Migration based velocity analysis in 2D laterally heterogeneous media

Hervé Chauris, Mark Noble, Gilles Lambaré and Pascal Podvin, École des Mines de Paris*

Summary

We present a new method based on Migration Velocity Analysis (MVA) to estimate 2D velocity models with no assumption on the reflector geometry nor on the background velocity field. In the prestack depth migrated volume, locally coherent events are automatically picked without interpretation and characterized by their positions and two slopes. The velocity is estimated by optimizing the flatness of these events in the Common Image Gathers. We show how to compute the gradient of the cost function *via* ray tracing. The efficiency of the method is illustrated by inverting the velocity field on a 2D synthetic data set using a local optimization algorithm. We also show that two different kinds of methods, Differential Semblance Optimization formulated in the migrated domain and Stereotomography or slope tomography in time data space, are equivalent.

Introduction

We address the problem of the estimation in 2D of the background or velocity macro model by MVA. Such methods basically use the flatness of events in Common Image Gathers (CIGs) as a criterion for velocity quality. Most of approaches without picking and valid in any 2D velocity field with the introduction of a cost function on CIGs, require a global optimization process with thus no feasible 3D extensions for realistic data sets. Only Differential Semblance Optimization method seems to be the only method to converge with a local approach, but an efficient gradient computation still has to be developed (Symes, 1998; Chauris and Noble, 1999).

When picking is introduced in the depth migrated domain, many other approaches have been proposed. However, the updating formulae to invert the velocity model are generally based at least on one of the three following assumptions: laterally invariant velocity, small offset and/or horizontal reflectors. Only one approach, valid for any 2D velocity field was proposed by (Liu, 1997), who related the perturbations of the reflector depth to the perturbations of the velocity model. In this method, the macro model consists both of velocity and interfaces. Continuous events have to be picked at some sparse CIG locations in a layer stripping approach requiring the interpretation of reflectors and the *a priori* knowledge of the exact depth of the reflector at the locations where CIGs are selected.

We present a new approach valid for any 2D velocity field with the picking of locally coherent events in the depth migrated domain, where the interpretation of continuous reflectors is not needed anymore. The estimation of velocity is based on flattening locally coherent events treated as uncorrelated events (*i.e.* not attached to a particular reflector). We are not only interested in the perturbations of the depth of an event but in the variations of its slopes too. The macro model only consists in smooth velocities, as the method does not require to introduce interfaces.

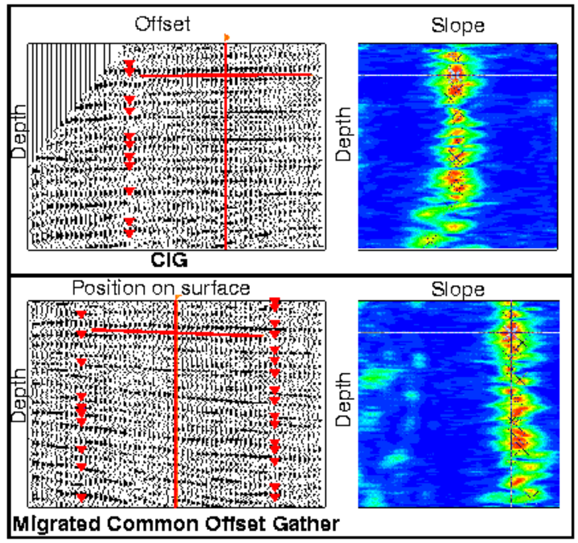

Fig. 1: Depth migrated images of real data with CIG and Common Offset Gather (left); Semblance panels (right) to determine local coherency and thus the position of the events and their two associated slopes. In this case, slopes are almost horizontal. For a given migrated trace, local slant stacks measure at every depth the local coherence in two panels. This operation is totally automatic and only requires $15min$ for a realistic 2D data set. No interpretation is introduced as there is no need for defining and following interfaces. An equivalent tool for the time domain was developed by (Billette et al., 1998).

