Diversity and Coding Gains of Threshold-Based Generalized Selection Combining

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Abstract—In this paper, we study the asymptotic performance of absolute-threshold and normalized-threshold-based generalized selection combining (AT-GSC and NT-GSC) schemes over generalized fading channels for high average signal-to-noise ratio (ASNR). By evaluating the asymptotic moment generating function (MGF) of the threshold-based GSC (T-GSC) output SNRs, we derive the diversity and combining gains for AT- and NT-GSCs with a large class of modulation formats and versatile fading conditions, including different types of fading channels and non-identical SNR statistics across diversity branches. Our analytical results reveal that the diversity gains of AT- and NT-GSC are equivalent to that of maximum ratio combining (MRC), and the differences in the combining gains for different threshold values, and modulation and diversity formats, are derived using the concept of the modulation factor.

I. INTRODUCTION

Generalized selection combining (GSC) diversity [1]–[4], [6], also known as hybrid selection/maximal-ratio combining (H-S/MRC) [2], has recently attracted a lot of research interest in many applications, such as wide-band CDMA and ultra-wide bandwidth (UWB) communication [7].

The performance evaluation of various forms of GSC diversity in different fading channels has attracted a lot of research interest in the past few years [2]–[4], [6], [8], [9]. For example, besides the conventional GSC, the minimum selection-based GSC [8] and signal threshold-based GSC (T-GSC) [10]–[13] schemes have been regarded as practical forms of GSC receivers. For example, in the single-antenna Rake receiver branch selection operation in T-GSC can be implemented using digital signal processing (DSP) to avoid a variable hardware complexity.

The performance of T-GSC over different fading channels has been studied recently [11]–[13]. In this paper, we try to provide general analytical results for the effects of different modulation formats and generalized fading conditions on threshold-based GSC receivers for an asymptotic scenario of high average signal-to-noise ratio (ASNR). High ASNR behaviors of fading channel receivers have attracted much attention in the literature [14]–[16], because they offer insights into the performance trends of a receiver when the noise diminishes and thus provide useful design guidelines, see e.g. [15] for the space-time code design.

To our knowledge, the achievable asymptotic gains of threshold-based GSC schemes are still not known yet, though such a knowledge will provide a useful guideline for designing better T-GSC receivers. For example, when a joint diversity and channel error correction coding (ECC) scheme needs to be employed to a multipath channel, one may reduce the input branch SNR threshold to increase the coding gain advantage provided by T-GSC diversity instead of increasing the complexity of the ECC scheme. There is a tradeoff between the complexities for T-GSC diversity combining and ECC processing. To understand how much coding gain can be reaped by using T-GSC (by adjusting the threshold) when maintaining a diversity gain, and at what complexity, it is important to gain such knowledge in the system design stage.

In order to understand the achievable performance of T-GSC schemes for high SNRs, and the relation between the diversity gain, the coding gain (or termed the combining gain), and the combining complexity, a concise and analytical result will be very useful. In this paper, we derive the asymptotic error and outage probabilities for AT- and NT-GSC schemes with different modulations and selection thresholds, and obtain the asymptotic diversity and combining gains. Our results demonstrate that T-GSC schemes achieve the same diversity gain as MRC for the error and outage probabilities. To quantify the performance gaps between different modulation formats and GSC thresholds, we use the concept of modulation factors defined in [16], and derive the relevant combining gains. Consequently, the performance gaps (combining gains) between different diversity schemes, modulation formats, and combining thresholds can be analytically predicted in a unified framework.

Throughout this paper, we use the superscripts *, T and H to denote the complex conjugate, transpose, and conjugate transpose, respectively. The subscript l denotes the lth element of a vector. E[x] is the expected value of x, and CN(\mu, \Sigma) denotes the complex Gaussian distribution with mean \mu and variance (or covariance matrix) \Sigma.

