Triangular Self-Convolution Window With Desirable Sidelobe Behaviors for Harmonic Analysis of Power System

He Wen, Zhaosheng Teng, and Siyu Guo

Abstract—Weak harmonic components can easily be obscured by nearby strong harmonics due to the spectral leakage in the power system. To obtain a window suitable for solving the problem, the Triangular self-convolution window (TSCW) is constructed, with the Triangular window being the parent window to take advantages of its narrow major lobe and simple computation. A TSCW-based phase difference correction algorithm for calculating the power system signal parameters, such as frequency, phase, and amplitude, is presented in this paper. The TSCW has a low peak sidelobe level, a high sidelobe rolloff rate, and a simple spectral representation. Leakage errors and harmonic interferences are thus considerably reduced by weighting samples with the TSCW. The TSCW-based phase difference correction algorithm is free of solving high-order equations, and the overall method can easily be implemented in embedded systems. The effectiveness of the method proposed was analyzed by means of computer simulations and practical experiments for multifrequency signals without noise and with quantization noise.

Index Terms—Discrete Fourier transform (DFT), power system harmonic, spectral leakage, Triangular self-convolution window (TSCW).

I. INTRODUCTION

THE WIDESPREAD use of nonlinear loads in power systems is the main cause of harmonic distortion in both power line voltage and current. The existence of these harmonics deteriorates the quality of delivered energy and affects the safe and economical operations of a power system. There is thus a necessity to analyze the harmonic components and reduce them to an acceptable level [1], [2].

Various approaches have been presented to detect harmonics [3]–[5], e.g., least-squared method, Kalman filter, Newton method, Prony method, artificial neural network, and discrete Fourier transform (DFT). Conventionally, the DFT, which can efficiently be calculated using fast Fourier transform (FFT), is the most commonly used technique for power system harmonic analysis because of its simplicity and easy implementation [6], [7]. By using FFT, the estimation of signal harmonics can be very accurate with synchronous sampling [8]. However, it is not easy to obtain synchronous sampling even when using the discrete phase-locked loop technique [9], because the fundamental frequency may vary with time, and interharmonics may exist in the power system. The direct application of the FFT has inherent performance limitations, such as spectral leakage and picket fence effect, due to the asynchronous sampling and the finite sampling record [10], [11]. The undesired effects of the spectral leakage can be minimized by weighting the time samples by a suitable window function [12], and the picket fence effect can be reduced by adopting interpolation algorithms [13].

To suppress the spectral leakage, different windows [14], e.g., the Rectangular window, the Triangular window, the Hanning window, the Hamming window, the Blackman window, the Blackman-Harris window [15], the Rife-Vincent window [16], the Nuttall window [17], the polynomial windows [18], the flat-top window [19], the Rectangular convolution window [20], and the extremely flat-top convoluted windows [21], [22], have been introduced or used in the windowed FFT algorithms. Compared with the direct application of the FFT under asynchronous sampling, the windowed interpolation algorithms can in some degree increase the accuracy of the harmonic analysis [8], [10], i.e., the measurement accuracy of the harmonic frequencies, amplitudes, and phases. Although the windowing domain is thought to be quite mature, some attempts have been made to build new windows [23], [24] or to propose new parametric windows [25]. There are mainly two approaches to build new optimal windows with desirable characteristics:

1) By adjusting coefficients of classical windows. Harris [15] gave a comprehensive review and comparison of the different window functions in his classical paper. Later, Nuttall [17] proposed a family of optimal classical cosine windows with good sidelobe behaviors. Salvatore and Trotta [19] presented two classes of flat-top windows, i.e., the fast-decaying class provides the high-sidelobe rolloff rate, and the minimum-sidelobe class exhibits the low-peak sidelobe level. Cortés et al. [26] proposed the fast-decaying minimum-sidelobe flat-top windows by making a tradeoff between the sidelobe rolloff rate and the peak sidelobe level. There are also some flexible parametric windows, with the best known being the Kaiser–Bessel and the Dolph–Chebyshev windows. These two windows and the other new windows can be derived by following a systematic window parameterization approach proposed by Desbiens and Tremblay [25] to achieve a specific asymptotic decrease rate of the sidelobe.

2) By convolutions of parent windows. This approach produces new windows by performing self-convolution or...
cross convolution of parent windows. This method was first reported by Harris [15] (although not in such an affirmative sense [22]). Xianzhong and Gretsch [20] presented the quasi-synchronous window with a small sidelobe by multiple convolutions of the same or different conventional windows. Similar procedures were also used in [21], [22], and [27]. Pan [27] proposed a new window by convolution of an adjustable window to obtain large sidelobe attenuations. Reljin et al. [21], [22] redesigned the window coefficients of known flat-top windows and proposed the extremely flat-top convoluted windows. By applying self-convolution on these redesigned flat-top windows, the extremely flat-top convoluted windows can be obtained, which are of greater sidelobe attenuation and higher sidelobe rolloff rate compared with parent windows.

