Intuitionistic Smooth Bitopological Spaces and Continuity

Jin Tae Kim and Seok Jong Lee
Department of Mathematics, Chungbuk National University, Cheongju, Korea

Abstract
In this paper, we introduce intuitionistic smooth bitopological spaces and the notions of intuitionistic fuzzy seminterior and semiclosure. Based on these concepts, the characterizations for the intuitionistic fuzzy pairwise semicontinuous mappings are obtained.

Keywords: Intuitionistic, Smooth bitopology

1. Introduction and Preliminaries
Chang [1] introduced the notion of fuzzy topology. Chang’s fuzzy topology is a crisp subfamily of fuzzy sets. However, in his study, Chang did not consider the notion of openness of a fuzzy set, which seems to be a drawback in the process of fuzzification of topological spaces. To overcome this drawback, Šostak [2, 3], based on the idea of degree of openness, introduced a new definition of fuzzy topology as an extension of Chang’s fuzzy topology. This generalization of fuzzy topological spaces was later rephrased as smooth topology by Ramadan [4].

Çoker and his colleague [5, 6] introduced intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets which were introduced by Atanassov [7]. Mondal and Samanta [8] introduced the concept of an intuitionistic gradation of openness as a generalization of a smooth topology.


Lim et al. [12] defined the term “intuitionistic smooth topology,” which is a slight modification of the intuitionistic gradation of openness of Mondal and Samanta, therefore, it is different from ours.

In this paper, we introduce intuitionistic smooth bitopological spaces and the notions of intuitionistic fuzzy \((T_i, T_j)\)-(r, s)-seminterior and semiclosure. Based on these concepts, the characterizations for the intuitionistic fuzzy pairwise \((r, s)\)-semicontinuous mappings are obtained.

\(I\) denotes the unit interval \([0, 1]\) of the real line and \(I_0 = (0, 1]\). A member \(\mu\) of \(I^X\) is called a fuzzy set in \(X\). For any \(\mu \in I^X\), \(\mu^c\) denotes the complement \(1 - \mu\). By \(\hat{0}\) and \(\hat{1}\) we denote constant mappings on \(X\) with value of \(0\) and \(1\), respectively.

This is an Open Access article distributed under the terms of the Creative Commons Attribution Non-Commercial License (http://creativecommons.org/licenses/by-nc/3.0/) which permits unrestricted non-commercial use, distribution, and reproduction in any medium, provided the original work is properly cited.
Let $X$ be a nonempty set. An intuitionistic fuzzy set $A$ is an ordered pair

$$A = (\mu_A, \gamma_A)$$

where the functions $\mu_A : X \rightarrow I$ and $\gamma_A : X \rightarrow I$ denote the degree of membership and the degree of nonmembership, respectively, and $\mu_A + \gamma_A \leq 1$. Obviously, every fuzzy set $\mu$ in $X$ is an intuitionistic fuzzy set of the form $(\mu, 1-\mu)$. $I(X)$ denotes a family of all intuitionistic fuzzy sets in $X$ and “IF” stands for intuitionistic fuzzy.

**Definition 1.1.** (4) A smooth topology on $X$ is a mapping $T : I^X \rightarrow I$ which satisfies the following properties:

1. $T(\emptyset) = T(\text{1}) = 1$.
2. $T(\mu_1 \lor \mu_2) \geq T(\mu_1) \land T(\mu_2)$.
3. $T(\mu_1) \land T(\mu_2) \geq T(\mu_1 \lor \mu_2)$.

The pair $(X, T)$ is called a smooth topological space.

**Definition 1.2.** (11) A system $(X, T_1, T_2)$ consisting of a set $X$ with two smooth topologies $T_1$ and $T_2$ on $X$ is called a smooth bitopological space.

**Definition 1.3.** (5) An intuitionistic fuzzy topology on $X$ is a family $T$ of intuitionistic fuzzy sets in $X$ which satisfies the following properties:

1. $\emptyset, 1 \in T$.
2. If $A_1, A_2 \in T$, then $A_1 \cap A_2 \in T$.
3. If $A_i \in T$ for each $i$, then $\bigcup A_i \in T$.

The pair $(X, T)$ is called an intuitionistic fuzzy topological space.

## 2. Intuitionistic Smooth Bitopological Spaces

Now, we define the notions of intuitionistic smooth topological spaces and intuitionistic smooth bitopological spaces.

**Definition 2.1.** An intuitionistic smooth topology on $X$ is a mapping $T : I(X) \rightarrow I$ which satisfies the following properties:

1. $T(\emptyset) = T(\text{1}) = 1$.
2. $T(A \cap B) \geq T(A) \land T(B)$.
3. $T(A \cup B) \geq T(B)$.

The pair $(X, T)$ is called an intuitionistic smooth topological space.