Locally coherent picked migrated events

Fig. 1 presents on the left part two migrated panels, a CIG and a Common Offset Gather, extracted from the migration of a 2D real data set. From this illustration, it is clearly easier to automatically determine lots of locally coherent events, than to follow continuous reflectors in the two panels. For this reason, we are only interested in locally coherent migrated events.

For a 2D data set, local coherence is sought simultaneously in two sections of the migrated volume (x, z, h), where h is the half offset. A locally coherent picked event will thus be characterized by five parameters (x, z, ξ, φ, h) (Fig. 2), where x, z are the event location in depth, ξ its apparent geological dip (as measured in the Common Offset Gather), φ its residual slope (as measured in the CIG). For computation, we also introduce the half-aperture an-

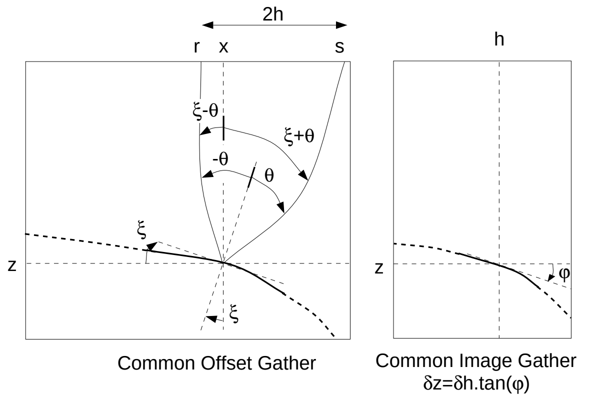

Fig. 2: The 5 values characterizing a locally coherent event in the prestack depth migrated domain: (x, z, ξ, φ, h).

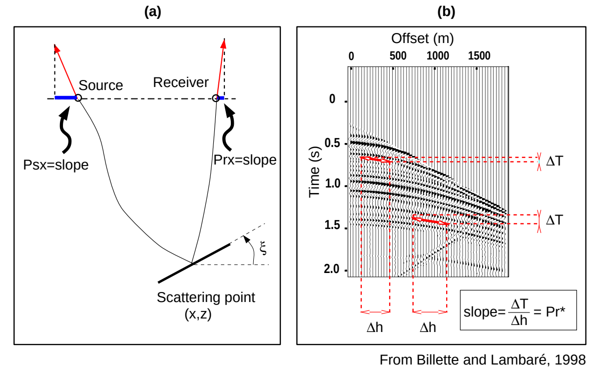

Fig. 3: Two definitions of the slope: (a) in ray theory, the slope is the horizontal component of the tangent slowness vector. (b) in the time domain, the slope is the tangent of locally coherent reflectors. The two slopes equal in the exact velocity field.

gle θ between the two specular rays starting from the picked event. We can also define $\theta_s = \xi + \theta$ and $\theta_r = \xi - \theta$, the two vertical angles for each ray. By definition and for each locally coherent event,

$$\tan(\varphi) = \left.\frac{\partial z}{\partial h}\right|_x \quad (1)$$

$$\tan(\xi) = \left.\frac{\partial z}{\partial x}\right|_h \quad (2)$$

Clearly, φ must be 0 (all picked events are flat) when the velocity model is correct so that our cost function is

$$f[u] = \frac{1}{2}\sum_{picks}\left(w\tan(\varphi)\right)^2 \quad (3)$$

where u is the migration slowness model and w a weighted coefficient defined below. After convergence, (x, z) will represent the actual location of the reflection/diffraction point in depth and ξ the real geologic dip of the corresponding reflector.

Optimization method

From a general point of view, velocity estimation on picked events may be understood as following:

(1) Prestack depth migration of the 2D data set for all offsets or shot points. Velocity model used for migration is the tested velocity field.
(2) Picking of locally coherent events, (*i.e.* $\xi(x, z, u)$ and $\varphi(x, z, u)$) in the migrated volume. If φ is not equal to 0 for all picked events, the velocity model used for migration should be updated.
(3) Ray tracing from all picked events up to the surface to compute the gradient of the cost function and update the velocity field. The two specular rays start symmetrically with respect to the normal to the dip defined by ξ and reach the surface with the offset associated to the picked event. Go to step (1) for another iteration with the updated velocity model.