To reduce the error of the picket fence effects, the windowed interpolation FFT (WIFFT) algorithms have been employed. For example, Grandke used the Hanning window in WIFFT [12], and Offelli and Petri employed other cosine windows [28]. WIFFT algorithms with dual-spectrum-line [12], [13], [28] or multispectrum line [29]–[31] were presented and applied to enhance the computation precision for harmonic analysis in the power system. However, when the window function is complex, for instance, the high-order combined cosine windows, the WIFFT algorithms involve computationally expensive procedures of solving high-order equations [32]. Some approaches have been proposed to solve the problem. Ta-Peng et al. [33] afforded the optimization of spectrum analysis for signal harmonics by adjusting frequency resolution and frequency shift. Yang and Ding [32] gave a discrete phase difference correction algorithm suitable for all kinds of symmetrical windows. Zhu [34] presented a practical numerical method for the exact calculation of harmonics/interharmonics using an adaptive window width.

However, the performance of the windowed FFT algorithms is affected by the window function [35], [36]. It has been shown that a narrow major lobe is related to good frequency resolution, whereas a low-peak sidelobe level and a high-sidelobe rolloff rate are indicative of a small spectral leakage [37], [38]. As a result, for the multicomponent signals, the harmonic interference and the spectral leakage may not effectively be suppressed by the combined cosine windows or Rectangular convolution windows for their slow sidelobe rolloff rate. In particular, when analyzing narrowband signals, weak harmonic components can easily be obscured by nearby strong harmonics due to the spectral leakage [39], and the use of classic windows may not achieve satisfactory measurement results.

Windows with great sidelobe attenuation and high-sidelobe rolloff rate can sufficiently reduce the spectral leakage and the harmonic interference [22]. An efficient approach to construct such a window is to use self-convolution of a classic window in time domain or, equivalently, self-multiplication of its spectral function. The major lobe width of the convolution window is proportional to the major lobe width of the parent window. As earlier mentioned, the major lobe width of a window is a determinant factor for the resulting frequency resolution or the ability of the window to distinguish close harmonic components. A narrow major lobe width is thus desirable, which means that the major lobe width of the parent window should also be narrow. Therefore, the Triangular window, having a narrow major lobe width and simple functions in both time and frequency domains, is chosen as the parent window, and the Triangular self-convolution window (TSCW) can subsequently be obtained. A TSCW with a reasonable order, or the number of self-convolutions of the Triangular window, gives a much lower peak sidelobe level along with higher sidelobe rolloff rate compared with classic windows. The TSCW can achieve better performance for applications where the spectral leakage is a main problem, such as the weak harmonic analysis of the power system. In addition, the TSCW-based phase difference correction algorithm is presented for calculating the power system signal parameters, i.e., frequency, phase, and amplitude. The proposed method has the major advantages that it is free of solving high-order equations and can get exact solutions, where white noises are present, and some harmonic components are extremely weak, making the method a good choice for real-time applications. These advantages are further enhanced by the efficient implementation of FFT and the preferable sidelobe behaviors of the described TSCW.

The organization of this paper is as follows: The TSCW is defined in Section II. The spectral characteristics of the TSCW are introduced in Section III. The TSCW-based phase difference correction algorithm is presented in Section IV. Experiment results are given in Section V, and a conclusion is given in Section VI.

II. TSCW

A. TSCW Function

The discrete-time Triangular window of length, or width, $M$ is obtained as

$$w_T(m) = \begin{cases} \frac{2m}{M^2}, & \text{for } m = 0, 1, 2, \ldots, \frac{M}{2} - 1 \\ \frac{2M - 4 - 2m}{M^2}, & \text{for } m = \frac{M}{2}, \ldots, M - 2 \\ 0, & \text{for } m = M - 1 \end{cases}$$

(1)

where $M$ is usually set to $2^i$, with $i$ being a natural number, for the sake of implementation of the FFT.

Following the method of convolutions of the parent windows [15], [20]–[22], the $p$th-order TSCW is constructed by the $p - 1$ self-convolutions of $p$ instances of the Triangular window as in (2), shown at the bottom of the next page, where $p$ is the number of the Triangular windows, henceforth called the order of the TSCW, or simply the window order.

From (1), it can be seen that the length of the result sequence of the convolution $w_T(m) * w_T(m)$ is $2M - 1$. Zero padding can be applied on the result sequence to get the second-order TSCW with length of $2M$. Similarly, for the $p$th-order TSCW resulting from $p - 1$ discrete convolutions and padded with $p - 1$ zeros, the $p$th-order TSCW with length of $N = pM$ can be obtained.

B. Frequency Response of the TSCW

The discrete-time Fourier transform (DTFT) of the Triangular window with length $M$ and sampling periods $T$, or simply
be expressed as

\[ W_T(\omega_s) = \frac{2e^{-jM\omega_s/2}}{M} \left( \frac{\sin(M\omega_s/4)}{\sin(\omega_s/2)} \right)^2 \]  

(3)

where \( \omega_s = 2\pi/T_s \) denotes the continuous angular frequency, and \( \omega_s \in [-\pi, \pi] \).

