Step 2: Minimize Risk, Regular Revenue, and Hazmat Revenue

Consider the new regular toll bounds, \( L_{\text{new}}, U_{\text{new}} \).

Apply modified EDO to obtain 2nd step solution \( \tau^{II}, v^{II}, t^{II}, x^{II} \).

Figure 3: The 2-Step EDO algorithm

either a global or a local solution to the problem. We limit our attention to situations where the LINGO solver can find either a global or a local solution within 24 hours. For simplicity, all arcs are tollable without a specified upper limit. For this reason, in both heuristics the initial lower and upper bounds on the arc tolls are set to zero and a very large number, respectively.

We note that \((GTS)\) is a bi-objective problem with two (possibly) conflicting terms, i.e., risk and revenue. There is typically no unique solution to a Multi-Objective optimization problem, and most often multiple optimal solutions can be obtained by improving one objective and deteriorating the other. A full scale Multi-Objective analysis is not considered. Instead we numerically investigate the efficiency of the heuristics building our comparisons based upon the total objective function values \((27)\) assuming \( \varphi = 1 \). An important result not explicitly presented in the table is that for some cases there exists a slight trade-off between the obtained risk and revenue values. These differences are quite acceptable as long as their corresponding total objective function values are close.

As demonstrated by the objective gap, the dual-toll policy obtained from 2-Step EDO algorithm yields nearly identical total objective values for most of the cases when compared with LINGO. For the Modified EDO heuristic, the solution diverges from LINGO in almost all cases (except for the 10-node test problem), implying that the 2-Step EDO outperforms the Modified EDO in terms of the solution efficiency. Note the dramatically different and erroneous solution calculated by Modified EDO in cases 1 and 3 for the 4-node network as it is in all cases for the 8-node network as well as cases 23 and 26 for the 15-node network.

Our experiments demonstrate that for larger problems the solution status using LINGO remains either local or unknown after many hours. Nonetheless, to further test the proposed heuristics for more complicated networks, among such cases (using \( ^{\dagger} \)) we report those where a local optimal solution is obtained by LINGO after 24 hours. It is evident from Table 4 that unlike the Modified
EDO, the 2-Step EDO algorithm yields, in a reduced computing time, either nearly identical local optimal solutions, ex., cases 9 and 26, or more attractive solutions, ex., cases 7, 8, 10 and 11, when compared to LINGO.

We conclude that the LINGO approach is computationally very demanding and infeasible for any but very small problems when compared to either of the heuristics, and its computation time exponentially grows with the network size, congestion and its complexity. For both heuristics, we observe that most of the computational effort is in solving the UE problem (R) using the Frank-Wolfe algorithm. It is also obvious that the Modified EDO algorithm, despite generating low quality solutions, consistently requires fewer iterations of Golden Section to satisfy the same stopping tolerance ($\Delta = 0.2$), indicating that, in general, it dominates the 2-Step EDO in terms of computational efficiency. Nonetheless, we notice that there exists cases, ex., 7, 9 and 16, wherein the trend is reversed. This occurs when less Frank-Wolfe iterations are undertaken by the 2-Step EDO to obtain the UE solution.

The overall conclusion of our numerical tests is that the 2-Step EDO is markedly more efficient than the Modified EDO algorithm in terms of solution quality. Also, for networks with significant congestion and complexity which cannot be handled by LINGO, the 2-Step EDO appears to be promising as a useful algorithm that can obtain quite good approximate solutions in significantly less amount of time. Therefore, for finding a locally optimal solution to problem (GTS), we suggest using the 2-Step EDO approach.

We note from Table 4 that there are zero hazmat revenue terms for all the cases. Thus the toll revenue reflects toll collected from regular vehicles. We observe that, in majority of the cases, the most risky arcs are subject to toll, and hazmat carriers can always take a detour without paying tolls due to the size of the network. Here, hazmat tolls serve to discourage the carriers from using the risky arcs, misleading one to conclude that the dual-toll policy acts as the road ban policy which completely prohibits hazmat carriers from the use of road segments. We now develop an illustrative example to demonstrate that the dual-toll policy is in fact not restrictive, and can produce flexible solutions to the carriers.

