

Filling the Teaching Gap between Electromagnetics and Circuits

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Abstract: Electrical engineers normally are taught electromagnetism in an electromagnetics course (e.g. in [1-2]), and circuit analysis in an independent course (e.g. in [4-6]). Circuits are dominated by Kirchhoff's laws, while electromagnetics is dominated by Maxwell's equations. However, the correspondence between two sets of equations is not immediately perceived and this creates some uncertainty in the young electrical or electronic engineer, which may grow with the doubt that Kirchhoff's laws may be somewhat laws of the nature independent of the laws of electromagnetism. This paper has the purpose of supplying teaching material that may be used to fill the gap, and therefore be taught either at the end of an electromagnetics or at the beginning of a circuit course. It exploits large parts of the paper published in a conference [8], but also contains significant enhancements. The paper first shows simple distributed parameter systems, whose behaviour follows Maxwell's equations, and then shows that they, under given assumptions, can be modelled as circuits, whose behaviour is governed by Kirchhoff's laws.

Key words: Circuit analysis, Kirchhoff's equations, Maxwell's equations, teaching.

Nomenclature

| | |
|----------------------|--|
| B | magnetic flux density (or magnetic induction) |
| D | electric flux density |
| E | electric field |
| J | current density field (or areic current field) |
| J_d | displacement current density field (or areic displacement current field) |
| H | magnetic field |
| ϵ | dielectric constant |
| ρ | electric resistivity |
| μ | magnetic permeability |
| ψ | flux linkage |
| c | charge density (or volumic charge) |
| AC or a.c. | alternating current |
| DC or d.c. | direct current |
| EMF | electromotiveforce |
| KVL | Kirchhoff's Voltage Law |
| KCL | Kirchhoff's Current Law |

1. Introduction

During past centuries the electromagnetism theory has seen the basic laws first (such as Gauss's, Ampère's, Faraday's, Ohm's) to be discovered in an

integral, macroscopic way, then to be expressed in a differential form that results in equations that, while having as a consequence the integral versions from which they derive, are useful extensions of them. The most important effort in this rationalization of the basic laws was from Maxwell, and therefore the resulting equations are called Maxwell's equations (in differential form).

Rather independently, the basic circuit laws, known as Kirchhoff's Current Law and Kirchhoff's Voltage Law, have been postulated and widely used.

Kirchhoff's laws are taught to be applicable to circuits, which can be fuzzily defined as electromagnetic systems composed by lumped components connected to each other by thin conductor lines (= wires).

The two equation sets (Kirchhoff's and Maxwell's) however, are independently introduced, so it is not always clear what is the rationale behind the postulation of Kirchhoff's laws in circuits, or, equivalently, what are the hypotheses that allow a physical, three-dimensional, system to be modeled and studied as a circuit (governed by Kirchhoff's laws)

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[1-7].

Ref. [8] was therefore conceived with the purpose of filling the knowledge gap between electro-magnetics and circuit theory, so that the relation of the two approaches is clarified. This paper reproduces parts of the results of Ref. [8], having in mind the need to enhance, taking advantage of what was there discussed, the way teachers teach the fundamentals on which circuits, as a concept, are based.

1.1 Graphical Conventions

To facilitate understanding the logical distinction we want to make between electromagnetic systems having a “circuitual shape” (i.e. being constituted by lumped components connected to each other by thin lines), we make a graphical distinction: in case of physical systems we reproduce the cross-sectional size of the lines, while in case of circuits we do not (Fig. 1).

1.2 Maxwell's and Other Relevant Electromagnetism Equations

Although very well known, here the four Maxwell's equations are reproduced in their integral form, so that they constitute an easy reference when reading of the remainder of the paper. The symbols are those reported in Section 1, and used throughout the paper.

$$\left\{ \begin{array}{l} \oint_{l(S)} \mathbf{E} \cdot d\mathbf{l} = - \iint_{S(l)} \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S} \\ \iint_{S(v)} \mathbf{D} \cdot d\mathbf{S} = \iiint_{v(S)} c \, dv \\ \oint_{l(S)} \mathbf{H} \cdot d\mathbf{l} = \iint_{S(l)} (\mathbf{J} + \mathbf{J}_d) \cdot d\mathbf{S}; \quad \mathbf{J}_d = \frac{d\mathbf{D}}{dt} \\ \oiint_S \mathbf{B} \cdot d\mathbf{S} = 0 \quad \forall \text{ closed } S \end{array} \right. \quad (1)$$

In addition to Maxwell's equations the pointwise Ohm's and continuity equations are reminded, because

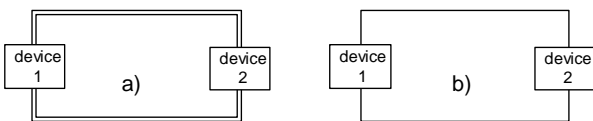


Fig. 1 Graphical convention adopted for conductors: (a) devices connected by real-connecting lines; (b) devices connected by idealized lines.

of their importance for the paper¹.

