Transmit Antenna Selection in MIMO-OFDM Systems: Bulk versus Per-tone Selection

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Abstract—This paper provides an exact diversity gain analysis for transmit antenna selection in MIMO-OFDM systems with linear receivers, a topic scarcely addressed in prior literature. In a frequency-selective fading (as opposed to a frequency-flat fading) channel one of either two methods of antenna selection may be applied. We name these methods bulk selection and per-tone selection. From an implementation perspective each method presents a different complexity-performance tradeoff. Through rigorous mathematical derivation, we demonstrate that both methods realize the same diversity order. Moreover, the coding gain realized by the two selection methods is also analyzed and compared through simulation. Overall, this paper confirms the advantage of antenna selection technology in MIMO-OFDM systems, and provides comprehensive guidelines for antenna selection strategies in realistic scenarios, from both the performance and complexity perspectives.

Index Terms—MIMO, Antenna Selection, OFDM, Diversity Order, Coding Gain

I. INTRODUCTION

A NТЕNNA selection in multiple-input multiple-output (MIMO) systems has been widely studied as a cost-effective, performance enhancing technology [1]. The cost savings are realized by having fewer RF (radio-frequency) chains than radiating elements at transmitter and/or receiver. An RF chain is typically an order of magnitude more expensive than the radiating element. The RF chains are switched into the radiating elements based on (complete or partial) channel state information (CSI).

Antenna selection techniques may be combined with transmit diversity schemes [2], [3] or with spatial multiplexing [4], [5]. The former focuses on improving the quality and reliability of reception while the latter focuses on (linearly) increasing data rate. For frequency-flat fading channels, performance analysis for antenna selection with transmit diversity has been addressed in [6]. Diversity analysis for transmit antenna selection with spatial multiplexing was addressed only recently in [7], [8] – it was shown that with a linear receiver, the achievable diversity order is \((N_T - L + 1)(N_R - L + 1)\), where \(N_T\) and \(N_R\) represent the number of transmit and receive antennas respectively, and \(L < N_T\) is the number of RF chains at the transmitter.

In many wireless systems today (e.g. WLAN, WiMAX) MIMO technology is combined with OFDM (Orthogonal Frequency Division Multiplexing) to avoid inter-symbol interference in frequency-selective fading channels. Due to the frequency-selective nature of the channel transmit antenna selection in a MIMO-OFDM system will fall under one of two selection approaches – (i) Bulk selection: the selected antenna subset is the same for all tones [9], [10], [11]), and (ii) Per-tone selection: a different antenna subset is selected for each tone [12].

The per-tone solution may be interpreted as baseband switching rather than RF chain switching. Naturally, per-tone switching does not benefit from the cost savings of having fewer RF chains than transmit antennas. Per-tone switching may also be interpreted as a form of linear precoding through antenna hopping.

Performance analysis for antenna selection in MIMO-OFDM has been scarcely addressed in the literature. The few publications on this topic have focused on experimental results [10], [11], algorithm design [12] or implementation aspects [9], [13]. In particular, [13] provides a preliminary comparison between bulk and per-tone selection through numerical results. To the best of our knowledge, there has been no study on the achievable diversity order or coding gain of these two selection approaches. The contributions reported in this paper can be summarized as follows:

- We derive the diversity order for both bulk and per-tone transmit antenna selection for spatial multiplexing based MIMO-OFDM.
- We show that both selection methods realize the same diversity order for three commonly considered coding schemes – joint space-frequency encoding/interleaving (joint coding), frequency-only encoding/interleaving (horizontal coding), and uncoded\(^1\) transmission.
- Finally, we present analytical and empirical results on the coding gain realized by the two selection approaches.

Overall this paper confirms the advantages of antenna selection technology in MIMO-OFDM systems, and provides comprehensive guidelines for choosing among the various antenna selection strategies.