According to the convolution theorem in the frequency domain, the DTFT of the \( p \)th-order TSCW with length of \( N = pM \) is

\[ W_{T-p}(\omega_s) = \frac{2p e^{-jM\omega_s/2}}{M^p} \left( \frac{\sin(M\omega_s/4)}{\sin(\omega_s/2)} \right)^{2p} \]  

(4)

The DFT of the \( p \)th-order TSCW is obtained from the DTFT by evaluating (4) at a discrete set of equally spaced frequencies \( \omega_k = 2k\pi/N \), for \( N = pM \) and \( k = 0, 1, 2, \ldots N - 1 \), and can be written as

\[ W_{T-p}(k) = \frac{2p e^{-jk\pi}}{M^p} \left( \frac{\sin(\pi k/(2p))}{\sin(\pi k/N)} \right)^{2p} \]  

(5)

### III. Spectral Characteristics of the TSCW

#### A. Spectral Characteristics

The major lobe width of a window function is defined as the distance between zero crossings in the window spectrum (in angular frequency). Because the DFT of the \( p \)th-order TSCW is symmetric with respect to the origin, the major lobe width of the \( p \)th-order TSCW is the doubled spectral distance between the origin and the nearest zero point on either side to the origin in the spectrum.

From (5), the following conditions must be satisfied to make \( |W_{T-p}(k)| \) zero:

\[ \pi k/(2p) = d\pi, \quad \text{for} \quad d = \pm 1, \pm 2, \ldots \]  

(6)

When \( k = 2pd \), the conditions in (6) are satisfied, and thus, \( |W_{T-p}(k)| = 0 \). Since \( k \in [0, N-1] \), the nearest zero point to the right of the origin is \( k = 2p \) given by \( d = 1 \), and its spectral distance to the origin is \( 4p\pi/N \). The major lobe width of the \( p \)th-order TSCW with length of \( N = pM \) is thus given by

\[ B_W = 8p\pi/N = 8\pi/M. \]  

(7)

Referring to (7), one can see that the major lobe width of a TSCW is equal to that of the original Triangular window used in the convolutions. If the length of the \( p \)th-order TSCW is fixed, then the value of \( M \) is inversely proportional to the order of the TSCW, since \( N = pM \). The major lobe width of a TSCW with a fixed length of \( N \) is thus determined by the order of the TSCW. The higher the order of the TSCW, the wider the major lobe.

\[ w_{T-p}(n) = \begin{cases} w_T(m) * w_T(m) \cdots * w_T(m), & \text{for } n = 0, 1, 2, \ldots, pM - p - 1, pM - p \\ 0, & \text{for } n = pM - p + 1, pM - p + 2, \ldots, pM - 1 \end{cases} \]  

(2)

Fig. 1 shows the magnitude frequency responses derived from (5) of the first-order TSCWs to the fourth-order TSCWs with a Triangular window length of \( M = 128 \). It can be seen in Fig. 1 that the peak sidelobe level and the sidelobe rolloff rate of a TSCW are proportional to the order of the window. The sidelobe performance of the TSCW is thus rapidly elevated with the increase of its order.

#### B. Comparison With Classic Windows

Figs. 2 and 3 show the magnitude frequency responses of the second- and fourth-order TSCWs with \( M = 128 \), the Hanning and Hamming windows with length of \( N = 256 \), and the Blackman and Blackman–Harris windows with length of \( N = 512 \).

The major lobe width of the TSCW is subject to the length of the Triangular window used. For the second-order TSCW, its
major lobe width is two times that of the Hanning and Hamming windows of the same length, and for the fourth-order TSCW, it is two times that of the Blackman and Blackman–Harris windows of the same length. In addition, with the increase of the window order, the sidelobe performances, such as peak sidelobe level and sidelobe rolloff rate, of the TSCW rapidly improve and are superior to those of the combined cosine windows of the same length.

Comparisons of the major lobe width, the peak sidelobe level, and the sidelobe rolloff rate between the second- and fourth-order TSCWs and the various classical windows are listed in Table I.

From Figs. 1–3 and Table I, when the sequence length is fixed to \( N \), with the increase of the window order, the peak sidelobe level of the TSCW decreases, and the sidelobe rolloff rate rapidly increases. This performance improvement is particularly significant at frequencies satisfying the conditions in (6), i.e., being multiples of the major lobe width. The frequency responses of the TSCW at these frequencies are of the lowest peak sidelobe level, and the average sidelobe level of the TSCW is lower than that of the classical windows with the same length, which means that the TSCW can more effectively suppress the spectral leakage.

IV. PHASE DIFFERENCE CORRECTION ALGORITHM BASED ON THE TSCW

A. Preliminary Considerations

Take the analysis of a multicomponent harmonic signal as an example. The time-domain representation of the sampled signal is

\[
x(i) = \sum_{h=1}^{H} \left\{ A_h \exp \left[ j \left( 2\pi h f_0 i / f_s + \varphi_h \right) \right] \right\},
\]

for \( i = 0, 1, \ldots, +\infty \) \( (8) \)

where \( h \) is the order of a harmonic, \( H \) is the total number of harmonic components, \( f_0 \) is the fundamental frequency, \( f_s \) is the sampling rate, and \( A_h / \varphi_h \) is the amplitude/phase of the \( h \)th harmonic. In the analysis, the harmonic order \( h \) can be regarded as a known quantity, since the upper bound of \( h \), i.e., \( H \), can be estimated, and then all the harmonic components by \( h = 1 \) to \( H \) can be traversed. \( f_s \) is also a preset constant. The task is thus to find out \( f_0 \), \( A_h \), and \( \varphi_h \) from the observed signal \( x(i) \).