Let $(X, T)$ be an intuitionistic smooth topological space. For each $r \in I_0$, an $r$-cut

$$T_r = \{ A \in I(X) | T(A) \geq r \}$$

is an intuitionistic fuzzy topology on $X$.

Let $(X, T)$ be an intuitionistic fuzzy topological space and $r \in I_0$. Then the mapping $T^r : I(X) \rightarrow I$ defined by

$$T^r(A) = \begin{cases} 1 & \text{if } \mu = 0, 1, \\ r & \text{if } A \in T - \{0, 1\}, \\ 0 & \text{otherwise} \end{cases}$$

becomes an intuitionistic smooth topology on $X$.

**Definition 2.2.** Let $A$ be an intuitionistic fuzzy set in intuitionistic smooth topological space $(X, T)$ and $r \in I_0$. Then $A$ is said to be

1. IF $T$-r-open if $T(A) \geq r$,
2. IF $T$-r-closed if $T(A^c) \geq r$.

**Definition 2.3.** Let $(X, T)$ be an intuitionistic smooth topological space. For $r \in I_0$ and for each $A \in I(X)$, the IF $T$-r-interior is defined by

$$T\text{-int}(A, r) = \bigcup \{ B \mid B \subseteq A, T(B) \geq r \}$$

and the IF $T$-r-closure is defined by

$$T\text{-cl}(A, r) = \bigcap \{ B \mid A \subseteq B, T(B^c) \geq r \}.$$

**Theorem 2.4.** Let $A$ be an intuitionistic fuzzy set in an intuitionistic smooth topological space $(X, T)$ and $r \in I_0$. Then

1. $T\text{-int}(A, r)^c = T\text{-cl}(A^c, r)$,
2. $T\text{-cl}(A, r)^c = T\text{-int}(A^c, r)$.

**Proof.** It follows from Lemma 2.5 in [13].

**Definition 2.5.** A system $(X, T_1, T_2)$ consisting of a set $X$ with two intuitionistic smooth topologies $T_1$ and $T_2$ on $X$ is called a intuitionistic smooth bitopological space(ISBTS for short). Throughout this paper the indices $i, j$ take the value in $\{1, 2\}$ and $i \neq j$.

**Definition 2.6.** Let $A$ be an intuitionistic fuzzy set in an ISBTS $(X, T_i, T_j)$ and $s \in I_0$. Then $A$ is said to be

...
Theorem 2.7. Let $A$ be an intuitionistic fuzzy set in an ISBTS $(X, T_1, T_2)$ and $r, s \in I_0$. Then the following statements are equivalent:

1. $A$ is an IF $(T_1, T_2)$-$(r, s)$-semiopen set.
2. $A^c$ is an IF $(T_1, T_2)$-$(r, s)$-semiclosed set.
3. $T_j$-cl($T_j$-int($A, r), s)$ $\supseteq A$.
4. $T_j$-int($T_j$-cl($A^c, r), s)$ $\subseteq A^c$.

Proof. (1) $\Rightarrow$ (2) Let $A$ be an $(T_1, T_2)$-$(r, s)$-semiopen set. Then there is an IF $T_i$-r-open set $B$ in $X$ such that $B \subseteq A \subseteq T_i$-cl($B, s)$, Thus $T_j$-int($B^c, s)$ $\subseteq A^c \subseteq B^c$. Since $B^c$ is IF $T_i$-r-closed in $X$, $A^c$ is an IF $(T_1, T_2)$-$(r, s)$-semiclosed set in $X$.

(2) $\Rightarrow$ (1) Let $A^c$ be an IF $(T_1, T_2)$-$(r, s)$-semiclosed set. Then there is an IF $T_i$-r-closed set $B$ in $X$ such that $T_j$-int($B, s)$ $\subseteq A^c \subseteq B$. Hence $B^c \subseteq A \subseteq T_j$-cl($B^c, s)$. Because $B^c$ is IF $T_i$-r-open in $X$, $A$ is an IF $(T_1, T_2)$-$(r, s)$-semiopen set in $X$.

(1) $\Rightarrow$ (3) Let $A$ be an IF $(T_1, T_2)$-$(r, s)$-semiopen set in $X$. Then there exist an IF $T_i$-r-open set $B$ in $X$ such that $B \subseteq A \subseteq T_j$-cl($B, s)$, Since $B$ is IF $T_i$-r-open, we have $B = T_i$-int($B, r) \subseteq T_i$-int($A, r$). Thus

$$T_j$-cl($T_i$-int($A, r), s)$ $\supseteq T_j$-cl($B, s)$ $\supseteq A$.