The following section is devoted to the computation of the gradient of the cost function. We basically have to understand firstly the relations between the two picked angles in the depth domain and other slopes in the time data (seismograms), and secondly how to relate perturbations of depth or angle to perturbations of the velocity field. All equations and velocity updating formulae are valid for any 2D velocity field. The physical meaning is easier when the migration scheme depends on shot position instead of offset. Therefore, we will first present the results in this case, and after in the offset case used for applications. As mathematical computations are rather involved, we only mention the final results and try to provide a physical understanding of the different steps. Paraxial ray theory (Farra and Madariaga, 1987) is largely used to compute the needed expressions.

Cost function for Common Shot migration

In the Common Shot case, the summation during prestack depth migration is done over the receiver position. The position associated to a given scattering point where energy focuses belongs to the isochron in the depth domain and its envelope (derivatives with respect to r). After some computations, this imaging condition means that whatever the tested migration velocity model,

$$p_{rx} = p_r^* \quad (4)$$

p_{rx} is the horizontal component of the slowness vector, calculated at the surface by ray tracing. p_r^* corresponds to the slope (tangent of the reflected arrival) in the time data where energy comes from (Fig. 3). As p_r^* is determined by the data, it is thus independent of the tested velocity field used for migration. As we will see, the equivalent relation for the source $p_{sx} = p_s^*$ is true only when the velocity field is exact. After computations, using the fact that the focused event belongs to an isochron and its envelope, we obtain

$$\tan(\varphi) = \frac{p_s^* - p_{sx}}{2u\cos(\theta)\cos(\xi)} \quad (5)$$

where $u(x,z)$ is the slowness value at the scattering point. $\cos(\theta)\cos(\xi)$ never equals to 0, except for vertical dips and grazing rays never accounted for in migration. Thus, $p_s^* = p_{sx}$ for the exact velocity field (*i.e.* $\tan(\varphi) = 0$). Velocity estimation is thus equivalent to adjusting horizontal slowness provided by ray tracing to actual slopes of the seismic event in the (time) data space. We thus define in equation (3) $w = 2u\cos(\theta)\cos(\xi)$ allowing an easier computation of the gradient.

$$f[u] = \frac{1}{2}\sum_{picks}[2u\cos(\theta)\cos(\xi)\tan(\varphi)]^2 \quad (6)$$

$$= \frac{1}{2}\sum_{picks}(p_{sx} - p_s^*)^2 \quad (7)$$

We first calculate the gradient of the cost function and then discuss the physical meaning of the cost function.

Gradient computation

As p_s^* is independent of the tested velocity field, the gradient of the cost function can be formulated as

$$\frac{\partial f[u]}{\partial u} = 2\sum_{picks} u\cos(\theta)\cos(\xi)\tan(\varphi)\left.\frac{\partial p_{sx}}{\partial u}\right|_{x,s} \quad (8)$$

In a given velocity model, the perturbations of the final conditions δs, δr, δp_{sx} and δp_{rx} can be expressed using paraxial ray theory (Farra and Madariaga, 1987) from the perturbations of initial conditions δz, $\delta\theta_s$ and $\delta\theta_r$ and of the slowness field δu. To compute the gradient of the cost function, we precisely have to know $\delta z(\delta u)$, $\delta\theta_s(\delta u)$ and $\delta\theta_r(\delta u)$ for small perturbations. The perturbation of the depth of the event has already been established by (Liu, 1997) for any 2D velocity field. The two last equations are determined by two conditions on surface:

(1) The source ray reaches exactly the same source (Common Shot Gather): $\delta s = 0$, giving $\delta\theta_s$.
(2) The receiver ray reaches the surface with exactly the same horizontal slowness vector: $\delta p_{rx} = 0$ (imaging condition (4)), giving $\delta\theta_r$.

