SPIN-FLIP SCATTERING VERSIONS OF THE RUDERMAN-KITTEL AND DZYALOSHINSKY-MORIYA INTERACTIONS

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Abstract. – We derive two classes of spin-flip versions of the Ruderman-Kittel and Dzyaloshinsky-Moriya interactions, defined for one-electron and collective spin-flip dynamics, and relate these couplings to the high and low temperature regions of EPR in the $T>T_g$ regime of transition metal spin glasses.

Studies of the behavior of dilute transition metal spin glasses have involved the properties of the electron-mediated RKKY and DM pair interactions

$$S^{AB} = \mathbf{S}_A \cdot \mathbf{S}_B$$

and $$D^{A\lambda B} = \hat{\mathbf{R}}_A \cdot \hat{\mathbf{R}}_B \left(\hat{\mathbf{R}}_A \times \hat{\mathbf{R}}_B\right) \cdot (\mathbf{S}_A \times \mathbf{S}_B)$$

where, in the latter case, the vectors $\hat{\mathbf{R}}_i$ locate the spins $\mathbf{S}_i$ with respect to a spin orbit scattering site. However, these interactions are defined as the effects of impurity spin-electron spin $(-\Gamma\mathbf{S}\cdot\mathbf{s})$ and electron spin-orbit $(\lambda\mathbf{l}\cdot\mathbf{s})$ perturbations on the ground state of an electron gas. In experimental situations such as EPR, the application of a rotating magnetic field $\mathbf{H}(t)$ induces a current in the itinerant electron system and it would be of interest to determine the effects of these same perturbations when they scatter a current. A natural formulation for a problem of this type is that of a scattering cross-section, and since the physical mechanism underlying EPR is the angular momentum change associated with spin rotations, we calculate the contributions to the electron spin-flip scattering cross-section for an assortment of multisite scattering processes.

As a starting point, we use the cross-section formulation of Asik, Ball, and Slichter [1]:

$$\sigma = (4\pi\hbar v_F)^{-1} \mathcal{N}(E_F) \times \times \int d\Omega_{k_F} \int d\Omega_{k'_F} \left|\left\langle \mathbf{k}_F + |V| \mathbf{k}'_{F-} \right\rangle\right|^2 \quad (1)$$

where v_F, E_F, k_F, are the Fermi velocity, energy, and wavevector, $\mathcal{N}$ is the one-electron density of states for one spin direction, and V is a perturbation inducing spin-flip scattering from $\left|\mathbf{k}'_F\right\rangle$ to $|\mathbf{k}_F\rangle$. Used originally for a discussion of the effects of spin-orbit scattering at non-magnetic impurity sites, equation (1) has wide validity. With the use of $\mathbf{l}\cdot\mathbf{s}$ and a VBS wave function, it reproduces Yafet's well-known result [2]; with the use of $\mathbf{S}\cdot\mathbf{s}$ and plane waves, it yields a cross-section leading to the Overhauser relaxation rate [3]. In these cases, the perturbations have acted as "single-site" effects. Multisite scattering events can be discussed by introducing sets of intermediate states into equation (1); i.e., one may consider second and higher order scattering matrix elements in lieu of the first order element $V_{kk'}$ in the equation.

We consider three cases: that for which an electron spin-flips through two spin scattering sites labeled A and B, that for which site B is instead a non-magnetic spin-orbit scatterer located at the origin, and a third order example involving two spin scattering sites and a non-magnetic spin-orbit site. The respective contributions to the cross-section are, to leading order in $1/R$,

$$\sigma_{AB} = \frac{\pi^5}{k_F^2}\left(\frac{3\Gamma}{4E_F}\right)^4 \left(\frac{\cos k_F R_{AB}}{k_F R_{AB}}\right)^2 \times \times \left\{\left(\mathcal{S}^{AB}\right)^2 - \left(\mathcal{S}^{AB}\right)_z^2\right\} \quad (2)$$

$$\sigma_{A\lambda} = \frac{45\pi^3}{4k_F^2}\left(\frac{\Gamma\lambda}{\Delta E_F}\sin^2\eta(k_F)\right)^2 \left(\frac{\cos k_F R_A}{k_F R_A}\right)^2 (S_A)_z^2 \quad (3)$$

