Diffraction and colorless gluon-clusters in high-energy hadron-hadron and lepton-hadron collisions

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Abstract

The role played by colorless gluon-clusters in the optical-geometrical approach to high-energy inelastic diffractive scattering is discussed. A simple analytical expression for the t-dependence of the integrated single diffractive cross section (t is the four-momentum transfer squared) is derived. Comparison with the existing data and predictions for future experiments are presented.

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Diffraction phenomena observed in optics can be used as “an instrument” to determine the unknown wavelengths of incident waves from the known geometrical structures of scatterers and vice versa. This has been pointed out and demonstrated by von Laue and his collaborators in their celebrated paper[1] eighty-five years ago. Based on this idea, systematic experiments[2] have been performed in the 1950's and 1960's to measure the sizes of various nuclei by using high-energy hadron-beams (as short-wavelength hadronic waves). In this sense, high-energy particle-accelerators can be considered as “supermicroscopes”[3].

Recently, the observation[4] of large-rapidity-gap events in deep-inelastic electron-proton scattering has initiated much interest on “diffraction” and/or “inelastic diffractive scattering” in high-energy collision processes; and in this connection, results of many new experiments and new analyses have been reported[5]. Having the above-mentioned applications of diffraction phenomena in mind, it is natural to ask whether or what the present-day “supermicroscopes”[3] tell us in this respect. In particular, can we understand the existing “diffractive inelastic scattering” data[5] in terms of optical concepts? If yes, what role do the colorless objects play, the “exchange” of which is supposed[6] to be responsible for such scattering processes?

In the present paper, we try to answer these questions, and discuss in particular the t-dependence of the inelastic differential cross-sections $d\sigma/dt$ where t is the invariant momentum-transfer. For this purpose, it is useful to recall and/or to note the following:

(I.)Diffraction is associated with departure from geometrical optics caused by the finite wavelength of light. Hence, to describe such phenomena, concepts and methods relevant for ray-tracing, as well as those related to wave propagations, are needed. In particular, Frauenhofer diffraction can be observed by placing a scatterer in the path of propagation of light (the wavelength of which is less than the linear dimension of the scatterer), where the light-source and the detecting screen are very far from the scatterer. The parallel incident light-rays can as usual be considered as plane waves (characterized by a set of constants $\vec{k}_0, w_0 \equiv |\vec{k}_0|$, and u_0 say, which denote the wave vector, the frequency and the amplitude of a component of the electromagnetic field respectively). After the scattering, the scattered field can be written in accordance with Huygens' principle as

$$u_P = \frac{e^{ikR}}{R} f(\vec{k}_0, \vec{k}). \tag{1}$$

Here, u_P stands for a component of the field originating from the scatterer, $\vec{k}$ is the wave vector of the scattered light in the direction of observation, $w \equiv |\vec{k}|$ is the corresponding frequency, R is the distance between the scatterer and the observation point P, and $f(\vec{k}_0, \vec{k})$ is the (unnormalized) scattering amplitude which describes the change of the wave vector in the scattering process. By choosing a coordinate system in which the z-axis coincides with the incident wave vector $\vec{k}_0$, the scattering amplitude can be expressed as follows

$$f(\vec{q}) = \frac{1}{(2\pi)^2} \iint_{\Sigma} d^2\vec{b}\, \alpha(\vec{b})\, e^{-i\vec{q}\cdot\vec{b}} \, . \tag{2}$$

Here, $\vec{q} \equiv \vec{k} - \vec{k}_0$ determines the change in wave vector due to diffraction; $\vec{b}$ is the position vector which marks (in the xy-plane) the point B where the diffractive scattering takes place, hence it is the "impact parameter"; $\alpha(\vec{b})$ is the "profile function" associated with scattering process. The integration extends over the region Σ in the xy-plane in which $\alpha(\vec{b})$ is different from zero. In those cases in which the scatterer is symmetric about the scattering axis (here the z-axis), Eq.(2) can be expressed as

$$f(\vec{q}) = \frac{1}{2\pi} \int_0^{\infty} b\, db\, \alpha(b) J_0(qb) \tag{3}$$

by using an integral representation[7] of J_0. It should be mentioned that, while many well-known results in optics follow directly from Eqs.(2) and (3), some of the observed phenomena (e.g. $|\vec{k}| = |\vec{k}_0| = \omega_0 = \omega$) are closely related to the fact that the scatterers are usually macroscopic and steady objects. Note in particular that $\Delta\vec{k} \equiv \vec{k}_0 - \vec{k}$ and $\Delta\omega \equiv \omega_0 - \omega$ are *not* always small (compared to $\vec{k}_0$ and ω_0) in the general case — especially when the scatterers are time-dependent objects in the microscopic world. This implies that Eqs.(2) and (3) are valid also when the incident waves are *not* scattered *elastically*.

