NEURAL NETWORKS AS A BASIS FOR QUANTUM ASSOCIATIVE NETWORKS

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Abstract

We have got a lot of experience with computer simulations of Hopfield's and holographic neural net models. Starting with these models, an analogous quantum information processing system, called quantum associative network, is presented in this article. It was obtained by translating an associative neural net model into the mathematical formalism of quantum theory in order to enable microphysical implementation of associative memory and pattern recognition. In a case of successful quantum implementation of the model, expected benefits would be significant increase in speed, in miniaturization, in efficiency of performance, and in memory capacity, mainly because of additionally exploiting quantum-phase encoding.

Keywords:
Hopfield model, associative neural network, quantum associative network, holography, quantum computing

1 Introduction

In previous articles [1, 2, 3] we have presented in detail how and where the mathematical formalism of associative neural network models by Hopfield [4] and Haken [5] is *analogous* to the mathematical formalism of quantum theory (similar comparative studies can be found in [6, 7, 8]). In this article, an original and fundamental information processing system is presented, and an example of a quantum neural-network-like information processing "algorithm" is described in a way understandable for computer scientists, physicists and neuroscientists. We have chosen this "algorithm", simple but nevertheless effective, technically realizable, and roughly (on global scale) biologically plausible [9, 10], as a convenient one for presenting Hopfield-network-like quantum information dynamics at a *fundamental* level. By saying Hopfield-like we mean a system that is based on the Hopfield model of neural networks or spin glass systems (i.e., amorphous assemblies of spins or small "magnets"), respectively.

By extensive simulations of the quantum-implementable Hopfield model and its generalizations using various concrete data sets, we have effectively realized parallel-distributed content-addressable memory, selective associative reconstruction or recognition of patterns memorized in a compressed form, and even some limited capability for predictions based on a learned data set [10, 11, 12, 13]. We have found that the results depend very much on the correlation structure of a specific set of input patterns. In other words, beside the "hardware" (implementation) and the "software" ("algorithm"), also the "virtual software" (i.e., the input-data correlation structure) is essential [14].

In section 2 we will present conventional associative neural nets in a form which will be in section 4 translated to quantum formalism in a straight-forward way. We will start from a Hopfield-like neural net model in the approximation of linear activation function (central, "non-saturated" section of the sigmoid function), because this is sufficient for associative processing and memorization, and suitable for our aim to implement information dynamics in a natural quantum system.

2 Associative neural networks

From now on, the terms like neural, neuron, synaptic connections, memory, association, learning, etc., will always be used in the formal sense of the present artificial neural net theory.

In our starting Hopfield-like neural net model, the dynamical equation for neuronal activities

$$q_i(t_2 = t_1 + \delta t) \;=\; \sum_{i=1}^{N} J_{ij}\; q_j(t_1) \qquad or \qquad \vec{q}(t_2) = \mathbf{J}\;\vec{q}(t_1) \qquad (1)$$

is coupled with the Hebb dynamical equation for synaptic connections

$$J_{ij} \;=\; \sum_{k=1}^{P} v_i^k\; v_j^k \qquad or \qquad \mathbf{J} = \sum_{k=1}^{P} \vec{v}^k\; \vec{v}^{kT} \qquad (2).$$

q_i is the current activity of the i^{th} neuron. The state of the network with N neurons is described by: $\vec{q} = (q_1, ..., q_N)$. v_i^k is the activity of the neuron i when taking part in encoding the k^{th} memory pattern $\vec{v}^k$. Equation (1) describes each neuron's summation of signals from all other neurons weighted by the strengths of their synaptic connections J_{ij}. Equation (2) prescribes that the strength of each synaptic connection is increasing when its two neurons are both active or both inactive, and decreases if their activity is not correlated. This is the experimentally-supported Hebb learning rule [9, 10]. The neuronal correlations belonging to each individual pattern, which are simultaneously stored in the network, are then summed over all (P) stored patterns. $\mathbf{J}$ is appropriately called "memory matrix".