For tractability, consider Test network 1 of Suwansirikul et al. (1987) with the data being as in Tables 6 and 7. Let arcs 1, 2 and 3 be subject to toll charges and assume that the upper bounds on regular tolls and hazmat tolls are 50 and 100, respectively. From Table 7 we note that only one route exists for hazmat shipments 1 and 3, whereas the hazmat carriers of shipment 2 can be directed to either of the two existing routes (i.e., route I: arc 2 or route II: arcs 1 and 3) by the network administrator. For this test network, problem (GTS) is solved using the 2-Step EDO algorithm under two different cases. In case 1, we let the population of arc 3 be at its original value, i.e., $\rho_3 = 200$. As shown in Table 8, in this case, the minimum value of the objective function (27) is obtained when there is no toll charged to hazmat carriers. In fact, due to parameter values of each arc and the congestion stemming from the regular vehicles in arc 2, the route made up of arcs 1 and 3 is not only less risky when compared to arc 2 but also is shorter in terms of travel time, and consequently is selected by shipment 2. This property obviates the need to charge any toll as
<table>
<thead>
<tr>
<th>Case</th>
<th>Solution Type</th>
<th>LINGO</th>
<th>Modified EDO</th>
<th>2-Step EDO</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Global</td>
<td>2469.86</td>
<td>2945.94</td>
<td>2469.86</td>
</tr>
<tr>
<td>2</td>
<td>Global</td>
<td>7150.91</td>
<td>8303.92</td>
<td>7322.19</td>
</tr>
<tr>
<td>3</td>
<td>Global</td>
<td>3389.86</td>
<td>3857.11</td>
<td>3389.86</td>
</tr>
<tr>
<td>4</td>
<td>Global</td>
<td>8070.92</td>
<td>8766.06</td>
<td>8444.92</td>
</tr>
<tr>
<td>5</td>
<td>Global</td>
<td>10707.51</td>
<td>10718.75</td>
<td>10934.49</td>
</tr>
<tr>
<td>6</td>
<td>Global</td>
<td>80913.27</td>
<td>173736.15</td>
<td>80914.86</td>
</tr>
<tr>
<td>7</td>
<td>Local</td>
<td>163788.1</td>
<td>490756.92</td>
<td>81026.94</td>
</tr>
<tr>
<td>8</td>
<td>Local</td>
<td>124858.4</td>
<td>536943.91</td>
<td>81102.14</td>
</tr>
<tr>
<td>9</td>
<td>Local</td>
<td>113760.7</td>
<td>761750.14</td>
<td>114085.94</td>
</tr>
<tr>
<td>10</td>
<td>Local</td>
<td>118068.7</td>
<td>239783.27</td>
<td>116180</td>
</tr>
<tr>
<td>11</td>
<td>Local</td>
<td>273323.5</td>
<td>568796.57</td>
<td>368686.4</td>
</tr>
<tr>
<td>12</td>
<td>Global</td>
<td>3.96974E7</td>
<td>3.97155E7</td>
<td>3.96808E7</td>
</tr>
<tr>
<td>13</td>
<td>Global</td>
<td>1626.21E5</td>
<td>1629.97E5</td>
<td>1629.97E5</td>
</tr>
<tr>
<td>14</td>
<td>Global</td>
<td>1632.15E5</td>
<td>1634.4E5</td>
<td>1634.18E5</td>
</tr>
<tr>
<td>15</td>
<td>Global</td>
<td>1626.98E5</td>
<td>1630.20E5</td>
<td>1630.20E5</td>
</tr>
<tr>
<td>16</td>
<td>Global</td>
<td>1630.24E5</td>
<td>1634.4E5</td>
<td>1630.20E5</td>
</tr>
<tr>
<td>17</td>
<td>Global</td>
<td>5669.20E4</td>
<td>5672.79E4</td>
<td>5670.43E4</td>
</tr>
<tr>
<td>18</td>
<td>Global</td>
<td>2305.34E5</td>
<td>2308.72E5</td>
<td>2309.83E5</td>
</tr>
<tr>
<td>19</td>
<td>Global</td>
<td>2319.45E5</td>
<td>2324.62E5</td>
<td>2325.87E5</td>
</tr>
<tr>
<td>20</td>
<td>Global</td>
<td>100196.95</td>
<td>169497.65</td>
<td>108656.77</td>
</tr>
<tr>
<td>21</td>
<td>Local</td>
<td>143418.95</td>
<td>174183.54</td>
<td>171064.71</td>
</tr>
<tr>
<td>22</td>
<td>Local</td>
<td>190541.59</td>
<td>180316.26</td>
<td>169340.49</td>
</tr>
<tr>
<td>23</td>
<td>Local</td>
<td>314151.08</td>
<td>399844.79</td>
<td>328904.86</td>
</tr>
</tbody>
</table>