$$\mathbf{E} = \rho \mathbf{J} \quad (2)$$

$$\oiint_{S(v)} \mathbf{J} \cdot d\mathbf{S} = - \frac{\partial}{\partial t} \iiint_{v(S)} c \, dv \quad (3)$$

In case of systems where it is known that all quantities are constant (i.e. DC systems), all time derivatives become zero, and the first and third Maxwell's equation become:

$$\left\{ \begin{array}{l} \oint_l \mathbf{E} \cdot d\mathbf{l} = 0 \quad \forall \text{ closed } l \\ \oint_{l(S)} \mathbf{H} \cdot d\mathbf{l} = \iint_{S(l)} \mathbf{J} \cdot d\mathbf{S} \end{array} \right. \quad (4)$$

and continuity equation becomes as well:

$$\oiint_S \mathbf{J} \cdot d\mathbf{S} = 0 \quad \forall \text{ closed } S \quad (5)$$

2. Distributed Systems and Circuits

Electromagnetism studies electrostatic (i.e. related to effects of the presence of charge in given portions of space) and electrodynamic, or magnetic (i.e. related to moving charges) phenomena.

It extends to phenomena related to interaction of the previous two, since in time-varying systems they are closely related to each other.

The use of Maxwell's equations or other electromagnetism tools either in integral or differential form has proven too demanding to analyze systems composed by different electromagnetic subsystems.

Consider the simple system shown in Fig. 2.

It is formed by an electric sine-wave voltage generator feeding two lamps with the interposition of a couple of wires, which are represented “thick”, because in a physical system they have not only a length but also a width.



Fig. 2 A simple electromagnetic system containing wires connecting lumped components.

¹ The continuity equation is not independent of Maxwell's equations: it can be easily derived taking the divergence of both members of the third of (1), and introducing c in the result taking it from the second one.

The analysis of this system would be greatly simplified if, instead of having to analyze simultaneously the whole system using Maxwell's equations (differential equations to be applied at any point of space taking into account all boundary conditions), we are able to write independent equations of the involved individual, lumped, components and link them by some additional congruence equations.

This approach can be referred to, for the time being, as the circuit or circuital approach.

A qualitative analysis of Fig. 2 shows that the generator connects to the lamps through long wires, while short, vertical connections are present at the two sides of the system. Therefore a hypothetical approximation of the system of Fig. 2 could be as shown in Fig. 3a: the connections are shown using thin wires, to evidence their connecting role, while the parts of the original system to be modeled individually are enclosed in boxes or circles. In Fig. 3b, a further evolution of the system is shown, in which the components are substituted with symbols indicating specific mathematical modeling of the considered components: ideal resistors for line and loads, ideal generator for the generator.

The circuital approximations of Fig. 3 are composed only by circuit elements (generator, lamps or resistors, transmission line box) and ideal wires. All physicists and electrical engineers already know very well that this “lumpization” of electromagnetic systems constituted by components joined by conductor wires is possible, but rarely the rationale behind this conversion is investigated.

In the following sections it is shown that the conversion of spatially-distributed physical systems into circuits is possible, under certain hypotheses, which also determine the choice on how to make the transformation, and imply some limitations.

In search of the implementation of the conversion into circuit of any system governed by the electromagnetism equations, better is to start with the simplest case, i.e. when all quantities do not vary with

time. By traditional nomenclature these systems are referred to as “direct current” systems.²

3. Applicability of Kirchhoff's Current Law in d.c. Circuits

Consider a region of space, able to exchange charge between its interior and exterior. In case we want to analyze a system by means of the technique of conversion into a circuit, it is rather obvious that this charge exchange occurs only by means of discrete “channels” constituted by the wiring entering the surface, while charge exchange in regions not occupied by wires is neglected.

Therefore it is reasonable to put forward the following:

ASSUMPTION 1: any charge flow is neglected anywhere outside circuit elements, except than within conductor wires.

Consider the region of space V surrounded by surface S (Fig. 4a).

In the drawings of Fig. 4, all the conductor wires converging into the volume V are considered (and shown).

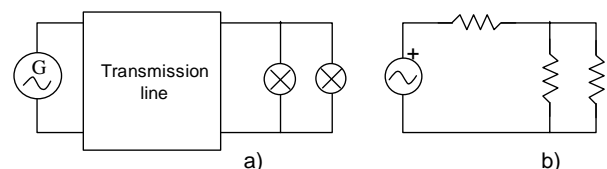


Fig. 3 Circuital approximation of system of Fig. 2.

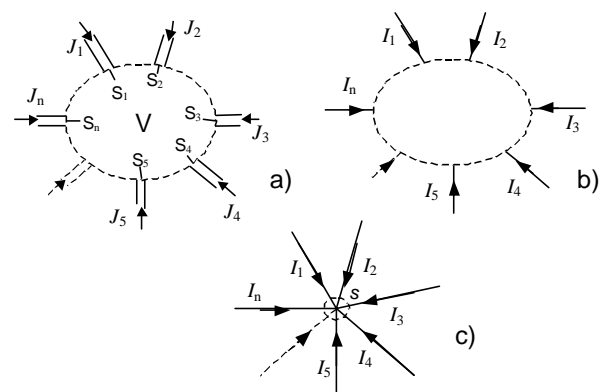


Fig. 4 A confined region of space (a) that can bring to a generalized node (b) and a node (c).