Alternatively, one can regulate the Hopfield emergent collective computation by minimization of the Hamiltonian energy function

$$H \;=\; -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\; J_{ij}\; q_i\; q_j \;=\; -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{P}\; v_i^k\; v_j^k\; q_i\; q_j \qquad (3).$$

This system of equations is enough for exhibiting parallel-distributed processing of input data. This is one of the simplest "algorithms" useful for theoretical brain modeling [4] as well as for data processing [10, 12], and is quantum implementable.

Let us rewrite the system (1) & (2) into continuous description of activities of neurons and synapses at position $\vec{r}$ and time t:

$$q(\vec{r}_2, t_2 = t_1 + \delta t) = \int J(\vec{r}_1, \vec{r}_2)\; q(\vec{r}_1, t_1)\; d\vec{r}_1 \qquad (1b),$$

$$J(\vec{r}_1, \vec{r}_2) \;=\; \sum_{k=1}^{P}\; v^k(\vec{r}_1)\; v^k(\vec{r}_2) \qquad (2b).$$

The memory recall is done by $\vec{q}_{output} = \mathbf{J}\vec{q}_{input} = \mathbf{J}\vec{q}'$. This can be analyzed as follows:

$$q(\vec{r}_2, t_2) = \int J(\vec{r}_1, \vec{r}_2)\ q'(\vec{r}_1, t_1)\ d\vec{r}_1 = \int [\sum_{k=1}^{P} v^k(\vec{r}_1)\ v^k(\vec{r}_2)]\ q'(\vec{r}_1, t_1)\ d\vec{r}_1 =$$

$$= [\int v^1(\vec{r}_1)\ q'(\vec{r}_1, t_1)\ d\vec{r}_1] v^1(\vec{r}_2) + [\int v^2(\vec{r}_1)\ q'(\vec{r}_1, t_1)\ d\vec{r}_1] v^2(\vec{r}_2) + ...$$

$$... + [\int v^P(\vec{r}_1)\ q'(\vec{r}_1, t_1)\ d\vec{r}_1] v^P(\vec{r}_2) =$$

$$= \ A\ v^1(\vec{r}_2)\ +\ B \qquad where\ \ A\ \doteq\ 1\ \ ('signal'),\ B\ \doteq\ 0\ \ ('noise') \qquad (4).$$

In (4) we had to choose such an input $\vec{q}'$ that is more similar to $\vec{v}^1$, for example, than to any other $\vec{v}^k, k \neq 1$. At the same time, the input $\vec{q}'$ should be almost orthogonal to all the other $\vec{v}^k, k \neq 1$. In that case, $\vec{q}$ converges to the memory pattern-qua-attractor $\vec{v}^1$, and the other terms give a negligible sum B, as it is shown in the last row of (4). Thus, $\vec{v}^1$ is recalled.

One can describe the above recall process in another, equivalent way using the projection-coefficients $C^k(t) = \int v^k(\vec{r}) q(\vec{r}, t) d\vec{r}$ to show the similarity to quantum theory and synergetics:

$$q(\vec{r}, t) = \sum_{k=1}^{P} C'^k(t) v^k(\vec{r}) = \sum_{k=1}^{P} \left(\int v'^k(\vec{r}) q'(\vec{r}, t) d\vec{r} \right) v^k(\vec{r}) =$$

$$= \left(\int v^1(\vec{r}) q'(\vec{r}, t) d\vec{r} \right) v'^1(\vec{r}) + ... + \left(\int v^P(\vec{r}) q'(\vec{r}, t) d\vec{r} \right) v'^P(\vec{r})\ =\ A\ v'^1(\vec{r})\ +\ B \qquad (4b)$$

where $A \doteq 1$ ('signal') and $B \doteq 0$ ('noise'). (The first row of equalities in (4b) follows, among others, from Haken's neurosynergetic model [5] where C^k are called order parameters.)

3 Holographic neural networks

If we introduce the *oscillatory* time-evolution of neuronal activities, we get more biologically plausible dynamics [15, 5, 8]. Neural net variables then become complex-valued: $q_j = q_{j0} e^{i\varphi_j}$; φ_j is the phase. This is more akin to quantum dynamics, and to holography also. However, in quantum theory, complex-valued formalism is essential, but in the network of coupled oscillatory neurons complex-valued formalism is just mathematically more convenient.