(II.) The idea of using optical and/or geometrical analogies to describe high-energy hadron-hadron collisions at small scattering angles has been discussed by many authors many years ago[8]. In particular, it has been pointed out by Byers and Yang[8] that not only the experimental findings in elastic scattering but also those in exchange reactions indicate the usefulness of optical-geometrical concepts for the description of such processes. It is shown that $\pi^- + p \to \pi^0 + n$ can be described in an eikonal approach in which the incident pion is pictured as point-like when it goes through the spatially extended proton "droplet" and coherently excites the latter. To be more precise, the counterparts of α [in Eq.(3)] are calculated from a potential model, and the corresponding $f(\vec{q})$'s are obtained from the equations corresponding to Eq.(3). The t-dependence of $d\sigma/dt$ with $t \equiv |\vec{q}|^2$ can then be obtained by evaluating the corresponding $|f(\vec{q})|^2$'s. The idea of "coherent droplet" has been extended by Chou and Yang[8] to describe "diffractive dissociation" of hadrons. Having seen these, it seems natural to ask: Can optical-geometrical analogy be used to describe diffractive scattering processes in general?

(III.) In connection with the observation[4] of large rapidity gap events in deep-inelastic electron-proton scattering, it is proposed[9] that "the colorless objects" (which dominate such collision events) are colorless gluon-clusters formed by interacting soft-gluons, and the latter can be described by the general theory of Bak, Tang and Wiesenfeld (BTW)[10] for open dynamical complex systems in which the basic interactions between the members of such systems are local. Theoretical arguments and experimental indications in support of such a statistical approach to QCD-systems have been presented[9]. It is pointed out in particular that the interactions prescribed by the QCD-Lagrangian and the non-conservation of gluon-numbers are indicative of the usefulness of treating interacting soft-gluon system as open dynamical system with many degrees of freedom. We recall, BTW[10] and many other authors[11] have pointed out and demonstrated by computer simulations that a wide class of open complex systems evolve into self-organized critical (SOC) states which are barely stable. A local perturbation of a critical state may propagate (i.e. it may pass this effect to some of its nearest neighbors, and then to the next-nearest neighbors, and so on in a domino effect) over all length scales, like an avalanche. Such a domino effect eventually terminates after a total time T, having reached a final amount of dissipative energy and having effected a total spatial extension S. The quantity S is called by BTW[10] the size, and the quantity T the lifetime of the avalanche (also known as the BTW-clusters). The distributions D_S of the size S, and the distribution D_T of the lifetime T of BTW-clusters in such open dynamical systems obey power laws: $D_S(S) \sim S^{-\mu}$, and $D_T(T) \sim T^{-\nu}$, where μ and ν are positive real constants. Such power-law scaling behaviors are consequences of SOC. In fact they can be, and have been, considered[10,11] as "the fingerprints" of SOC states and BTW-clusters.

An analysis[9] of the existing data[5,12] for diffractive DIS and those for diffractive photoproduction[5,13] has been carried out; and it is seen in particular that the probability density for a colorless gluon-cluster of size S and lifetime T to exist in such processes indeed show power-law behaviours mentioned above. Furthermore, it is shown[9] that the exponent μ can be readily determined from the data[5,12,13]. In fact, the extracted numerical value of μ are 1.95$\pm$0.12 and 1.98$\pm$0.07 respectively. Also the extracted value for ν is found to be approximately 2. The existing diffractive pp and $\bar{p}p$ scattering data[14,15] has been also analyzed by using exactly the same method. The results show that the corresponding size-distributions for pp and $\bar{p}p$ collisions at different incident energies can also be described by $D_S(S) \sim S^{-\mu}$, and the extracted value for μ are approximately the same[16]. These results strongly suggest that the size- and the lifetime-distribution of the colorless gluon-clusters are universal and robust.

