Digital Convergence, Pricing Strategies and Firms' Profits in the Telecommunication and Entertainment Media Industry

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Abstract—Digital convergence of telecommunication and entertainment media services drove former network monopolists into a prisoner's dilemma forcing them to enter each others markets. In this reciprocal duopoly firms now offer their services in a bundle, commonly known as Triple Play. I investigate whether bundling is indeed a profitable pricing strategy for these services and if it can facilitate market power leverage. I will show that bundle pricing serves as a powerful leverage device for one firm in this industry. This is achieved through a quality sorting effect accruing as the firms wish to shield themselves from increased price competition in the market for bundles. Thereby, one firm emerges as the high-quality, high-profit provider in both markets, whereas the competing firm has to settle for low qualities and profits.

Index Terms—digital convergence, service pricing, bundling, market power leverage

I. INTRODUCTION

DIGITAL Convergence has revolutionized the entire telecommunication and entertaining media industry. In Germany, for example, today former telecommunication monopolist Deutsche Telekom AG offers not only internet and telephony services over her network, but also digital TV (IPTV). On the contrary, TV broadcasters, such as regional cable network monopolists, have also invested in network digitization in order to augment their product portfolio by a telephony and internet service themselves. From a more theoretical perspective, digital convergence has created a prisoner’s dilemma which forced these companies to enter each others markets: Each firm finds it profitable to capture a share of the competitor’s market and hence no one can commit not to do so. Consequently, both firms will enter each others markets, a structure which I refer to as a reciprocal duopoly because monopolies have transformed into duopolies through reciprocal entry. The peculiar feature of this market structure is that each firm originates from a home market where it is considered to have some additional market power over her competitor. Moreover, it is widely believed that each firm’s home service (i.e. telephony and web access for the telecommunication firm and TV for the cable provider) is superior to that of the competitor’s, e.g. in terms of reliability, customer service, transfer speeds, video quality or content. Although the observed product differentiation might also be due to a rather technical nature at first, I argue that there is also a long-lasting economic intuition which will prevail even if technical difficulties should be overcome in a matured industry. More precisely, I consider this industry to be vertically differentiated, such that firms may deliberately choose different service qualities in order to weaken price competition. Each firm will then make use of its home market advantage by offering a service quality higher than that of its competitor, because high-quality products are generally associated with higher profits. In practice providers often employ an additional business strategy which has become known under the buzz word Triple Play. That is, firms often sell their services in a bundle only, although it would be technically possible to offer each service separately. For example, if a customer wants to use the cable company’s telephony service he will also have to sign up for a TV contract - the firm’s home product. Likewise, Deutsche Telekom makes the provision of its IPTV service conditional upon the purchase of its telephony service. I employ a formal game-theoretic model to investigate whether bundling is indeed a profitable pricing strategy and under which circumstances it might be part of an equilibrium strategy. My main result is that bundle pricing serves as a powerful market leverage device through which one firm may carry its home market advantage over to the secondary market. This is achieved through a quality-sorting effect which emerges as firms seek to deterify price competition by specializing on providing either the high- or low-quality service in both markets, thereby leaving the high-end provider better and the low-end competitor worse off than under separate pricing. Furthermore, I can show that this result is robust with respect to different cost functions, as long as the costs of service quality stem from fixed costs mainly. More specifically, I consider a three-stage game in