² Better name would have been “constant operation systems” or “steady-state systems”.

Let us now consider the continuity Eq. (3).

$$\oint_{S(v)} \mathbf{J} \cdot d\mathbf{S} = -\frac{\partial}{\partial t} \iiint_{v(S)} c \, dv$$

where c is the spatial charge density, or charge per unit of volume.

Since by hypothesis we are “in d.c.,” meaning that in the considered systems all quantities are constant, charge density is also constant and therefore in this case the continuity equation becomes Eq. (5).

Because of Assumption 1 conductive currents are possible only within wires. So the integral of Eq. (5) is simplified since \mathbf{J}_t is non-zero only through S_k , that are the intersections of S with conductor wires.

$$\int_S \mathbf{J} \cdot d\mathbf{s} = \sum_{k=1}^n \int_{S_k} \mathbf{J} \cdot d\mathbf{s} = \sum_{k=1}^n (-I_k) = 0$$

where $I_k = \int_{S_k} \mathbf{J} \cdot d\mathbf{s}$

$$\text{Thus: } \sum_{k=1}^n I_k = 0$$

That is KCL for the region V .

Fig. 4b shows another representation of the same system of Fig. 4a. In this paper, however, there is a logical distinction between the two: the thin wires of Fig. 4b are idealized wires, based on special assumptions. At this point of the paper, the only assumption beyond the symbol of idealized wires is Assumption 1.

Obviously enough, the demonstration proposed, referred to the scheme of Fig. 4a, contained what is normally called in circuit terminology a “generalized node”, is applicable also in the scheme of Fig. 4c containing a conventional node: it is just necessary to consider a tiny surface S around the connection of some wires.

CONCLUSION 1: in a physical system operating in DC for which Assumption 1 is applicable the KCL applies.

4. Applicability Kirchhoff's Voltage Law in d.c. Circuits

Consider the system displayed in Fig. 5.

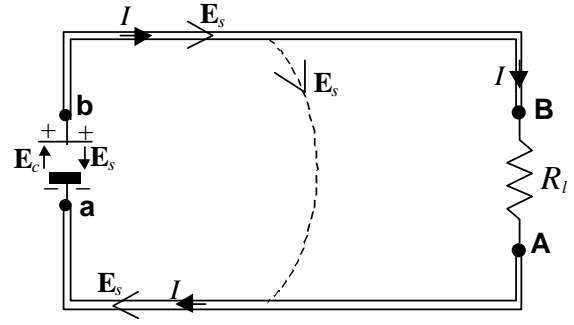


Fig. 5 Basic system with charge flow and chemical and electrostatic fields.

It is composed by an electrochemical battery (at the left side of the figure) connected through physical wires to a load resistor R_l at the right side.

Let us first imagine that there is some positive charge located at the upper terminal of the battery and an equal amount of negative charge in the lower one. These charges would create an electric field \mathbf{E}_s in the space around them: inside conductors it is longitudinal, while outside it has a different orientation (one possible force line is shown dashed in figure), but has no relevance for analysis of DC systems since by effect of Assumption 1, charge movement is allowed only within conductor wires.

Any individual charge present in the conductors (i.e. an electron) would then circulate in the conductor loop of the system, and finally offset the initial charge accumulated at the two battery terminals, and so in a very short time the conductor loop would be neutral and no more charge could flow.

During this flow, the energy received by the charge by effect of \mathbf{E}_s is dissipated during the transit, by effect of the energy loss occurring during charge movement in conductor materials, i.e. where Ohm's law applies.

It is a well-known fact that the battery is able to cause continuous charge flow in the circuit. Although the actual behavior of an electrochemical battery is very complex, it can be modeled for the purposes of this study as a system able to “pump” charges pushing them from its negative electrode towards its positive one by means of an inner electric “chemical” field \mathbf{E}_c that this way lets charges flow. Any charge looping in

the system of Fig. 5, obtains energy when it goes through the battery, by combined effect of \mathbf{E}_c and \mathbf{E}_s , which is later and delivered (dissipated) when the charge flows through the external circuit.

The energy supplied by \mathbf{E}_c to the charge equals the energy dissipated during the flow. Since in the entire loop \mathbf{E}_c has a net contribution to the work transferred to the charge, it is a non-conservative field.