Cost function for Common Offset migration

In this case, the approach is very similar. The sum of the two horizontal slownesses (at source and receiver) is constant and independent of the velocity model used for migration:

$$p_{sx} + p_{rx} = p_s^* + p_r^* \quad (9)$$

Equation (5) is equivalent in this case to

$$2u\cos(\theta)\cos(\xi)\tan(\varphi) = [p_s^* - p_{sx}] - [p_r^* - p_{rx}] \quad (10)$$

and once again the cost function can be defined as

$$f[u] = \frac{1}{2}\sum_{picks}[2u\cos(\theta)\cos(\xi)\tan(\varphi)]^2 \quad (11)$$

$$= 2\sum_{picks}(p_{sx} - p_s^*)^2 = 2\sum_{picks}(p_{rx} - p_r^*)^2 \quad (12)$$

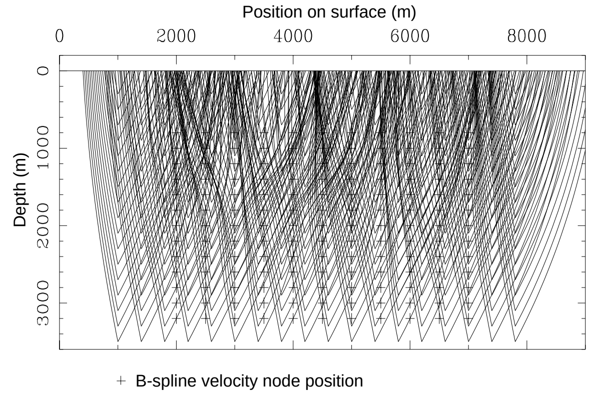

Fig. 4: Ray segments in the exact velocity model. B-spline nodes are positioned every $500m$ in x, from 2000 to $7000m$ and every $200m$ in z from depth 800 to $3200m$. Less ray segments have influence on velocity nodes in the deeper part.

As before, the criterion compares calculated and observed slopes, even if with its formulation, no slope in the time data is picked. The gradient of the cost function with respect to velocity is obtained as before with two conditions, giving $\delta\theta_s$ and $\delta\theta_r$:

(1) The distance between the two points where the rays cross the surface is the offset associated to the locally coherent event (Common Offset Gather): $\delta s - \delta r = 0$.
(2) The summation of the horizontal slowness vectors at the surface is constant: $\delta p_{sx} + \delta p_{rx} = 0$ (imaging condition: (9)).

DSO and Stereotomography

Equations (6) and (7) are two equivalent definitions of the cost function, but have different physical meanings. In (6), the velocity model is estimated by flattening events in CIGs, minimizing horizontal derivatives. The method can thus be related with Differential Semblance Optimization (DSO) (Symes, 1998; Chauris and Noble, 1999). The major advantage compared to previous work is the CPU-efficient computation of the gradient of the cost function developed here. Conversely, (7) minimizes the differences between observed and calculated slopes and thus belongs to stereotomographical method (Billette et al., 1998). However, we know here more precisely where the information comes from. Moreover, the picking should be easier as done in the migrated domain. Such developments allowed to unify totally different approaches for velocity estimation: Migration Velocity Analysis and slope tomography.

Preliminary tests

To test the validity of the method, we get rid of the picking and the migration steps as if they were perfect. Data are here only represented by small reflector segments with their specific dips. To know how to migrate the small reflectors and $\tan(\varphi)$ value in a given velocity model, we shot in the exact velocity field two rays until the surface, starting symmetrically around the normal of the dip and reaching the surface. Each picked event is thus associated to $(s, r, t_{s,r}, p_s^*, p_r^*)$. The position and the slope of a migrated event is given by the following conditions: the specular rays reach the given source and receiver locations with the total traveltime $t_{s,r}$ and associated slopes on surface $p_s^* + \Delta p$ and $p_r^* - \Delta p$ because of the imaging condition (9). This is the principle of migration and the perturbations do not have to be small (no linearization): the method is valid even for migration velocity models really far from the solution. The slope in the CIG is given by (10).