II. SYSTEM MODEL AND MATHEMATICAL PRELIMINARIES

We consider a MIMO-OFDM system with \(K\) tones. The \(N_R \times N_T\) channel matrix for the \(k\)-th (\(0 \leq k \leq K - 1\)) tone is represented by \(\mathbf{H}^{(k)}_1 = [\mathbf{h}_1^{(k)}, \mathbf{h}_2^{(k)}, \ldots, \mathbf{h}_T^{(k)}]\). The elements of \(\mathbf{H}^{(k)}_1\) are assumed to be i.i.d. zero-mean circularly symmetric complex Gaussian. The coherence bandwidth of the channel is assumed to span \(N_C\) consecutive tones, so that \(\mathbf{H}^{(k_1)}_1\) and \(\mathbf{H}^{(k_2)}_1\) are independent if the two angles \(2\pi k_1/K\) and \(2\pi k_2/K\) are spaced larger than \(2\pi N_C/K\) on the unit circle. Therefore there are approximately \(M = \left\lceil \frac{K}{N_C} \right\rceil\) groups of independently fading tones. In order to make the problem analytically tractable we assume block fading in the frequency domain (as is commonly assumed in the time domain [14]),

\(^1\)Uncoded transmission does not preclude temporal coding. Temporal encoding is allowed in all three schemes. The nomenclature used applies only to the space and frequency dimensions.

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i.e., the channel holds constant over $N_C$ consecutive tones and then changes in an independent fashion in the next block. Without loss of generality we shall assume henceforth that $M = K$.

Assuming $L$ of $N_T$ transmit antennas are selected, corresponding to the $j$-th subset of possible choices for the $k$-th tone, the resulting $N_R \times L$ channel matrix is denoted by $\{\mathbf{h}(k)_{l,j}\} = [\mathbf{h}(k)_{1,j}, \mathbf{h}(k)_{2,j}, \ldots, \mathbf{h}(k)_{L,j}]$. In the case of bulk selection, the same $L$ antennas are selected for all $K$ tones, and connected to $L$ RF chains by a switch. In per-tone selection, the $L$ antennas are selected separately for each subcarrier (or coherence bandwidth – recall the block fading assumption). Since different antenna subsets may be chosen across the transmission bandwidth in the latter case, $N_T$ RF chains are required. In both cases $L$ data streams are simultaneously transmitted over the selected antennas. Throughout this paper we assume that perfect CSI is available to the receiver for the purpose of data reception.

With a zero-forcing (ZF) receiver, the post-processing channel gain for the $l$-th ($1 \leq l \leq L$) spatial stream in the $k$-th tone is proportional to the squared Euclidean norm of the projection of the $l$-th selected column vector $\mathbf{h}(k)_{l,j}$ onto the null-space of the remaining (selected) channel column vectors, denoted by $f_r(k)_{l,j}$. Assuming unit thermal noise power at each receive antenna $f_r(k)_{l,j}$ is also the post-processing signal-to-noise ratio (SNR). Since an MMSE (minimum mean squared error) receiver realizes the same diversity gain as a ZF receiver [8], we focus on the ZF receiver knowing that the results are equally applicable to an MMSE receiver.

The diversity order is defined as the magnitude of the slope of the average frame error probability $P_e(\rho)$ plotted as a function of SNR on a log-log scale in the high SNR regime [15]:

$$d = \lim_{\rho \to \infty} \frac{-\log P_e(\rho)}{-\log \rho},$$

where $\rho$ is the average (pre-processing) SNR at a receive antenna. We adopt the operator $\cong$ as defined in [15], to denote exponential equality, i.e., we write $f(\rho) \cong \rho^{-b}$ to represent $\lim_{\rho \to \infty} \frac{-\log f(\rho)}{-\log \rho} = b$. Equivalently (for the sake of convenience in this paper), we use $f(x) \equiv x^b$ to represent $\lim_{x \to 0} \frac{\log f(x)}{\log x} = b$. Note that the post-processing SNR for a communication link can typically be represented as $\rho_{post} = \rho g$, where $g$ is the effective channel gain. Since outage dominates error probability at high SNR [15], [16], for a SISO (single-input single-output) channel we can write

$$P_e(\rho) \cong P_{out}(\rho) = Pr(\log_2(1 + \rho_{post}) < R_0) = F_g((2^{R_0} - 1)/\rho),$$

where $R_0$ is the transmission rate (in bps/Hz) and $F_g(x) = Pr(g \leq x)$ is the cumulative density function (CDF) of the random variable $g$. As shown in [16], when $x \to 0$: $F_g(x) = \alpha x^d + o(x^{d+1})$. Hence (1) may be written with $P_{out}(\rho) = \left[\frac{\rho}{(2^{R_0} - 1)\alpha^{1/d}}\right]^d + o(\rho^{-1(d+1)})$, so that the diversity order is $d$ and the coding gain is $G = \frac{1}{(2^{R_0} - 1)\alpha^{1/d}}$ (c.f. [16]).