(3) $\Rightarrow$ (1) Let $T_j$-cl($T_i$-int($A, r), s)$ $\supseteq A$ and take $B = T_i$-int($A, r$). Then $B$ is an IF $T_i$-r-open set and

$$B = T_i$-int($A, r)$ $\subseteq A$$
$$ \subseteq T_j$-cl($T_i$-int($A, r), s)$
$$ = T_j$-cl($B, s)$.

Hence $A$ is an IF $(T_1, T_2)$-$(r, s)$-semiopen set.

(3) $\Leftrightarrow$ (4) It follows from Theorem 2.4.

Theorem 2.8. Let $A$ be an intuitionistic fuzzy set in an ISBTS $(X, T_1, T_2)$ and $r, s \in I_0$. Then

1. If $A$ is IF $T_1$-r-open in $(X, T_1)$, then $A$ is an IF $(T_1, T_2)$-$(r, s)$-semiopen set in $(X, T_1, T_2)$.
2. If $A$ is IF $T_2$-s-open in $(X, T_2)$, then $A$ is an IF $(T_2, T_1)$-$(s, r)$-semiopen set in $(X, T_1, T_2)$.

Proof. (1) Let $A$ be an IF $T_1$-r-open set in $(X, T_1)$. Then $A = T_1$-int($A, r)$. Thus we have

$$T_2$-cl($T_1$-int($A, r), s)$ $\supseteq T_2$-cl($A, s)$ $\supseteq A$.

Hence $A$ is IF $(T_1, T_2)$-$(r, s)$-semiopen in $(X, T_1, T_2)$.

(2) Similar to (1).

The following example shows that the converses of the above theorem need not be true.

Example 2.9. Let $X = \{x, y\}$ and let $A_1, A_2, A_3,$ and $A_4$ be intuitionistic fuzzy sets in $X$ defined as

$$A_1(x) = (0.1, 0.7), \ A_1(y) = (0.7, 0.2);
A_2(x) = (0.6, 0.2), \ A_2(y) = (0.3, 0.6);
A_3(x) = (0.1, 0.7), \ A_3(y) = (0.9, 0.1);$$
and
$$A_4(x) = (0.7, 0.1), \ A_4(y) = (0.3, 0.6).$$

Define $T_1 : I(X) \rightarrow I$ and $T_2 : I(X) \rightarrow I$ by

$$T_1(A) = \begin{cases} 1 & \text{if } A = 0.1, \\ \frac{1}{2} & \text{if } A = A_1, \\ 0 & \text{otherwise}; \end{cases}$$
and
$$T_2(A) = \begin{cases} 1 & \text{if } A = 0.1, \\ \frac{1}{3} & \text{if } A = A_2, \\ 0 & \text{otherwise}. \end{cases}$$

Then $(T_1, T_2)$ is an ISBT on $X$. Note that

$$T_2$-cl($T_1$-int($A_3, \frac{1}{2}$), $\frac{1}{3}$) $= T_2$-cl($A_1, \frac{1}{3}$) $\supseteq A_3$$
and
$$T_1$-cl($T_2$-int($A_4, \frac{1}{3}$), $\frac{1}{2}$) $= T_1$-cl($A_2, \frac{1}{2}$) $\supseteq A_4.$

Hence $A_3$ is IF $(T_1, T_2)$-$(\frac{1}{3}, \frac{1}{2})$-semiopen and $A_4$ is IF $(T_2, T_1)$-$(\frac{1}{4}, \frac{1}{2})$-semiopen in $(X, T_1, T_2)$. But $A_3$ is not an IF $T_1$-$\frac{1}{2}$-open set in $(X, T_1)$ and $A_4$ is not an IF $T_2$-$\frac{1}{3}$-open set in $(X, T_2)$.

Theorem 2.10. Let $(X, T_1, T_2)$ be an ISBTS and $r, s \in I_0$. Then the following statements are true:
(1) If \( \{ A_k \} \) is a family of IF \((T_i, T_j)\)-(r, s)-semiopen sets in \( X \), then \( \bigcup A_k \) is IF \((T_i, T_j)\)-(r, s)-semiopen.

(2) If \( \{ A_k \} \) is a family of IF \((T_i, T_j)\)-(r, s)-semiclosed sets in \( X \), then \( \bigcap A_k \) is IF \((T_i, T_j)\)-(r, s)-semiclosed.

**Proof.** (1) Let \( \{ A_k \} \) be a collection of IF \((T_i, T_j)\)-(r, s)-semiclosed sets in \( X \). Then for each \( k \),

\[
A_k \subseteq T_j-\text{cl}(T_i-\text{int}(A_k, r), s).
\]

So we have

\[
\bigcup A_k \subseteq \bigcup T_j-\text{cl}(T_i-\text{int}(A_k, r), s) \subseteq T_j-\text{cl}(\bigcup A_k, r), s).
\]

Thus \( \bigcup A_k \) is IF \((T_i, T_j)\)-(r, s)-semiclosed.