Sutherland, Manger, Souček and others [16] have successfully *simulated* such a quantum-like generalization of Hopfield neural networks with concrete data sets and impressive applications. Therefore, let us roughly present their *holographic* neural-net model [17] as a step forward to neuro-quantum brain models and hypothetical hybrid computers.

A sequence of k input–output vector pairs is used for learning. Each input and output component oscillates with a specific activity amplitude and a specific phase (in the exponent). Thus, the k^{th} input–output pair is:
the input vector $\vec{s}^k = (s_1^k e^{i\theta_1^k}, s_2^k e^{i\theta_2^k}, ..., s_N^k e^{i\theta_N^k})$, $i = \sqrt{-1}$, and
the output vector $\vec{o}^k = (o_1^k e^{i\varphi_1^k}, o_2^k e^{i\varphi_2^k}, ..., o_M^k e^{i\varphi_M^k})$.

To encode these patterns, we let them "interfere" in a way that is mathematically equivalent to optical interference of the "object wave" and the "reference wave". In the real (optical) holography, the "object wave" is those which is directly reflected from the object whose three-dimensional image will be stored in the hologram. "In the object wave," the information about the object form is encoded or modulated. The "reference wave" is the second wave reflected from the object, but it is indirect (a reflection in a mirror is made, for example). These two waves, carrying the modulated scene information from two different angles or points of view, are then made to interfere on the hologram-plate. The constructive or deconstructive interference determines the local rates of optical transparency of the hologram-plate, so that it encodes the pattern-correlations (i.e., memory components in our interpretation). Mathematically, a small portion of the wave is equivalent to activity of a neuron, and wave-interference is equivalent to interaction between assemblies of neurons with oscillating activities. This is our holography–neural-net analogy.

Using a Hebb-like expression, analogous to equation (2), we calculate the "connection strength between two neurons h and j"

$$J_{hj} = \sum_{k=1}^{P} s_h^k \, o_j^k \, e^{-i(\theta_h^k - \varphi_j^k)} \qquad or \qquad \mathbf{J} = \sum_{k=1}^{P} \vec{o}^{\,k} \, (\vec{s}^{\,k*})^{\dagger} \qquad (5).$$

Every J_{hj} encodes the superposition of local input–output amplitude correlations and the phase-angle differences in the exponent. Thus, the matrix of all amplitude-correlations multiplied with corresponding phase-differences represents a "hologram," i.e. the memory. Actually, Sutherland interprets phases as the "real" information and the corresponding amplitudes as their confidential levels. He reports significant improvement of performance in comparison to conventional neural nets because of this [16].

In the real holography, an image is reconstructed from the hologram by sending a new reference wave (it may be carrying partial information only – that's the so-called key) through the hologram plate. In Sutherland's simulated "neural" holography, a stored pattern (say, $k = 1$) can be reconstructed, analogously to equation (4), from the "neural hologram" by "interacting" the key-input with the memory:

$$\vec{o}\,' = \mathbf{J}\vec{s}\,' \;\Leftrightarrow\; \sum_{h=1}^{N} J_{hj} s'_h e^{i\theta'_h} = \sum_{h=1}^{N}\sum_{k=1}^{P} s_h^k o_j^k e^{-i(\theta_h^k - \varphi_j^k)} s'_h e^{i\theta'_h} = o_j^1 e^{i\varphi_j^1} \qquad (6).$$

We had to choose an input s'_h that is similar to s_h^1, and θ'_h that is similar to θ_h^1. In such a case, $s'_h s_h^1 \doteq 1$ (if s are normalized) and $e^{-i\theta_h^1} e^{i\theta'_h} \doteq 1$. The "holographic" recall method is evidently similar to the Hopfield neural recall, satisfying similar conditions. [1]

Analogous holographic associative memories can be implemented optically [18], acoustically, quantum-electronically, quantum-biologically [19, 6, 8, 20]. Various kinds of holography are a bridge between neural net models and quantum information systems, similarly to those exploiting quantum coherent states [21], quantum dots [22], quantum information channels [23, 24], etc. (see also [25]).