### A. Antenna Selection for Frequency-Flat Fading Channels

For frequency-flat fading channels, antenna selection for spatial diversity includes [6] selection combining, hybrid selection-maximum ratio combining (HS-MRC) and antenna selection with space-time block coding (AS-STBC). For selection combining (i.e., $L = 1$), the effective post-processing channel gain is $g = \max ||\mathbf{h}||^2$, where $||\mathbf{h}||^2$ is chi-squared distributed with $2N_R$ degrees of freedom. Using order statistics one can show $F_g(x) = x^{N_R N_T} + o(x^{N_R N_T + 1})$, with the corresponding diversity order and coding gain given by:

$$d_{SD}^{(flat)} = N_T N_R, \quad G_{SD}^{(flat)} = \frac{(N_R)^{1/N_R}}{2^{R_0} - 1}.$$  

The extensions to HS-MRC and AS-STBC follow similarly.

The error performance analysis of transmit antenna selection for spatial multiplexing is much more involved, given the inter-dependent nature of the post-processing data stream channel gains. For uncoded transmission with a linear receiver it has been shown [8], [7]

$$d_{SM}^{(flat)} = (N_T - L + 1)(N_R - L + 1).$$

### B. Preliminaries

Before proceeding, we state the following Lemmas, which facilitate the analysis in the subsequent section. Proofs for the lemmas can be found in the appendix.

**Lemma 1:** In an $N_R \times L$ i.i.d. Gaussian frequency-flat fading MIMO channel with zero forcing receiver ($L$ data streams and $N_R \geq L$), with optimal spatial encoding/interleaving, the system diversity order equals $d = N_R - L + 1$.

**Lemma 2:** In an $N_R \times N_T$ i.i.d. Gaussian MIMO channel with ZF receiver, when $L \leq N_T$ transmit antennas are selected to transmit $L$ data streams ($N_R \geq L$), with optimal spatial encoding/interleaving, the system diversity order equals $d = (N_T - L + 1)(N_R - L + 1)$.

The two Lemmas indicate spatial encoding does not enhance diversity gain over an uncoded system, notwithstanding antenna selection. We extend the discussion to MIMO-OFDM in the following section.

### III. DIVERSITY ORDER FOR TRANSMIT ANTENNA SELECTION IN MIMO-OFDM SYSTEMS

#### A. Bulk Selection

With bulk antenna selection, the contribution from each transmit antenna to the received signal is a function of all $M$ channels, corresponding to the independently fading blocks. In MIMO-OFDM wireless systems such as WLAN and WiMAX, data is encoded/interleaved over space and frequency. For bulk selection with joint space-frequency coding we state the following theorem:

**Theorem 1:** In an $N_R \times N_T$ MIMO-OFDM system with $M$ blocks of independently fading tones, each exhibiting i.i.d. Gaussian distribution across space, when $L \leq N_T$ transmit antennas are selected using bulk selection, and $L$ space-frequency encoded spatially multiplexed data streams are received with a zero-forcing receiver ($N_R \geq L$) in each tone, the achievable diversity order is $d = M(N_T - L + 1)(N_R - L + 1)$.