and

$$\sigma_{A\lambda B} = \frac{\pi^5}{k_F^2}\left(\frac{45}{32}\frac{\Gamma^2\lambda}{\Delta E_F^2}\sin^2\eta(k_F)\right)^2 \times \times \left[\frac{\sin\left[k_F R_A + \eta(k_F)\right]}{k_F R_A}\frac{\sin\left[k_F R_B + \eta(k_F)\right]}{k_F R_B}\right]^2 \times \left\{\left(\mathcal{D}^{A\lambda B}\right)^2 - \left(\mathcal{D}^{A\lambda B}\right)_z^2\right\} \quad (4)$$

where Δ and η are the half width and phase shift associated with the VBS on the spin-orbit site, and $\mathcal{S}^{AB}$ and $\mathcal{D}^{A\lambda B}$ are the RKKY and DM torque vectors

$$\mathcal{S}^{AB} = \mathbf{S}_A \times \mathbf{S}_B$$

and

$$\mathcal{D}^{A\lambda B} = \hat{\mathbf{R}}_A \cdot \hat{\mathbf{R}}_B \left\{\mathbf{S}_A \times \left[\mathbf{S}_B \times \left(\hat{\mathbf{R}}_A \times \hat{\mathbf{R}}_B\right)\right] + \left[\left(\hat{\mathbf{R}}_A \times \hat{\mathbf{R}}_B\right) \times \mathbf{S}_A\right] \times \mathbf{S}_B + (\mathbf{R}_A \times \mathbf{R}_B)\,\mathbf{S}_A \cdot \mathbf{S}_B\right\}.$$

The z axis is perpendicular to the rotation plane of $\mathbf{H}(t)$.

The generic coupling forms $\mathcal{T}^2 - \mathcal{T}_z^2$ of equations (2) and (4) may be taken as the forms the RKKY and DM interactions assume when the relevant perturbations spin-flip scatter a current. Equation (3) has no "pair interaction" equivalent but resembles a "single-site" anisotropic contribution, although it is a two-center effect.

Equations (2) to (4) may be used to demonstrate a concentration dependence in the Korringa term of the high temperature EPR linewidth observed in CuMn-

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class materials. For example, the contribution of σ_{AB} to the Korringa relaxation rate is given by

$$1/\tau \simeq 0.13\pi^5 x \frac{\mathcal{N}(E_F) V_F}{k_F^2} \left(\frac{\Gamma}{E_F}\right)^4 S(S+1) k_B T \quad (5)$$

where x is the concentration of impurity spins. In general, with y the concentration of non-magnetic spin-orbit scatterers and including the familiar concentration-independent single site term, the Korringa rate has the form

$$1/\tau = (\alpha + \beta x + \gamma y + \delta xy ...) k_B T. \quad (6)$$

This result may not be too important as the high T linewidth as observed by Mozurkewich *et al.* [4] is a bottlenecked result. We now demonstrate the existence of another set of electron-mediated couplings which arise when the electron gas responds *collectively* to the rotating field.

Collective effects in the form of spin waves arise when particles of the itinerant system undergo repeated Coulomb exchange scattering in regions where there is a population difference between spin down and spin up particles. These effects are already known in pure iron [5] and in palladium alloys [6]. In CuMn-class materials, we consider the exchange scattering between itinerant electrons, driven by the rotating field against the magnetic impurities, and those which are screening the impurity charge at a given instant. Of necessity, this mechanism is defined only at the impurity sites so that the collective effects described below cannot exist in the pure host. One studies the time dependence of the electron density fluctuations, specifically the equation

$$i\hbar \dot{s}_{+1}(\mathbf{q}) = [s_{+1}(\mathbf{q}),\ H] \quad (7)$$

where $s_{+1}(\mathbf{q}) = -\left(1/\sqrt{2}\right) \Sigma \mathbf{p} a^{+}_{\mathbf{p}-\mathbf{q},+} a_{\mathbf{p}-}$, and the Hamiltonian H includes the kinetic energy, Zeeman interactions, Coulomb scattering, and spin and spin-orbit impurity scattering.