II. SIGNAL AND SYSTEM MODEL

Consider a signal model of a single transmitter and an L-branch diversity receiver. The received signal at the lth branch is given by \( y_l = h_l d + n_l \), where \( d \) is the data symbol with \( E[|d|^2] = 1 \), \( n_l \sim CN(0, N_0) \), and \( h_l \) is the complex channel gain which may follow Rayleigh, Rician, Nakagami-m, or Weibull distributions. The SNR at the lth branch is given by \( \gamma_l = |h_l|^2/N_0 \).
TABLE I
ASYMPTOTIC PARAMETERS OF THE SNR OVER FADED CHANNELS. IN
THE TABLE, THE SNR PDF \( f(\gamma) \) IS DECOMPOSED TO
\( f(\gamma) \simeq a\gamma^t \left( b/\gamma^{t+1} \right)^\gamma t \), WHERE \( \gamma \) IS THE INPUT ASNR PER
BRANCH. \( \Gamma(x) \) IS THE GAMMA FUNCTION.

<table>
<thead>
<tr>
<th>Fading types</th>
<th>( t, a, b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rayleigh</td>
<td>( t=0, a=1/\gamma, b=1 )</td>
</tr>
<tr>
<td>Nakagami-( q )</td>
<td>( t=0, a=1/2\gamma^2, b=1/2\gamma )</td>
</tr>
<tr>
<td>Rician</td>
<td>( t=0, a=(1+K)e^{-K/\gamma}, b=(1+K)e^{-K} )</td>
</tr>
<tr>
<td>Nakagami-( m )</td>
<td>( m-1, a=m/(\Gamma(m)\gamma^m), b=m/(\Gamma(m)) )</td>
</tr>
<tr>
<td>Weibull</td>
<td>( t=2, a=(\gamma_1/2)^{\gamma_2/2}, b=m/(\Gamma(1+2/\gamma_1)) )</td>
</tr>
</tbody>
</table>

For AT-GSC \( (\gamma_0, L) \), branches with SNR exceeding \( \gamma_0 \) are selected for combining, where \( \gamma_0 \) is the preset SNR threshold. The output SNR is given by
\[ \gamma_{\text{ATGSC}} = \sum_{l=1}^{L} \gamma_{l,\text{ATGSC}}(\gamma_0), \]
where \( \gamma_{l,\text{ATGSC}}(\gamma_0) = \begin{cases} \gamma_l & \gamma_l \geq \gamma_0 \\ 0 & \gamma_l < \gamma_0 \end{cases} \)

For NT-GSC \( (\gamma_0, L) \), branches with normalized SNR (defined as the SNR divided by the SNR of the strongest branch) exceeding \( \gamma_0 \) is selected for combining, where \( \gamma_0 (0 \leq \gamma_0 \leq 1) \) is the pre-specified threshold. We have
\[ \gamma_{\text{NTGSC}} = \sum_{l=1}^{L} \gamma_{l,\text{NTGSC}}(\gamma_0), \]
where \( \gamma_{l,\text{NTGSC}}(\gamma_0) \) is the NT-GSC \( (\gamma_0, L) \) output SNR at the \( l \)th branch, and
\[ \gamma_{l,\text{NTGSC}}(\gamma_0) = \begin{cases} \gamma_l & \gamma_l \geq \gamma_{\text{th}}(\gamma_0) \\ 0 & \gamma_l < \gamma_{\text{th}}(\gamma_0) \end{cases} \]
Here, \( \gamma_{\text{th}}(\gamma_0) \) is the largest SNR among all the \( L \) branches.

For high ASNR, if the error probability can be written as [14]
\[ P_{\text{err}} = \left(G_e \bar{\gamma}\right)^{-G_d} + o(\bar{\gamma}^{-G_d}) \]
where \( \bar{\gamma} \) is the input ASNR per branch, and \( o(x) \) satisfies the property that \( \lim_{x\to0} \frac{o(x)}{x} = 0 \), then we call \( G_e \) the combining gain, and \( G_d \) the diversity gain, which manifests the effective diversity order, see e.g. [14]. For outage performance of GSC with a predefined SNR threshold \( \gamma_{\text{thr}} \), if the asymptotic outage probability [defined as \( P_{\text{out}}(\gamma_{\text{thr}}) = \Pr(\gamma_s < \gamma_{\text{thr}}) \)] can be expressed as [14], [16]
\[ P_{\text{out}}(\gamma_{\text{thr}}) = \left(O_c \bar{\gamma}^{\gamma_{\text{thr}}}ight)^{-O_d} + o(\bar{\gamma}^{-O_d}), \]
then we call \( O_c \) and \( O_d \) the outage combining gain and diversity gain, respectively.