Table 4: Performance of the 2-Step EDO vs. Modified EDO and LINGO. Table 5 provides additional information for each case.

*a* Denotes the total toll collected from regular and hazmat users. However, hazmat toll revenue is zero in all the cases.

*b* Indicates the gap between the total objective function value (27) of the corresponding heuristic and LINGO.

‡Indicates the cases where the corresponding heuristic provides better solutions than the local optima found by LINGO.
Table 5: Details of the understudy network for each case of Table 4. Additional information is found in Figure 4 and Tables 6 and 7 (4-node); Tables B.1 and B.2 in the supplementary document (8-node); Tables B.3 and B.4 (10-node); and Tables B.5 and B.6 (15-node).

Figure 4: An illustrative example: Test network 1 of Suwansirikul et al. (1987)

the decision of hazmat carriers follows the administrator’s desired solution.

Now let the arc 3 population increase to 600 for case 2. The UE solution remains nearly identical to case 1 maintaining the same relation between the travel times of routes I and II. That is, with no hazmat tolls, shipment 2 still tends to select route II. However, with the excessive population exposure in arc 3, route II now becomes very risky. To redirect the carriers of shipment 2 while considering their own preferences, a toll is charged on arc 1, indicating that now route I becomes shorter in terms of travel time, and is selected by carriers of shipment 2. Note the routes undertaken by shipment 2 for cases 1 and 2 in Table 8.

As mentioned by Marcotte et al. (2009), one disadvantage of the toll problem is that tolls are not necessarily set on risky arcs, and consequently may not be fair to all carriers. In this example, although arc 3 is considered as more risky, arc 1 is being charged and all the hazmat carriers cost is incurred by shipment 1 who does not go through the populated (risky) area. Indeed, they pay
Table 6: Arc attributes for the illustrative 4-node example in Figure 4, with $\rho_a$: the population density along arc $a$ and $c_a(v_a) = A_a + B_a(v_a/l_a)^4$

<table>
<thead>
<tr>
<th>Arc $a$</th>
<th>$A_a$</th>
<th>$B_a$</th>
<th>$l_a$</th>
<th>$\rho_a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>0.6</td>
<td>40</td>
<td>200</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>0.3</td>
<td>50</td>
<td>150</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>0.9</td>
<td>40</td>
<td>200</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>0.75</td>
<td>40</td>
<td>400</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>0.45</td>
<td>40</td>
<td>250</td>
</tr>
</tbody>
</table>

Table 7: Trip information for the illustrative example

Table 8: Comparison of results for the illustrative example under the two cases

6 Case Study on Sioux Falls Network

As a basis for our case study, we use a network based on the roads of the city of Sioux Falls, South Dakota, consisting of 24 nodes and 76 arcs, as shown in Figure 5. For consistency, we use the same network parameters of Suwansirikul et al. (1987, Table X) as the input parameters of the case study.
Figure 5: Sioux Falls Network

travel delay function $c_{ij}(v_{ij})$. We also consider the original 552 OD pairs for the regular vehicles with the corresponding travel demands as displayed in Suwansirikul et al. (1987, Table XI). The Census site information provides an effective framework for estimating each arc $(i,j)$ population. Since there is no reliable record of the actual OD pairs used by the hazmat carriers in the region, hazmat shipments including origin, destination and demand are randomly generated assuming that all trucks contain the same hazmat type and pose the same level of exposure to the network.

