Fast Varying Channel Estimation in Downlink LTE Systems

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Abstract—This paper tackles the problem of fast varying channel estimation in LTE systems. Particular attention is given to downlink transmission where OFDMA is used as the multiple access technique for the air interface. In OFDMA, fast variation of the channel results in significant inter-carrier interference (ICI) which cannot be mitigated with conventional estimation and equalization techniques. In this paper, we investigate the polynomial modeling of rapidly time varying channels. Specifically, we propose a time-domain estimation method and a low-complexity frequency-domain estimation method both of which leverage the special structure of the approximated channel matrix to compute ICI channel components. It is shown by simulations that these two methods provide a considerable performance improvement over conventional methods by means of improved ICI mitigation. We further study the robustness of these two methods to both partial and full channel profile knowledge. It has been shown that the frequency-domain approach has much lower complexity and better robustness compared to the time domain approach.

Index Terms—channel estimation, intercarrier interference, OFDM, polynomial expansion.

I. INTRODUCTION

With the continuously increasing demand on high data rates on every application of mobile radio technology, emerging technologies like LTE (long term evolution) and WiMAX (Worldwide Interoperability for Microwave Access) are advancing in order to respond to the needs for future mobile wireless access systems. One of the common aspects of LTE and WiMAX is the orthogonal frequency-division multiple access (OFDMA) adopted in the air-interface of both as multiple access technique for the downlink transmission. Because of its favorable features, OFDMA also appears as the potential candidate for 4G mobile cellular systems.

OFDMA is based on orthogonal frequency-division multiplexing (OFDM) and therefore inherits also the drawbacks of OFDM transmission. In OFDM based systems, one of the most challenging problems is the estimation of the channel in fast-varying conditions. Indeed, for rapidly varying channels, the variation of the channel within one OFDM symbol destroys the orthogonality between the subcarriers and introduces inter-carrier interference (ICI). Accordingly, simple estimation and one tap equalization are not sufficient and this necessitates more sophisticated estimators for such situations.

Basis expansion modeling (BEM) is one of the possible ways to approximate the time-variation of the channel within a certain time window and has recently taken a lot of attention for fast-varying channel estimation [1]. Basically, this method reduces the complexity as the problem is reduced to estimating the basis coefficients. Among the existing BEMs, a particular attention has been given to the polynomial BEM (P-BEM) [2, 3, 4] for relatively low Doppler spreads. It mainly consists of approximating the channel variation during a certain time window by a polynomial function. Although, P-BEM has an attractive performance when the whole band is available for estimation, it is shown in [2] that with a sparse pilot distribution as in LTE we can not estimate directly ICI terms. In [2], authors have investigated the improvement provided by using first order polynomial approximation over two successive OFDM symbols to estimate the channel variation based on given pilot distributions. Then, the obtained initial time domain estimates are transformed into the frequency domain where all the modeling components are computed. A similar approach but a time domain version (aiming at finding all time domain modeling parameters) has been proposed in [5] for two successive OFDM symbols and generalized to multiple OFDM symbols in [4] with a polynomial modeling of degree dependent on the number of OFDM symbols used in the estimation. All these approaches have shown interesting results but they are not applicable for practical pilot distributions as in LTE.

In this paper, we consider the pilot distribution defined in the LTE specifications and we focus on the polynomial modeling to estimate the channel for all the OFDM symbols in a given resource block (RB). In particular, we propose a so-called double (polynomial) expansion modeling (DEM) of the channel. This consists of applying a first order (linear) modeling during one OFDM symbol duration and a second order modeling to approximate the channel variation between 8 OFDM symbols. We further analyze the structure of the channel matrix in the frequency domain as in [2] but based on a 2nd order polynomial modeling and we derive an algorithm for the computation of ICI terms directly in the frequency domain without any need of estimating all the time domain modeling parameters. Finally, we propose a lower complexity all-in frequency domain estimator suitable especially for OFDMA systems. In fact, time domain modeling seeks estimating the whole channel response, while for OFDMA systems only a part of the channel frequency response is needed. This algorithm mainly seeks to estimate the needed part of the
channel matrix directly in the frequency domain without any time domain estimation. We also compare these two algorithms in terms of complexity and robustness to channel profile knowledge.

The rest of this paper is organized as follows. Section II presents the general system model, the pilot distribution in LTE, and a brief review of BEM. In section III, we analyze the structure of the channel matrix based on second order P-BEM. Then, Section IV describes the algorithms based on the application of DEM over several OFDM symbols. Simulation results are presented in Section V and we conclude the paper in Section VI.