Therefore analysis of this circuit can be made starting from the supposed simultaneous presence of electrostatic, conservative (\mathbf{E}_s) and chemically induced, non-conservative (\mathbf{E}_c) fields in the battery:

$$\mathbf{E}_t = \mathbf{E}_s + \mathbf{E}_c \quad (6)$$

The charge movement in the loop is determined by the presence of the whole field \mathbf{E}_t , not only \mathbf{E}_s ; therefore the Ohm's law is to be written:

$$\mathbf{E}_t = \rho \mathbf{J}$$

and, taking the loop integral of both sides:

$$\oint \mathbf{E}_t \cdot d\mathbf{l} = \oint \rho \mathbf{J} \cdot d\mathbf{l} \quad (7)$$

The left part of Eq. (7) is:

$$\oint \mathbf{E}_t \cdot d\mathbf{l} = \int_a^b \mathbf{E}_c \cdot d\mathbf{l} \quad (8)$$

because of the conservativity of \mathbf{E}_s and the absence of \mathbf{E}_c outside the battery.

The integral of right part of Eq. (7) may be computed neglecting the resistance of conductor wires in comparison to battery and load resistances.

$$\begin{aligned} \int_a^b \rho \mathbf{J} \cdot d\mathbf{l} + \int_b^B \rho \mathbf{J} \cdot d\mathbf{l} + \int_B^A \rho \mathbf{J} \cdot d\mathbf{l} + \int_A^a \rho \mathbf{J} \cdot d\mathbf{l} &\cong \\ &\cong \int_a^B \rho \mathbf{J} \cdot d\mathbf{l} + \int_B^A \rho \mathbf{J} \cdot d\mathbf{l} = (R_B + R_l)I \end{aligned} \quad (9)$$

while the latter equality is justified by the relations, for both resistive components:

$$\int \rho \mathbf{J} \cdot d\mathbf{l} = \int \rho(l) J(l) dl = \int \rho(l) \frac{I}{S(l)} dl = I \int \rho \frac{dl}{S(l)} = RI$$

where it has been exploited that, as a consequence of the continuity equation, current I does not depend on the integral variable l .

Substituting Eqs. (8) and (9) into Eq. (7) gives:

$$V_e = \oint \mathbf{E}_c \cdot d\mathbf{l} = (R_B + R_l)I \quad (10)$$

where the quantity V_e , defined by means of \mathbf{E}_c , is called “electromotive force” (EMF) of the circuit (subscript e stands for electromotive)³.

Eq. (10) is a usual expression of Ohm's law for one-loop system, and may be considered to be the result of application of KVL to the circuit of Fig. 6, that assumes the role of equivalent circuit of system of Fig. 5.

The utilization of KVL in this circuit is now validated by means of the Maxwell's and Ohm's equations.

Although rather obvious, it is important to stress that the result obtained is not just linked to the presence of an electrochemical battery. Several possibilities exist to create devices that in its inside “pump” changes from its negatively charged terminal to its positively charged one, i.e. they make charges move through the external circuit through the electric field created in the conductor by the charges located at the terminals of the pumping devices.⁴

Let us now consider a more complicated electric system shown in Fig. 7.

It contains several loops, resistors and several batteries. Moreover, it is not electrically isolated from the outside world: because of the connections at the corners of its loops.

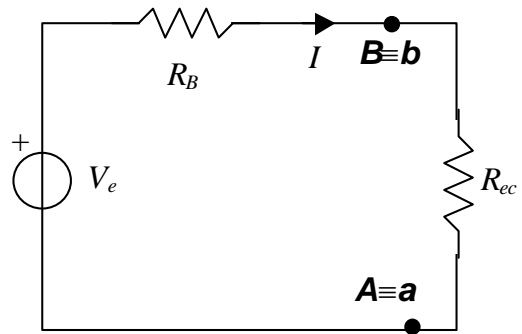


Fig. 6 Equivalent circuit of the system shown in Fig. 5.

³ The term electromotive force is maintained for its worldwide use; it is however apparent that it is partially confusing, since the quantity to which it refers is not a force in the physical sense.

⁴ Significant sources of constant electromotive force are fuel cells, photovoltaic cells; electric machines operating as source are sources of time varying electromotive forces.

In the whole system considered, including the parts not shown in figure, there exists in principle a field \mathbf{E}_s caused by the charge accumulated at all the battery terminals (considering also those outside the shown part of the system), and the corresponding current density field $\mathbf{J} = \sigma \mathbf{E}_s$, where, obviously, any point of space has \mathbf{E}_s , \mathbf{J} , σ of its own.

However, because of Assumption 1, there is no interest in considering the fields present outside the conductor wires.

The direction of electric and current density fields inside the conductor is parallel to their axes, but its orientation is not known a priori (this lack of knowledge has prevented the possibility of reporting the vector arrows in Fig. 7).

In Fig. 7 three possible loops may be considered: L1, L2 and L3. Let us concentrate, without loss of generality, on loop L1.

When a generic charge q goes along loop L1 in the system shown in Fig. 7, the work of the electrostatic field \mathbf{E}_s on it is null, because of the conservative nature of the electrostatic field.