(IV.) The diffraction picture we propose has two basic ingredients: First, the beam particles ($\gamma^\star$, γ, $\overline{p}$ or p in Fig.1) are considered as high-frequency waves going through a medium. Second, the medium consists of a system of virtual (space-like) color-singlet gluon-clusters which are inside *and* outside the proton in form of a "cluster cloud"[9]. For sufficiently large momentum transfer, the struck clusters can be carried away by the beam particle. Unlike hadrons, such clusters are avalanches which have neither a typical size, nor a typical lifetime, nor a given static structure. Their size- and lifetime-distributions obey simple power-laws, indicating the existence of SOC. This means, in the diffraction processes discussed here, "the sizes of the scattering screens" are in general different in different scattering events. The properties of the scatterer are determined by SOC and confinement

— consistent with the basic ideas of QCD.

We model this picture quantitatively in the rest frame of the proton target. We choose a right-handed Cartesian coordinate with its origin O at the center of the proton and the z-axis in the direction of the incident beam-particle which is considered point-like as it goes through the xy-plane at point $B \equiv (0, b)$. That is, the incident beam and the center O of the proton determine the scattering plane (yz-plane) of the collision event, where the distance $\overline{OB}$ is the corresponding impact parameter b. Since we are dealing with inelastic scattering (where the momentum transfer also in the longitudinal direction can be large) it is possible to envisage that the scattering takes place effectively at the point B, where it meets colorless gluon-clusters. The latter are avalanches initiated by local perturbations [local gluon-interactions associated with absorption or emission of one or more gluon(s), say] of SOC states in systems of interacting soft gluons. Since gluons carry color, the interactions which lead to the formation of *colorless* gluon-clusters must take place inside the confinement region of the proton. This means, while a considerable part of such colorless objects can be outside the proton, the location A where such an avalanche is initiated *must* be *inside* the proton. That is, in terms of $\overline{OA} \equiv r$, $\overline{AB} \equiv R_A(b)$, and proton's radius r_p, we have $r \leq r_p$ and $[R_A(b)]^2 = b^2 + r^2 - 2br\cos\angle BOA$. For a given impact parameter b, it is useful to know the distance $R_A(b)$ between B and A, as well as "the average squared distance" $\langle R_A^2(b)\rangle = b^2 + a^2$, $a^2 \equiv 3/5\, r_p^2$, which is obtained by averaging over all allowed locations of A in the confinement region. That is, we can model *the effect of confinement* in cluster-formation by picturing that all the avalanches in particular those which contribute to scattering events characterized by a given b are initiated from an "effective initial point" $\langle A_b\rangle$. Since an avalanche is a dynamical object, it may propagate within its lifetime in any one of the 4π directions away from $\langle A_b\rangle$. (Note: avalanches of the same size may have different lifetimes as well as different shapes. The location of an avalanche in space-time is referred to its center-of-mass.)

In order to find the amplitude $\alpha(b)$ in Eqs.(2) and (3), we recall the following: (i) SOC dictates that there are avalanches of all sizes (S_i's) and that the probability amplitude of finding an avalanche of size S_i can be obtained from the probability-distribution $D_S(S_i) = S_i^{-\mu}$ where experiments show[9] $\mu \approx 2$. (ii) QCD implies that the interactions between the constituents of an avalanche (a colorless gluon cluster) is stronger than those between the avalanches. Hence the struck avalanche can be "carried away" by the incident particle. Geometrically, the relative chance for the latter to strike an avalanche of size S_i (on the plane perpendicular to the incident axis) is $S_i^{2/3}$. (iii) Having the above-mentioned isotropic propagation of avalanches from $\langle A_b \rangle$ in mind, the relative number-densities at different b-values can be readily determined. Since for a given b, the distance in space between $\langle A_b \rangle$ and $B \equiv (0, b)$ is $(b^2+a^2)^{1/2}$, the number of avalanches which pass a unit area on the shell of radius $(b^2+a^2)^{1/2}$ centered at $\langle A_b \rangle$ is proportional to $(b^2+a^2)^{-1}$, provided that (because of causality) the lifetimes (T's) of these avalanches are not shorter than $\tau_{\min}(b)$. The latter is the time interval for an avalanche to travel from $\langle A_b \rangle$ to B (with the velocity of light, say), hence $\tau_{\min}(b) \propto (b^2+a^2)^{1/2}$. This means, only those avalanches having lifetimes $T \geq \tau_{\min}(b)$ can contribute to such an event. Recall that avalanches are due to SOC[9–11] and thus the chance for an avalanche of lifetime T to exist is $D_T(T) \propto T^{-\nu}$ with $\nu \approx 2$. Hence, by integrating T^{-2} over T from $\tau_{\min}(b)$ to infinity, we obtain the fraction associated with those whose lifetimes satisfy $T \geq \tau_{\min}(b)$: This fraction is a constant times $(b^2+a^2)^{-1/2}$. The amplitude $\alpha(b)$ can now be obtained from the probability amplitude for avalanche-creation mentioned in (i) by taking the weighting factors mentioned in (ii) and (iii) into account. The result is:

$$\alpha(b) \propto \sum_i S_i^{-1/3} (b^2+a^2)^{-3/2} \quad . \tag{4}$$

which can be written[9,16] as const.$(b^2+a^2)^{-3/2}$.