Notation: We use upper (lower) bold face letters to denote matrices (column vectors), $(\cdot)^T$ and $(\cdot)^H$ denote transpose and complex conjugate transpose operators, respectively. $\otimes$ represents the Kronecker product. We denote an $N \times N$ identity matrix by $I_N$. Furthermore, we use $X_{i,k}$ to indicate the $(i+1, k+1)_{th}$ entry of the matrix $X$ and $\text{diag}(x)$ to indicate a diagonal matrix with $x$ on its diagonal.

II. SYSTEM MODEL

A. General System Model

OFDMA is a multiple access technique based on OFDM, where subcarriers might be associated to different users. In an OFDM system, the data symbols collected in symbol vector $s$ are first transformed from the frequency domain to the time domain using an IFFT. Then, a CP consisting of the last $\text{CP}$ symbols is added before the parallel to serial converter. At the receiver side, a serial to parallel conversion is applied, CP is removed and an FFT operation is performed to obtain the received symbols in the frequency domain.

Then, the expression of the received vector in the frequency domain for the $(p+1)_{th}$ OFDM symbol can be expressed as

$$r^{(p)} = F H^{(t)}(p) F^H s^{(p)} + z^{(p)} = G^{(p)} s^{(p)} + z^{(p)},$$

where $H^{(t)}(p)$ and $G^{(p)}$ represent the channel matrices respectively in time and frequency domains for the $(p+1)_{th}$ OFDM symbol; $F$ denotes the FFT operation whose elements are given by: $F_{i,k} = \frac{1}{\sqrt{N}} \exp(-j\frac{2\pi ik}{N})$, and $z$ is the complex additive white Gaussian noise vector. A simple diagram of such a transmission is depicted in Fig. 1.

For an FFT size of $N$, it can be easily shown that $H^{(t)}(p)$ has the entries

$$H_{i,k}^{(t)} = h_{\text{mod}(i-k,N),p(N+L_{\text{CP}})+i},$$

where $h_{l,n}$ represents the $(l+1)_{th}$ tap of the channel impulse response at instant $n$ and $L_{\text{CP}}$ is the CP length. We note that for a finite channel length $L$, $h_{l,n} = 0$ for $l > L - 1$. In the sequel, for the sake of clarity, the index $p$ will only be used where necessary.

Due to the time variation of the channel during the OFDM symbol, the frequency domain channel matrix $G$ would not be diagonal. The estimation problem in this case is extended to finding the full matrix $G$ (or the desired part of it corresponding to the frequency band allocated to a given user). In fact, it will have the entries

$$G_{i,k} = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} H_{k,n} e^{j\frac{2\pi n(k-i)}{N}},$$

where $\{H_{k,n}\}_{k=0,...,N-1}$ denote the Fourier transform of the channel impulse response $\{h_{l,n}\}_{l=0,...,L-1}$ at the time instant $n$ for a channel length of $L$ and are given by

$$H_{k,n} = \frac{1}{\sqrt{N}} \sum_{l=0}^{L-1} h_{l,n} \exp\left(-j\frac{2\pi kl}{N}\right).$$

B. LTE Context and Pilot Distribution

Our study is mainly based on the downlink LTE systems where OFDMA has been adopted as the multiple access technique. Here, resources (in both time and frequency dimensions) are associated to users based on subframes of 2 RBs as depicted in Fig. 2. A simple RB may consist of 6 or 7 OFDM symbols (depending on the CP length) and 12 subcarriers.

As for reference signals, a sparse pilot distribution in both frequency and time dimensions has been specified. In particular, the first and the fifth OFDM symbols of each RB are defined to contain pilots, and in frequency dimension, the subcarriers with indices multiples of 6 are used for carrying pilots. This pilot distribution is shown in the Fig. 2 for an RB having 7 OFDM symbols.

C. Basis Expansion Modeling (BEM)

In this section, we recall how a BEM can be used to estimate the variations of the channel. For more detailed description reader may refer to [1]. The idea behind BEM is to express the $N$ channel samples as a function of $(Q+1)$ basis functions approximating the variation of the channel during a specific period. In this modeling, for every channel tap, we write

$$h_l = Bh_l^0 + \eta_l,$$

where $h_l = [h_{l,0}, \ldots, h_{l,N-1}]^T$ contains the $N$ channel samples corresponding to the $(l+1)_{th}$ tap and $h_l^0 =$
Similarly all basis coefficients in a single column vector as full preamble case) or based on only a part of it (i.e., in the $F$-domain). Here, we use $B_{p,q} = (p/N)^q$ to express the polynomial modeling.