\( x(i) \) is then truncated into a finite sequence

\[
x_N(n) = \sum_{h=1}^{H} \left\{ A_h \exp \left[ j \left( 2\pi h f_0 n / f_s + \varphi_h \right) \right] \right\},
\]

for \( n = 0, 1, \ldots, N - 1 \) \( (9) \)

\( x_N \) is subsequently weighted by using the TSCW \( w_{T-p}(n) \) of length \( N \). The spectrum of \( x_N(n) \cdot w_{T-p}(n) \) obtained by using the DFT is given by

\[
X_N(k) = \sum_{h=1}^{H} \left\{ A_h \exp \left[ j \varphi_h \right] W_{T-p}(k - h k_0) \right\},
\]

for \( k = 0, 1, \ldots, N - 1 \) \( (10) \)

where \( k_0 = f_0 N / f_s \) and \( h k_0 \) represents the normalized frequency, i.e., \( h k_0 = h f_0 / \Delta f \). The unit of the normalized frequency is \( \Delta f = f_s / N \), which is actually the frequency resolution under the sampling rate and the signal sequence length. In fact, any window can be used to truncate the signal sequence for DFT computation, but as shall be seen, the characteristics of a specific window can make a difference in the measurement performance.

Now, given the DFT sequence, if synchronous sampling is employed, i.e., \( k - h k_0 \) = 0, then the unknown harmonic parameters \( f_0 \), \( A_h \), and \( \varphi_h \) can easily be retrieved by a direct peak search procedure. For the case as shown in Fig. 4, however, asynchronous sampling is used, and the spectral lines deviate by an unknown displacement of \( \Delta k_h = k_h - h k_0 \) (\( -1 \leq \Delta k_h \leq 1 \)) from the exact peaks corresponding to the harmonic components, which makes the foregoing approach unavailable. Obviously, the parameters cannot accurately be computed from a single spectral line. Interpolation can be used to estimate the parameters [12], [13], [28]–[31], but as mentioned in Section I, this method usually involves solving high-order equations. Another approach, i.e., the phase difference correction algorithm based on the TSCW, is then given.

Let us introduce a second signal sequence \( x_{N+L} \) of the same length as \( x_N \) that is also truncated from the original signal \( x(i) \) but with a time delay of \( L / f_s \), i.e.,

\[
x_{N+L}(n) = \sum_{h=1}^{H} \left\{ A_h \exp \left[ j \left( 2\pi h f_0 (n + L) / f_s + \varphi_h \right) \right] \right\},
\]

for \( n = 0, 1, \ldots, N - 1 \) \( (11) \)

where the range of \( L \) is set as \( 0 < L \leq N \) to keep the correlation of the two sequences.

The DFT of this signal is thus given by

\[
X_{N+L}(k) = \sum_{h=1}^{H} \left\{ A_h \exp \left[ j \left( 2\pi h f_0 L / f_s + \varphi_h \right) \right] W_{T-p}(k - h k_0) \right\},
\]

for \( k = 0, 1, \ldots, N - 1 \) \( (12) \)

By comparing (10) and (12), it can be seen that the time delay causes phase differences in the spectrums. The phase differences are relevant to the harmonic order \( h \), the fundamental
TABLE I
COMPARISONS OF WINDOW CHARACTERISTICS WITH THE LENGTH OF N