III. ASYMPTOTIC MGFs OF T-GSC OUTPUT SNRS
For different fading channel types, the expressions for the polynomial approximation of the probability density functions (PDFs) were given in [14], [16]. Here, for completeness of this paper, we list them in Table I.

A. AT-GSC \( (\gamma_0, L) \)
Define \( f_{\gamma_l}(x) \) as the PDF of \( \gamma_l \) and \( F_{\gamma_l}(x) = \int_0^x f_{\gamma_l}(y)dy \) as the cumulative distribution function (CDF) of \( \gamma_l \), respectively. For AT-GSC \( (\gamma_0, L) \), the MGF of the output SNR is given by [11]
\[ \Phi_{\text{ATGSC}}(s) = E[\exp(-s\gamma_{\text{ATGSC}})] = \prod_{l=1}^{L} \Phi_{\gamma_l}(s) \]
where \( \Phi_{\gamma_l}(s) \) is the MGF of \( \gamma_l \), and is given by \( \Phi_{\gamma_l}(s) = F_{\gamma_l}(s) + \Phi_{\gamma_l}(s, \gamma_0) \), and \( \Phi_{\gamma_l}(s, y) = \int_0^{\infty} f_{\gamma_l}(x)e^{-sy}dx \) is the truncated MGF of \( \gamma_l \) [6]. Note that \( \Phi_{\gamma_l}(s, 0) = \Phi_{\gamma_l}(s) \) and \( \Phi_{\gamma_l}(0, x) = 1 - F_{\gamma_l}(x) \).
We express the PDF at branch \( l \) as \( f_{\gamma_l}(x) = a_l\gamma_{\text{th}}^t + o(\gamma_{\text{th}}^t) = b_l\gamma_{\text{th}}^t + e(\gamma_{\text{th}}^t) \) for high ASNR. The truncated MGF can be approximated as \( \Phi_{\gamma_l}(s, x) = a_l \Gamma(t_l + 1, s\gamma_{\text{th}})/s^{t_l+1}, \)
where \( \Gamma(t_l, x) = \int_x^{\infty} e^{-u}u^{t_l-1}du \) is the complementary incomplete Gamma function. Also, the CDF \( F_{\gamma_l}(x) \) is approximated as \( F_{\gamma_l}(x) = a_t\gamma_{\text{th}}^{t_l + 1}/(t_l + 1) \).

By substituting the asymptotic result above into (3), we obtain
\[ \Phi_{\text{ATGSC}}(s) \simeq \prod_{l=1}^{L} a_l \left[ \gamma_{\text{th}}^{t_l + 1}/t_l + 1 + \left(\Gamma(t_l + 1, s\gamma_{\text{th}})/s^{t_l+1}\right) \right] \]
\[ = (s\gamma_{\text{th}})^{-L} \sum_{l=1}^{L} \left[ b_l \gamma_{\text{th}}^{t_l + 1}/t_l + 1 + \left(\Gamma(t_l + 1, s\gamma_{\text{th}})/s^{t_l+1}\right) \right] \]
\[ \simeq \prod_{l=1}^{L} b_l \gamma_{\text{th}}^{t_l + 1}/t_l + 1 \sum_{l=1}^{L} \left(\gamma_{\text{th}}/s\right)^{t_l + 1} \gamma_{\text{th}} \]
(4)

When \( \gamma_0 = 0 \), all the \( L \) branches are selected. In this case, we have \( \Gamma(t_l + 1, s\gamma_{\text{th}}) = \Gamma(t_l + 1) \), and (4) reduces to
\[ \Phi_{\text{ATGSC}}(s) \simeq \prod_{l=1}^{L} b_l \gamma_{\text{th}}^{t_l + 1}/t_l + 1 \left(\gamma_{\text{th}}/s\right)^{t_l + 1} \]
which is equivalent to the asymptotic MGF of MRC output SNR [14], [16], as expected.