4 Quantum associative networks

We have discussed a Hopfield-like neural-net or spin-glass model in a specific form which is quantum implementable. Now we will explicitly present its quantum relative, with *phases* φ *implicitly incorporated in the wave-functions* $\Psi = A\,exp(i\varphi)$ (A is the amplitude). The neural-net equations will just be translated into quantum formalism in such a way that the information-processing capabilities will be preserved. From the very beginning, let us put attention to the following correspondence scheme between the neural (left) and quantum variables (right) ("$\Leftrightarrow$" means "corresponds to"):

$$q \Leftrightarrow \Psi, \quad q' \Leftrightarrow \Psi', \quad v^k \Leftrightarrow \psi^k, \quad J \Leftrightarrow G \;\; or \;\; \mathbf{J} \Leftrightarrow \mathbf{G},$$

[1] For details, improvements and generalizations of this simplest holographic neural-net scheme, as well as their applications, see [16, 17].

$$equation\ (1) \Leftrightarrow equation\ (7), \quad (2) \Leftrightarrow (8), \quad (4) \Leftrightarrow (9), \quad (4b) \Leftrightarrow (9b).$$

The equations in pairs are *mathematically equivalent*, because the *collective* dynamics in neural and quantum complex systems are similar, in spite of different nature of neurons and their connections on one hand, and quantum "points" $\Psi(\vec{r})$ and their "interactions" described by $G(\vec{r}_1, \vec{r}_2)$ on the other. Variable with a prime means that its quantitative value is close to the variable without prime.

Taking into account this correspondence list, all the descriptions of neural and holographic memory (re)construction is valid for the quantum case also. First, we encode patterns into specific quantum states. Then, interference of these quantum states is produced, thus exhibiting a sort of quantum memory. At the end, in order to reconstruct a "quantum pattern", the quantum state, described by the so-called wave-function, has to be "collapsed" as it will be explained later.

Now we will describe the mathematical formalism of this procedure which follows from the fundamentals of quantum theory as described in any textbook on quantum mechanics [26]. The author's quantum Hopfield-like network model combines the dynamical equation for the quantum state [27]

$$\Psi(\vec{r}_2, t_2) = \int\int G(\vec{r}_1, t_1, \vec{r}_2, t_2)\ \Psi(\vec{r}_1, t_1)\ d\vec{r}_1\ dt_1 \qquad or \qquad \Psi(t_2) = \mathbf{G}\ \Psi(t_1) \tag{7}$$

and the expression for the parallel-distributed interactive transformation of the quantum system [27]

$$G(\vec{r}_1, t_1, \vec{r}_2, t_2) = \sum_{k=1}^{P} \psi^k(\vec{r}_1, t_1)^*\ \psi^k(\vec{r}_2, t_2) \qquad or \qquad G(\vec{r}_1, \vec{r}_2) = \sum_{k=1}^{P} \psi^k(\vec{r}_1)^*\ \psi^k(\vec{r}_2) \qquad (8).$$

ψ^k are eigen-wave-functions, energetically stable quantum states. Let us assume that we have succeeded to *encode some information* in them. Eigen-wave-functions have then become "quantum patterns". In such a case, the interaction matrix $\mathbf{G}$ (so-called propagator) constitutes the *quantum memory.*

Like in neural nets, the process of learning is a result of, metaphorically speaking, a web of quantum interactions "between formal quantum neurons or qubits", described by $\Psi(\vec{r}, t)$. Interactions are described by weights $G(\vec{r}_1, t_1, \vec{r}_2, t_2)$ which are elements of the Hebb-like quantum memory matrix $\mathbf{G}$.