6.1 The Two-Stage Process

To investigate the benefits of the dual-toll policy for the case where the leader’s objective is to minimize the population exposure and every link is considered as tollable, we solved problem (TS) by the two-stage process depicted in Figure 2. To obtain the solutions to both problems (MR) and (18), Java was used to code the Modified Frank-Wolfe algorithm while the inverse optimization problems RTP($\tilde{\nu}$) and HTP($\tilde{x}$) as well as problem (19) were solved, using CPLEX 12.5.1. We performed all experiments on an Intel(R) Processor 2.99 GHz computer.

To estimate parameters $\sigma_R$ and $\sigma_H$, we noted the Value of Time (VoT) parameter which converts the time into monetary units. Hence, a reasonable assumption is to set $\sigma_R$ and $\sigma_H$ to the inverse of the VoT of regular drivers and hazmat carriers, respectively. Various measures of VoT can be found in the literature each incorporating different travel cost factors. In particular, Litman (2009) estimates the travel time cost of a regular vehicle as $20.4$/hr incorporating fuel cost, maintenance
cost and insurance cost whereas Park et al. (2014) consider the operational cost of a truck to be \$24.44/hr assuming the similar travel cost factors. According to this, the parameters $\sigma_R$ and $\sigma_H$ were set to 0.05 and 0.04, respectively.

In order to better analyze the performance of the dual-toll policy, we compute the population exposure (risk), regular total travel delay and hazmat total travel delay for the unregulated network where no toll is charged to regular vehicles and hazmat trucks, i.e., no-toll case. For doing so, we identify the untolled UE flow pattern of regular vehicles, $v^{\text{no-toll}}$, satisfying the following VI:

$$
\sum_{(i,j) \in A} c_{ij}(v^{\text{no-toll}})^T (v_{ij} - v^{\text{no-toll}}_{ij}) \geq 0 \quad \forall v \in V,
$$

as well as the untolled hazmat flow pattern, $x^{\text{no-toll}}$, by solving the shortest path problem as below:

$$
x^{\text{no-toll}} = \arg \min_{x \in X} \sum_{s \in S} \sum_{(i,j) \in A} n^s c_{ij}(v^{\text{no-toll}}) x_{ij}^s.
$$

Consequently, under the no-toll case, the network performance can be characterized by the following measures:

$$
z(v^{\text{no-toll}}, x^{\text{no-toll}}) = \sum_{s \in S} \sum_{(i,j) \in A} n^s c_{ij}(v^{\text{no-toll}}) \rho_{ij} x_{ij}^s \text{ no-toll},
$$

$$
D_R(v^{\text{no-toll}}) = \sum_{(i,j) \in A} c_{ij}(v^{\text{no-toll}}) v^{\text{no-toll}}_{ij},
$$

$$
D_H(v^{\text{no-toll}}, x^{\text{no-toll}}) = \sum_{s \in S} \sum_{(i,j) \in A} n^s c_{ij}(v^{\text{no-toll}}) x_{ij}^s \text{ no-toll},
$$

where $z$, $D_R$, and $D_H$ denote the risk, regular vehicles delay and hazmat trucks delay, respectively.

For the dual-tolled case, we let $(v^*, x^*)$ denote the minimum-risk flow solution obtained from the post-iteration step, and $(\tau^*, t^*)$ be the dual-toll policy determined by RTP($v^*$) and HTP($x^*$).

Equipped with the above definitions, we are now ready to compare the dual-tolled network performance with its no-toll counterpart. In particular, we compute the percentage change in risk, regular travel delay and hazmat travel delay using the following metrics:

$$
\% \text{ Change of Risk} = \frac{z(v^*, x^*) - z(v^{\text{no-toll}}, x^{\text{no-toll}})}{z(v^{\text{no-toll}}, x^{\text{no-toll}})} \times 100
$$

$$
\% \text{ Change of Delay (Regular)} = \frac{D_R(v^*) - D_R(v^{\text{no-toll}})}{D_R(v^{\text{no-toll}})} \times 100
$$

$$
\% \text{ Change of Delay (Hazmat)} = \frac{D_H(v^*, x^*) - D_H(v^{\text{no-toll}}, x^{\text{no-toll}})}{D_H(v^{\text{no-toll}}, x^{\text{no-toll}})} \times 100
$$

For the dual-tolled problem, we also report the total amount of tolls collected from regular drivers and hazmat carriers, namely, $\tau^T v^*$ and $t^T x^*$.