1I use the term market power very broadly here but will formalize its notion later in the text.
2This seems to be a natural assumption in the context of network industries, where scale economics are rather prominent since each additional customer induces near zero marginal costs, whereas (fixed) costs of e.g. network maintenance are very high. A related argument justifies to focus on a two-firm economy because high sunk costs constitute an insurmountable entry barrier to firms not controlling a network of their own.
which firms first choose their pricing strategy (i.e. bundle- or separate pricing), then decide upon their service quality levels in all markets and finally compete in prices. Thus, this paper relates to two major strands of economic literature: vertical product differentiation and bundling. The basic structure of the game employed here, where firms in a duopoly decide first on quality levels and then on prices, owes much to [34] and [35]. These early contributions derive the consumers’ quality choice from a direct utility function relating different preferences to differences in income. Instead, [37], Section 2.1.1] considers an indirect utility function, which introduces a heterogeneous taste parameter that can be interpreted as the marginal rate of substitution between income and quality. Thus, higher income corresponds to higher tastes for quality and in this vein [37] was able to capture the notion of the earlier papers by the same simple (indirect) utility function which I will employ here. These classical contributions have all affirmed that in equilibrium firms will differentiate their products and that the firm producing the higher quality will earn greater profits than the low-quality provider. As [19] points out, this high-quality advantage is hardly surprising, since all authors have assumed either zero or small and decreasing costs of quality improvement. Although, subsequently some authors ([3], [6], [24], [29]) have confirmed the high quality advantage for specific cost functions, [19] is able to generalize this result to all cost functions which are increasing and convex in the quality chosen, but independent of the output. Furthermore, he shows that if firms choose their quality sequentially in Stackelberg fashion, then the Stackelberg leader will always select the product of higher quality. This result is important for the present context because it explains that the incumbent can exploit its home market advantage by establishing itself as the high-quality provider. The literature on bundling, on the other hand, has at first been primarily concerned with monopolized markets. The seminal papers of [1], [22], [31] and [32] highlighted bundling as a price discrimination device for multi-product monopolists. Later, [38], in an effort to reestablish the previously discredited leverage theory, considered a market structure in which a multi-product firm holds a monopoly in one product market, but faces imperfect competition in the other. With some noticeable exceptions this basic market structure has subsequently been in the focus of attention and by it scholars have shown various means through which bundling may facilitate market power leverage. Among these were entry deterrence ([9], [20], [25], [38]), economics of aggregation ([5]), cost savings ([4], [10], [30]), informational leverage ([13]), reduction of rivals’ innovation incentives ([14]), (tacit) collusion ([33], [36]) and competition mitigation ([8], [11], [38]). In this paper, I show that bundling might also achieve market leverage through a quality sorting mechanism. I deviate from [38]’s standard market structure by assuming a duopoly in both markets. This assumption is per se not new to the bundling literature (e. f. [2], [17], [18], [21], even in the context of the Triple Play industry ([15], [28]). In my model, however, each firm has a home market in which it enjoys a strategic advantage. Moreover, none of the above analyses is concerned with market leverage since each firm’s market power is lessened considerably when its primary market is a duopoly. Also in my setting, ex-ante it is not clear whether bundle pricing may facilitate market leverage because the leverage efforts of either firm counteract. Nevertheless, I can show that the quality sorting mechanism of bundling is powerful enough to serve as a leverage device in vertically differentiated industries, even if market power is rather limited. The remainder of this article is structured as follows. Section II will introduce the formal model which serves as the basis of my analysis. In particular, I will consecutively follow the logic of backwards induction by deriving equilibrium conditions for the separate- and bundle pricing regime first and eventually deducing the equilibrium pricing strategy of each firm. To conclude, I will comment on the results in the light of the leverage theory in Section III.

II. THE MODEL

There are two established firms i = 1, 2 in the industry whose home (or primary) markets are denoted by m = A, B, respectively. I assume that firms have already entered each other’s home markets (reciprocal entry) and offer exactly one service for each market. Firms play a three-stage game: In the first stage, firms decide whether to sell their products as a bundle or separately. I will consider both simultaneous and sequential decision making. In the second stage of the game, firms choose whether they will offer a high- or low-quality service for each market, where quality levels are exogenously given by q1 ≥ q2 > 0. Finally, in the third stage, firms simultaneously set prices p ∈ R+. The solution concept is that of subgame perfectness. It is at the heart of this model for each firm to have a home market in which it can exercise some additional market power over her competitor. A first-mover-advantage seems to capture this very adequately. Indeed, the incumbent has been in the market before and should therefore be able to decide upon his service quality prior to the entrant. In this case, a standard result of the vertical differentiation

3See also [26] for a more elaborate argument. Therein the corresponding direct counterpart of [37]’s indirect utility function is constructed and shown that the underlying preference relation satisfies reflexivity, transitivity, completeness and local nonstasis.

2See [12] for an explicit solution to the model in [37].

1On the contrary, if quality improvement induces an increase of marginal cost (at a higher rate than consumer’s willingness to pay) [23] affirms that the low-quality provider earns greater profits.

5In a nutshell, the leverage hypothesis suggests that a firm with market power in its primary market could use bundling as a device in order to gain an advantage in a secondary market. The hypothesis has for a long time been dismissed on the grounds of the Chicago critique (cf. e.g. [7], [16], [27]), which, however, implicitly assumed a perfectly competitive secondary market and constant returns-to-scale technology.

6These papers have mainly investigated whether pure bundling (offering just the bundle) or mixed bundling (offering the goods as a bundle and individually) will emerge as an equilibrium strategy. Here I consider pure bundling only and will therefore omit further details of this literature.

7More specifically I take the firms’ entry decision as given and sunk, such that exit is prohibitively costly. Thus, I rule out any aspects related to strategic entry deterrence nor will I address the question whether entry should have occurred in the first place.