(2) It follows from (1) using Theorem [2.7].

**Definition 2.11.** Let \((X, T_i, T_j)\) be an ISBTS and \( r, s \in I_0 \). For each \( A \in I(X) \), the IF \((T_i, T_j)\)-(r, s)-semiinterior is defined by

\[
(T_i, T_j)-\text{sint}(A, r, s) = \bigcup \{ B \in I(X) \mid B \subseteq A, B \text{ is IF } (T_i, T_j)-(r, s)\text{-semiopen}\}
\]

and the IF \((T_i, T_j)-(r, s)\)-semiclosure is defined by

\[
(T_i, T_j)-\text{scl}(A, r, s) = \bigcap \{ B \in I(X) \mid A \subseteq B, B \text{ is IF } (T_i, T_j)-(r, s)\text{-semiclosed}\}.
\]

Obviously, \((T_i, T_j)-\text{scl}(A, r, s)\) is the smallest IF \((T_i, T_j)-(r, s)\)-semiclosed set which contains \( A \) and \((T_i, T_j)-\text{sint}(A, r, s)\) is the greatest IF \((T_i, T_j)-(r, s)\)-semiopen set which is contained in \( A \). Also, \((T_i, T_j)-\text{scl}(A, r, s) = A\) for any IF \((T_i, T_j)-(r, s)\)-semiclosed set \( A \) and \((T_i, T_j)-\text{sint}(A, r, s) = A\) for any IF \((T_i, T_j)-(r, s)\)-semiopen set \( A \).

Moreover, we have

\[
T_i-\text{int}(A, r) \subseteq (T_i, T_j)-\text{sint}(A, r, s) \subseteq A \subseteq (T_i, T_j)-\text{scl}(A, r, s) \subseteq T_i-\text{cl}(A, r).
\]

Also, we have the following results:

(1) \((T_i, T_j)-\text{scl}(\emptyset, r, s) = \emptyset, (T_i, T_j)-\text{scl}(1, r, s) = 1\).

(2) \((T_i, T_j)-\text{scl}(A, r, s) \supseteq A\).

(3) \((T_i, T_j)-\text{scl}(A, r, s) \subseteq (T_i, T_j)-\text{scl}(B, r, s) \subseteq (T_i, T_j)-\text{scl}(A \cup B, r, s)\).

(4) \((T_i, T_j)-\text{scl}(T_i, T_j)-\text{scl}(A, r, s), r, s) = (T_i, T_j)-\text{scl}(A, r, s)\).

(5) \((T_i, T_j)-\text{sint}(\emptyset, r, s) = \emptyset, (T_i, T_j)-\text{sint}(1, r, s) = 1\).

(6) \((T_i, T_j)-\text{sint}(A, r, s) \subseteq A\).

(7) \((T_i, T_j)-\text{sint}(A, r, s) \cap (T_i, T_j)-\text{sint}(B, r, s) \supseteq (T_i, T_j)-\text{sint}(A \cap B, r, s)\).

(8) \((T_i, T_j)-\text{sint}(T_i, T_j)-\text{sint}(A, r, s), r, s) = (T_i, T_j)-\text{sint}(A, r, s)\).

**Theorem 2.12.** Let \( A \) be an intuitionistic fuzzy set in an ISBTS \((X, T_i, T_j)\) and \( r, s \in I_0 \). Then we have

(1) \((T_i, T_j)-\text{sint}(A, r, s)^c = (T_i, T_j)-\text{scl}(A^c, r, s)\).

(2) \((T_i, T_j)-\text{scl}(A, r, s)^c = (T_i, T_j)-\text{scl}(A^c, r, s)\).

**Proof.** (1) Since

\[
(T_i, T_j)-\text{sint}(A, r, s) \subseteq A \text{ and } (T_i, T_j)-\text{sint}(A, r, s)^c \text{ is IF } (T_i, T_j)-(r, s)\text{-semiopen in } X, A^c \subseteq (T_i, T_j)-\text{sint}(A, r, s)^c \text{ and } (T_i, T_j)-\text{sint}(A, r, s)^c \text{ is IF } (T_i, T_j)-(r, s)\text{-semiclosed. Thus}
\]

\[
(T_i, T_j)-\text{scl}(A^c, r, s) \subseteq (T_i, T_j)-\text{scl}(T_i, T_j)-\text{scl}(A^c, r, s)^c, r, s) = (T_i, T_j)-\text{scl}(A, r, s)^c.
\]

