THE PARALLEL GENETIC ALGORITHM FOR ELECTROMAGNETIC INVERSE SCATTERING OF A CONDUCTOR

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A Parallel Genetic Algorithm is presented to solve the inverse scattering problem which is formulated as an optimal problem where the cost function to be minimized is the energy norm of the residual, i.e. the difference between the estimated and observed field, and the parameters are the unknown control points for using a spline curve to construct the shape of the object conductor. This approach is computationally heavy since the direct problem needs to be solved in every optimization iteration in order to compute an estimated field. Experiments demonstrate our PGA provides an efficient method for such a problem.

Keywords: Inverse scattering; Parallel genetic algorithm; Moment method and cubic spline

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I. INTRODUCTION

The inverse scattering problem under consideration is to determine the shape of the conductor cylinder given the knowledge of an incident electromagnetic plane wave, which is of particular interest in many areas of industrial research such as non-invasive measurement, system identification and remote
sensing. As a basic problem, it has been heavily studied in both theoretical and numerical aspects. However, this inverse scattering problem is difficult to solve, especially from a numerical viewpoint because like most other inverse problems, the inverse scattering problem is both ill-posed and nonlinear. Typically, the problem can be formulated as a non-linear optimization problem. Many optimization methods have been applied to such problems such as the traditional optimization method and evolution optimization method, respectively [1, 2].

Compared with the traditional optimization method, the genetic algorithm is more suitable for such a problem because it does not require derivative or other auxiliary information. However, like other iterative methods, the object function evaluation in the inverse scattering problem is a time consuming computing task due to the forward problem involved in it, which often makes it difficult to use the genetic algorithm in a real time application. To speed up the computation, we present one method based on the parallel genetic algorithm to reconstruct the shape of a perfectly conducting cylinder illuminated by transverse electric (TE) waves.

In this article, the remaining parts are organized as below. In Section II, the theoretical foundation and numerical computing for the inverse scattering is introduced. In Sections III and IV, the parallel genetic algorithms and how they are applied to the reconstruction problem are presented. The numerical results of our methods are presented in Section V. Finally, some discussions are presented in Section VI.

II. SCATTERING PROBLEM

Here we focus our attention on the two-dimensional scattering problem. As illustrated in Figure 1, a perfectly conducting cylinder is excited by an incident TE wave without the component of $E$. The impressed field induces surface currents on the conducting cylinder, which in turn produce a scattering field. The field induced by the surface current can be formulated specialized to the cylinder surface as below by [3]

$$\frac{J(r)}{2} - \frac{k}{4j} \int_C J(r') \cos \alpha' H^{(2)}_1(k|\mathbf{r} - r'|) d\mathbf{l}' = -H^1_2(r), \quad r \in C,$$

where $J(r)$ is the induced surface magnetic current density, $H^1_2(r)$ is the scattered magnetic field, and $H^{(2)}_1(\cdot)$ is the Hankel function of the second kind of order one.
Now the forward scattering problem can be depicted as: given the impressed field and the shape of the conducting cylinder, then calculate the scattering field. This problem has been studied both by theoretic and numerical calculations. For the numerical calculation, the moment method is most often used to solve the forward problem. In this paper, the forward problem was solved by the moment method [4]. The comparison between the numerical computation and the exact solution for a circular cylinder shows that the numerical computation is a very good approximation to the exact solution.

Then the inverse scattering problem can be cast as: given the measured scattering field, then reconstruct the shape of the conducting cylinder. This problem can be solved by reformulating it as an optimization problem:

$$\min_f = \left( \frac{1}{M} \sum_{n=1}^{M} |H_s^{\text{measured}}(r_n) - H_s^{\text{calculated}}(r_n)|^2 \right)^{1/2} + \alpha/A(X(s)), \quad (2)$$

FIGURE 1 Cylinder excited by TE wave.
where $M$ is the total number of the measurement points, $H_s^{\text{measured}}(r)$ and $H_s^{\text{calculated}}(r)$ are the measured field and the calculated field on the measurement point $r$, respectively. $A(X(s))$ is the area determined by the closed contour $X(s)$, which is a regular term for the minimization problem. $\alpha$ is the regular coefficient whose value varies from 0.001 to 1.

Most of the shape of a conductor can be modelled by means of a spline curve, so the shape function in this paper is expressed based on the cubic spline:

$$X(s) = (x(s), y(s)), \quad s \in [0, 1],$$

where $x(s)$ and $y(s)$ are the spline curves determined by control points.

### III. PARALLEL GENETIC ALGORITHMS

The genetic algorithm (GA) is a search and optimization technique based on the evolutionary principle of natural chromosomes [5]. Specifically, the evolution of chromosomes due to the actions of crossover, mutation and natural selection of chromosomes based on Darwin's survival-of-the-fittest principles being artificially simulated results in a robust search and optimization procedure. The parallel genetic algorithm (PGA) as a new member of GA introduced by H. Mühlenbein, etc. [6–8]. In the PGA, a new operator — Migration Operator is incorporated, which makes the changes in a population come not only from the classical operators: crossover and mutation, but also from the new species into the population and makes the PGA to be a more natural model of the natural system. In the PGA, first several subpopulations are generated in isolated states, then the subpopulations evolve independently for a certain time, then the sub-populations share their best individuals with others.