B. NT-GSC
For NT-GSC \( (\gamma_0, L) \) the MGF of the output SNR is derived as [13]
\[ \Phi_{\text{NTGSC}}(s) = \sum_{l=1}^{L} \int_0^{\infty} f_{\gamma_l}(y) \exp(-sy)dy \]
\[ \prod_{l=1}^{L} \left[ F_{\gamma_l}(\gamma_0, y) + \Phi_{\gamma_l}(s, \gamma_0) - \Phi_{\gamma_l}(s, y) \right]dy \]
(6)
where \( \Phi_{\gamma_l}(s, \gamma_0) = \int_0^{\gamma_0} f_{\gamma_l}(x)e^{-sx}dx \) is the truncated MGF and \( F_{\gamma_l}(\gamma_0, y) = \int_0^{\gamma_0} f_{\gamma_l}(x)dx \) is the CDF of \( \gamma_l \), respectively.
For high ASNR, we may approximate \( \Phi_{l, \text{NT-GSC}}(s) \) as
\[
\Phi_{l, \text{NT-GSC}}(s) \simeq \frac{b_l}{(\gamma g_l)(t_l+1)} \left[ \frac{\Gamma(t_l + 1, s \gamma b_l)}{s^{t_l+1}} \right] + \frac{\Gamma(t_l + 1, s \gamma b_l)}{s^{t_l+1}}. 
\]

Thus, we have
\[
\Phi_{l, \text{NT-GSC}}(s) \simeq \sum_{l=1}^{L} \int_0^\infty \frac{b_l}{(\gamma g_l)(t_l+1)} \exp(-sy) \cdot \prod_{l \neq l}^{L} \frac{b_l'}{(\gamma g_l')(t_l'+1)} \left[ \frac{\Gamma(t_l' + 1, s \gamma b_l')}{s^{t_l'+1}} \right] dy 
\]

Using a change of variable that \( \theta = sy \), we get
\[
\Phi_{l, \text{NT-GSC}}(s) \simeq \frac{F(b, t, \gamma h)}{(\gamma h)^-\sum_i^{L}(t_i+1)} 
\]
where \( b = [b_1, \ldots, b_L]' \), \( t = [t_1, \ldots, t_L]' \), \( \gamma = 1/L \), and \( \gamma_l \) is the total input ASNR divided by \( L \), and
\[
F(b, t, \gamma h) = \prod_{l=1}^{L} \frac{b_l}{(\gamma g_l)(t_l+1)} \sum_{l=1}^{L} \int_0^\infty \exp(-x) \cdot \prod_{l \neq l}^{L} \frac{b_l'}{(\gamma g_l')(t_l'+1)} \left[ \frac{\Gamma(t_l' + 1, \gamma h b_l)}{s^{t_l'+1}} + \Gamma(t_l' + 1, \gamma h b_l) \right] dx. 
\]

In (10), \( g_l = \gamma_l/\gamma \). Obviously \( \sum_{k=1}^{L} g_k = L \).

IV. ERROR AND OUTAGE PERFORMANCE ANALYSIS

Using our new asymptotic MGF results, we derive the combining gain and diversity gain for the error probability and outage probability of AT- and NT-GSC schemes in generalized fading channels, as given next.

A. AT-GSC (\( \gamma_{th}, L \))

By using the integral form of the bit error probability (BEP) expression for BPSK we get
\[
P_{\text{BPSK,AT-GSC}}(\gamma) \simeq \frac{1}{\pi} \int_0^{\pi/2} \Phi_{\text{AT-GSC},\gamma} \left( \frac{1}{\sin^2 \theta} \right) d\theta = \frac{1}{\pi} \int_0^{\pi/2} \Phi_{\text{AT-GSC},\gamma} \left( \frac{1}{\sin^2 \theta} \right) d\theta \cdot \gamma^\sum_i^{L}(t_i+1)
\]
where \( \Phi_{\text{AT-GSC},\gamma} \) is obtained from right handside (RHS) of (4) by replacing variable \( s \) with \( 1/\sin^2 \theta \), and \( \Phi_{\text{AT-GSC},\gamma} \) is obtained by further setting \( \gamma = 1 \), and thus
\[
\Phi_{\text{AT-GSC},\gamma}(s|\gamma = 1) = s^{-\sum_{i=1}^{L}(t_i+1)} \prod_{l=1}^{L} b_l g_l^{-1}(t_l+1) \cdot \left[ \frac{(\gamma h s)^{t_l+1}}{t_l+1} + \Gamma(t_l + 1, \gamma h s) \right]. 
\]

We can extract the factor in (11) which is directly related with the BPSK modulation format as
\[
\tilde{G}_{\text{BPSK,AT-GSC}}(b, t, \gamma_{th}) = \frac{1}{\pi} \int_0^{\pi/2} \Phi_{\text{AT-GSC},\gamma} \left( \frac{1}{\sin^2 \theta} \right) d\theta 
\]
We define \( \tilde{G}_{\text{BPSK,AT-GSC}}(b, t, \gamma_{th}) \) as the modulation factor for BPSK with AT-GSC (\( \gamma_{th}, L \)).