Consequently, the analysis carried out with system of Fig. 5 can be repeated for any loop of Fig. 7. The Ohm's law gives:

$$\oint_{L1} \mathbf{E}_t \cdot d\mathbf{l} = \oint_{L1} \rho \mathbf{J} \cdot d\mathbf{l} \quad (11)$$

The left part of Eq. (11) is:

$$\oint_{L1} \mathbf{E}_t \cdot d\mathbf{l} = \int_{a1}^{b1} \mathbf{E}_{c1} \cdot d\mathbf{l} + \int_{a2}^{b2} \mathbf{E}_{c2} \cdot d\mathbf{l} \quad (12)$$

because \mathbf{E}_s is conservative.

Again, the integral of right part of Eq. (11) may be computed neglecting the resistance of conductor wires in comparison to battery and load resistances.

$$\oint_{L1} \rho \mathbf{J} \cdot d\mathbf{l} = (R_{B1} + R_1)I_1 + (R_{B2} + R_2)I_2 \quad (13)$$

where I_1 and I_2 can both be computed as $\oint_S \mathbf{J} \cdot d\mathbf{S}$ using any cross section of the system branches containing R_1 and R_2 respectively.

The above analysis perfectly replicates that made for the simpler case of Fig. 5; therefore if the following definitions are adopted:

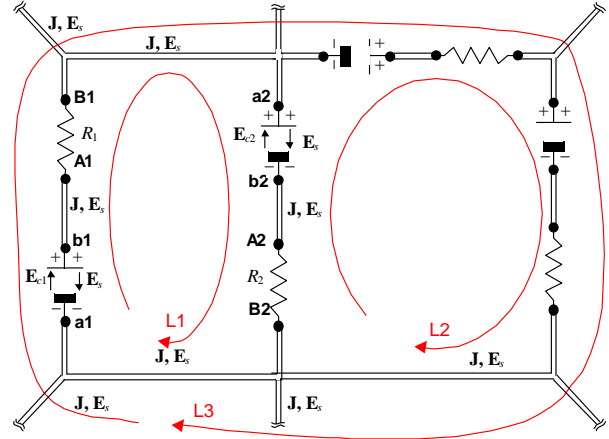


Fig. 7 A multiple-loop, multiple source, multiple resistor system.

$$V_{e1} = \left| \int_{a1}^{b1} \mathbf{E}_{c1} \cdot d\mathbf{l} \right| = \int_{a1}^{b1} \mathbf{E}_{c1} \cdot d\mathbf{l}$$

$$V_{e2} = \left| \int_{a2}^{b2} \mathbf{E}_{c2} \cdot d\mathbf{l} \right| = - \int_{a2}^{b2} \mathbf{E}_{c2} \cdot d\mathbf{l}$$

the loop L1 of the system of Fig. 7 can be studied as reported in the upper part of Fig. 8, and the entire system of Fig. 7 can be studied using the equivalent circuit shown in the bottom part of Fig. 8.

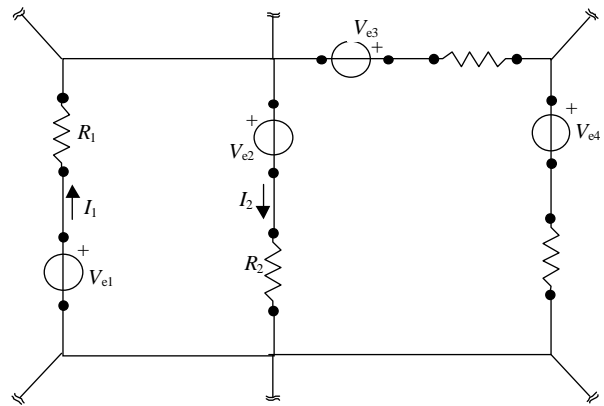
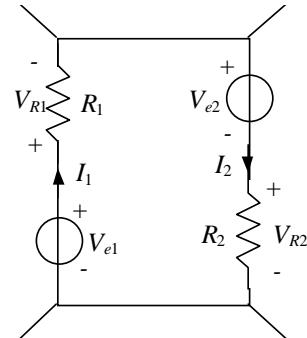


Fig. 8 Equivalent of loop L1 of the system shown in Fig. 7 (top) and of the full system of Fig. 7 (bottom).

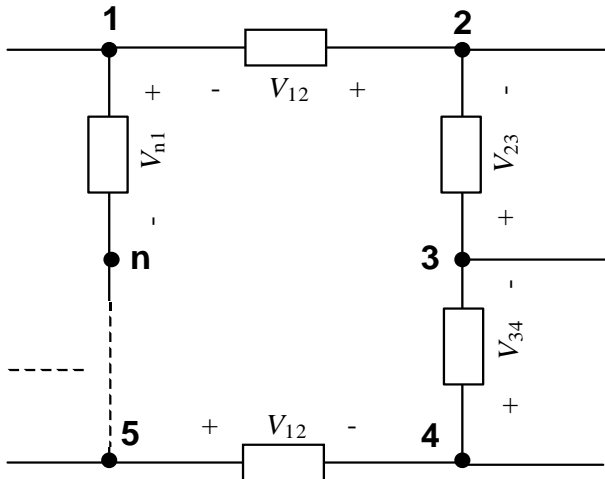


Fig. 9 One-loop, multiple-resistor circuit.