**Proof:** Using the same approach as Appendix B, if the $j$-th antenna subset $U_j$ is selected, optimal joint space-frequency encoding designed to extract full spatial diversity gain should realize an effective SNR that equals the summation of the
post-processing channel gains of the ZF receiver over all data streams and independent tones. Maximizing this summation will maximize the diversity gain. We may upper- and lower-bound outage on this output SNR as follows:

\[
L_B \leq \Pr \left( \max_j \sum_{k=1}^M \sum_{l=1}^L R_{l,j}^{(k)} \leq x \right) \leq U_B, \tag{4}
\]

where

\[
L_B = \Pr \left( \max_j \max_k \sum_{l=1}^L R_{l,j}^{(k)} \leq x \right)
\]

\[
U_B = \Pr \left( \max_j \max_k \sum_{l=1}^L R_{l,j}^{(k)} \leq x \right).
\]

Since \(U_B \leq L_B\), we have

\[
\Pr \left( \max_j \sum_{k=1}^M \sum_{l=1}^L R_{l,j}^{(k)} \leq x \right) \leq U_B. \tag{5}
\]

Now, by interchanging the maximization operations, exploiting tone independence, and using Lemma 2 we can show

\[
U_B = \Pr \left( \max_j \sum_{k=1}^M \sum_{l=1}^L R_{l,j}^{(k)} \leq x \right) \leq x^M(N_T-L+1)(N_R-L+1), \tag{6}
\]

which concludes the proof.

As a special case of bulk selection with space-frequency encoding we state the following theorem which pertains to horizontal (or frequency only) encoding, i.e., codewords do not span multiple spatial streams but are free to span across multiple tones.

**Theorem 2:** For the same setting as in Theorem 1, bulk selection with horizontal coding achieves the same diversity order as joint space-frequency encoding.

**Proof:** With horizontal encoding the diversity order is constrained by the lowest SNR across spatial streams. A diversity maximizing selection criterion is to select the antenna subset which maximizes the weakest stream SNR [8], and the maximum diversity order can be determined by considering the outage probability \(\Pr \left( \max_j \min_k \sum_{l=1}^M R_{l,j}^{(k)} \leq x \right)\).

Since horizontal encoding is a special case of space-frequency encoding it follows from Theorem 1 that the diversity order must be upper-bounded by \(M(N_T-L+1)(N_R-L+1)\). To lower-bound the diversity order, consider the inequality

\[
\Pr \left( \max_j \min_k \sum_{l=1}^M [R_{l,j}^{(k)}] \leq x \right) \leq \hat{U}_B, \tag{7}
\]

where

\[
\hat{U}_B = \Pr \left( \max_j \min_k \sum_{l=1}^M [R_{l,j}^{(k)}] \leq x \right) \leq \Pr \left( \max_j [\max_k R_{l,j}^{(k)}] \leq x \right) = \Pr \left( \max_j \min_k R_{l,j}^{(k)} \leq x \right) \leq x^M(N_T-L+1)(N_R-L+1).
\]

The exponential inequality follows from \([8],[7]\). Finally, for the case of uncoded MIMO-OFDM we state the following theorem.

**Theorem 3:** For the same setting as in Theorems 1 and 2, bulk selection with uncoded transmission achieves the diversity order \(d = (N_T-L+1)(N_R-L+1)\).

**Proof:** Since the diversity gain is limited by the lowest SNR across all tones and data streams, the maximum diversity order is determined by the outage probability \(\Pr \left( \max_j \min_k R_{l,j}^{(k)} \leq x \right)\) which may be upper-bounded by \(\Pr \left( \max_j \min_k R_{l,j}^{(k)} \leq x \right) \leq x(N_T-L+1)(N_R-L+1)\) [8], [7]. Now, let \(\hat{D}_{l,j} = \min_k R_{l,j}^{(k)}\). We can show \(\Pr(D_{l,j} \leq x) \leq \Pr(P_{l,j}^{(k)} \leq x)\), \(\forall k\). Replacing \(R_{l,j}\) in Proposition V of [8] by \(\hat{D}_{l,j}\), we see (not explicitly shown due to space limitation) that the diversity order is lower-bounded by \((N_T-L+1)(N_R-L+1)\), which concludes the proof.

From Lemmas 1 and 2, and Theorems 1, 2 and 3, we conclude the following. **With a linear receiver, irrespective of antenna selection, spatial encoding does not realize (spatial) diversity gain while frequency encoding realizes (frequency) diversity gain.** This conclusion has important implications for practical MIMO-OFDM code construction and performance analysis.

**B. Per-Tone Selection**

As explained earlier, per-tone selection transmits data streams over different RF chains in each tone. We state the following theorem for per-tone selection.