The quantum memory recall is realized by the *wave-function "collapse"*:

$$\Psi(\vec{r}_2, t_2 = t_1 + \delta t) = \int G(\vec{r}_1, \vec{r}_2)\ \Psi'(\vec{r}_1, t_1)\ d\vec{r}_1 = \int [\sum_{k=1}^{P} \psi^k(\vec{r}_1)^* \psi^k(\vec{r}_2)]\ \Psi'(\vec{r}_1, t_1)\ d\vec{r}_1 =$$

$$= [\int \psi^1(\vec{r}_1)^* \Psi'(\vec{r}_1, t_1) d\vec{r}_1] \psi^1(\vec{r}_2) + [\int \psi^2(\vec{r}_1)^* \Psi'(\vec{r}_1, t_1) d\vec{r}_1] \psi^2(\vec{r}_2) + ...$$

$$... + [\int \psi^P(\vec{r}_1)^* \Psi'(\vec{r}_1, t_1) d\vec{r}_1] \psi^P(\vec{r}_2) =$$

$$= \ A\ \psi^1(\vec{r}_2)\ +\ B \qquad where\ \ A\ \doteq\ 1\ \ ('signal'),\ B\ \doteq\ 0\ \ ('noise') \qquad (9).$$

In (9) we had to choose such an "input" Ψ' that is *more similar* to ψ^1, for example, than to any other $\psi^k, k \neq 1$. At the same time, the "input" Ψ' should be *almost orthogonal* to all the other $\psi^k, k \neq 1$. In that case, Ψ converges to the quantum "pattern-qua-attractor" ψ^1, and the other terms give a negligible sum B, as it is shown in the last row of (9). Thus, the memory pattern ψ^1 is recalled (measured). If the condition, well known from the Hopfield model simulations, that "input" must be similar to one stored pattern (at least more than to other stored patterns) is not satisfied, then there is no single-pattern recall.

The above process can be described in another, equivalent way:

$$\Psi(\vec{r},t) = \sum_{k=1}^{P} c'^k(t)\psi^k(\vec{r}) = \sum_{k=1}^{P} \left(\int \psi'^k(\vec{r})^* \Psi'(\vec{r},t) d\vec{r} \right) \psi^k(\vec{r}) =$$

$$= \left(\int \psi^1(\vec{r})^* \Psi'(\vec{r},t) d\vec{r} \right) \psi'^1(\vec{r}) + ... + \left(\int \psi^P(\vec{r})^* \Psi'(\vec{r},t) d\vec{r} \right) \psi'^P(\vec{r}) \; = \; A \; \psi'^1(\vec{r}) \; + \; B \quad (9b)$$

where $A \doteq 1$ ('signal') and $B \doteq 0$ ('noise'). (The first row of equalities in (9b) takes into account the quantum superposition principle.)

Let us describe the memory recall procedure again:
In the quantum as well as neural case, the new input was let to interact with memory. Mathematically, the input vector was multiplied with the memory matrix (**G** or **J**). Or locally, as in eqs. (9) or (4), the components describing activities of formal neurons (or qubits), which encode the input, were multiplied with the connection weights, determined by (8) or (2), which encode the memory (correlations of all possible learned patterns). Because the Hebb-like memory matrix is a superposition of single-pattern auto-correlations, we can expand the above multiplication into a series. Each term of this series corresponds to a stored pattern "interacting" with the input. If we choose the input similar to one stored pattern, the corresponding term is amplified ("resonance"), and simultaneously, the others are suppressed. The "resonant" term gives the output; other terms represent the "noise".

However, in contrast to neural nets, a non-artificial quantum system obeys the closure relation $\sum_k \psi_k(\vec{r}_1,t)^* \psi_k(\vec{r}_2,t) = \delta(\vec{r}_1 - \vec{r}_2)$ [26] which is probably not too restrictive under certain conditions related to (quasi)orthogonality (discussed in [28]).

If the condition of similarity would not be satisfied, the output would be a superposition of all stored patterns and the input, i.e. an un-intelligible "mixture" of patterns. On the other hand, if the input is identical to a part of a stored pattern, we get pattern-recognition, i.e. the stored pattern is exactly recalled from memory as triggered by the input. Usually, the network has to do some prediction in novel, but similar, circumstances, therefore inputs should be just similar to the learned ones. If they are too similar, we get a trivial case; if they are not similar, we get too much "noise".