At this point a more general result is obvious:
in any loop of any DC circuit the sum of all electromotive forces (integrals of inner, non conservative fields) equals the sum of all resistances multiplied by the correspondent currents. (14)

Although in DC systems electromotive forces have a very different physical interpretation than voltages across resistors, as shown above, Statement (14) can be expressed in a more general form.

if any loop is gone through in a clockwise direction (or, equivalently in counter clockwise direction) the sum of all voltages rises (or, equivalently, the sum of all voltage drops) is zero. (15)

Statement (15) is the well known KVL. Although it has been derived considering the fields inside the conductors, it assumes a form that is immediately usable in circuits, to state useful relations between circuit quantities.

The KVL can be seen to be the circuital version of conservativity of electrostatic field.

Indeed it states that any loop implies null voltage sum, which is the equivalent of the notion that any circuit integral of elementary work of electrostatic field is null. Furthermore it is also equivalent to state that in any circuit is possible to define a definite voltage value for any node, a fact that again recalls the correspondent characteristic of electrostatic (or any conservative field).

The equivalence of KVL and possibility to define given potentials to circuit nodes can be easily shown considering Fig. 9. Here different lumped components (having arbitrary inner behavior) are connected to each other in a loop inserted in a larger circuit (in the most general case any node may have a wire connected with it and with other electric circuit components).

The voltages across their terminals are named after the terminals themselves, and they are considered according to the polarity references (positive voltage rises) shown in Fig. 9.

If a voltage can be defined for each node, then, considering that the $(n+1)$ th node is indeed the 1-st node, it is:

$$\sum_{k=1}^n V_{k,k+1} = \sum_{k=1}^n (V_{k+1} - V_k) = \sum_{k=2}^n V_k + V_1 - \sum_{k=1}^n V_k = 0$$

Let us summarize the procedure followed in this paragraph:

- It has been seen that the current continuously flowing in a simple DC circuit is consequence of the presence of a non-conservative field in one or more “forcing” component; as a consequence of this field, the terminals of the component wherein this field is present are able to remain differently charged;
- The charge difference at the terminals of a forcing component causes the presence of electrostatic field in the conductors outside it, whose potential difference at these terminals is equal to its electromotive force;
- This electrostatic field is obviously conservative and so voltages across circuit terminals are independent of the path considered, and thus a potential can be defined for each node;
- The presence of node potentials is necessary and sufficient condition for the KVLs.

The gist of this process is: forcing components to determine an electrostatic (conservative) field in the conductors outside them, so potentials of individual circuit points can be defined and therefore the KVL applies.

This works in any case, if we consider that the circuits are totally separated from the outside world,

except from interactions that may occur inside circuit elements (such as in the battery of Fig. 7)

Therefore the following is put forward:

ASSUMPTION 2: any interaction of the considered physical circuit with the outside world is made only inside circuit elements. No interaction with wires and space between wires is supposed to occur.

Then, the following conclusions can be made.

CONCLUSION 2: in a physical system operating in DC for which Assumption 2 is applicable, the KVL applies.

CONCLUSION 3: in a physical system operating in DC for which Assumptions 1 and 2 are applicable, the KCL and KVL apply, and therefore it can be studied as a circuit.

5. Extension of Kirchhoff's Laws to Time-Variable Circuits

In the previous paragraphs it was seen that in circuits operating with constant quantities, that by tradition are called DC circuits, the KVL is a direct consequence of the electrostatic field present in the circuit, and KCL is a consequence of continuity equation.

In this section, these concepts will be expanded to cover also time varying circuits.

5.1 Extension of Kirchhoff's Current Law

When we draw circuits we imagine that no current can flow between circuit wires. In other words, circuits have not real wires, but ideal ones, which are such that current (either conductive or displacement current) can flow only inside wires, while cannot in the free space around them. Naturally displacement currents can flow inside circuit elements, e.g. capacitors.

This justifies making the following assumption.

ASSUMPTION 3: the displacement current $\partial \mathbf{D} / \partial t$ is neglected anywhere, outside circuit elements.

This is independent from Assumption 1 that referred to conduction currents, i.e. currents of the only type present in DC circuits.

By Assumptions 1 and 3 conductive currents are possible only within wires and displacement currents only through capacitor armatures. So the integral of Eq. (5) is simplified since only through surfaces $S_k J_t$ is non-zero, where S_k are the intersection of S with conductor wires or capacitors.

$$\int_S \mathbf{J}_t \cdot d\mathbf{s} = \sum_{k=1}^n \int_{S_k} \mathbf{J}_t \cdot d\mathbf{s} = \sum_{k=1}^n I_k = 0$$

That is the KCL.

The concept is exemplified in Fig. 10, where the sums are to be performed with k going from 1 to 6 (left-side circuit) or 1 to 7 (right-side circuit).

CONCLUSION 4: in a physical system for which Assumptions 1 and 3 are applicable, the KCL applies.