If we collect all the channel taps in a single column vector as $h = [h_{0,0}, \ldots, h_{L-1,0}, \ldots, h_{0,N-1}, \ldots, h_{L-1,N-1}]^T$ and similarly all basis coefficients in a single column vector as $h_b = [h_{0,0}, \ldots, h_{L-1,0}, \ldots, h_{0,Q}, \ldots, h_{L-1,Q}]^T$, then neglecting the modeling error we obtain

$$h = (B \otimes I_L)h_b.$$  

(6)

As a result, after some algebra, the received symbol vector can be expressed in terms of the BEM as

$$r = \sum_{q=0}^{Q} D_q \Delta_q s + z$$  

(7)

where

$$D_q = Fdiag(b_q)F^H$$

and

$$\Delta_q = diag(F_L[h_{0,q}, \ldots, h_{L-1,q}]).$$

Here, $F_L$ collects the first $L$ columns of the matrix $\sqrt{N}F$, and $b_q$ represents the $(q + 1)^{th}$ column of the BEM matrix $B$.

If we define $D = [D_0, \ldots, D_Q]$ and $S = I_{Q+1} \otimes (diag(s)F_L)$ then we obtain

$$r = DS\hat{h} + z$$  

(8)

Using (8), channel estimators can be derived either based on the whole knowledge of the input symbol vector $s$ (i.e., the full preamble case) or based on only a part of it (i.e., in the presence of pilot signals). We note clearly that BEM simplifies the estimation as the problem is reduced to estimating $h_b$ of size $(Q + 1)L$ rather than estimating all the $NL$ channel coefficients of $h$.

### III. Analysis of the $G$ Matrix

In this section, we analyze the structure of the $G$ matrix assuming that the channel variation over one RB in the time domain can be approximated by a polynomial modeling. Here, we use $2^{nd}$ degree polynomial modeling to approximate the channel variation over 8 OFDM symbols. In this case, the expression of the channel variation for each tap and for the $(p + 1)^{th}$ OFDM symbol can be expressed as

$$h_{i,n}^{(p)} = h_{i,0} + (pT_0 + N/E)[\epsilon_i + (pT_0 + N/E)^2\mu_i],$$  

(9)

where $0 \leq n \leq N - 1$ and $T_0 = 1 + Lc_p/N$; $h_{i,0}$, $\epsilon_i$ and $\mu_i$ become the basis coefficients for the $(l + 1)^{th}$ tap representing the variation of the channel in a certain time window (i.e., over several OFDM symbols). From (9), one can easily obtain

$$H_{i,n}^{(p)} = H_{i,0} + (pT_0 + N/E)E_i + (pT_0 + N/E)^2U_i,$$  

(10)

where $E_i$’s and $U_i$’s are simply the Fourier transforms of $\epsilon_i$’s and $\mu_i$’s, respectively. Then, using (10) in (3), the coefficients of the diagonal terms in the $G$ matrix can be found as

$$G_{i,i}^{(p)} = \sqrt{N}(H_{i,0} + pT_0E_i + (pT_0)^2U_i)$$

$$+ \frac{N - 1}{2\sqrt{N}}(E_i + 2pT_0U_i)$$

$$+ \frac{(N - 1)(2N - 1)}{6N\sqrt{N}}U_i.$$  

(11)

Using the properties of Fourier transform of power series, the coefficients of the non diagonal terms can be written as

$$G_{i,k}^{(p)} = C_e(mod((k - i), N))E_k$$

$$+ C_u(mod((k - i), N))U_k$$

$$+ 2pT_0C_e(mod((k - i), N))U_k,$$  

(12)

where $C_e$ and $C_u$ are constants defined as

$$C_e(n) = -\frac{1}{\sqrt{N}}, \frac{1}{1 - \exp \left( \frac{i2\pi n}{N} \right)}.$$  

(13)

$$C_u(n) = -\frac{1}{\sqrt{N}}, \frac{1}{1 - \exp \left( \frac{i2\pi n}{N} \right)}$$

$$+ \frac{2}{N (1 - \exp \left( \frac{i2\pi n}{N} \right))},$$  

(14)

with $n \neq 0$.

Comparing the diagonal terms obtained from (11) for different OFDM symbols, we can observe that the estimation problem is reduced to finding 3 diagonal estimates in order to find whole channel modeling parameters in the frequency domain. On the one hand, this simply means that once we have $G_{i,i}^{(p)}$s for 3 OFDM symbols (i.e., for 3 values of $p$), we can simply compute the corresponding $E_i$ and $U_i$ as

$$U_i = \left[ \frac{4}{12\sqrt{NT_0}}(G_{i,i}^{(p)} - G_{i,i}^{(0)}) - (G_{i,i}^{(4)} - G_{i,i}^{(0)}) \right]$$  

(15)

$$E_i = \frac{(G_{i,i}^{(4)} - G_{i,i}^{(0)})}{4\sqrt{NT_0}} - U_i\frac{N - 1}{N} + 4T_0$$  

(16)
On the other hand, (12) can be used to calculate the ICI terms using the computed $E_i$’s and $U_i$’s directly in the frequency domain. Based on these observations, it can be inferred that a sufficiently accurate knowledge of the diagonal terms of 3 OFDM symbols allows us to estimate all the coefficients of the $G$ matrices.