8The choice of q2 ≻ q1 will be motivated later in the text and is not crucial for the quality sorting effect to hold. In fact, as one will see later, the assumption has been made to ensure existence of subgame equilibria other than the desired.
literature is that the first-mover will choose to provide the high-quality service, because it is associated with the higher revenues, whereas the entrant has to settle for the low-quality, low-revenue service. When pursuing a bundling strategy, however, firms create a joint market on which ex-ante neither firm is regarded to have an advantage. Thus, in order not to forestall any results within the otherwise symmetric framework, only the simultaneous choice of qualities is feasible here. On the contrary, comparing different game structures when switching from a separate pricing to a bundling regime might dilute some of the more subtle effects underlying this transition. A way out of this dilemma - which I will pursue here - is to assume simultaneous quality choice under all regimes. To be able to account for a first-mover advantage under separate pricing anyway, I will simply associate the incumbent firm with the high-quality product exogenously there. I assume that firms’ costs of quality improvement fall on fixed costs only. Yet, notice that this assumption does not neglect the existence of marginal costs per se, but rather suggest that marginal costs are not influenced by a firm’s service quality choice. In particular, consider the following cost function for each service:

\[ C(D_{mi}, q_{mi}) = bD_{mi} + cq_{mi} \]

where \( D_{mi} \) denotes the demand of service \( q_{mi} \), \( b \) is the constant marginal costs and \( c > 0, \alpha > 1 \) are parameters of the fixed cost function, characterizing its scale and elasticity. Clearly, if marginal costs are quality independent, they have no influence on the service quality and merely result in a linear mark-up on prices. Thus, for expostional clarity, I can w.l.o.g. normalize \( b \) to zero. There is a continuum of consumers normalized to mass 100 who have a positive valuation for exactly one service type from each market \( m = A, B \). More specifically, consumers differ in their marginal willingness to pay for quality, \( \theta \), and value a service of quality \( q \) at \( V(\theta, q)^{\theta} q \).

Consequently, consumers with a relatively low \( \theta \) do not value quality enough in order find it reasonable to purchase a rather expensive high-quality service, which consumers with a relatively high \( \theta \) would still find attractive. In contrast to horizontal product differentiation models, however, at equal prices all consumers prefer the service of higher quality. In addition, I allow for the possibility that consumers may have a different taste of quality for different service types, i.e. \( \theta^{A} > 0, \theta^{B} < 0 \).

Moreover, \( \theta \) is uniformly i.i.d. from the unit interval. As a limit case, I assume that tastes are uncorrelated across service types, such that consumers are uniformly distributed on the unit square which is spanned by \( \theta^{A} \) and \( \theta^{B} \).

Finally, in order to be able to isolate the strategic effect of bundling alone, I deliberately neglect any scope economics or consumption dependencies, i.e. complementarity or substitutability across services, such that a consumer’s total valuation is linearly separable his valuation for each service. A separate Pricing Subgame

As a point of departure, let us first investigate the subgame that occurs if both firms choose the separate pricing strategy. Under this regime firms assign a separate price to each of their two services such that consumers can compile an individual service package, possibly containing services of different firms. Clearly, under this regime there is no economic link between the markets which could influence firms’ or consumers’ decisions. Hence, by the home market advantage, the incumbent firm of market \( m \), say \( i \), will provide the high-quality product in \( m \), while the entrant, say \( j \), must content itself with offering the low-quality service here. Consequently, under separate pricing each firm will earn high profits in its home and low profits in its secondary market. Due to the reciprocal market structure, firms cannot transport their home market advantage over to the foreign market and thus both firms will earn identical overall profits. I have assumed a rather general cost function in order to show that my results are robust to variations of its parameters. However, to make the analysis yet tractable, I have to consider discrete quality levels as a sacrifice. If firms were to chose quality levels from a continuous set, they would most naturally chose only those qualities which allow them to operate profitably. Consequently, a minimum feasibility requirement one can make is that the high-quality firm earns positive profits under separate pricing. In this vein, I find a constraint governing the relationship between the parameters of the cost function, on the one hand, and the feasible quality levels, on the other.

**Lemma 1 (Feasibility Constraint).** In all feasible settings the high-quality firm must earn positive profits under a separate pricing regime. This is ensured when

\[ cq_{H}^{-1} < 25 \frac{44}{3} \mu \]

where \( \mu = \frac{q_{L}}{q_{H}} \in (0, \frac{1}{4}) \).

It is important to see that the term \( cq_{H}^{-1} \), which will be in the center of my analysis, may be interpreted as a measure of how prominent cost considerations are in the firms’ quality decision process. In particular, \( cq_{H}^{-1} \) may be seen as a convexity measure of the cost function and \( c \) as the general magnitude of costs. For low values of \( cq_{H}^{-1} \), costs rise only slowly with quality because either convexity is mild or costs are not included in the analysis in any scope.