By inserting this result into Eq.(3), we obtain the corresponding amplitude in momentum space:

$$f(q) = \text{const.} \int_0^\infty b\, db \, \left(b^2+a^2\right)^{-3/2} J_0(qb) \tag{5}$$

This integral can be worked out analytically[17], and it follows (from $|f(q)|^2$ with $|t| \equiv |\vec{q}|^2$) that the integrated single diffractive cross section is:

$$\frac{d\sigma}{dt} = C \exp\left(-2a\sqrt{|t|}\right) \ . \tag{6}$$

Here $a^2 \equiv \frac{3}{5} r_p^2$ and C is an unknown normalization constant. Furthermore, since this t-dependence of $d\sigma/dt$ follows from analogy to diffraction phenomena and properties of BTW-clusters based on rather general requirements (SOC, confinement, causality, and interactions prescribed by the basic QCD-Lagrangian), its validity is expected to be universal and robust. In particular, it is expected that Eq.(6) should not be sensitive to incident energies, and not be sensitive to the quantum numbers of the projectiles.

To compare this result with experiments, we first take the available $d\sigma/dt$ data[14,15] for pp and $p\overline{p}$ scattering, and plot them against $\sqrt{|t|}$. Such a semilogarithmic plot is shown in Fig.2. Since the data points lay approximately on a straight line, we perform a least-square-fit to determine the slope and find $a = 3.4 \pm 0.01\,\mathrm{GeV}^{-1}$ with $\chi^2/\mathrm{NDF} = 6/136$. Next, we calculate a from its definition $a^2 \equiv \frac{3}{5}\, r_p^2$ by using the experimental value[18] $\langle r_p^2 \rangle \approx (0.81\,\mathrm{fm})^2$. The result is $a \approx 3.2\,\mathrm{GeV}^{-1}$. Furthermore, a comparison with the most recent HERA-data[5,13,19] for $\gamma^\star p$ and γp collisions has also been made, and the result is shown in Fig.3. Further experiments for hadron-, lepton- and photon-induced reactions would be useful to see whether the regularities shown in Eq.(6) and in Figs.(2) and (3) will remain to be true for other values of incident energy, momentum-transfer, as well as for other projectiles and/or targets.

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FIGURES

Fig. 1. The reactions considered in this paper, together with the definitions of the relevant kinematical variables.

Fig. 2. The $d\sigma/dt$ data taken from Refs.[14,15] are plotted against $\sqrt{|t|}$ in the measured kinematical range. The solid line is our result. The dashed and dot-dashed lines show the conventional fits $d\sigma/dt \propto \exp\left[Bt + Ct^2\right]$. The former is the UA4-fit to their data [15] with $B = 8.0 \pm 0.1\,\mathrm{GeV}^{-2}$ and $C = 2.3 \pm 0.1\,\mathrm{GeV}^{-4}$. The latter is a fit to the same expression for all the data points in this figure, where $B = 5.7 \pm 0.1\,\mathrm{GeV}^{-2}$ and $C = 0.8 \pm 0.1\,\mathrm{GeV}^{-4}$.

Fig. 3. The $d\sigma/d|t|$ data from Refs.[5,13,19] for $\gamma^\star$ and γ induced reactions are plotted against $\sqrt{|t|}$. They are shown by circles and squares respectively. Here, the empty circles and squares are the data from Ref.[5] while the solid ones are those from Refs.[13,19]. The solid lines stand for our result as given by Eq.(6) with $a = 3.4\,\mathrm{GeV}^{-1}$.