For some special cases, such as Rayleigh, Rician and Nakagami-\( q \) fading channels \( (t_1 = \cdots = t_L = 0) \), (13) can be simplified to
\[
\tilde{G}_{\text{BPSK,AT-GSC}}(b, t, \gamma_{th}) = \prod_{l=1}^{L} b_l g_l^{-1}(t_l+1) 
\]
\[
\cdot \int_0^{\pi/2} \sum_{k=0}^{L} (L)_{\gamma_{th}}^{L-k} \sin^{2k} \theta \exp(-\gamma_{th}k/\sin^2(\theta)) d\theta 
\]
which can be evaluated efficiently using a Gauss Chebyshev quadrature (GCQ).

The asymptotic BEP for AT-GSC (\( \gamma_{th}, L \)) with BPSK can be rewritten as
\[
P_{\text{BPSK,AT-GSC}}(\gamma) \simeq \tilde{G}_{\text{BPSK,AT-GSC}}(b, t, \gamma_{th}) \gamma^{-\sum_i^{L}(t_i+1)} 
\]
By comparing (15) with (1), we get the diversity gain for BPSK modulation with AT-GSC (\( \gamma_{th}, L \)) as \( G_d = \sum_{i=1}^{L}(t_i+1) \), and the combining gain as
\[
G_{\text{BPSK,AT-GSC}} = \left( \tilde{G}_{\text{BPSK,AT-GSC}}(b, t, \gamma_{th}) \right)^{-1/G_d} 
\]
The modulation factors \( \tilde{G}_{M,\text{AT-GSC}}(b, t, \gamma_{th}) \) for \( M \)-ary modulation formats may be evaluated using the same approach shown above. In general we have the following results.

**Proposition 1** The diversity gain of AT-GSC (\( \gamma_{th}, L \)) of a modulation format is equal to that of the \( L \)-fold MRC and is given by \( G_d = \sum_{i=1}^{L}(t_i+1) \), and the combining gain is given by
\[
G_{M,\text{AT-GSC}} = \left( \tilde{G}_{M,\text{AT-GSC}}(b, t, \gamma_{th}) \right)^{-1/G_d} 
\]
where \( \tilde{G}_{M,\text{AT-GSC}}(b, t, \gamma_{th}) \) is the modulation factor of that format, and \( b, t \) are the channel parameters defined in Table I.

The outage probability for AT-GSC (\( \gamma_{th}, L \)) is defined as
\[
P_{\text{out} \gamma_{th}} = \int_0^{\gamma_{th}} f_{\text{AT-GSC}}(\gamma) d\gamma, \quad \text{where } f_{\text{AT-GSC}}(\gamma) \text{ is the PDF of the AT-GSC (\( \gamma_{th}, L \)) output SNR, and } \gamma_{th} \text{ is the specified SNR threshold.}
\]
The explicit expression of \( f_{\text{AT-GSC}}(\gamma) \) may be obtained by using the \( L \)-fold convolution of \( f_{\text{AT-GSC}}(\gamma) = F_{\gamma}(\gamma_{th}) \delta(\gamma - \gamma_{th}) + f_{\gamma}(\gamma|\gamma > \gamma_{th}) \), which is, however, complicated to use for outage probability evaluation. As a simpler approach, we can use the inversion Laplace transform (ILT) of (4), and re-write (4) as \( \Phi_{\text{AT-GSC}}(s) = \Phi_{\text{AT-GSC},\gamma} \left( \frac{1}{s^{t+1}} \right) \)
Using numerical inversion of the Laplace transform of the CDF of \( \gamma_{\text{AT-GSC}} \) [17], and the trapezoidal summation, an
efficient formula to evaluate $P_{\text{G},\text{NT-GSC}}(\gamma_{\text{th}}, \gamma_{\text{ot}}|\bar{\gamma} = 1)$ can be obtained, which is, however, omitted here for brevity. The asymptotic outage probability can be written as

$$P_{\text{G},\text{NT-GSC}}(\gamma_{\text{th}}, \gamma_{\text{ot}}) \simeq P_{\text{G},\text{AT-GSC}}(\gamma_{\text{th}}, \gamma_{\text{ot}}|\bar{\gamma} = 1)^{-\gamma_{\text{G}} G_d}$$  \hspace{1cm} (18)$$