**Theorem 4:** In an \(N_R \times N_T\) MIMO-OFDM system, with \(M\) blocks of independently fading tones, each exhibiting i.i.d. Gaussian distribution across space, when \(L \leq N_T\) transmit antennas are selected using per-tone selection, and \(L\) spatially multiplexed data streams are received with a zero forcing receiver \((N_R \geq L)\) in each tone, the best achievable diversity order is the same as stated for bulk selection in Theorems 1, 2 and 3 under the assumptions of joint space-frequency encoding, horizontal encoding and uncoded transmission respectively.

**Proof:** For joint space-frequency encoding, the maximum diversity order is given by

\[
\Pr \left( \sum_{k=1}^M \max_j \sum_{l=1}^L R_{l,j}^{(k)} \leq x \right) \leq \Pr \left( \max_j \sum_{k=1}^M \sum_{l=1}^L R_{l,j}^{(k)} \leq x \right) \leq x^M(N_T-L+1)(N_R-L+1), \tag{8}
\]

where the final exponential equivalence follows from Lemma 2. We omit the proofs for horizontal encoding and the uncoded cases due to space limitations. These proofs follow in straightforward fashion.

**IV. CODING GAIN ANALYSIS**

In the previous section we demonstrated that bulk selection and per-tone selection realize the same diversity gain when
a linear receiver is applied. However, diversity order is by itself not an accurate measure of error rate. To bridge this gap, in the following, we analyze coding gain (based on outage probability). We consider the case of \( L = 1 \), where the precise coding gain can be derived in closed form. The outage probability for bulk selection with no frequency encoding is

\[
P_{\text{out}}^{\text{bulk}} = Pr \left( \max_{t} \min_{k} \log_{b} (1 + \rho \| h_{k}^{(t)} \|^2) \leq R_{0} \right),
\]

which when \( \rho \to \infty \) simplifies to

\[
P_{\text{out}}^{\text{bulk}} = \left( \frac{(NR)^{1/N_R}}{(2R_{0} - 1)M^{1/N_R}} \right)^{-N_T/N_R} + o(\rho^{-(N_T/N_R+1)}).
\]

Hence the coding gain for bulk selection is

\[
G_{\text{SD}}^{\text{bulk}} = \frac{(NR)^{1/N_R}}{(2R_{0} - 1)M^{1/N_R}}.
\]

Due to space restrictions, for per-tone selection we state the coding gain without proof

\[
G_{\text{SD}}^{\text{per-tone}} = \frac{(NR)^{1/N_R}}{(2R_{0} - 1)M^{1/N_R}}.
\]

Comparing (11) with (12), we have \( G_{\text{SD}}^{\text{per-tone}} / G_{\text{SD}}^{\text{bulk}} = M^{(N_T-1)/N_TN_R} \). As an example, if \( N_T = 4, N_R = 2 \) and \( M = 8 \), per-tone selection realizes a coding gain advantage of 3.4 dB over bulk selection, although they both realize the same diversity. In general, higher the frequency selectivity (i.e., larger \( M \)), larger will be the gap in coding gain between the two approaches. The coding gain derivation with frequency encoding is more mathematically involved and is omitted here due to space limitations.

Note that when \( M \) is extremely large (for instance in a wideband system with very large delay spread), the sum capacity of all the tones normalized by the number of tones approaches a constant (analogous to the channel hardening effect in [17] but in the frequency domain). Bulk antenna selection will then present marginal (if at all any) performance gain. Finally, for antenna selection in spatial multiplexing based MIMO systems, the precise coding gain is hard to derive, given that the problem reduces to order statistics amongst inter-dependent random variables, an underdeveloped topic by itself [18]. Simulation results in the following section will shed light on the coding gain gap between the two selection approaches.

In conclusion, bulk selection realizes the same diversity gain as per-tone selection, allowing fewer RF chains, at the expense of coding gain. With industry focused on RF “system on chip (SoC)” solutions, integrating additional RF chains may not be as expensive in the years to come. Furthermore, RF switching for bulk selection may introduce both attenuation and delay [10]. Therefore for wideband applications with high frequency selectivity per-tone selection is a more attractive option.