IV. PROPOSED ALGORITHMS

Below, we first present the double expansion modeling algorithm and we explain how time modeling of channel variation can be used to ease the channel estimation in the time domain. And then we explain how the presented method can be applied solely in the frequency domain. We also provide a complexity comparison between these two methods and we finally discuss the importance of channel profile knowledge.

A. BEM Based Estimation

As a first step, we apply directly a basis expansion on the channel and try to estimate the basis coefficients just using the assumed pilot distribution. It has been shown in [2] that a pilot distribution with similar spacing is not sufficient to calculate directly the variation of the channel which is causing the ICI. But it permits to obtain sufficiently accurate estimates of the diagonal terms for further processing.

By separating the input symbol matrix $S$ defined in Section II.C as a sum of the matrix $S_p$ containing pilots and a matrix $S_d$ containing data symbols [1], we can rewrite (8) as

$$r = DS_p h^b + DS_d h^b + z.$$  \hspace{1cm} (17)

Then, the corresponding linear minimum mean squared error (MMSE) filter $W_{LMMSE}$ leading to the estimates $\hat{h}^b = Wr$ with minimum MSE can be obtained as [1, 2]

$$W_{LMMSE} = R_h h^H (\Pi R_h h^H + \Pi_x + N_0 I_N)^{-1},$$ \hspace{1cm} (18)

where $R_h b = E[hh^H]$ is the autocorrelation matrix between the basis expansion coefficients, and $N_0$ denotes the noise spectral density. In (18), $\Pi = DS_p$ depends on the pilots and $R_x = D E[S_d h^b h^b H S_d^H]D^H$ is calculated using the assumed statistical properties of the channel and data.

From these time domain estimates, first the diagonal terms of the frequency domain channel matrices corresponding to the 1$st$, 5$th$, and 8$th$ OFDM symbols are computed. Then, using (11) and (12), the whole channel matrices are computed for the whole assigned RB. In the sequel, we will mainly focus on the estimation based on a single RB but the extension of the algorithms to a subframe for different cases is trivial.

B. Frequency Domain Estimation

As discussed in the previous sections, only the estimation of the diagonal terms of the frequency domain channel matrices corresponding to the 1$st$, 5$th$, and 8$th$ OFDM symbols are needed to compute all the matrices for the whole RB. This simply motivates the direct estimation of these terms in the frequency domain.

For time-invariant channels, channel estimation in the frequency domain has been widely investigated. In [6], the authors use LS estimates at the pilot locations, then use MMSE interpolation to find all estimates over the bandwidth of interest. In our estimation procedure, we adopt this interpolation method by considering the ICI as additive noise to get sufficiently accurate estimates of the desired diagonal terms. Then, we proceed in the same manner as before in computing all other terms. This approach is particularly practical for OFDMA systems, as we just need to estimate the assigned frequency coefficients and not the whole channel frequency response. It is also worth noting that the performance of this algorithm depends on the number of pilots used, so a complexity-performance trade-off could be made.

C. Complexity Analysis

For the BEM based approach, the MMSE matrix can be calculated offline as it only depends on the known pilots and their positions. In this case, the complexity is reduced to a matrix multiplication and then FFT processing to get the coefficients in the frequency domain before computing all components. The complexity will be then in the order of $O((Q + 1)LN + N\log N)$. We note here that time domain estimation aims at estimating whole channel response while resources may be attributed to users in the frequency domain as in OFDMA so that only some frequency components are needed. Another point to be added, in this case, is that decision directed methods as in [2] typically require the inversion of large data-dependent matrices and so they are not practical from an implementation point of view.

For the full frequency domain approach, the complexity depends on the equalizer used which defines the required frequency domain coefficients. In our case, we suppose that the user subcarriers are only interfered by the $M$ (which is set to 6 in the simulations) nearest subcarriers and so we consider estimating in each OFDM symbol a square matrix of size $(N_i = 2M + 12)$. If we use $N_p$ pilots then this approach introduces a complexity on the order of $O(N_iN_p)$. It is worth noting that, in this case, decision directed approaches might be used to enhance the performance with equal additional complexity.