The results of our numerical experiments under different numbers of regular and hazmat OD
Table 9: Dual-toll performance under different numbers of Regular and Hazmat OD pair with first-stage problem as (MR)

<table>
<thead>
<tr>
<th>Case</th>
<th>No. of OD pairs</th>
<th>% Change of Risk</th>
<th>% Change of Delay</th>
<th>Tolls Collected</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Regular Hazmat</td>
<td></td>
<td></td>
<td>Regular Hazmat</td>
</tr>
<tr>
<td>1</td>
<td>10 2</td>
<td>−25.35</td>
<td>304.43</td>
<td>16.63</td>
</tr>
<tr>
<td>2</td>
<td>10 5</td>
<td>−15.07</td>
<td>84.33</td>
<td>10.81</td>
</tr>
<tr>
<td>3</td>
<td>10 10</td>
<td>−12.65</td>
<td>214.77</td>
<td>11.97</td>
</tr>
<tr>
<td>4</td>
<td>20 10</td>
<td>−12.64</td>
<td>170.32</td>
<td>11.96</td>
</tr>
<tr>
<td>5</td>
<td>20 15</td>
<td>−8.43</td>
<td>116.91</td>
<td>8.55</td>
</tr>
<tr>
<td>6</td>
<td>20 20</td>
<td>−6.23</td>
<td>94.04</td>
<td>11.35</td>
</tr>
<tr>
<td>7</td>
<td>50 20</td>
<td>−6.68</td>
<td>65.84</td>
<td>10.68</td>
</tr>
<tr>
<td>8</td>
<td>100 20</td>
<td>−7.27</td>
<td>207.81</td>
<td>8.28</td>
</tr>
<tr>
<td>9</td>
<td>200 20</td>
<td>−12.55</td>
<td>1679.94</td>
<td>2.61</td>
</tr>
<tr>
<td>10</td>
<td>400 20</td>
<td>−41.60</td>
<td>31709.63</td>
<td>-33.20</td>
</tr>
<tr>
<td>11</td>
<td>552 10</td>
<td>−63.36</td>
<td>18471.56</td>
<td>-60.30</td>
</tr>
</tbody>
</table>

pairs are reported in Table 9. From Table 9, it is evident that there is a significant trade-off between the no-toll and dual-tolled scenarios in terms of risk exposure and travel delay. That is, the exposure to hazmat transportation can be significantly reduced under the dual-toll policy. However, this requires the regular vehicles to incur an increase in their travel time. Note the negative % changes of risk, and positive % changes of travel delay for regular drivers. Although one may predict a similar behavior for hazmat carriers’ travel delay, from case 11 we notice that when all regular OD pairs are considered, i.e., the congested network case, a higher decrease in the risk can be achieved at the expense of substantial increase in regular travel delay, whereas the hazmat travel delay is reduced to less than the half of its value in the no-toll case.

It is important to note that the change of delay for hazmat carriers is affected by the network congestion, i.e., the number of regular OD pairs considered. In particular, for the cases with lower congestion, we observed that the dual-toll policy detours the hazmat carriers into longer routes which are not being used by regular vehicles. Therefore, it leads to an increase in the carriers travel delay since the congestion induced by regular vehicles in the no-toll case is negligible. Cases with positive % change of delay for hazmat carriers refer to such situation. On the other hand, when the network is highly congested, the dual-toll deviates hazmat carriers into less congested routes to mitigate the risk exposure, and consequently results in a decrease in the travel delay they experience.

One can also see from Table 9 that the reduction of risk is often achieved by collecting tolls from the regular vehicles. Nevertheless, when the number of hazmat OD pairs is large enough (above 15 for the network considered), owing to the similar reasoning described in Section 5.2 for the illustrative example, such reduction is obtained by collecting tolls from both types of traffic.