12I will discuss these latter parameters in more detail shortly.
13I distinguish specific services from service types, where the latter denote all services offered on a particular market, independent of their providers and thus independent of quality.
14By this, I implicitly assume that the market is not covered in equilibrium because there will always be some consumers who do not value a given service at its price. This assumption has mainly been made to avoid case differentiations and is not crucial for the the main implications of this model. Incidentally, it also seems reasonable that there are always some consumers which refrain from buying a certain product.
15I will argue later that uncorrelated tastes are actually a worst-case scenario for the quality sorting effect to occur.
16Of course, on each market the competitively supplied services are demand substitutes.
17I thereby implicitly assume that services from different producers are compatible across markets.
18Notice that firms will never choose to offer services of the same quality in equilibrium because this lack of differentiation would otherwise dissipate all profits in the subsequent Bertrand stage.
19The reader is referred to the appendix for a precise analytical analysis.

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[Image 0x0 to 612x792]
are generally small, or both. Thus, firms will be able to operate profitably here and high-quality providers generally earn more than their low-quality competitors. As $cq^e_H$ increases, cost considerations become more prevalent, until eventually costs are so dominant that no firm finds it profitable to continue business. Consequently, for what follows, it makes sense to cut off the feasible values of $cq^e_H$ at the bound identified by lemma 1.

B. Bundle Pricing Subgame

In contrast to the separate pricing regime, which did not evoke any cross-market effects, bundling creates externality on the other market because both bundles compete for the same customers. Although consumers tastes for quality are uncorrelated such that there is demand in every ‘niche’ of the market, we will see that this externality acts as a quality-sorting mechanism for almost all feasible settings. To fix ideas, set firm 1 as the high- and firm 2 as the low-quality provider in market $A$.\(^\text{20}\)

Then, my main result is that the high-quality incumbent of market $A$ can leverage his quality dominance over to the secondary market, $B$, thus providing the high-quality product for both markets and earning greater profits than under separate pricing. This result is robust for all feasible model parameters. Keeping quality assignments fixed for market $A$, it is now at the core of this paper to investigate the quality choice of firms in market $B$. Certainly, if bundling had no effect on the firms’ quality decision, firm 1 (2) would choose $q_L$ ($q_H$) in equilibrium again. Denote this scenario by $LH$ and notice that it implies equal profits for both firms. However, if bundling will indeed serve as a quality sorting mechanism, then firm 1 should be able to establish itself as the high-quality seller in market $B$, possibly even forcing the other firm into providing the low-quality service in both markets (scenario $HL$). On the other hand, if firm 2 chose $q_H$ in market $B$ as well (scenario $HH$), bundles became closer substitutes, price competition intensified and hence ex-ante it is not clear whether firms’ profits increase in comparison to the separate pricing regime.

| BUNDLE PRICING SUBGAME: QUALITY DECISIONS IN MARKET B |
|-----------------|-----------------|
| $q_H$ | $q_L$ |
| $\Pi_1^b(HH), \Pi_2^b(HH)$ | $\Pi_1^b(HL), \Pi_2^b(HL)$ |
| $\Pi_1^b(LH), \Pi_2^b(LH)$ | $\Pi_1^b(LL), \Pi_2^b(LL)$ |

Similar holds if both firms offer a low service-quality in market $B$ (scenario $LL$). Next, I will determine which of the four scenarios will be an equilibrium of the bundle pricing subgame, which is summarized by table 1. First I give constraints which ensure that a price equilibrium exists in each of the four scenarios.

**Lemma 2 (Price Equilibrium Feasibility Constraints).** Price equilibria exist only if quality levels are sufficiently differentiated. Scenario HL is feasible for $q_H > 1.77 q_L$, scenario HH for $q_H > 2.31 q_L$, scenario LH for $q_H > q_L$, and scenario LL for $q_H > 3.73 q_L$.

The reader may now understand why $q_H > 4 q_L$ is actually an unfavorable assumption, because it ensures the existence of all four scenarios of the bundle pricing subgame. As $q_L$ approaches $q_H$ further, the equilibria where both firms provide the same service quality in market $B$ gradually cease to exist. This is intuitively clear, since services must be sufficiently differentiated in market $A$ if firms fail to distinguish their services in market $B$. Scenarios HL and LH, on the contrary, continue to hold under significantly less service differentiation. Next, I derive conditions for the four relations $\alpha - \delta$ depicted in figure 1, which govern each firm’s quality choice in market $B$, given the choice of its opponent and therefore determine the pure strategy equilibria of the bundle pricing subgame.