The 2D synthetic velocity model consists of the sum of a constant gradient model and 2D perturbations represented by $13 * 11$ cubic cardinal B-spline nodes. Horizontal distance between the nodes is $500m$ and vertical distance $200m$ from depth 800 to $3200m$. 306 small reflectors were evenly distributed in the background, every $300m$ in x and $200m$ in z for the exact velocity model. Initial dip was fixed arbitrarily to 20°. Fig. 4 presents the ray trajectories in the exact velocity model, needed to obtain for each event $(s, r, t_{s,r}, p_s^*, p_r^*)$ so as to migrate these events in a given velocity model.

We inverted a synthetic 2D velocity defined by 143 velocity nodes, starting from the exact constant gradient velocity model. We used a local optimization process, here the steepest descent algorithm. The aim was to prove the efficiency of the method using a local approach. After 30 iterations, we converge to a model which shows great similarities with the exact velocity model (Fig 5a and b). The cost function decreased from 1 to 0.05, also proving the success of the method. Moreover, the total CPU computational cost for the 30 iterations was only 15 minutes on a UltraSparc 10. The zones where the resolution is not the best correspond to parts covered with too few rays. To accelerate to convergence rate, we modified the gradient of $f[u]$ with a linear function only depending on depth. Indeed, more rays travel the shallower zones compared to the deeper zones.

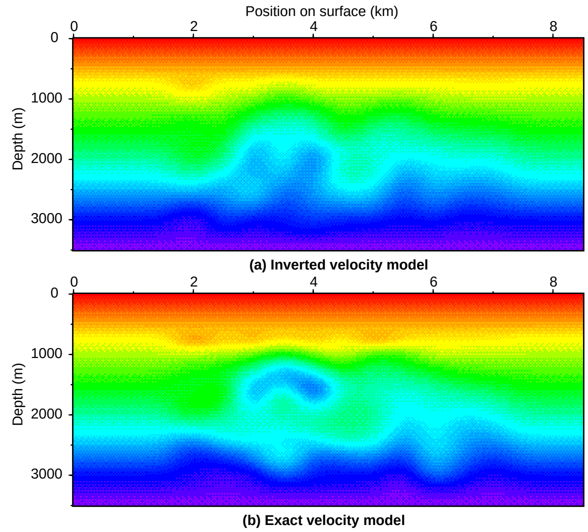

Fig. 5: Inverted and exact velocity models, showing great similarities and proving the success of the inversion. The deepest part is less well resolved, as less ray segments cross it. The initial velocity model was a constant gradient field and 143 B-spline nodes were simultaneously inverted.

Conclusions

The method we presented here is based on MVA valid for any 2D velocity field. The theoretical part allows to know how reflector shape and depth depend on velocity. We also showed that *a priori* different methods for velocity estimation are in fact equivalent. The success of the method has been illustrated on a 2D synthetic data set. Further developments will include the picking of locally coherent events and tests on real data sets.

Acknowledgmen ts

This work was partly funded by the European Commission within the JOULE project, 3D-Focus (contract JOF3-CT97-0029).

References

Billette, F., Lambaré, G., and Podvin, P., 1998, Stereotomography with automatic picking: application to the Marmousi dataset: 68^{th} Annual SEG Meeting and Exposition, Soc. Expl. Geophys., Expanded Abstracts, 1317–1320.

Chauris, H., and Noble, M., 1999, 2D velocity macro model estimation from seismic reflection by local Differential Semblance Optimization: applications on synthetic and real data sets:, Submitted to Geophys. J. Int.

Farra, V., and Madariaga, R., 1987, Seismic waveform modeling in heterogeneous media by ray perturbation theory: J. Geophys. Res., **92**, 2697–2712.

Liu, Z., 1997, An analytical approach to migration velocity analysis: Geophysics, **62**, 1238–1249.

Symes, W. W., 1998, High frequency asymptotics, differential semblance and velocity estimation: 68^{th} Annual SEG Meeting and Exposition, Soc. Expl. Geophys., Expanded Abstracts, 1616–1619.