So far, our results indicate that the dual-toll policy, motivating the regular and hazmat traffic to use divergent routes, is an effective tool for risk-mitigation which produces attractive solutions for
Table 10: Dual-toll performance under different numbers of Regular and Hazmat OD pair, with first-stage problem as equation (31) and the weight parameters as \( \omega_1 = 0.5, \omega_2 = 0.5, \) and \( \omega_3 = 0 \)

<table>
<thead>
<tr>
<th>No. of OD pairs</th>
<th>% Change of Risk</th>
<th>% Change of Delay</th>
<th>Tolls Collected</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case</td>
<td>Regular</td>
<td>Hazmat</td>
<td>Regular</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>2</td>
<td>-25.12</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>5</td>
<td>-14.95</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>10</td>
<td>-12.57</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
<td>10</td>
<td>-12.53</td>
</tr>
<tr>
<td>5</td>
<td>20</td>
<td>15</td>
<td>-8.85</td>
</tr>
<tr>
<td>6</td>
<td>20</td>
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<tr>
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<td>-10.61</td>
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<tr>
<td>10</td>
<td>400</td>
<td>20</td>
<td>-27.10</td>
</tr>
<tr>
<td>11</td>
<td>552</td>
<td>10</td>
<td>-24.39</td>
</tr>
</tbody>
</table>

The regulator. This implies to the consistency of our numerical results with the theory we established in this paper. However, since the regulator does not directly incorporate the users’ viewpoint in its decision-making process, some users may have to incur unbearable time and financial burdens under the dual-tolled network. Note the substantially increased travel delay as well as the total tolls paid by regular vehicles in Table 9. Clearly, such policies would not be feasible to the users and the regulator has to ensure their satisfaction in the implementation phase. Hence, to obtain solutions that can be more acceptable to the network users, we suggest replacing the (MR) problem in the first-stage, i.e., equation (17), with a new administrator’s problem of the following form:

\[
\min_{v \in V, x \in X} J(v, x) = \omega_1 \sum_{s \in S} \sum_{(i,j) \in A} n^s c_{ij}(v_{ij}) \rho_{ij} x_{ij}^s + \omega_2 \sum_{(i,j) \in A} c_{ij}(v_{ij}) v_{ij} + \omega_3 \sum_{s \in S} \sum_{(i,j) \in A} n^s c_{ij}(v_{ij}) x_{ij}^s
\]

with problems (R) and (H) being the same as in Section 3.2 in the second-stage, and \( \omega_1, \omega_2, \omega_3 \in [0,1] \), \( \sum_i \omega_i = 1 \), the weight parameters, addressing the relative significance of each term.

As one can note, other than the risk exposure term, in equation (31), we incorporate the regular total travel delay, also known as total network congestion, and the hazmat total travel delay, hoping to obtain implementable solutions wherein the administrator compromises its risk mitigation targets for the sake of the users’ travel time.

To evaluate the efficiency of the provided dual-toll under the new administrator’s problem, we repeated all the experiments of Table 9, with equation (17) being replaced with (31) in the two-stage process of Figure 2. Note that due to having similar structural properties as (MR), an approximate solution to equation (31) can still be found by the Modified Frank-Wolfe algorithm. Table 10 presents the results of our computational tests based on the aforementioned metrics.

An interesting observation, when comparing each problem instance of Table 10 with its counterpart in Table 9, is that a slightly less reduction of risk is achieved with no increase in regular travel.
delay and slightly higher increase in hazmat travel delay. In fact, with a few number of regular OD pairs considered, the congestion is very low leading to no change in the UE flow pattern and no tolls being collected from regular vehicles. However, now hazmat carriers have to incur higher travel times to avoid risky arcs. For cases with higher congestion, i.e., starting from case 7, the dual-toll policy has the advantage of directing the regular vehicles to the less risky arcs while reducing their travel delay at the expense of viable tolls paid by the drivers. Similar argument to low congestion cases applies for hazmat carriers travel delay. Nonetheless, when the network is highly congested, i.e., cases 10 and 11, hazmat travel delay also decreases since hazmat carriers are directed into less congested areas compared with the no-toll case.