Comparing (18) with (2), we have the outage diversity gain as $O_d = G_d$, and the combining gain as

$$O_{c,\text{AT-GSC}} = [P_{\text{G},\text{AT-GSC}}(\gamma_{\text{th}}, \gamma_{\text{ot}}|\bar{\gamma} = 1)]^{-1/G_d \gamma_{\text{ot}}}.$$  \hspace{1cm} (19)$$

B. NT-GSC

For BPSK NT-GSC ($\eta_{\text{th}}, L$) we have

$$P_{\text{BPSK,NT-GSC}}(\bar{\gamma}) \simeq \frac{1}{\pi} \int_0^{\pi/2} \Phi_{\text{NT-GSC},\bar{\gamma}} \left( \frac{1}{\sin^2 \theta} \right) d\theta$$  \hspace{1cm} (20)$$

where $\Phi_{\text{NT-GSC},\bar{\gamma}} \left( \frac{1}{\sin^2 \theta} \right)$ is obtained from RHS of (9) by replacing variable $s$ with $\frac{1}{\sin \theta}$. Since $s^{-\sum_{l=1}^L (t_l + 1)}$ is the only factor in (20) that is involved with the modulation formats, we define the BPSK modulation factor for NT-GSC ($\eta_{\text{th}}, L$)as

$$G_{\text{BPSK,NT-GSC}} = \frac{1}{\pi} \int_0^{\pi/2} (\sin \theta)^2 \left( \sum_{l=1}^L (t_l + 1) + 0.5 \right) d\theta$$  \hspace{1cm} (21)$$

The asymptotic BEP can be written as

$$P_{\text{BPSK,NT-GSC}}(\bar{\gamma}) \simeq F(b, t, \eta_{\text{th}}) G_{\text{BPSK,NT-GSC}} \gamma^{-\sum_{l=1}^L (t_l + 1)}$$  \hspace{1cm} (22)$$

where $F(b, t, \eta_{\text{th}})$ is given by (10). By comparing (22) with (1), we get the diversity gain for BPSK modulation with NT-GSC ($\eta_{\text{th}}, L$) as $G_d = \sum_{l=1}^L (t_l + 1)$, and the combining gain as

$$G_{\text{BPSK,NT-GSC}} = \left( F(b, t, \eta_{\text{th}}) \frac{1}{2 \sqrt{\pi}} \left( \Gamma(G_d + 0.5) \right) \right)^{-1/G_d}$$  \hspace{1cm} (23)$$

Notice that for a given linear modulation format, its modulation factor $G_{\text{M,NT-GSC}}$ for NT-GSC ($\eta_{\text{th}}, L$) is equal to that of conventional GSC ($L_c, L$) derived in [16], and this factor is independent of $L_c$, $\eta_{\text{th}}$ and channel parameters, but only depends on $L$ and the modulation format used. This result shows that the T-GSC scheme achieves the same diversity gain as the MRC on generalized fading channels, but the combining gain differs by a constant which depends on $\gamma_{\text{th}}, \eta_{\text{th}}, L$ and the channel parameters $\{b_i, t_i\}_{i=1}^L$. We also note that for a large class of modulation formats, the following proposition holds.

**Proposition 2** The diversity gain of NT-GSC ($\eta_{\text{th}}, L$) for each modulation format is equal to that of the $L$-fold MRC and is given by $G_d = \sum_{l=1}^L (t_l + 1)$, and the combining gain is given by

$$G_{\text{NT-GSC}} = \left( F(b, t, \eta_{\text{th}}) G_{\text{M,NT-GSC}} \right)^{-1/G_d}$$  \hspace{1cm} (24)$$

where $G_{\text{M,NT-GSC}}$ is the modulation factor of the considered $M$-ary modulation format with NT-GSC.