**Lemma 3 (Quality Equilibrium Constraints).** The constraints $\alpha - \delta$ can be well approximated by:

- $\Pi_1^b(LH) > \Pi_2^b(LL) \Rightarrow cq^e_H < 29.43 - 27.65 \mu - \mu^e$ \(\text{for } 1 - \mu^e > 0\)
- $\Pi_1^b(HH) > \Pi_2^b(LH) \Rightarrow cq^e_H < 19.74 - 24.26 \mu - \mu^e$ \(\text{for } 1 - \mu^e > 0\)
- $\Pi_1^b(LL) > \Pi_2^b(HL) \Rightarrow cq^e_H < 17.16 - 18.37 \mu - \mu^e$ \(\text{for } 1 - \mu^e > 0\)
- $\Pi_1^b(LH) > \Pi_2^b(LL) \Rightarrow cq^e_H < 6.71 - 10.16 \mu - \mu^e$ \(\text{for } 1 - \mu^e > 0\)

where $\mu = \frac{q_L}{q_H}$.

First, see that the bounds identified by lemma can be uniquely ranked in terms of $cq^e_H$ as $\alpha > \beta > \gamma > \delta$, for all $\mu$ and $e$. Hence, the equilibria of the bundle pricing subgame depend only on the cost considerations given by $cq^e_H$ and are independent of the precise relationship between $q_H$ and $q_L$. Consequently, the results of the present analysis would remain valid if firms chose their quality levels endogenously. Furthermore, notice that $\alpha$ is less restrictive than the feasibility bound identified by lemma 1. Therefore, inequality $\alpha$ holds for all feasible settings and is never binding. For very small values of $cq^e_H$, i.e. $\delta$ is binding, the $HH$ scenario is the only equi-
librium. Here costs have only small impact on the quality decision and thus both firms strive towards offering the high-quality service. When cost considerations become more prominent, however, scenario HL is the robust unique equilibrium outcome of the bundle pricing subgame for all remaining settings. To see this, consider inequality $\beta$ binding at first, such that constraints $\gamma$ and $\delta$ are violated. It is easy to see that now firm 2 will always provide the low quality service in market B, irrespective of firm 1's choice. I call this the low-quality commitment effect. Likewise, because constraints $\alpha$ and $\beta$ hold, firm 1 will provide the high-quality service in market B as a dominant strategy—this is the high-quality commitment effect. Hence, at this intermediate range of $c_{lf}$, scenario HL emerges as an equilibrium in dominant strategies. Yet scenario HL continues to be the unique equilibrium of the subgame even if the high- and low equilibrium effect do not hold simultaneously. For example, scenario HL remains the unique equilibrium if costs drop such that condition $\delta$ (and thus the low-quality commitment effect) is violated, because firm 2 still favors the HL scenario outcome over the HH scenario outcome there. Likewise, if costs rise such that condition $\beta$ (and thus the high-quality commitment effect) is violated, firm 1 would still choose HL over LL.

Corollary 4 (Equilibria of the Bundle Pricing Subgame). If service quality costs are negligible, both firms will provide a high-quality service in market B. If service quality costs are non-negligible, the quality sorting effect of bundling forces firms to specialize on providing either a low- or high service quality in both markets.

C. Equilibrium Pricing Strategies

To conclude, I can now determine firms’ equilibrium pricing strategies in the first stage of the game. First, it is important to understand that firm 1 can force a de-facto bundle pricing regime upon firm 2 by unilaterally deciding to bundle. This feature is peculiar to the two-firm economy assumed here where customers have a positive valuation for only one service of each type.\(^2\) Second, lemma 5 shows that firm 1 indeed has a strong incentive to bundle for all feasible cost functions.

Lemma 5 (Profitability of Bundle Pricing). Under the bundle pricing regime firm 1 earns strictly greater profits than under separate pricing iff scenario HH or HL obtains. In scenario LH both firms earn less than under separate pricing. In scenario HL firm 2 is worse off under bundling compared to separate pricing.

The HL scenario is so appealing to the firms because it allows them to effectively shield themselves from the aggressive Bertrand price competition by segmenting the market into low- and high-quality buyers. By the high-quality advantage, in scenario HL firm 1 is much better and firm 2 worse off than under the separate pricing strategy, of course. Conversely, if under the bundle pricing regime firms would have continued to provide a high-quality service in their home market and a low-quality service in their secondary market (LH scenario), price competition would have intensified compared to the separate pricing strategy because bundles became relatively close substitutes. Thus, both firms would rather price their products separately than choosing LH under a bundle pricing strategy. In those settings for which scenario HH obtains, cost considerations have only little impact on firms’ decisions. In fact, costs are so small that firm 1 cannot prevent firm 2 from participating of the high-quality advantage itself, which is in turn so large that it even recoups the losses incurred from intense price competition. This eventually leads to higher profits than under separate pricing. Thus, firm 1 can achieve market leverage by choosing a bundling strategy in the first stage of the game. Moreover, this outcome is invariant with respect to simultaneous or sequential moves.