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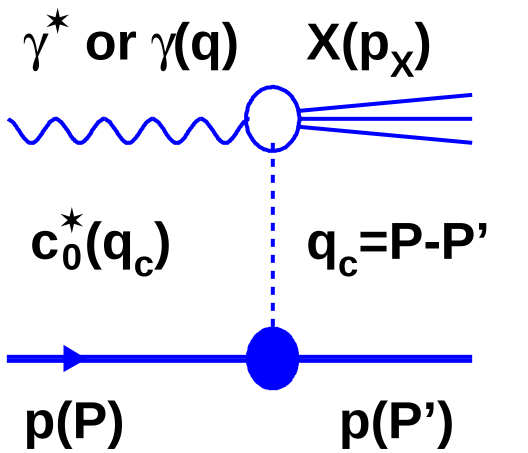


$$Q^2 = -q^2, \quad W^2 = (q+P)^2$$

$$p_X^2 = (q+q_c)^2 = M_X^2$$

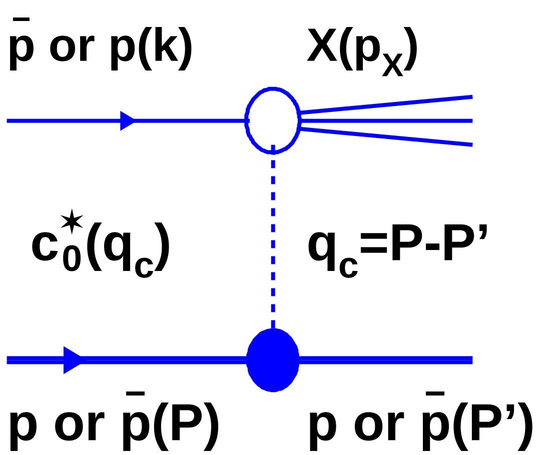
$$t = q_c^2 \ (\leq 0)$$

$$x_P = \frac{qq_c}{qP} \approx \frac{M_X^2 + Q^2}{W^2 + Q^2}$$

$$P^2 = P'^2 = M^2$$

$\gamma^\star + p \rightarrow X + p$ or

$\gamma + p \rightarrow X + p$ for $|t|$ ($\leq 4M^2$, say) $\ll W^2$



$$k^2 = M^2, \quad s = (k+P)^2$$

$$p_X^2 = (k+q_c)^2 = M_X^2$$

$$t = q_c^2 \ (\leq 0)$$

$$x_P = \frac{kq_c}{kP} \approx \frac{M_X^2}{s}$$

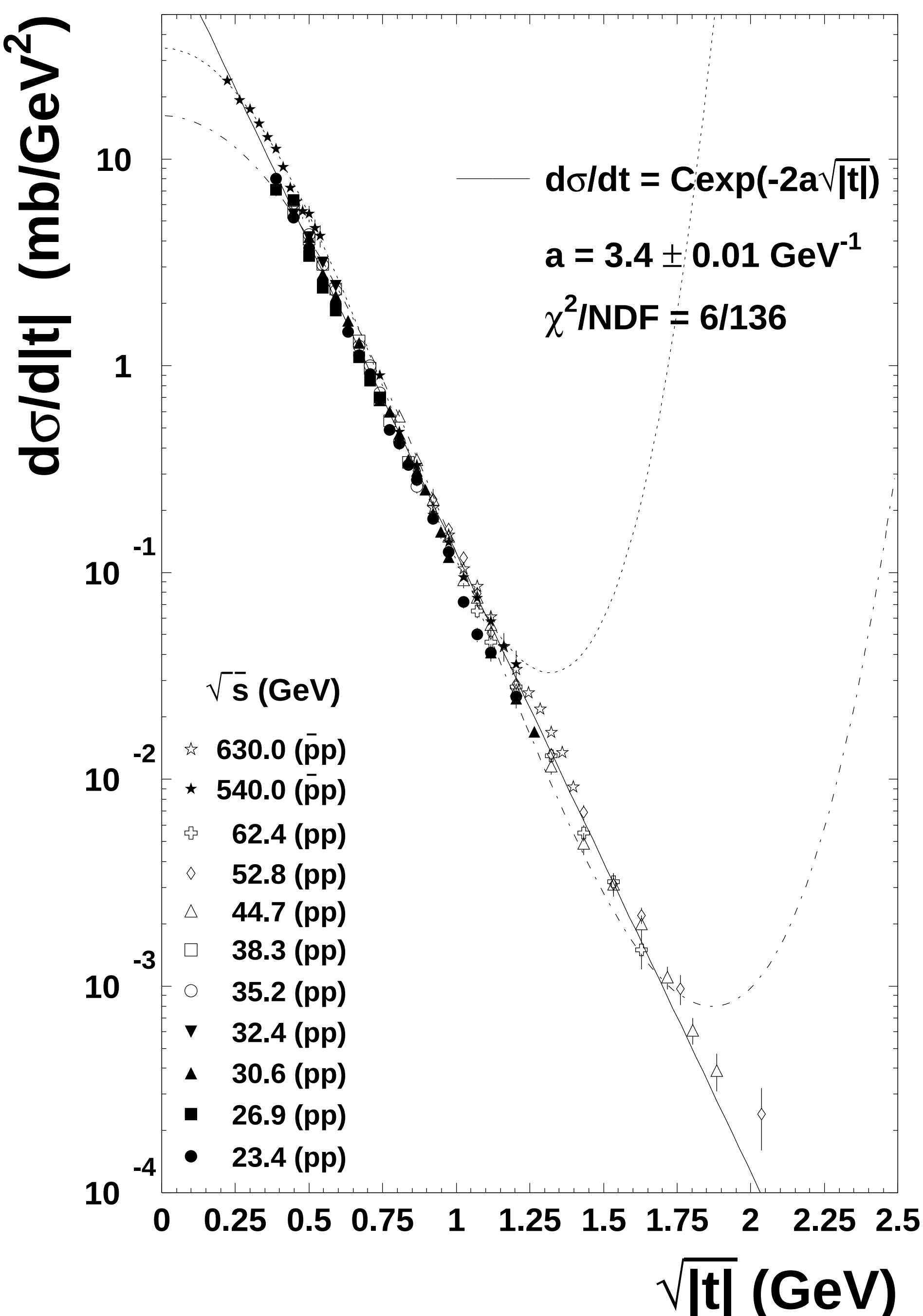