V. NUMERICAL RESULTS

Antenna selection for MIMO-OFDM has been incorporated into the IEEE 802.11n high throughput WLAN draft standard [9]. In this section, we provide simulation results comparing bulk and per-tone selection in the 802.11n environment, with \( N_T = 4, N_R = L = 2 \) and 20 MHz transmission bandwidth. Two data streams are transmitted from the two selected antennas with \( K = 64 \) tones, 64-QAM modulation and \( r = 2/3 \) convolutional coding (space and frequency interleaving is applied as in the draft standard). The two antenna selection approaches are simulated along with a \( 2 \times 2 \) non-selection scheme. Figure 1 shows the packet error rate (each packet is comprised of 8000 bits) as a function of SNR over a typical non-line-of-sight channel with 15ns rms delay spread (large coherence bandwidth) for bulk and per-tone selection as well as non-selection. Figure 2 simulates a more frequency-selective channel with 150ns rms delay spread. Clearly, bulk and per-tone selection exhibit the same diversity order as reflected by the slope of the error rate curves. From Lemma 1 the optimal diversity for the non-selection based system is \( M(N_R - L + 1) \). Therefore bulk and per-tone selection should achieve at most \( N_T - L + 1 \) times higher diversity, according to Theorems 1 and 4. The coding gain difference is however larger with increased frequency selectivity, thereby confirming the analysis presented in this paper. Bulk selection achieves significant gain over non-selection with low frequency selectivity, but the gain diminishes with increasing selectivity.

VI. CONCLUSION

In this paper, we derived the diversity order and coding gain achieved by transmit antenna selection for MIMO-OFDM systems with linear receivers. The diversity analysis applies to joint space-frequency encoding, horizontal encoding and uncoded transmission. We show that both selection schemes realize the same diversity gain under the various encoding options, but differ in the amount of coding gain realized. Our analysis was verified through simulations in a realistic IEEE 802.11n WLAN setting. Overall, this paper provides a comprehensive guideline for antenna selection strategies in MIMO-OFDM systems and provides important insights into
codeword construction and performance analysis for MIMO-OFDM.

Appendix

A. Proof of Lemma 1

Proof: With a ZF receiver, the equivalent channel can be viewed as an \(L \times L\) diagonal matrix \(\text{diag}\{\sqrt{R_1}, \ldots, \sqrt{R_L}\}\), where \(R_l\) is the squared Euclidean norm of the projection of \(\mathbf{h}_l\) onto the null space of the remaining columns of the MIMO channel matrix. With a repetition code, which is diversity optimal (although a better code will realize greater coding gain), the effective channel gain is \(\sum_{l=1}^{L} R_l\), and it follows that

\[
Pr\left(\sum_{l=1}^{L} R_l \leq x\right) \leq Pr\left(R_1 \leq x\right) \approx x^{N_R-L+1},
\]

(13)

\[
Pr\left(\sum_{l=1}^{L} R_l \leq x\right) \geq Pr\left(L \max_{i} R_i \leq x\right) \approx x^{N_R-L+1}.
\]

(14)

The final exponential equivalence comes from the fact that SNR-maximization ordering in a successive interference cancellation (SIC) receiver does not improve diversity over from the non-ordered case [19]. Lemma 1 is hence proved.

B. Proof of Lemma 2

Proof: There are totally \(N_U\) possible antenna subsets \(U_1, \ldots, U_{N_U}\). For each subset \(U_j\), following the proof of Lemma 1 above, the achievable diversity order is determined by diversity optimal encoding, which results in an equivalent channel gain of \(\sum_{l=1}^{L} R_{l,(j)}\). Therefore an antenna selection criterion maximizing diversity gain is \(\arg \max_j \sum_{l=1}^{L} R_{l,(j)}\).

We can state the following

\[
Pr\left(\max_{j} \sum_{l=1}^{L} R_{l,(j)} \leq x\right) \leq Pr\left(\max_{j} \min_{l} R_{l,(j)} \leq x/L\right) \leq x^{(N_T-L+1)(N_R-L+1)},
\]

(15)

where the final exponential equivalence in (15) follows from Proposition V in [8], and the final exponential equivalence in (16) follows from Theorem 1 in [7]. The proof is complete by combining (15) and (16).

References