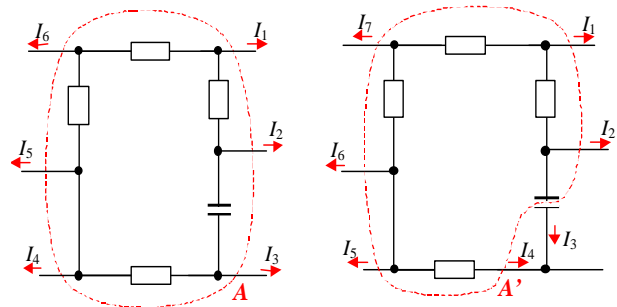


Fig. 10 Sample surfaces A and A' showing continuity equation and KCL in a sample circuit.

5.2 Issues with Long Lines: Metacircuits

Let us now try to use the theory introduced up to know in a system composed by three subsystems: two of them have a couple of terminals that are the only way to interchange conductive current with the outside world, and do not allow exchange of displacement currents (these will be called lumped components). These are connected to a distributed system, wherein conductive and displacement current circulate.

To fix ideas, let the two lumped components be a generator of sinusoidal EMF (just as the one used in the first example) and a ohmic resistor, while the distributed system is a transmission line constituted by two conductors and the surrounding space (top of Fig. 11).

In a system of this kind it may be unacceptable to disregard the effects of the magnetic field through the loop created by the three subsystems; very often, it is also not possible to disregard the effect of displacement currents between wires.

The issue is complex and requires more space than just a paper paragraph. A rather thorough analysis is shown in Appendix A of Ref. [7]. Here, just to clarify the issue, a simplified discussion is proposed, in which we neglect the effects of displacement currents. We just mention here that this gives acceptable results for 50/60 Hz power lines typically for lines having lengths up to a few tens of km.

In this example, where the transmission line has a much greater length than the distance between the two conductors, the magnetic field in the loop can be assumed as being equal in shape to the magnetic field created by two indefinite length wires. It can also be assumed that the effects of the magnetic field in space between the transmission line and the lumped components can be neglected. Finally, we assume that the magnetic induction between the two conductors is due only to the current flowing in the conductors themselves.

Under these hypotheses, and neglecting displacement currents, the behavior of the transmission line can be described using equations that do not involve knowledge of what happens outside it, and therefore can be substituted by a lumped component (block “*L*” in bottom-left part of Fig. 11).

The part of the circuit inside *L* is governed by the first of (1), and its integral consequence, the Faraday’s law; therefore the system of Fig. 11 is described by:

$$v = (R + R_L)i(t) + v_B(t) \quad (16)$$

where,

$$v_B(t) = \frac{d\psi}{dt} = L \frac{di}{dt}$$

and *L*, self-inductance of the circuit, is the proportionality coefficient between current and the flux linkage it creates.

Rather obvious, Eq. (16) can be interpreted as the

KVL of the circuit reported in the bottom-right part of Fig. 11, from which, then, *i(t)* can be determined when *v(t)* is known, and vice-versa.

It is important to stress that we just demonstrated how to evaluate *v*, or, if we want, $v - v_u = R + i(t) + v_B(t)$, while we did not mention other voltages. Indeed the bottom-right of Fig. 11 cannot be used to evaluate of other voltages, such as for example $v_{AD}(t)$ or $v_{AC}(t)$. This because Faraday’s EMF of loops containing simultaneously a point from the couple C-D and one from the couple A-B does depend on the loop geometry, which in turn is a consequence of the magnetic field is non-conservative.

This can be visualized clearly considering for example a measuring system of voltage v_{AD} in the physical system (Fig. 12). It is apparent that any change of the position of a voltmeter that would be intended to measure v_{AD} would change the area of the loop composed by the conductor *AD* and the measuring wire, and therefore the electromotive force generated according to Faraday’s law (first Eq. of (1)).

More explicitly, if we consider the two contours *c*₁ (*D-A-v*₁-*D*) and *c*₂ (*D-A-v*₂-*D*) we get applying (first Eq. of (1)):

$$\begin{aligned} \oint_{c_1} \mathbf{E} \cdot d\mathbf{l} &= - \iint_{A(c_1)} \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{A} \Rightarrow v_1 - R_{AD}i = - \frac{d\psi_{A(c_1)}}{dt} \\ \oint_{c_2} \mathbf{E} \cdot d\mathbf{l} &= - \iint_{A(c_2)} \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{A} \Rightarrow v_2 - R_{AD}i = - \frac{d\psi_{A(c_2)}}{dt} \Rightarrow \\ v_2 - v_1 &= - \frac{d(\psi_{A(c_2)} - \psi_{A(c_1)})}{dt} \Rightarrow v_2 \neq v_1 \end{aligned}$$

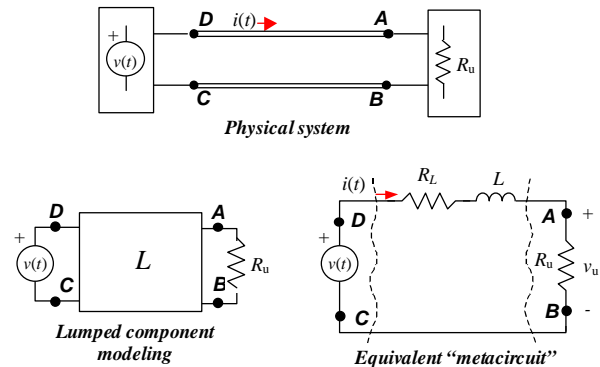


Fig. 11 Physical systems containing long lines can be represented by a lumped component model or a metacircuit.