D. Channel Profile Knowledge

For the presented algorithms, MMSE estimation is used to estimate the channel values either in the time or in the frequency domain. For reliable estimates, MMSE estimator requires knowledge about the channel characteristics in order to calculate the correlation matrices. Therefore, one may need to know the channel profile in order to improve the estimation. As an optimum solution, the channel profile can be estimated (see, e.g., [7]) but the complexity of the estimator may not be affordable for practical applications. Therefore, the mostly used approach in practice is the assumption of the channel profile as rectangular [8] and the estimation of the channel length if necessary (see, e.g., [9] and [10] for possible solutions).
In this paper, we consider both of the solutions for channel profile knowledge to be feasible and we test the robustness of both the time-domain DEM-based algorithm and the frequency domain one to channel profile mismatches.

V. SIMULATION RESULTS

In this section, we analyze the performance of the proposed algorithms under the assumptions explained above. In the simulations, the number of subcarriers is set to \( N = 256 \) where 180 of them are used for transmission and the others will be considered as symmetric guard subcarriers. The symbol duration is taken to be \( 66.7 \mu s \) with a normal CP length which is equal to 19 samples. For data symbols, 64-QAM constellation is adopted while for pilots QPSK modulation is used assuming equal energy on the whole FFT grid. We use an improved Jake’s model [11] to generate the extended Typical Urban ETU channel profile [12]. A maximum Doppler spread of 300 Hz is considered and it is supposed to be perfectly known at the receiver. Therefore, with an inter-subcarrier spacing of 15 kHz, the considered normalized Doppler spread to the OFDM symbol duration is equal to 0.02. The above mentioned simulation parameters are compatible with the ones defined for LTE systems [13]. We further consider that the user is associated to a single subframe and a zero forcing (ZF) equalizer is used for all the cases.

First, we investigate the performance of the proposed ICI computation algorithm based on the channel matrix structure obtained using the polynomial modeling. Fig. 3, shows the normalized MSE curves of the diagonal terms and the non-diagonal terms as a function of received SNR which is defined as the ratio of received signal energy per symbol to the noise spectral density \( N_0 \). The results are obtained after using the pilot-based BEM estimation. We remind here that the pilot-based BEM approach is efficient only in estimating the diagonal terms, therefore ICI terms are deduced by using the special structure of the \( G \) matrix as discussed in section III. We observe that by leveraging the \( G \) matrix structure, ICI components can be estimated accurately as their normalized MSE goes down below \( 10^{-2} \) for an SNR above 30 dB.

Fig. 4 gives more insights on the performance of the DEM based estimation method. The pilot-based BEM curve showed with a dashed-line represents the performance without ICI computation and by using one-tap conventional equalization. On the other hand, the curves corresponding to the performances after calculating ICI terms are depicted by dashed-dotted in case of an assumed rectangular channel profile and by dotted in case of perfect channel profile knowledge. We also put the performance with perfect channel state information (CSI) with solid line. We observe a clear improvement in symbol error rate (SER) performance of the applied algorithm compared to the pilot-based BEM. More specifically, the effect of ICI resulting in an error floor is significantly decreased and a performance closer to the perfect channel estimation is achieved in the case of perfect profile knowledge. From Fig. 4, we can also conclude that the time domain approach is highly sensitive to profile mismatches since we may have considerable improvement by estimating the channel profile compared to estimating only its length.

In Fig. 5, we trace the SER curves using the full frequency domain estimation as described in Section IV.B. We present the one-tap equalizer based on the estimation of the diagonal terms using 2D interpolation, then we show the curves after ICI computation based on the proposed algorithm. We can observe an obvious performance improvement of the applied algorithm in terms of ICI mitigation. In addition, this method has much lower complexity compared to the first method particularly for OFDMA systems. It is also worth noting that this frequency domain approach shows more robustness to channel profile mismatches as there is no significant performance difference between perfect profile knowledge and an assumed rectangular one.

VI. CONCLUSION

In this paper, we investigated a polynomial modeling of rapidly time varying channels. We proposed a time domain method and a low-complexity frequency domain estimation method particularly for downlink LTE systems. These methods leverage the special structure of the approximated channel matrix to compute ICI components. It has been shown by simulations that these two methods provide a considerable performance improvement over conventional methods by means of improved ICI mitigation. We further studied the robustness of these two methods to both partial and full channel profile knowledge. It has been shown that the frequency domain approach has much lower complexity and better robustness compared to the time domain approach.

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Fig. 4. SER curves using the time domain DEM method.

Fig. 5. SER curves using the frequency domain method.

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