From that \( A^c \subseteq (T_i, T_j)-\text{scl}(A^c, r, s) \) and \((T_i, T_j)-\text{scl}(A^c, r, s)^c \subseteq A \) and \((T_i, T_j)-\text{scl}(A^c, r, s)^c \) is IF \((T_i, T_j)-(r, s)\)-semiclosed, \((T_i, T_j)-\text{scl}(A^c, r, s) \) is IF \((T_i, T_j)-(r, s)\)-semiclosed. Thus we have

\[
(T_i, T_j)-\text{scl}(A^c, r, s)^c = (T_i, T_j)-\text{sint}(T_i, T_j)-\text{scl}(A^c, r, s)^c, r, s) \subseteq (T_i, T_j)-\text{sint}(A, r, s).
\]

Hence

\[
(T_i, T_j)-\text{sint}(A, r, s)^c \subseteq (T_i, T_j)-\text{scl}(A^c, r, s).
\]
Therefore

\((T_i, T_j)\text{-sint}(A, r, s)^e = (T_i, T_j)\text{-scl}(A^e, r, s)\).

(2) Similar to (1).

3. Continuity in Intuitionistic Smooth Bitopology

We define the notions of IF pairwise \((r, s)\)-semicontinuous mappings in intuitionistic smooth bitopological spaces, and investigate their characteristic properties.

**Definition 3.1.** Let \(f : (X, T) \to (Y, U)\) be a mapping from an intuitionistic smooth topological spaces \(X\) to an intuitionistic smooth topological spaces \(Y\) and \(r \in I_0\). Then \(f\) is called an IF \(r\)-continuous mapping if \(f^{-1}(B)\) is IF \(T\)-open in \(X\) for each IF \(U\)-open set \(B\) in \(Y\).

**Definition 3.2.** Let \(f : (X, T_1, T_2) \to (Y, U_1, U_2)\) be a mapping from an ISBTS \(X\) to an ISBTS \(Y\) and \(r, s \in I_0\). Then \(f\) is said to be **IF pairwise \((r, s)\)-continuous** if the induced mapping \(f : (X, T_1) \to (Y, U_1)\) is an IF \(r\)-continuous mapping and the induced mapping \(f : (X, T_2) \to (Y, U_2)\) is an IF \(s\)-continuous mapping.

**Definition 3.3.** Let \(f : (X, T_1, T_2) \to (Y, U_1, U_2)\) be a mapping from an ISBTS \(X\) to an ISBTS \(Y\) and \(r, s \in I_0\). Then \(f\) is said to be **IF pairwise \((r, s)\)-semicontinuous** if \(f^{-1}(A)\) is an IF \((T_1, T_2)-(r, s)\)-semiopen set in \(X\) for each IF \(U_1\)-open set \(A\) in \(Y\) and \(f^{-1}(B)\) is an IF \((T_2, T_1)-(s, r)\)-semiopen set in \(X\) for each IF \(U_2\)-open set \(B\) in \(Y\).

**Remark 3.4.** It is obvious that every IF pairwise \((r, s)\)-continuous mapping is IF pairwise \((r, s)\)-semicontinuous. But the following example shows that the converse need not be true.

**Example 3.5.** Let \((X, T_1, T_2)\) be an ISBTS as described in Example 2.9 Define \(U_1 : I(X) \to I\) and \(U_2 : I(X) \to I\) by

\[
U_1(A) = \begin{cases} 
1 & \text{if } A = 0, 1, \\
0 & \text{otherwise}; 
\end{cases}
\]

and

\[
U_2(A) = \begin{cases} 
1 & \text{if } A = 0, 1, \\
\frac{1}{3} & \text{if } A = A_4, \\
0 & \text{otherwise}. 
\end{cases}
\]

Then \((U_1, U_2)\) is an ISBTS on \(X\). Consider a mapping \(f : (X, T_1, T_2) \to (X, U_1, U_2)\) defined by \(f(x) = x\) and \(f(y) = y\). Then \(f\) is IF pairwise \((\frac{1}{2}, \frac{1}{3})\)-semicontinuous. But \(f\) is not an IF pairwise \((\frac{1}{2}, \frac{1}{3})\)-continuous mapping.

**Theorem 3.6.** Let \(f : (X, T_1, T_2) \to (Y, U_1, U_2)\) be a mapping from an ISBTS \(X\) to an ISBTS \(Y\) and \(r, s \in I_0\). Then the following statements are equivalent:

1. \(f\) is IF pairwise \((r, s)\)-semicontinuous.
2. \(f^{-1}(A)\) is an IF \((T_1, T_2)-(r, s)\)-semiclosed set in \(X\) for each IF \(U_1\)-closed set \(A\) in \(Y\) and \(f^{-1}(B)\) is an IF \((T_2, T_1)-(s, r)\)-semiclosed set in \(X\) for each IF \(U_2\)-closed set \(B\) in \(Y\).
3. For each intuitionistic fuzzy set \(B\) in \(Y\),

\[
T_2\text{-int}(T_1\text{-cl}(f^{-1}(B), r), s) \subseteq f^{-1}(U_1\text{-cl}(B, r))
\]

and

\[
T_1\text{-int}(T_2\text{-cl}(f^{-1}(B), s), r) \subseteq f^{-1}(U_2\text{-cl}(B, s)).
\]

4. For each intuitionistic fuzzy set \(A\) in \(X\),

\[
f(T_2\text{-int}(T_1\text{-cl}(A, r), s)) \subseteq U_1\text{-cl}(f(A), r)
\]

and

\[
f(T_1\text{-int}(T_2\text{-cl}(A, s), r)) \subseteq U_2\text{-cl}(f(A), s).
\]

**Proof.**

1. \(\Leftrightarrow\) (2) Trivial.