We have constructed a solution to equation (7) in the random phase approximation which includes in first order, single-site spin scattering, and in higher order, the same multisite processes that lead to equations (2) thru (4). The associated cross-section is essentially the sum of the one-electron terms enhanced by the factor $\left|1 - U\Gamma_{\mathbf{q}\omega}\right|^2$ where U is the Coulomb integral and $\Gamma_{\mathbf{q}\omega}$ is the transverse Lindhard function but with the quasi-particle spectrum modified by impurity scattering.

By following Izuyama *et al.* [5], we have obtained the following spin wave spectrum:

$$\hbar\Omega_{\mathbf{q}} = \hbar\omega_{\mathbf{q}} + \hbar\omega_{\mathbf{q}}^{(\mathrm{imp})} \quad (8)$$

where $\hbar\omega_{\mathbf{q}}$ is identical in form to the impurity-independent acoustic branch of [5], and the impurity term is given by

$$\begin{aligned} \hbar\omega_{\mathbf{q}}^{(\mathrm{imp})} &= -\Gamma (S_A)_z + \frac{\zeta\Gamma}{2k_F}\left(\frac{N_- + N_+}{N_- - N_+}\right) \times \\ &\times \frac{\sin 2k_F R_{AB}}{R_{AB}^2} \sin\frac{\mathbf{q}}{2}\cdot\mathbf{R}_{AB} C^{AB}(\mathbf{q}\cdot\mathbf{R}_{AB}) \\ &- \frac{5}{12}\frac{\zeta\Lambda}{k_F}\left(\frac{N_- + N_+}{N_- - N_+}\right) \frac{\cos\left[k_F (R_A + R_B) + 2\eta(k_F)\right]}{R_A R_B R_{AB}} \\ &\times \sin k_F R_{AB} \sin\frac{\mathbf{q}}{2}\cdot\mathbf{R}_{AB} C^{A\lambda B}(\mathbf{q}\cdot\mathbf{R}_{AB}) \end{aligned} \quad (9)$$

where

$$\zeta = k_F^2\Gamma / (8\pi E_F N),\ \Lambda = 15\Gamma\lambda \sin^2 \eta(k_F) / (8k_F\Delta);$$

N being the number of host sites/volume and N_σ being the number of electrons of spin state σ of the screening charge. The couplings C have the generic form

$$C^{ij\cdots}(\mathbf{q}\cdot\mathbf{R}) = \mathcal{T}_z^{ij\cdots} \cos\frac{\mathbf{q}}{2}\cdot\mathbf{R} + T^{ij\cdots} \sin\frac{\mathbf{q}}{2}\cdot\mathbf{R} \quad (10)$$

where $\mathcal{T}^{ij\cdots}$ and $T^{ij\cdots}$ are the torques $\mathcal{S}$, $\mathcal{D}$ and scalars S, D defined earlier. There is no contribution to the spectrum from the two-center process where one of the sites is a non-magnetic spin-orbit impurity.

Equation (8) represents a spectrum of excited states localized at spin site A where A has a neighboring spin scatterer at B and a spin-orbit scatterer at the origin. The coupling terms are given only to leading order in $1/R$. At temperatures sufficiently low that the couplings represented by the states are not disrupted by phonons, these states can be occupied by absorption of photons from the rotating field. The cross-section then undergoes a resonant enhancement. In this regime, the electron system is dominated by the localized collective modes and the spin wave states, now occupied, represent the couplings. Note that the "AB" and "AλB" terms of $\hbar\Omega$ are acoustic modes so that the couplings represented by $\mathcal{S}_z^{AB}$ and $\mathcal{D}_z^{A\lambda B}$ will not appear in EPR unless the rotating field is spatially non-uniform. However, three-spin-center terms $\sim 1/R^3$ are optical and these will be discussed in a future publication. For the anisotropic terms, the third spin can be the spin of a magnetic spin-orbit scatterer. One may raise the question whether a single-site scattering from a magnetic spin-orbit impurity can contribute to transverse relaxation. Traditionally, $\mathbf{l}\cdot\mathbf{s}$ leads to longitudinal effects, and the situation is not clear in this new case.

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