<table>
<thead>
<tr>
<th>Window type</th>
<th>Window Length</th>
<th>Major lobe width</th>
<th>Peak side lobe level (dB)</th>
<th>Side lobe roll-off rate (dB/oct)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangular</td>
<td>N</td>
<td>4π/N</td>
<td>-13</td>
<td>6</td>
</tr>
<tr>
<td>Triangle</td>
<td>N</td>
<td>8π/N</td>
<td>-27</td>
<td>12</td>
</tr>
<tr>
<td>Hanning</td>
<td>N</td>
<td>8π/N</td>
<td>-31</td>
<td>18</td>
</tr>
<tr>
<td>Hamming</td>
<td>N</td>
<td>8π/N</td>
<td>-43</td>
<td>6</td>
</tr>
<tr>
<td>Blackman</td>
<td>N</td>
<td>12π/N</td>
<td>-59</td>
<td>18</td>
</tr>
<tr>
<td>Blackman-Harris</td>
<td>N</td>
<td>16π/N</td>
<td>-92</td>
<td>6</td>
</tr>
<tr>
<td>Nuttall window (Max decay 3-term)</td>
<td>N</td>
<td>12π/N</td>
<td>-46</td>
<td>30</td>
</tr>
<tr>
<td>Blackman-Nuttall [40] (Min level 4-term)</td>
<td>N</td>
<td>16π/N</td>
<td>-98</td>
<td>6</td>
</tr>
<tr>
<td>Nuttall window (Max decay 4-term)</td>
<td>N</td>
<td>16π/N</td>
<td>-61</td>
<td>-40</td>
</tr>
<tr>
<td>Rife-Vincent(1) (4-term)</td>
<td>N</td>
<td>16π/N</td>
<td>-61</td>
<td>18</td>
</tr>
<tr>
<td>Rife-Vincent(III) (4-term)</td>
<td>N</td>
<td>16π/N</td>
<td>-74</td>
<td>12</td>
</tr>
<tr>
<td>Fast decaying 5-term flat-top [19]</td>
<td>N</td>
<td>20π/N</td>
<td>-57</td>
<td>42</td>
</tr>
<tr>
<td>Min sidelobe 5-term flat-top [19]</td>
<td>N</td>
<td>20π/N</td>
<td>-90</td>
<td>18</td>
</tr>
<tr>
<td>New Min sidelobe 5-term flat-top [26] (5-term)</td>
<td>N</td>
<td>20π/N</td>
<td>-95</td>
<td>6</td>
</tr>
<tr>
<td>New fast decaying min sidelobe flat-top [26] (5-term)</td>
<td>N</td>
<td>20π/N</td>
<td>-79</td>
<td>30</td>
</tr>
<tr>
<td>The second order TSCW</td>
<td>N=2M</td>
<td>16π/N (8π/M)</td>
<td>-52</td>
<td>24</td>
</tr>
<tr>
<td>The fourth order TSCW</td>
<td>N=4M</td>
<td>32π/N (8π/M)</td>
<td>-104</td>
<td>48</td>
</tr>
</tbody>
</table>

The first term on the right-hand side provides the best approximation of the hth harmonic component when asynchronous sampling is used, whereas the second term stands for the leakage impacts of the other harmonics. Since TSCW has a low peak sidelobe level and a high rolloff rate, the second term is usually negligible. This leaves us with the following equation:

\[ X_N(k_h) = A_h \exp(j\varphi_h)W_{T-p}(k_h - h k_0). \]  \((14)\)

The same logic can be applied in (12), which results in

\[ X_{N+L}(k_h) = A_h \exp\left(j(2\pi h f_0 L/f_s + \varphi_h)\right)W_{T-p}(k_h - h k_0) \]  \((15)\)

and the phases of the kth spectral line of the spectrum of x_N and x_{N+L} are

\[ \varphi_h = \varphi_h + \pi\Delta k_h \]  \((16)\)

\[ \varphi'_h = \varphi_h + \pi\Delta k_h + 2\pi h f_0 L/f_s. \]  \((17)\)

From (16) and (17), the phase difference \(\Delta \varphi_h = \varphi'_h - \varphi_h\) between the kth spectral lines of the spectrums of the sequences x_N and x_{N+L} can be simplified as

\[ \Delta \varphi_h = 2\pi h f_0 L/f_s. \]  \((18)\)

As \(-1 \leq \Delta k_h \leq 1, 2\pi\Delta k_h L/N\) should be in the range of \([-2\pi, 2\pi]\). However, because the periods of \(\varphi'_h\) and \(\varphi_h\) are both \(2\pi\), the \(\Delta \varphi_h\) is probably too big or small to make \(2\pi\Delta k_h L/N\)
out of the range \([-2\pi, 2\pi]\). Hence, \(\Delta \phi_h\) should be adjusted as follows:

\[
\Delta \phi_h = \begin{cases} 
\Delta \phi_h + 2\pi, & \Delta \phi_h - 2\pi k_h L/N < -2\pi \\
\Delta \phi_h - 2\pi, & \Delta \phi_h - 2\pi k_h L/N > 2\pi
\end{cases}
\]

(19)

which will produce a result in the range \(-2\pi \leq 2\pi \Delta k_h L/N \leq 2\pi\).

Then, for \(hk_0 = hf_0/\Delta f\) and \(\Delta k_h = k_h - hk_0\), the displacement term \(\Delta k_h\) can be obtained as

\[
\Delta k_h = k_h - hk_0 = k_h - \Delta \phi_h N/(2\pi L).
\]

(20)

Thus, from (18), the frequency of the \(h\)th harmonic is

\[
f_h = hf_0 = \Delta \phi_h f_s/(2\pi L).
\]

(21)

From (14), the amplitude and phase of the \(h\)th harmonic are respectively given by

\[
A_h = \left|X_N(k_h)\right|/(W_{TSCW}(k_h - hk_0))
\]

(22)

\[
\varphi_h = \arg[X_N(k_h)] - \arg[W_{TSCW}(k_h - hk_0)]
\]

(23)

where \(\left|X_N(k_h)\right|\) and \(\arg[X_N(k_h)]\) are the magnitude and phase of the \(k_h\)th spectral line of the spectrum of \(x_N\), respectively. In addition, \(W_{TSCW}(k_h - hk_0)\) and \(\arg[W_{TSCW}(k_h - hk_0)]\) can be obtained by (5) and (20).