It is important to notice that for all problem instances of Table 10 we consider $\omega_3 = 0$ due to the assumption that congestion induced by hazmat carriers is negligible. Consequently, the dual-tolled transportation network is in favor of regular vehicles. Note the positive % change in hazmat carriers travel delay as well as the paid tolls. In a real application, being aware of the potential risk imposed by hazmat trucks to the network, it might be acceptable to the corresponding transportation companies to use more expensive routes. However, more attractive solutions for hazmat carriers can be obtained by considering a nonzero $\omega_3 \in (0, 1]$, at the cost of deteriorating risk mitigation targets and regular vehicles' viewpoint. Nonetheless, it is clear that more attention can be always drawn to the risk and network congestion objectives as long as $\omega_3 < \omega_1$ and $\omega_3 < \omega_2$.

In summary, our numerical experiments have shown that with the inclusion of the network users' travel delay within the first-stage problem, i.e., equation (31), other than congestion minimization, the risk is significantly reduced and is only slightly higher than that obtained under equation (17). Therefore, the two-stage process with the new administrator’s problem becomes more interesting for implementation because it produces far less expensive solutions for the network users (regular vehicles in our case) which are yet attractive to the regulator.

6.2 The 2-Step EDO

We now present our numerical results for the more general case where the administrator’s objective is not solely to minimize the population exposure, but also a fraction of the network users’ costs is minimized in the objective function. In problem (TS), it was assumed that all road segments were subject to charge by tolls. In real-world situations, however, it is possible that some of them have to stay toll-free due to technical or economical reasons. When this is the case, we solve problem (GTS) using the 2-Step EDO heuristic depicted in Figure 3 as a true bilevel model.

For our testing, all 552 OD pairs with the corresponding travel demands as displayed in Suwansirikul et al. (1987, Table XI) were considered for the regular vehicles, whereas 10 hazmat trips including origin, destination and demand were generated randomly. Also, we assumed that 18 arcs are subject to charge by separate tolls for regular and hazmat users with 20 and 30 being the upper bounds on regular and hazmat tolls, respectively. These arcs which are most likely bridges, tunnels or highways are shown in Figure 6 as the ones connecting the dotted areas. That is, the arcs in the dotted areas are considered as toll-free.
Figure 6: Tollable arcs in the Sioux Falls Network are outside the dotted areas.

<table>
<thead>
<tr>
<th>Change of Risk (%)</th>
<th>Change of Delay (Regular) (%)</th>
<th>Change of Delay (Hazmat) (%)</th>
<th>Tolls collected (Regular)</th>
<th>Tolls Collected (Hazmat)</th>
</tr>
</thead>
<tbody>
<tr>
<td>−20.79</td>
<td>23.57</td>
<td>−14.33</td>
<td>78442.04</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 11: Performance of the dual-toll policy provided by the 2-step EDO algorithm

Also, we let $\sigma_R = 0.05, \sigma_H = 0.04$ as in the previous section, and $\varphi = 0.1$. With these assumptions, the 2-Step EDO heuristic was employed to obtain a dual-toll policy. The results are presented in Table 11 according to the similar metrics defined in Section 6.1.

It is important to note that due to the inclusion of a fraction of the carriers’ cost within the leader’s objective of problem $(GTS)$, the reduction in risk is not as high as the one produced by problem $(TS)$. Note the difference between the results of Table 11 and the ones in case 11 of Table 9. Further, when some road segments are free of toll charges, the optimal value of the risk objective function is likely to deteriorate, i.e., second best pricing. Nonetheless, it is evident from Table 11 that the exposure to hazmat transportation can be still substantially reduced by the dual-toll policy obtained from problem $(GTS)$. Also, for such reduction, the regular vehicles incur far less increase in their travel delay and paid tolls. Hazmat carriers, on the other hand, are directed into less congested routes and thus experience shorter travel times without paying tolls.