5 From neural to quantum nets – discussion

Equations (7), (8) and (9) come from basic quantum physics. Quantum interference is very fundamental and essential. With the Young experiment it was an early experimental predecessor of the Schrödinger quantum wave-mechanics and the Heisenberg quantum operator theory [26].

There is nothing "neural" (in biological sense) in the system of quantum equations (7) & (8). It is just *similar, according to their mathematical structure and coupling*, to the system of neural-net equations (1) & (2). Because we are certain that the neural system (1) & (2) realizes efficient information processing, we have taken the similar system of equations from the quantum-physics formalism in order to discover quantum Hopfield-like information dynamics. Using our computer simulation experience with the Hopfield model, we have just given a precise information-processing interpretation to a selected and deliberately connected set of quantum equations which are, each one alone, known to all physicists and well experimentally verified. (But many physicists do not know the Hopfield information dynamics, like many computer scientists do not know quantum theory. This was an additional motivation for the present correspondence review.)

Since the proposed "algorithm" is realizable in any complex system obeying similar collective dynamics, information processing of the same type could in principle be realized in many such (classical) systems, if we neglect some specific features arising from the nature of *individual* elements of the system. The "algorithm" is thus relatively universal, with exception of some specific characteristics to be discussed now.

There is a difference between the neural "algorithm" (1)-(2)-(4) and the quantum "algorithm" (7)-(8)-(9): Neural variables in equations (1)-(2)-(4) are real-valued, but quantum variables are complex-valued, because they implicitly incorporate the *phase*. Thus, a quantum quantity or expression usually incorporates, at least implicitly, the imaginary unit i and/or the asterisk denoting complex conjugation. [1] Neural versions of uncertainty principle, similar to Heisenberg's quantum uncertainty principle, can be found [29].

The quantum memory recall procedure is a special case of usual quantum measurements. The difference is that usual physical processes in our case have an informational interpretation, i.e. our eigen-wave-functions have a specific prescribed *meaning*. Let us describe the reconstruction of the selected quantum eigen-wave-function from the informational or cognitive point of view.

In quantum language, the quantum system was perturbed by the experimenter in a specific way. Informationally, the new input was let to interact with quantum memory. Mathematically, the input vector was multiplied with the memory matrix (the Green-function propagator). Or, the input-wave-components (as shown in (9), similarly to (4)) were multiplied with the propagator's components (correlations of all possible learned quantum patterns) which encode the memory. Because the propagator is a superposition of self-interferences of each eigen-wave-function, we can expand the above multiplication into a series. Each term of this series corresponds to an implicitly-present eigen-wave-function "interacting" with the input, i.e. the perturbation. If we choose the perturbation-pattern similar to one stored quantum pattern, the corresponding term is amplified ("resonance"), and simultaneously, the others are suppressed. The "resonant" term gives the output, i.e. the finally measured quantity corresponding to a selected eigen-wave-function, other terms represent uninteresting implicit states.

Like in neural nets, if the condition of similarity (or a high rate of orthogonality, respectively) would not be satisfied, the output would be a superposition of all stored quantum patterns and the perturbation (which is our input). This means, the output would be an un-intelligible interference of eigen-wave-functions. On the other hand, if similarity turns to identity (the input is identical to a stored quantum pattern), we get quantum collapse to the nearest stored quantum pattern. The quantum-pattern recall may work well also if the measurement-perturbation is just partially correlated with a stored eigen-wave-function. Quantum holography, like any other kind of holography, realizes the effects described in the last paragraphs, and they are often used in experimental practice [18, 19].

There is experimental evidence [6, 7, 30] that biological neural nets essentially cooperate, over synapto-dendritic and microtubular networks, with quantum systems in the brain [31]. So, quantum and neural nets are thus not merely independent levels with similar collective processing, but display together a sort of fractal-like dynamical structures [32].