Once \(k_h\) has been determined by a peak search procedure [28], the phase difference \(\Delta \phi_h\) between the \(k_h\)th spectral lines of the spectrums of the sequences \(x_N\) and \(x_{N+L}\) can be obtained. Then, the displacement term \(\Delta k_h\) can be calculated by (20). Finally, the frequency, amplitude, and phase can be determined by (21)-(23), respectively. The steps of the phase difference correction algorithm based on the TSCW are as follows.

Signal truncating: \(x_N\) and \(x_{N+L} \rightarrow\) TSCW weighting: \(x_N \cdot w_{TSCW}\) and \(x_{N+L} \cdot w_{TSCW} \rightarrow\) DFT: \(X_N(k)\) and \(X_{N+L}(k) \rightarrow\) Peak searching: \(k_h \rightarrow\) Phase difference calculating: \(\Delta \phi_h \rightarrow\) Displacement term correcting: \(\Delta k_h \rightarrow\) Parameter estimating: \(A_h, f_h,\) and \(\varphi_h\).

Given the time- and frequency-domain characteristics of the TSCW, the phase difference correction algorithm can then be applied on TSCW-weighted signals to achieve accurate estimation. This algorithm is free of solving high-order equations and is thus advantageous for efficient implementations in embedded systems.

**C. Discussion of the Window Length and Order**

As shown in (7), the major lobe width of the \(p\)th-order TSCW with length \(N\) is \(8p\pi/N\). Therefore, the minimum allowed difference between two adjacent frequency components \(|f_1 - f_2|\) should satisfy the following inequality:

\[
\frac{4\pi}{N} < \frac{2\pi|f_2 - f_1|}{f_s}
\]

(24)

where \(f_s\) is the sampling rate.

For a specific application, for example, a power harmonic analysis application, the right-hand side term of (24) is usually restricted by practical constraints. \(|f_2 - f_1|\) can often be estimated based on the measurement requirements and the nature of the signals under concern, and the sampling rate \(f_s\) is generally subject to the availability of device and the desirable performance–price ratio, etc. Therefore, this term can reasonably be determined independent of the values of \(p\) and \(N\). Thus, (24) is reduced to a constrain that the ratio of the window order and length \(p/N\) must obey.

As mentioned in Section III, the higher the window order \(p\), the better the sidelobe behaviors, and the more sufficiently the spectral leakage is eliminated, which leads to better measurement accuracy. From (24), however, a larger value of \(p\) implies a wider window, which increases the memory overhead and computing time. On the other hand, if a low-order TSCW is used, a small value of window length \(N\) will be sufficient to distinguish the interesting frequency components, which means the computation load can be decreased. However, this is at the cost of analysis accuracy. Obviously, a tradeoff between the desirable accuracy and the real-time capability of harmonic analysis, i.e., between the window order \(p\) and window length \(N\), is necessary.

Take our multifunctional three-phase harmonics ammeter for example. The ammeter measures the fundamental and the second to eleventh harmonic components. \(|f_1 - f_2|\) is set to 50 Hz, which is the fundamental frequency of power systems in China. The sampling rate is set to 2k samples/s, which satisfies the Shannon criterion. Calibration of the device requires that the energy accumulation period be less than 512 ms. Since the sampling period is 0.5 ms, the window length \(N\) should be no more than 1024. This is just an upper bound restriction on the window length. In the application, the adopted TMS320VC5502 DSP chip works at a core clock rate of CCLK = 300 MHz. With this setting, one round of processing six windowed signals of length \(N = 512\) from the six-channel input takes 140 ms, which leaves a sufficient time margin for other operations, such as communication and report preparing. In addition, 12 kB of the 64-kB RAM of the chip is allocated for the signals. If \(N = 512\) is used, then from (24), it can be seen that the maximum allowed order of the TSCW is 4. A measurement precision of 1% is required, and under this constraint, only the fourth-order TSCW is applicable. On the other hand, if \(N = 1024\) is used, then the maximum allowed order is 8, and better precision can be achieved. This window length, however, not only increases the processing time to about 280 ms, leaving much less time for the other operations, but also takes 24 kB of memory, which is rather expensive for a 64-kB RAM chip. The configuration of \(p = 4\) and \(N = 512\) is thus used in our application.

**V. Experimental Results**

**A. Application to Noisyless Signals**

**Comparison With WIFFT Algorithm:** To validate the effectiveness of the presented method, simulations are done in MATLAB. The signal model given in [7], [12], and [28], which has been used in many works to verify the soundness of algorithms, is adopted. The noiseless signal is

\[
x(n) = A_0 + A_1 \sin(2\pi h_1 f_0 n/f_s + \varphi_1) \\
+ A_3 \sin(2\pi h_3 f_0 n/f_s + \varphi_3)
\]

(25)
where $A_0 = 0.2$, $A_1 = 6$, $h_1 = 20.2$, $\varphi_1 = 0.1$, $A_3 = 1$, $h_3 = 60.6$, $\varphi_3 = 0$, $f_0 = 1/2.048$ Hz, and $f_s = 1000$ samples/s.