Proposition 6 (Pricing Strategies and Leverage). By choosing a bundle pricing strategy, the high-quality provider of market A can leverage its quality dominance over to his secondary market B and thereby earn greater profits than under a separate pricing strategy.

III. Conclusion

The previous analysis was based on a reciprocal duopoly setting with home markets, which is believed to have been constituted through digital convergence of communication and entertainment media services. In practice, these services are offered by former telecommunication and cable monopolists which sell them in a bundle—the so-called Triple Play package. I have investigated whether bundling of services is indeed a profitable pricing strategy, if it can facilitate market power leverage and if it emerges as an equilibrium strategy. To this extend, a three-stage game was considered, where in stage one firms decide whether to offer services in a bundle or separate and in stage two and three decide upon quality and price, respectively. I could show that bundle pricing serves as a powerful leverage device in this industry. This is achieved through a quality sorting effect accruing as the firms wish to shield themselves from increased price competition in the market for bundles. Thereby, one firm emerges as the high-quality, high-profit provider in both markets, whereas the competing firm has to settle for low qualities and profits. The leverage effect is said to be “powerful” because it holds under a number of worst-case assumptions. First, recall that market power is rather limited in the present framework because neither firm holds a monopoly position. Nevertheless, leverage is achieved under all feasible settings. Next, I have restricted the analysis to those settings for which price equilibria exist for all four possible scenarios of the bundle pricing subgame. As I have argued, alternative settings tend to strengthen my results. Finally, I have assumed consumers’ ‘taste’ for quality to be uncorrelated across service types. This, of course, is least appreciated by the quality sorting mechanism because demand is evenly spread out up to every corner of the market. If rather consumers’ taste was positively correlated, price competition under the LH scenario would intensify yet more and thus promote HL as the equilibrium outcome even further.

APPENDIX

A. Proof of Lemma 1

First I derive the price equilibrium under separate pricing:

\(^2\)Separate pricing, on the contrary, requires the consent of both firms.
The consumers indifferent between assigning to the high and low quality service in market A (B) are located at $\tilde{\sigma}_A = \frac{\Delta p_A}{\Delta q_A} = \frac{p_A - p_A^*}{q_A - q_A^*}$ and $\tilde{\sigma}_B = \frac{\Delta p_B}{\Delta q_B} = \frac{p_B - p_B^*}{q_B - q_B^*}$. Likewise, the consumer indifferent between buying the low-quality service and not buying at all in market A (B) satisfies $\tilde{\sigma}_A^{lq} = \frac{p_A^* - p_A}{q_A - q_A^*}$ and $\tilde{\sigma}_B^{lq} = \frac{p_B^* - p_B}{q_B - q_B^*}$. On each market, the profit of the incumbent i (high-quality) and entrant j (low-quality) firm is:

$$\Pi_i = 400 \left( \frac{q_H (q_H - q_L)}{4 (q_H - q_L)^2} - c \right), \quad (1)$$

$$\Pi_j = 100 \left( \frac{q_H q_J (q_H - q_L)}{4 (q_H - q_L)^3} - c \right).$$

In following I derive constraints for $\Pi_i > 0$, which rewrites to $c \frac{q_H}{4} < 400 \left( \frac{4 - \mu}{\mu} \right)^2$. (2)

To be able to compare this result with others obtained later, I will approximate the right hand side of this inequality linearly. To this extend, set $z(\mu) = 400 \left( \frac{4 - \mu}{\mu} \right)^2$ and notice that $\frac{\partial z}{\partial \mu} < 0$. Furthermore $z(0) = 25$ and $z(\frac{1}{4}) = \frac{64}{3} = 21.333$. Thus, the feasibility inequality may be approximated by $c \frac{q_H}{4} < 25 - 25 (25 - \frac{64}{3}) \mu$ and the lemma obtains.

**B. Proof of Lemma 2**

To avoid duplicism, w.l.o.g. I set $q_{A2} = q_H$ and $q_{A2} = q_L$. Of course all results also hold for the symmetric case. Then, the consumers indifferent between firm 1's and 2's bundle lie on the line $\tilde{\sigma}_B^{b} = \frac{\Delta p_B}{\Delta q_B} = \frac{q_H - q_H}{q_B - q_B}$, whereas the consumers indifferent between buying bundle 1 or 2 at all are located along $\tilde{\sigma}_B^{lq} = \frac{p_B^* - p_B}{q_B - q_B}$ and $\tilde{\sigma}_B^{lq} = \frac{p_B^* - p_B}{q_B - q_B}$, respectively.