Note that the ILT of $s^{-G_d}$ is given by $\gamma^{-G_d+1}/\Gamma(G_d)$. Thus, the PDF $f_{\text{NT-GSC}}(\gamma)$ is approximated by

$$f_{\text{NT-GSC}}(\gamma) \simeq F(b, t, \eta_{\text{th}}) \gamma^{-G_d+1}/\Gamma(G_d)$$  \hspace{1cm} (25)$$

The outage probability is obtained as $P_{\text{G,NT-GSC}}(\gamma_{\text{th}}, \gamma_{\text{ot}}) \simeq (F(b, t, \eta_{\text{th}})/\Gamma(G_d + 1)) (\gamma/\gamma_{\text{ot}})^{-G_d}$.\[2]\

**Proposition 3** The outage diversity gain of NT-GSC ($\eta_{\text{th}}, L$) is equal to that of the $L$-fold MRC and is given by $O_d = G_d = \sum_{l=1}^L (t_l + 1)$, and the outage combining gain is

$$O_{c,\text{NT-GSC}} = (F(b, t, \eta_{\text{th}})/\Gamma(G_d + 1))^{-1/G_d}$$  \hspace{1cm} (26)$$

In summary, for AT- and NT-GSC schemes the outage diversity gain is the same as the error rate diversity gain, that is $O_d = G_d$. Also, the outage combining gains are given by (19) and (26), respectively.

V. NUMERICAL RESULTS AND DISCUSSION

We present some numerical results on the asymptotic average error and outage probabilities of AT- and NT-GSC schemes. In Fig. 1, we show exact and asymptotic SEP results for 16-QAM with NT-GSC (with $\eta_{\text{th}} = \{0, 0.5, 1\}$) in an independent but not necessarily identically distributed (i.n.d.) Nakagami fading channel, where the $m$-parameter is given by $[3, 2.4, 1.8, 1.5]$ from the strongest branch to the weakest one; and the ASNRs decrease by 1.5 dB successively following the same branch order.

The result in Fig. 1 shows that the asymptotic error rate curves for NT-GSC become accurate at high ASNR. We also found that for AT-GSC the asymptotic BER matches the exact one well for high SNR, but this result is omitted here due to the space limitation.

The error rate combining gain $G_{\text{BPSK,AT-GSC}}$ for BPSK AT-GSC over an i.i.d. Nakagami-$m$ channel ($m = 2.1$ and $L = 1, \ldots, 10$) is given Fig. 2. Assuming the same channel
parameters, the outage probability combining gain $O_{c,NT-GSC}$ for NT-GSC is presented in Fig. 3. Fig. 2 shows that as $L$ increases, the combining gain $G_{BPSK,AT-GSC}$ decreases, but it approaches a constant as $L$ becomes large. The combining gain decreases significantly as the SNR threshold $\gamma_{th}$ increases, suggesting that in a practical system threshold $\gamma_{th}$ for AT-GSC should not be set very high to avoid excessive loss of the combining gain. Fig. 3 shows that the outage combining gain $O_{c,NT-GSC}$ increases with $L$, except the case of $\eta_{th} = 1$ (SC diversity), where the combining gain does not depend on $L$. The performance loss compared to the MRC case ($\eta_{th} = 0$) is non-uniform with $\gamma_{th}$. The loss increases fastest in the medium range of $\eta_{th}$ (e.g., from 0.2 to 0.4). Based on results in Figs. 2 and 3, one will be able to change the achieved T-GSC coding gain by adjusting thresholds $\gamma_{th}$ and $\eta_{th}$ for AT-GSC and NT-GSC, respectively, for a given diversity gain $G_d = Lm$. This can be co-designed with the ECC scheme to provide the required total coding gain.

VI. CONCLUSIONS

We have derived the asymptotic error and outage probabilities of AT- and NT-GSC schemes, in terms of the diversity and combining gains, over arbitrary independent diversity fading channels. Our results have demonstrated that the diversity gains of AT-GSC ($\gamma_{th}$, $L$) and NT-GSC ($\eta_{th}$, $L$) are the same as that of $L$-fold MRC given the same channel parameters. Also, the combining gain gaps between different modulation formats and different thresholds values $\gamma_{th}$ and $\eta_{th}$ can be analytically predicted. These results put new insights into the performance of T-GSC receivers and aid the joint design of diversity and channel coding schemes.

REFERENCES