The most popular migration model is the island model. First, each subpopulation evolves using a genetic algorithm, illustrated in Figure 2(b). After evolving one or several generations, some best individuals of subpopulations migrate to other subpopulations. Therefore, in a PGA, changes in a subpopulation come not only from itself evolving, but also from migrating some new individuals, which makes the PGA represent a better model of nature. The island model allows the individuals to travel to any other subpopulations, as illustrated in Figure 2(a). Compared with the sequential genetic algorithm, PGA introduces a communication overhead. In order to reduce the communication overhead, another mechanism is adopted more often called the stepping stone model. The stepping stone model reduces the number of messages by limiting
destinations to which emigrants may travel. In some ways, the island model represents a better model of nature at the cost of more communications.

In order to solve an optimization problem using PGA like all other genetic algorithms, we have to do the following three basic things:

1. Define a representation. We should use a representation that is minimal yet completely expressive. Our representation should be able to represent any solution to our problem, but at all possible we should design it so that it cannot represent infeasible solutions to our problem. If infeasible solutions are possible then the objective function must be designed to penalize them accordingly.

2. Define the genetic operators. The basic genetic operators include initialization, mutation, crossover and migration. The initialization operator defines how to create many initial solutions to the specific problem. The mutation operator tells
how to change some parts of a solution to produce a new solution to the problem. The crossover operator uses two existing solutions determine a new solution. In the PGA, the migration operation introduces several new operators:

- **Select the emigrants:** selecting the best individuals to migrate from each subpopulation,
- **Integrating the immigrants:** replacing the worst individuals with the immigrants,
- **Sending the emigrants** and **Receiving the immigrants:** sending and receiving individuals between subpopulations.

3. Define the objective function. Genetic algorithms are often more attractive than gradient search methods because they do not require complicated differential equations or a smooth search space. Genetic algorithms need only a single measure of how good a single individual is compared to other individuals.
With the above notations, we can describe the basic steps of a PGA as follows:

**Step 1** Use an initializing genetic operator to initialize the subpopulations.

**Step 2** In each subpopulation, select some chromosomes to crossover in order to produce a group of new chromosomes.

**Step 3** According to the mutation probability; mutate the offspring produced by the crossover operator.

**Step 4** Insert the offspring into the population. Determine whether the isolated evolution stopping criteria are satisfied. If satisfied, stop the isolated evolution; go to Step 5. Otherwise, go to Step 2.

**Step 5** Determine whether the stopping criteria are satisfied. If satisfied, stop the iteration; otherwise go to Step 6.

**Step 6** Use the migration operators to share best individuals, then go to Step 2.

## IV. RECONSTRUCTION OF CYLINDER SHAPE WITH A PGA

In this section, we present a PGA for extracting the cylinder shape. The input information is the measured magnetic field on \( M \) measurement points. We can select \( N \ (\geq 4) \) control points to represent the spline contour, the first 4 points are distributed with a fixed angle (90°) called the support point set which makes the contour always starlike in shape. The others are distributed with any angle on the plane and called the fitting point set which makes the contour fit as best the desired curve. Each chromosome consists of an even number of genes which can be expressed as: \( (r_1, r_2, r_3, r_4, r_5, \ldots, r_N, \theta_N) \), in which the first four are the distances between the control points and the center, the other gene pairs are the distance between the control points and the center and the angle between the x coordinate and the radial vector.

The parallel genetic algorithm for shape reconstruction can be described as follows:

### Part One: Parallelization

**Step 1** Parameters Setting:
- The number of subpopulations: How much subpopulations will we use?
- The number of individuals of a subpopulation
- Isolation time in generations: How long the subpopulations will self-evolve?

**Step 2** Send initialization messages to the subpopulations

**Step 3** Get the population from the subpopulations
**Step 4** Send step messages to the subpopulations

**Step 5** Get the population from the subpopulations and determine whether to repeat or stop, if repeat, continue to step 6.

**Step 6** Randomly set how many subpopulations will send individuals to others and how many best individuals will be sent. Our migration strategy is different from the island model and the stepping stone model. Who and how many subpopulations will migrate their bests individuals to others are randomly determined. This strategy is a better compromise between the island model and the stepping stone model, which not only reduces the communication costs but also makes the PGA behavior more close to nature.

**Step 7** Migration between the selected subpopulations and go to step 4.

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**Part Two: Evolution in Subpopulations**

Let $P$ denote the population with $M$ chromosomes $I_1, I_2, \ldots, I_M$. Let $L$ denote the size of the population. Let $NUM$ represent the number of generations. Let $N_r$ denote the number of chromosomes replaced in each generation. For convenience, let $N_r$ be an even number.

**Step 1** Get Initialization Message: To initialize each chromosome in subpopulation. The real-valued chromosome representation is used in our algorithm and the genes are generated uniformly within the variables upper and lower bounds. The real-valued representation provides a higher precision solution and is more efficient, especially when the algorithm is carried out in a message passing computer model.