Take the calculation of the $h_1$ harmonic component for example. As shown in Section IV, the signal in (25) is truncated into two sequences, i.e., $x_N$, which consists of samples $0–2047$, and $x_{N+L}$, which consists of samples $10–2057$. $x_N$ and $x_{N+L}$ are then windowed by using the second-order TSCW. Moreover, the spectrums of the windowed sequences are obtained by using the DFT.

1) The highest peak is found at $k_1 = 21$.
2) The phases of the twenty-first spectral line of the spectrum of $x_N$ and $x_{N+L}$ are $\phi_1 = 0.628836081$ and $\phi'_1 = 1.248564919$. Hence, the phase difference is $\Delta \phi_1 = 0.619728238$, and $2\pi k_1 L/N = 1.288543861$.
3) Formula (20) gives $\Delta k_1 = 0.799999985$.
4) From the spectrum of $x_N$, the magnitude and phase of the $h_1$ harmonic are retrieved at the two first spectral lines as $|X_N(k_1)| = 20.2792264590.5139$ and $\arg[X_N(k_1)] = 0.628836081$, respectively.
5) From (21)–(23), the frequency, amplitude, and phase can be calculated, respectively, as $h_1 f_0 = 9.863281257$, $A_1 = 6.00000069$, and $\varphi_1 = 0.099972$.

Table II illustrates the absolute errors obtained by the second- and fourth-order TSCWs with the length of $N = 2048$. In the same table, the absolute errors obtained by the WIFFT algorithms in [7] and [12] are also listed. Table II shows that the phase difference correction algorithm by using the fourth-order TSCW gives more accurate results than those of [7] and [12].

Reference [28] uses the same signal model as shown in (25). In [28], the signal length $N = 256$ is adopted, and the sampling rate $f_s$ is equal to 256 samples/s. Hence, the signal model used in [28] is

$$x(nT) = A_0 + A_1 \sin(2\pi f_1 nT + \varphi_1) + A_3 \sin(2\pi f_3 nT + \varphi_3)
\quad = 0.2 + 6 \sin(2\pi (20.2)n/256 + \varphi_1)$$

$$+ \sin(2\pi (60.6)n/256).$$

Table III illustrates the absolute errors obtained by different windows with the phase difference correction algorithm in this paper and the WIFFT algorithms in [28]. Table III shows that the accuracy obtained by the WIFFT algorithms with Max decay 4-term in [28] is almost the same as the

<table>
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<tr>
<th>Parameters</th>
<th>Exact Values</th>
<th>$N$</th>
<th>$A_0$</th>
<th>$A_1$</th>
<th>$h_1 f_0$</th>
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<td>7E-3</td>
<td>-2E-3</td>
<td>-6E-4</td>
<td>3E-3</td>
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<td></td>
<td>Hanning [12]</td>
<td>2048</td>
<td>1E-7</td>
<td>1E-8</td>
<td>1E-7</td>
<td>-1E-4</td>
<td>-5E-7</td>
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<td>Phase difference correction algorithm</td>
<td>The second order TSCW ($p=2, M=1024$)</td>
<td>2048</td>
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<td>7E-7</td>
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<td>The fourth order TSCW ($p=4, M=512$)</td>
<td>2048</td>
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<th>$h_1 f_0$</th>
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<tr>
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<td>256</td>
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<td>5E-7</td>
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<td>Rife-Vincent(I) (4-term)</td>
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<td>3E-9</td>
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<td>3E-6</td>
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<td>New fast decaying min sidelobe flat-top (5-term)</td>
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<tr>
<td>The second order TSCW ($p=2, M=128$)</td>
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<tr>
<td>The fourth order TSCW ($p=4, M=64$)</td>
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TABLE IV
SIMULATION RESULTS (ABSOLUTE ERRORS) OF ANALYSIS ON THE POWER SYSTEM SIGNAL

<table>
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<th>3rd</th>
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<td></td>
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<td>0.1 V</td>
<td>12 V</td>
<td>0.1 V</td>
<td>2.7 V</td>
<td>0.05 V</td>
<td>2.1 V</td>
<td>0.3 V</td>
<td>0.6 V</td>
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<td>WIFFT [6] (N=1024)</td>
<td>Hanning</td>
<td>3E-4</td>
<td>2E-3</td>
<td>2E-4</td>
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<td>1E-6</td>
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<tr>
<td></td>
<td>Blackman-Harris</td>
<td>2E-5</td>
<td>1E-4</td>
<td>1E-5</td>
<td>1E-5</td>
<td>9E-7</td>
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<tr>
<td>Phasedifference correction algorithm (N=512)</td>
<td>The fourth order TSCW</td>
<td>6E-6</td>
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<table>
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<tr>
<td>WIFFT [6] (N=1024)</td>
<td>Hanning</td>
<td>4E-4</td>
<td>7E-0</td>
<td>6E-3</td>
<td>1E0</td>
<td>6E-3</td>
<td>-</td>
<td>1E-3</td>
<td>5E-3</td>
<td>2E-3</td>
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<tr>
<td></td>
<td>Blackman-Harris</td>
<td>1E-5</td>
<td>7E-1</td>
<td>1E-3</td>
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<td>2E-4</td>
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<td>The fourth order TSCW</td>
<td>6E-6</td>
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<td>6E-4</td>
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</tr>
</tbody>
</table>

Rife-Vincent (I) (4-term) with the phase difference correction algorithm. It is worth noting that the accuracy obtained by the fourth-order TSCW (N = 256) with the phase difference correction algorithm is higher than those given by the other windows.