2. \(\Rightarrow\) (3) Let \(B\) be an intuitionistic fuzzy set in \(Y\). Then \(U_1\text{-cl}(B, r)\) is IF \(U_1\)-closed and \(U_2\text{-cl}(B, s)\) is IF \(U_2\)-closed in \(Y\). Hence by (2), \(f^{-1}(U_1\text{-cl}(B, r))\) is an IF \((T_1, T_2)-(r, s)\)-semiclosed set and \(f^{-1}(U_2\text{-cl}(B, s))\) is an IF \((T_2, T_1)-(s, r)\)-semiclosed set in \(X\). Thus we obtain

\[
T_2\text{-int}(T_1\text{-cl}(f^{-1}(B), r), s) \subseteq T_2\text{-int}(U_1\text{-cl}(f^{-1}(U_1\text{-cl}(B, r)), r), s) = f^{-1}(U_1\text{-cl}(B, r))
\]

and

\[
T_1\text{-int}(T_2\text{-cl}(f^{-1}(B), s), r) \subseteq T_1\text{-int}(U_2\text{-cl}(f^{-1}(U_2\text{-cl}(B, s)), s), r) = f^{-1}(U_2\text{-cl}(B, s)).
\]

3. \(\Rightarrow\) (4) Let \(A\) be an intuitionistic fuzzy set in \(X\). Then by (3), we have

\[
T_2\text{-int}(T_1\text{-cl}(A, r), s) \subseteq T_2\text{-int}(T_1\text{-cl}(f^{-1}(f(A)), r), s) \subseteq f^{-1}(U_1\text{-cl}(f(A), r))
\]

and

\[
T_1\text{-int}(T_2\text{-cl}(A, s), r) \subseteq T_1\text{-int}(T_2\text{-cl}(f^{-1}(f(A)), s), r) \subseteq f^{-1}(U_2\text{-cl}(f(A), s)).
\]

53 | Jin Tae Kim and Seok Jong Lee
and

$$T_1\text{-int}(T_2\text{-cl}(A, r)) \subseteq T_1\text{-int}(f^{-1}(f(A), s), r) \subseteq f^{-1}(U_2\text{-cl}(f(A), s)).$$

Hence

$$f(T_2\text{-int}(T_1\text{-cl}(A, r)), s) \subseteq U_1\text{-cl}(f(A), r)$$

and

$$f(T_1\text{-int}(T_2\text{-cl}(A, s), r)) \subseteq U_2\text{-cl}(f(A), s).$$

(4) ⇒ (2) Let A be any IF $$U_1$$-r-closed set and $$B$$ any IF $$U_2$$-s-closed set in Y. By (4), we obtain

$$f(T_2\text{-int}(T_1\text{-cl}(f^{-1}(A), r)), s) = U_1\text{-cl}(f(f^{-1}(A), r)) \subseteq U_1\text{-cl}(A, r) = A$$

and

$$f(T_1\text{-int}(T_2\text{-cl}(f^{-1}(B), s), r)) = U_2\text{-cl}(f(f^{-1}(B), s)) \subseteq U_2\text{-cl}(B, r) = B.$$

Hence

$$T_2\text{-int}(T_1\text{-cl}(f^{-1}(A), r), s) \subseteq f^{-1}(A)$$

and

$$T_1\text{-int}(T_2\text{-cl}(f^{-1}(B), s), r) \subseteq f^{-1}(B).$$

Therefore $$f^{-1}(A)$$ is an IF $$(T_1, T_2)$$-(r, s)-semiclosed set and $$f^{-1}(B)$$ is an IF $$(T_2, T_1)$$-(r, s)-semiclosed set in X.

**Theorem 3.7.** Let $$f : (X, T_1, T_2) \rightarrow (Y, U_1, U_2)$$ be a mapping from an ISBTs X to an ISBTs Y and $$r, s \in I_0$$. Then the following statements are equivalent:

1. f is IF pairwise $$(r, s)$$-semicontinuous.