**Step 2** Get Step Message:

1. Selection and crossover: We use the fitness proportional model to select chromosomes to reproduce offspring. To obtain $N_r$ children, we should select $N_r/2$ mothers and fathers respectively. Each pair of parents has two children. The crossover operator is described as follows: denote the parents are $X$ and $Y$, the children are $X'.gene(i) = pX.gene(i) + (1 - p)Y.gene(i)$ and $Y'.gene(i) = (1 - p)X.gene(i) + pY.gene(i)$, where $p = U(0, 1)$. So, $N_r$ new chromosomes are reproduced.

2. Mutation: For all the newly generated chromosomes, according to a mutation probability to mutate them. The mutation method is described as follows: randomly select one gene $m$, and set it equal to an uniform random number $U(a_m, b_m)$, where $a_m$ and $b_m$ are the lower and upper bounds of gene $m$. 
3. Replacement scheme: We use the newly generated $N_r$ chromosomes to replace those having worst fitness.

4. Local hillclimbing: Firstly, we compute the fitness of each chromosome and the fitness can be viewed as a distribution function. Then we compute the entropy of the distribution function. When the entropy does not change again, we decide that the genetic algorithm has reached a region of attraction of a good minimum and begin to implement local hillclimbing, and then go to step 3, if the number of generations $N$ is reached, we also go to step 3, otherwise, go to 1.

**Step 3** Get Migration Message: Send emigrations from selected subpopulations to destinations, and then delete the equivalent number of worst individuals in the destination subpopulations.

**Remark** The local hillclimbing method is the downhill Simplex method in multidimensions [9]. The best individual is the initial starting point.

V. NUMERICAL RESULTS

The efficiency and the accuracy of the present method was tested with numerical simulations. A perfect conducting cylinder in a free space is excited by a TE polarization plane wave. The wavelength of the incident wave $\lambda$ is 0.1 m. We will use the scattering field measured outside the conductor cylinder to reconstruct the shape of the object. In order to get sufficient information about the object, three incident waves with the incident angles of $\phi = 0^\circ, 120^\circ, \text{ and } 240^\circ$ are used and the measurement is carried out on a cycle of radius $5\lambda$ at equal spacing, respectively. In our experiment, the measured field on 30 measurement points is used. The shape of the object is constructed with 8 control points distributed at equal angles. The parameters for PGA are set as below:

- Number of subpopulations: 9
- Number of individuals of subpopulation: 40
- Generations in subpopulation self-evolution: 20
- Mutation probability: 0.05
- Crossover probability: 0.9

In the first example, the shape of the conductor cylinder is constructed with four control points: $(0.12, 0), (0,0.12), (-0.12, 0)$, and $(0, -0.12)$. In order to
FIGURE 3(a)

FIGURE 3(b)
evaluate the accuracy of the algorithm, the root mean square (rms) error was defined as below [1].

\[
\left[ \frac{1}{N'} \sum_{i=1}^{N'} \left( \frac{F_{\text{cal}}(\theta_i) - F(\theta_i)}{F(\theta_i)} \right)^2 \right]^{1/2},
\]

where \( N' \) is set to 80 and \( F_{\text{cal}} \) is the calculated shape function. The simulation results Figure 3(a) and Figure 3(b) show the reconstruction of the shape of conductor cylinder is excellent. Gaussian noises with zero means were added to the measured fields in order to test the ability of the shape reconstruction algorithm against random noise. Noise with normalized standard deviation: \( 10^{-5}, 10^{-4}, 10^{-3}, 10^{-2}, \) and \( 10^{-1} \) are used in the experiments. The results was showed in Figure 3(c).

In the second example, the shape of conductor is constructed using four control points: \((0.12, 0), (0, 0.08), (-0.09, 0), \) and \((0, -0.1)\). The results are shown in Figure 4(a), Figure 4(b) and Figure 4(c).

Our results show that a more accurate shape reconstruction potential was gained. The algorithm was implemented using C language with MPICH in LINUX environment. All the examples were computed with a 5-PC-cluster.
FIGURE 4(a)

FIGURE 4(b)
and the computing time varies from 40 min to 60 min due to the different initial parameters.

VI. DISCUSSION

The PGA given in this paper provides a means by which a more accurate shape reconstruction can be achieved. Two highly significant practical advantages of this method are: 1) to express the conductor cylinder shape by means of a spline curve. Firstly, spline approximations are broadly used in the industrial community, so a spline curve may produce more accurate shape reconstruction. Secondly, our method can easily keep the curve as a starlike shape, which makes the PGA more efficient. 2) The Parallel Genetic Algorithm provides not only a more accurate solution but also a speedup because the population evaluations were shared by several processes although we do not give a detailed comparison. These properties meet the need for fast shape reconstructions.

We did not fully investigate the extent to which the PGA could be used to improve the accuracy of optimization based conductor cylinder shape reconstruction, though the results have demonstrated that the PGA is more
efficient for such problems. The experimental results show that our approach is very promising to further our capability of conductor shape reconstruction.

References