It seems that, in the brain, neural networks trigger the collapse of quantum wave-function

[1] We have discussed some consequences of the complex-valued formalism elsewhere: [2] (sec. 4), [1].

and thus transform the quantum complex-valued, probabilistic dynamics into the neural (classical) real-valued, deterministic dynamics. So, neural nets serve as an interface between classical environment of the organism and its quantum basis (details in [1, 2, 7, 31]). The transformation of the complex-valued quantum dynamics into the real-valued classical dynamics during the collapse of the wave-function (a quantum sort of pattern recognition – remember eq. (9)) is very probably triggered by the system's interaction with environment [33].

If we try to implement Shannon-type information processing in quantum networks, some essential quantum features, manifested in the complex-valued formalism, are unavoidably partly eliminated [1]. However, nature has obviously always performed a sort of non-Shannon-type implicit-information processing (Bohm uses term "active information" [34]) where the "users of information" are not human observers, but natural systems (e.g., immune systems) themselves [33].

Holographic and other oscillatory neural networks [15, 5] base on complex-valued formalism like quantum theory, but they do not realize the deep essence of quantum dynamics, i.e. the quantum holism – indivisibility and continuous non-local interconnectedness or entanglement [34, 24]. However, these essentially-quantum features are broken and dynamics are thus discretized during quantum measurements. The implicit and multiple quantum process is useful for parallel-distributed computation of extremely high speed and capacity, but it has to be collapsed during the readout (i.e., recall) in order to obtain results of computation [35, 36]. During observation, quantum dynamics become more neural-net-like, e.g. the observable "activities of a network of quantum neurons" are real-valued. Neuro-quantum (classical-quantum) cooperation, as manifested in the collapse-readout, is thus essential for the brain which combines Shannon-type cognitive information and non-Shannon-type consciousness phenomenalism [37, 2, 32].

6 Conclusions

A quantum information processing "algorithm" (7)-(8)-(9) was presented. We have constructed it following the Hopfield and holographic neural net models which process information efficiently as tested by numerous computer simulations of ours [13] and of other modelers [4, 10, 16]. The central contribution of this paper is a proposed realization of *Hopfield-like information processing and content-addressable memory in quantum systems* on a fundamental level.

In parallel, some other recent approaches to implement the Hopfield model in quantum systems [38, 39] were developed independently from ours. Although they are more sophisticated, our model allows more straight-forward implementation in a usual, unconstrained quantum system (i.e., without artificial manipulations needed, apart from perturbational insertions of inputs into the system). These similar approaches as well as the fast-developing field of quantum computing [35, 36] raise the possibility for our Quantum Associative Networks being physically implemented in some form (in computers as well as in biological systems) [42]. However, one should mind the differences between quantum neural-like networks, which have a natural computing capability, and quantum computers based on relatively artificial implementation of logic gates in quantum devices.

Let us repeat the steps of our quantum "algorithm":

1. construction (encoding) of quantum "patterns" $\psi^k(\vec{r}, t)$;

2. interference-based construction of the quantum propagator $\mathbf{G}$ which then works as quantum memory ("sieve", projector) – equation (8);

3. quantum computing, which may be iterative – equation (7) (*output* = *memory* $*$ *input*); **4.** reconstruction of memory **G** in new circumstances (decoding) – equations (9) or (9b). Here, the usual quantum complete, orthonormal set of eigenstates ("patterns") is used in a *fuzzy* way. Instead of obeying exact orthogonality of quantum theory, we work with nearly orthogonal data.

There are numerous complex systems which are candidates for implementation of collective computation obeying similar "algorithms" as described here (for reviews see [32, 40, 7, 20]). Their advantage would be miniaturization and much greater speed, efficiency and computational capacity. The main source of significant improvements in performance in our model would be, like in holographic neural nets, as shown in [16], the use of *quantum phase* φ, hidden in $\psi(\vec{r},t) = A(\vec{r},t)e^{i\varphi(\vec{r},t)}$, for encoding and processing of information (details in [28, 42]).

In brain [6], neuro-quantum interaction seems to be essential for the "collapse"-readout of quantum computation results, and is connected to sub-consciousness–consciousness or memory–consciousness transitions of mental contents. Therefore, the proposed classical-quantum hybrid models, ours and those developed independently in parallel by others [37, 38, 39, 41, 22], are an alternative approach for computational neuroscience and quantum computing [35, 36, 42].

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