Power System Harmonic Analysis: The signal given in [6] is analyzed, whose amplitudes were actually measured in an electric power network. The signal is modeled as

\[ x(n) = \sum_{h=1}^{11} A_h \sin(2\pi f_h n / f_s + \phi_h). \]  (27)

The fundamental frequency \( f_0 \) is 50 Hz, and the sampling rate \( f_s \) is 3000 samples/s. The minimum allowed distance between two adjacent spectral components is \( |2f_0 - f_0| = 50 \). The algorithm presented in this paper only requires that relationship (24) be satisfied, i.e., \( N > 480 \), and a value of 512 is adopted to simplify the FFT computation.

All the parameters are shown in Table IV. The fourth-order TSCW with length of \( N = 4M = 512 \) is used. The simulation results of the proposed method and the results of WIFFT algorithms reported in [6] are given in Table IV. It should be noted that a signal length of 1024 was used in [6].

The simulation results in Table IV show that with the sequence length decreasing, the accuracy of the estimated amplitudes given by the TSCW-based phase difference correction algorithm is almost the same as that of the WIFFT algorithms based on the Hanning and Blackman–Harris windows. Meanwhile, the TSCW-based phase difference correction algorithm gives a higher phase estimation accuracy, satisfying the requirements for the harmonic analysis of the power system.

Furthermore, as shown in Table IV, the fundamental amplitude is about 2400 times those of the second harmonic, and there is another strong component, i.e., the third harmonic, which is present near the weak components. Because of the good sidelobe behaviors of the fourth-order TSCW, the spectral leakage and harmonic interferences are strongly suppressed. Hence, the power system signal parameters, including even-order harmonics and weak components, can be accurately determined.

B. Application to Signals Polluted by Noises

The Blackman window, the Blackman-Harris window, the Rife–Vincent (III) (4-term), and the fourth-order TSCW, all with a length of \( N = 512 \), are used when the signal formulated in (27) is superposed with a zero-mean white noise.

Only the simulation results of the fundamental frequency and the amplitude and phase of the second harmonic are given for clarity. The fundamental frequency is reported because its measurement precision determines the overall accuracy of the amplitude and phase estimation. On the other hand, the amplitude and phase estimations of a very weak harmonic component, i.e., the second harmonic, are used to demonstrate the good sidelobe behaviors of the TSCW. A weak second harmonic is also a common context in practical power systems.

These results are shown in Figs. 5–7.

As shown in Figs. 5–7, the relative errors by using the fourth-order TSCW are the lowest. For signal-to-noise ratios (SNRs) < 40 dB, the effects of the white noise are significant. For SNR > 30 dB, the accuracies achieved by the fourth-order TSCW are higher than those of the other windows. It should be noted that with a high noise level, the harmonic components, including
the weak component, can accurately be detected by using the fourth-order TSCW.

C. Measurement Experiment

A 16-bit A/D converter AD73360 and a 32-bit fixed-point processor TMS320VC5502 are used to develop a high-accuracy multifunctional three-phase harmonic ammeter. The ammeter weights the voltage and current signals by using the fourth-order TSCW with a length of 512, and applies the TSCW-based phase difference correction algorithm to calculate the parameters of the fundamental and the harmonics.

To comply with the real-time requirement on the harmonic measurement, the sampling rate of the A/D converter is set to 2k samples/s, and the core clock rate of the DSP processor is set to 300 MHz. The testing A-phase voltage data are shown in Table V, which demonstrates the high accuracy in real-world applications of the proposed method.

VI. Conclusion

Spectral leakage is always a main problem in applications such as the weak harmonic analysis of power system. To minimize the errors caused by spectral leakage, this paper chooses the Triangular window as the parent window, which has a narrow major lobe width and simple functions in both time and frequency domains, and constructs the TSCW. Compared with the combined cosine windows, the TSCW with a reasonable order has better sidelobe behaviors, which leads to a considerable reduction of leakage error and harmonic interference. Furthermore, the phase difference correction algorithm based on the TSCW has advantages, such as computational efficiency and easiness to implement in embedded systems. The results of the simulations and practical measurement indicate that the proposed method has a high accuracy and good antinoise ability. In addition, the proposed method can be advantageous to those who prefer the FFT methods for weak harmonic analysis in electric power systems without adopting phase-locked looped mechanisms.

Nevertheless, there are many aspects that need to be looked into. For example, although the phase difference correction algorithm based on the TSCW (with $N = 256$ and $N = 512$) has high accuracy, the number of samples $N$ is large and thus may increase the computational burden. In the future, the authors plan to revise the phase difference correction algorithm to lower its requirement on the number of samples $N$ while maintaining comparable harmonic analysis performance.

REFERENCES