In summary, the dual-toll obtained from the 2-Step EDO heuristic, can effectively encourage regular vehicles and hazmat carriers to use dissimilar routes, and thus can achieve considerable reductions in the associated transport risk and the travel delay experienced by hazmat carriers while collecting much less tolls from the users. The results imply that, by combining the risk and the paid tolls, problem $(GTS)$ can yield attractive solutions for the regulator which are also economically feasible to the users. Although, it might be perceived as less fair from regular users in
terms of the travel delay, when the leader’s objective of problem (GTS) also allows for minimizing the congestion induced by regular vehicles, the 2-Step EDO heuristic can produce more acceptable solutions to the regular vehicles at the cost of compromising the risk and the paid tolls.

7 Conclusion and Future Research

This paper has considered dual-toll setting, with a nonlinear BPR travel delay function, as an efficient policy for mitigating the population exposure to dangerous goods transportation. We demonstrated that the bilevel problem (TS) is equivalent to a two-stage problem with (MR) being in the first-stage and regular/hazmat inverse optimizations in the second-stage. It is important to stress that, under the nonlinear BPR function, a numerical solution to problem (MR) may be suboptimal; thus the existence of a nonnegative valid toll vector pair may not be guaranteed. Based upon analytical properties, this paper has proposed a post-iteration to the numerical solution to (MR) which always ensures obtaining a nonnegative dual-toll policy inducing the minimum hazmat risk. We suggested using the Modified Frank-Wolfe heuristic to determine an approximate solution to problem (MR). Further, to study the quality of this solution, we transformed the nonlinear problem (MR) to two MILPs that assure global optimality and serve to justify the approximate solution obtained by the heuristic. Numerical results show that the heuristic algorithm converges to the optimal solution of the administrator’s problem (MR) with reduced computational effort.

We also studied a more general dual-toll setting problem, namely (GTS), where the regulator wishes to minimize a combination of hazmat transport risk and paid tolls in a framework that permits constraints on toll charges. A 2-Step EDO heuristic is proposed to solve (GTS) as a true bilevel problem, by decomposing it into interacting univariate subproblems for each arc and applying simultaneous line searches. The (GTS) problem was also solved using LINGO as an exact method to verify heuristic results. Our experiments revealed that the 2-Step EDO is computationally more efficient than LINGO and converges to the local optimal solution of (GTS).

To illustrate the effectiveness of the dual-toll policies provided by problems (TS) and (GTS), we carried out additional experiments with the Sioux Falls problem. We observed that, under the dual-toll policy obtained from (TS), the exposure to hazmat transportation can be significantly reduced, yielding attractive scenarios for the regulator. While being effective in terms of risk mitigation, our numerical tests showed that such scenarios are substantially expensive from the users’ perspective in terms of both the travel delay and the paid tolls; thus they would likely not be acceptable by users in the implementation phase. To overcome this, we suggest incorporation of a fraction of the users’ travel delay within the administrator’s objective function. We demonstrated that the new (TS), while compromising risk-mitigation targets to minimize the users’ travel delay, is still able to achieve significant reductions in risk, and can produce win-win scenarios for both the regulator and the network users. Another observation relates to problem (GTS). Although combining the risk and the paid tolls most likely deteriorates the risk reduction when compared with (TS), we show that the solutions obtained under such dual-toll are not only likely to be attractive for the
regulator but also likely to be financially acceptable to users.

A challenging problem in hazmat transportation is equity in distributing the risk among the different population centers. Hence, a suggested future refinement of our model is to obtain a dual-toll setting policy which also includes equity concerns of the public. This may be achieved by adding a set of constraints for each zone that limits the level of exposure to hazmat transport risk to a certain value.

Another important issue pertinent to network regulation is the inevitable increase in the users' travel cost. Although the dual-toll setting model, by incorporating the network users’ cost concerns, can substantially reduce the overall travel cost imposed to the users, such benefit is not equitably distributed. In fact, a detailed analysis of the solution shows that, while being more fair to some users, some individuals have to incur high travel costs and sacrifice for the sake of total system satisfaction. Therefore, another fruitful direction is to integrate cost equity among the network users in the decision-making process.

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Supplementary Document

A supplementary document for this manuscript is available at http://www.chkwon.net/papers/dual_toll_supplementary.pdf

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