$$P^2 = P'^2 = M^2$$

$p + p \rightarrow X + p$ or

$\bar{p} + p \rightarrow X + p$ or

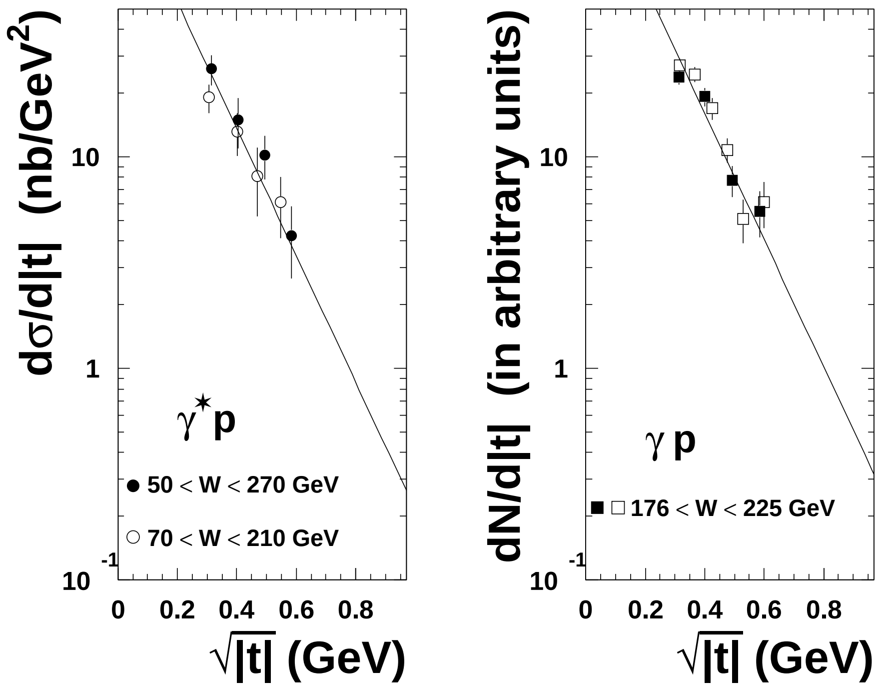
$p + \bar{p} \rightarrow X + \bar{p}$ for $|t|$ ($\leq 4M^2$, say) $\ll s$

Fig. 1



dσ/d|t| (mb/GeV²)
dσ/dt = Cexp(-2a√|t|)
a = 3.4 ± 0.01 GeV⁻¹
χ²/NDF = 6/136
√s (GeV)
630.0 (p̄p)
540.0 (p̄p)
62.4 (pp)
52.8 (pp)
44.7 (pp)
38.3 (pp)
35.2 (pp)
32.4 (pp)
30.6 (pp)
26.9 (pp)
23.4 (pp)
√|t| (GeV)

Fig. 2



dσ/d|t| (nb/GeV²)
10
1
10⁻¹
0 0.2 0.4 0.6 0.8
√|t| (GeV)
γ*p
50 < W < 270 GeV
70 < W < 210 GeV
dN/d|t| (in arbitrary units)
γ p
176 < W < 225 GeV

Fig. 3