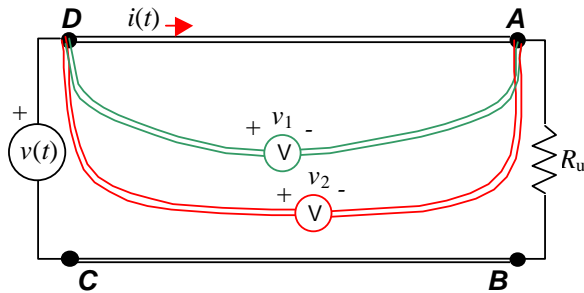


Fig. 12 Different measuring loops create different measures on the voltmeter V .

We can draw the following conclusions from the example.

- Since circuits have the characteristic that all voltages between nodes can be computed, it is not possible to determine a circuit completely equivalent to a system having long lines;
- Something similar to a circuit can be determined that has a behavior equivalent to the given system, when used only for separate determination of electrical quantities in the two ends of the given system. We call this modified version of the circuit concept metacircuit.

Since metacircuits are not the same thing to circuits, a specific graphic representation is advisable. That justifies the presence of the curved-dashed lines used in the bottom-right part of Fig. 11, illustrating a metacircuit.

6. Relationship with Electroquasistatics and Magnetoquasistatics

The link between electromagnetics and circuits has some connections with other approximations of electromagnetism (Maxwell's) equations usually considered [9]:

- Electroquasistatic approximation: it considers the \mathbf{B} variation sufficiently slow that the second term in the first of (1) to be zero. This implies \mathbf{E} being conservative since its work on any loop is zero.
- Magnetoquasistatic approximation: it involves considering the \mathbf{D} variation sufficiently slow that its time derivative is set to zero in the third of (1).

The relationships of these approximations to what is

done when circuits are used are not straightforward. However we can observe that:

- the electroquasistatic approximation means neglecting Faraday's induction law. We do this in true circuits, outside lumped components (i.e. between wires), but we do not in what we called metacircuits;
- the magnetoquasistatic approximation means neglecting the effects of displacement currents, which is what we do in this paper in circuits in the empty space between wires. This is not sufficient in some cases, for instance for long power lines. This special case was not dealt with in this paper, but analyzed in Appendix A of Ref. [7].

7. Summary

We can resume what has been obtained in this paper as follows.

- Assumptions 1 and 2 allow physical system operating in DC to be treated as circuit, for which KCL and KVL are assumed to be valid.
- Assumptions 1 to 4 allow a physical system to be treated as circuit, for which KCL and KVL are assumed to be valid.

Therefore we can now say that: under precisely stated assumptions, a system, which is composed by circuit elements and conducting wires, can usually be analysed by means of the mathematical-graphical tool called circuit. For them KCL and KVL are postulated to be valid (Circuit elements are subsystems that have electrical interaction to the rest of the system only through their terminals).

However, systems containing long lines cannot be treated as true circuits. For them we introduced a new concept, called, metacircuit.

All electric engineers know that when systems contain long lines, only equations relating quantities at each of its two ends to each other can be computed, and not "cross-quantities" such as cross-voltages. However, this distinction is always fuzzy. Indeed this paper, as well as Refs. [7] and [8] have shown that it is very important, and deserves a specific name and specific

graphic representation.

The approach described can be extended to other systems such as those containing transformers, electric machines, multipoles, etc.; however such a comprehensive analysis is out of the scope of this paper.

The general approach presented in the paper has been adopted in book [8].

8. Conclusions

This paper had the purpose of clarifying what circuits are, making a neat distinction between physical systems with long wires, which are three-dimensional systems governed by Maxwell's equations (we called them *circuitual systems*), and circuits, which are abstract graphical-mathematical entities, and are very easily treated using Kirchhoff's laws.

Kirchhoff's laws are not just a consequence of electromagnetics laws: to use them in substitution to electromagnetic laws, we need to add to them a few assumptions, a task that is normally not performed in textbooks.

To give a contribution to clarify how circuits relate to physical systems, this paper first states clearly that what we call circuits are a mathematical-graphical abstraction of physical systems having a circuitual shape. Then it shows which assumptions we need to add to the basic electromagnetics laws to allow circuits to describe physical electromagnetics systems, thus to use Kirchhoff's laws to analyze them.

The paper considers both stationary (DC) and

time-varying (AC) circuits. It shows an important limitation of AC circuits, which is overcome introducing the concept of metacircuits.

The approach proposed is clarifying for engineers and useful for teaching, and has been adopted in book [8].

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