2. For each intuitionistic fuzzy set A in X,

$$f((T_1, T_2)\text{-scl}(A, r, s)) \subseteq U_1\text{-cl}(f(A), r)$$

and

$$f((T_2, T_1)\text{-scl}(A, s, r)) \subseteq U_2\text{-cl}(f(A), s).$$

3. For each intuitionistic fuzzy set B in Y,

$$(T_1, T_2)\text{-scl}(f^{-1}(B), r, s) \subseteq f^{-1}(U_1\text{-cl}(B, r))$$

and

$$(T_2, T_1)\text{-scl}(f^{-1}(B), s, r) \subseteq f^{-1}(U_2\text{-cl}(B, s)).$$

(4) For each intuitionistic fuzzy set $$B$$ in Y,

$$f^{-1}(U_1\text{-int}(B, r)) \subseteq (T_1, T_2)\text{-int}(f^{-1}(B), r, s)$$

and

$$f^{-1}(U_2\text{-int}(B, s)) \subseteq (T_2, T_1)\text{-int}(f^{-1}(B), s, r).$$

**Proof.** (1) ⇒ (2) Let A be an intuitionistic fuzzy set in X. Then $$U_1\text{-cl}(f(A), r)$$ is IF $$U_1$$-r-closed and $$U_2\text{-cl}(f(A), s)$$ is IF $$U_2$$-s-closed in Y. Since f is IF pairwise $$(r, s)$$-semicontinuous, $$f^{-1}(U_1\text{-cl}(f(A), r))$$ is an IF $$(T_1, T_2)$$-(r, s)-semiclosed set and $$f^{-1}(U_2\text{-cl}(f(A), s))$$ is an IF $$(T_2, T_1)$$-(r, s)-semiclosed set in X. Hence

$$f((T_1, T_2)\text{-scl}(A, r), s) \subseteq (T_1, T_2)\text{-scl}(f^{-1}(U_1\text{-cl}(f(A), r)), r, s) \subseteq f^{-1}(U_1\text{-cl}(f(A), r))$$

and

$$f((T_2, T_1)\text{-scl}(A, s), r) \subseteq (T_2, T_1)\text{-scl}(f^{-1}(U_2\text{-cl}(f(A), s)), s, r) = f^{-1}(U_2\text{-cl}(f(A), s)).$$

Therefore

$$f((T_1, T_2)\text{-scl}(A, r, s)) \subseteq U_1\text{-cl}(f(A), r)$$

and

$$f((T_2, T_1)\text{-scl}(A, s, r)) \subseteq U_2\text{-cl}(f(A), s).$$

(2) ⇒ (3) Let $$B$$ be an intuitionistic fuzzy set in Y. Then by (2), we obtain

$$f((T_1, T_2)\text{-scl}(f^{-1}(B), r, s)) \subseteq U_1\text{-cl}(f(f^{-1}(B), r)) \subseteq U_1\text{-cl}(B, r)$$

and

$$f((T_2, T_1)\text{-scl}(f^{-1}(B), s, r)) \subseteq U_2\text{-cl}(f(f^{-1}(B), s)) \subseteq U_2\text{-cl}(B, s).$$

Hence

$$(T_1, T_2)\text{-scl}(f^{-1}(B), r, s) \subseteq f^{-1}(U_1\text{-cl}(B, r))$$

and

$$(T_2, T_1)\text{-scl}(f^{-1}(B), s, r) \subseteq f^{-1}(U_2\text{-cl}(B, s)).$$
Thus and

\[(T_2, T_1) \text{-scl}(f^{-1}(B), s, r) \subseteq f^{-1}(U_2\text{-cl}(B, s)).\]

(3) \Rightarrow (4) Let B be an intuitionistic fuzzy set in Y. Then by (3), we have

\[(T_1, T_2) \text{-scl}(f^{-1}(B^c), r, s) \subseteq f^{-1}(U_1\text{-cl}(B^c, r)).\]

and

\[(T_2, T_1) \text{-scl}(f^{-1}(B^c), s, r) \subseteq f^{-1}(U_2\text{-cl}(B^c, s)).\]

Hence

\[f^{-1}(U_1\text{-int}(B, r)) = (f^{-1}(U_1\text{-cl}(B^c, r)))^c \subseteq (T_1, T_2) \text{-scl}(f^{-1}(B^c), r, s)^c \]

\[= (T_2, T_1) \text{-sint}(f^{-1}(B), r, s).\]

and

\[f^{-1}(U_2\text{-int}(B, s)) = (f^{-1}(U_2\text{-cl}(B^c, s)))^c \subseteq (T_2, T_1) \text{-scl}(f^{-1}(B^c), s, r)^c \]

\[= (T_2, T_1) \text{-sint}(f^{-1}(B), s, r).\]