The locus of consumers indifferent between all three choices (if existent) is $L = (I_A, I_B)$, with $L_A = \tilde{\sigma}_A^{b} q_H - \tilde{\sigma}_A^{lq} q_B$, and $L_B = \tilde{\sigma}_B^{b} q_H - \tilde{\sigma}_B^{lq} q_B$. Scenario LH:

Let us start with the investigation of the LH scenario, because it represents a short-term transition stage between the separate pricing and the bundling regime which would obtain if firms did not alter quality-levels but only prices. Figure 2 shows firms' demand in this scenario. In particular, firms' revenue is given by

$$R_H^L = 100 \left( \int \tilde{\sigma}_B d\theta_A - \int \tilde{\sigma}_B d\theta_A \right) p_B^L \left( HL \right),$$

$$R_L^L = 100 \left( \int \tilde{\sigma}_B d\theta_A - \int \tilde{\sigma}_B d\theta_A \right) p_B^L \left( HL \right),$$

Solving these equations for optimal prices and setting $q_H = \mu q_H$ one finds the unique price equilibrium to be:

$$p_B^L \left( HL \right) = p_B^L \left( HL \right) = q_H \sqrt{2 - \mu + 3 \mu^2 - l - \mu \left( l - 3 \mu \right)} \left( 3 \right)$$

Notice that positive prices exist for all $\mu \in (0,1)$. However, prices decrease when services become less differentated, such that for $\mu = 1$, i.e. $q_H = q_L$, prices eventually drop to zero.

**Fig. 2. Bundle Pricing Subgame: Scenario LH**

**Scenario HL**:

Next, assume the quality-sorting effect holds and firm 1 emerges as the high- and firm 2 the low-quality provider in both markets. Then the demand pattern looks as in figure 3. Firm's revenues and optimal prices are now

$$R_H^L = 100 \left( \int \tilde{\sigma}_B d\theta_A - \int \tilde{\sigma}_B d\theta_A \right) p_B^L \left( HL \right),$$

$$R_L^L = 100 \left( \int \tilde{\sigma}_B d\theta_A - \int \tilde{\sigma}_B d\theta_A \right) p_B^L \left( HL \right),$$

$$p_B^L \left( HL \right) = q_H \left( 2 - 4u + 2u^2 - 9\phi_u - 18\phi_u \right),$$

where

$$\phi(\mu) = \sqrt{\frac{1}{3} \left( 11u^2 - 54u + 27 \right) \left( 17u^2 - 18u + 9 - 3 \sqrt{4u^2 - 6u + 3u^2} \right)}$$

See that $p_B^L \left( HL \right) > p_B^L \left( HL \right)$. However, in order for

The consumers outside option has been normalized to an utility level of zero.

Since each market is considered separately here, this is a standard result (cf. e.g. [24]).

Specifically, I assume $p_B^L \left( HL \right) > p_B^L \left( HL \right)$, because under the alternative $q_H < q_L$, assumption there exists no price equilibrium. Detailed proofs of this and following side notes are available from the author upon request.

24 In particular, firms' revenue

25 The figure assumes $\frac{\Delta p_B^L \left( HL \right)}{\Delta q} \leq 1$, which is the only setting for which a price equilibrium exists.
$p^*_H(\text{HL}) > 0$ it must hold that $\varphi > 0$, which is fulfilled iff

$$\mu > \frac{27 - 12\sqrt{3}}{11} \approx 0.565035483.$$ 

is less restricting and amounts to

$$\mu < \frac{10}{9} + \frac{17}{9\xi} - \frac{\xi}{9} \approx \frac{1}{2.30739},$$

where $\xi = \sqrt{269 + 27\sqrt{106}}$.

Fig. 3. Bundle Pricing Subgame: Scenario HL

Scenarios HH and LL:

Now consider the case where firms do not differentiate their products in market B. Say both firms choose $q_i \in \{q_H, q_L\}$ in market B, then their revenue functions are (cf. figure 4).

$$R^*_B(XX) = 100 \left(1 - L_A - \sum_{i \in A B} \hat{\theta}_{b_i} d\theta_A \right) p^*_H(XX)$$

where $\hat{\theta}_{b_i} = \frac{1}{\mu}$ if $q_i = q_H$ and $\mu$ if $q_i = q_L$.

and $\varphi_X$ is the positive real root of $(81\mu^3 - 162\mu^2 + 33\mu) \varphi_X^5 + (16 + 4\mu + 324\mu^2 - 216 \mu^3) \varphi_X^4 + (108 \mu^3 - 228\mu + 56) \varphi_X^3 + (48\mu^3 - 209\mu^2 + 80) \varphi_X^2 - 20\mu^3 + 56\mu^2 - 52\mu + 16 = 0$

if $\varphi_X = q_{HH}$ or $(81\mu^3 - 162\mu^2 + 33\mu) \varphi_X^5 + (16 + 4\mu + 324\mu^2 - 216 \mu^3) \varphi_X^4 + (108 \mu^3 - 228\mu + 56) \varphi_X^3 + (48\mu^3 - 209\mu^2 + 80) \varphi_X^2 - 20\mu^3 + 56\mu^2 - 52\mu + 16 = 0$