(4) \Rightarrow (1) Let A be any IF \(U_1\)-r-open set and B any IF \(U_2\)-s-open set in Y. Then \(U_1\text{-int}(A, r) = A\) and \(U_2\text{-int}(B, s) = B\). Hence

\[f^{-1}(A) = f^{-1}(U_1\text{-int}(A, r)) \subseteq (T_1, T_2) \text{-sint}(f^{-1}(A), r, s) \subseteq f^{-1}(A).\]

and

\[f^{-1}(B) = f^{-1}(U_2\text{-int}(B, s)) \subseteq (T_2, T_1) \text{-sint}(f^{-1}(B), s, r) \subseteq f^{-1}(B).\]

Thus

\[f^{-1}(A) = (T_1, T_2) \text{-sint}(f^{-1}(A), r, s)\]

and

\[f^{-1}(B) = (T_2, T_1) \text{-sint}(f^{-1}(B), s, r).\]

Hence \(f^{-1}(A)\) is an IF \((T_1, T_2)-(r, s)\)-semiopen set and \(f^{-1}(B)\) is an IF \((T_2, T_1)-(s, r)\)-semiopen set in X. Therefore \(f\) is IF pairwise \((r, s)\)-semicontinuous.

**Theorem 3.8.** Let \(f : (X, T_1, T_2) \rightarrow (Y, U_1, U_2)\) be a bijective mapping from an ISBTS X to an ISBTS Y and \(r, s \in I_0\). Then

\(f\) is IF pairwise \((r, s)\)-semicontinuous if and only if

\[U_1\text{-int}(f(A), r) \subseteq f((T_1, T_2) \text{-sint}(A, r, s))\]

and

\[U_2\text{-int}(f(A), s) \subseteq f(((T_2, T_1) \text{-sint}(A, s, r)).\]

for each intuitionistic fuzzy set \(A\) in \(X\).

**Proof.** Let \(A\) be an intuitionistic fuzzy set in \(X\). Since \(f\) is one-to-one, by Theorem 3.7, we have

\[f^{-1}(U_1\text{-int}(f(A), r)) \subseteq (T_1, T_2) \text{-sint}(f^{-1}(f(A)), r, s) \]

\[= (T_1, T_2) \text{-sint}(A, r, s)\]

and

\[f^{-1}(U_2\text{-int}(f(A), s)) \subseteq (T_2, T_1) \text{-sint}(f^{-1}(f(A), s, r) \]

\[= (T_2, T_1) \text{-sint}(A, s, r).\]

Because \(f\) is onto, we obtain

\[U_1\text{-int}(f(A), r) = f(f^{-1}(U_1\text{-int}(f(A), r))) \subseteq f(((T_1, T_2) \text{-sint}(A, r, s))\]

and

\[U_2\text{-int}(f(A), s) = f(f^{-1}(U_2\text{-int}(f(A), s))) \subseteq f(((T_2, T_1) \text{-sint}(A, s, r)).\]

Conversely, let \(B\) be an intuitionistic fuzzy set in \(Y\). Since \(f\) is onto, we obtain

\[U_1\text{-int}(B, r) = U_1\text{-int}(f(f^{-1}(B)), r) \subseteq f(((T_1, T_2) \text{-sint}(f^{-1}(B), r, s))\]

and

\[U_2\text{-int}(B, s) = U_2\text{-int}(f(f^{-1}(B)), s) \subseteq f(((T_2, T_1) \text{-sint}(f^{-1}(B), s, r)).\]

Because \(f\) is one-to-one, we have

\[f^{-1}(U_1\text{-int}(B, r)) \subseteq f^{-1}(f((T_1, T_2) \text{-sint}(f^{-1}(B), r, s))\]

\[= (T_1, T_2) \text{-sint}(f^{-1}(B), r, s)\]

and

\[f^{-1}(U_2\text{-int}(B, s)) \subseteq f^{-1}(f(((T_2, T_1) \text{-sint}(f^{-1}(B), s, r))\]

\[= (T_2, T_1) \text{-sint}(f^{-1}(B), s, r).\]
Therefore by Theorem 3.7, $f$ is an intuitionistic fuzzy pairwise $(r, s)$-semicontinuous mapping.

**Conflict of Interest**

No potential conflict of interest relevant to this article was reported.

**Acknowledgments**

This work was supported by the research grant of Chungbuk National University in 2012.

**References**


**Jin Tae Kim** received the Ph. D. degree from Chungbuk National University in 2012. His research interests include general topology and fuzzy topology. He is a member of KIIS and KMS.  
E-mail: kjtmath@hanmail.net

**Seok Jong Lee** received the M. S. and Ph. D. degrees from Yonsei University in 1986 and 1990, respectively. He is a professor at the Department of Mathematics, Chungbuk National University since 1989. He was a visiting scholar in Carleton University from 1995 to 1996, and Wayne State University from 2003 to 2004. His research interests include general topology and fuzzy topology. He is a member of KIIS, KMS, and CMS.  
E-mail: sjl@cbnu.ac.kr