In the LL scenario I must assume that $\mu < \frac{1}{2} - \frac{1}{\sqrt{3}} \approx 0.375205$

in order for $P_{q_H} < P_{q_L}$ to hold: A result, which I have forestalled in my assumptions. In the HH scenario this equilibrium condition

26The figure assumes again that $p^*_H(XX) > p^*_L(XX)$, which is the only setting for which I obtain a price equilibrium.

Fig. 4. Bundle Pricing Subgame: Scenario LL and HH

C. Proof of Lemma 3

I consider each of the relations subsequently.

HH vs. LH

Thus, $\Pi^*_H(\text{HH}) > \Pi^*_H(\text{LH})$ if $c < R^*_H(\text{HH}) - R^*_H(\text{LH})$. Denote that $R^*_H(\text{HH}) - R^*_H(\text{LH}) = q_{LH}(\mu)$, such that the constraint rewrites to $c q_{HH} < \frac{f}{1 - \mu^e}$. Because $\frac{\partial f}{\partial \mu} < 0$, in the interval $\mu e(0, \frac{1}{4})$, $f$ can be well approximated by $c q_{HH} < 19.7445 - 24.2604 \mu$.

HL vs. LL

Analogously, for firm 1 to choose $q_{HL}$ in market B here, the parameters of the cost function must satisfy $c < R^*_H(\text{HL}) - R^*_H(\text{LL})$ or $c q_{HH} < \frac{g(\mu)}{1 - \mu^e}$. Again $\frac{\partial g}{\partial \mu} < 0$, such that I obtain $c q_{HH} < \frac{29.4331 - 27.6484 \mu}{1 - \mu^e}$ as a linear approximation.

LV vs. HH

Here I derive constraints for $\Pi^*_L(\text{HH}) > \Pi^*_L(\text{LH})$. The inequality can be rewritten to $R^*_L(\text{HH}) - R^*_L(\text{LH}) < c$ or $c q_{HH} > \frac{h(\mu)}{1 - \mu^e}$

Here $\frac{\partial h}{\partial \mu} < 0$, $h(0) = 6.706$ and $\frac{h(\frac{1}{4})}{4} = 4.165$, such that I obtain the linear approximation $c q_{HH} > \frac{6.706 - 10.164 \mu}{1 - \mu^e}$.

LV vs LL

Finally consider $\Pi^*_L(\text{LH}) > \Pi^*_L(\text{LL})$. Here, firm 2 will choose the high quality service in market B, if $c < R^*_L(\text{LH}) - R^*_L(\text{LL})$ or $c q_{HH} > \frac{g(\mu)}{1 - \mu^e}$. Since $\frac{\partial g}{\partial \mu} < 0$, $g(0) = 17.157$ and $\frac{g(\frac{1}{4})}{4} = 12.6562$, I obtain $c q_{HH} > \frac{17.157 - 18.368 \mu}{1 - \mu^e}$ as a linear approximation.
Figure 5 shows how well the linear approximations of the functions $z(\mu)$, $f(\mu)$, $g(\mu)$, $h(\mu)$ and $k(\mu)$ are.

Fig. 5. Comparison of exact functions (solid lines) and linear approximations (dashed lines)

**D. Proof of Lemma 5**

Due to the high-quality advantage $\Pi_{H}^{1}(HL) > \Pi^{T} > \Pi_{H}^{1}(HL)$ trivially holds. To be sure that $\Pi_{H}^{1}(HH) > \Pi^{T}$, I set $R_{H}^{1}(HH) - R_{H}^{1} = q_{H} r(\mu)$, where $\frac{\partial r}{\partial \mu} < 0$. $r(0) = 11.939$ and $r(\frac{1}{4}) = 7.2120$, such that the inequality can be linearly approximated by $cq_{H}^{1} < 11.939 - 18.712 \mu$. Notice that this inequality is always fulfilled when HH is an equilibrium. Finally, for $\Pi_{H}^{1}(LH) < \Pi^{T}$, I must show that $q_{H} s(\mu) = R_{H}^{1}(LH) - R_{H}^{1} - R_{H}^{1} < 0$, since costs are identical in both scenarios. Here $\frac{\partial s}{\partial \mu} > 0$, and thus $s^{\text{max}} = s(\frac{1}{4}) = -6.4675$. Hence the inequality holds for all $\mu$.

**References**


