# Comparison of some tuning methods for integrating processes

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***Abstract*— There are several tuning methods available for PI controllers. The Magnitude Optimum Multiple Integration (MOMI) tuning method results in a very good closed-loop response for a large class of process models. The method calculates the PI controller parameters from the process model or from the open-loop or the closed-loop time response of the process. Recently, the MOMI tuning method has been extended for integrating processes. This paper is giving comparison of MOMI tuning method with some other PI controller tuning methods for integrating processes. It is shown that the MOMI method is comparable or even better than some of compared methods.**

## I. Introduction

Apart from standard tuning rules, such as Ziegler-Nichols, Cohen-Coon, Chien-Hrones-Reswick or refined Ziegler-Nichols rules, more sophisticated tuning approaches have been suggested so far. They are usually based on more demanding process identification algorithms [1,2,3,4].

One such method is the magnitude optimum method (MO) [7]. The MO method results in a very good closed-loop response for a large class of process models frequently encountered in the process and chemical industries [6].

Recently, the efficiency of the MO method has been additionally improved by using non-parametric approach in time-domain instead of using explicit parametric identification of the process. The method is based on multiple integrations of process input and output signals and is hence called the Magnitude Optimum Multiple Integration (MOMI) method [7]. The proposed approach uses information from a relatively simple experiment in time-domain while retaining all the advantages of the MO method.

However, deficiency of the MO method (and consequently the MOMI method) is that it is not suitable for integrating processes, since the MO criteria cannot be met with 1-degree-of-freedom (1-DOF) controller structure [9].

Recently it was shown that the MO criteria can be met when using 2-degrees-of-freedom (2-DOF) PI controller structure on integrating processes [9]. The aim of this paper is to compare the proposed MOMI method for integrating processes with some other PI controller tuning methods for integrating processes.

The paper is set out as follows. Section 2 describes the MOMI tuning method for integrating processes. Section 3 compares the proposed method with some other tuning methods for integrating processes. Conclusions are provided in section 4.

## II. MOMI tuning method for integrating processes

Figure 1 shows the process in a closed-loop configuration with a 2-DOF controller, where signals *r*, *u*, *d* and *y* represent a reference, a controller output, an input disturbance and a process output, respectively.

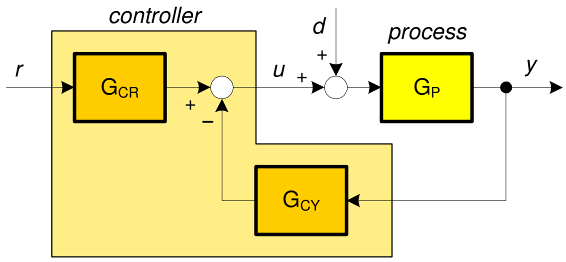


Fig. 1. Typical closed-loop configuration using a 2-DOF controller.

One possible controller design objective is to maintain the closed-loop magnitude (amplitude) as flat and as close to unity over as wide a frequency range as possible [6,7], as shown in Figure 2.

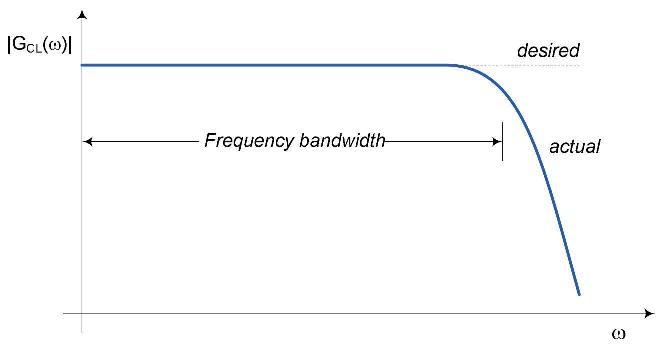


Fig. 2. The closed-loop amplitude (magnitude) response over frequency.

This technique is variously called magnitude optimum

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(MO), modulus optimum or Betragsoptimum, and results in a fast and non-oscillatory closed-loop time response for a large class of process models [6,7].

If $G_{CL}(s)$ is the closed-loop transfer function from the reference ($r$) to the process output ($y$):

$$G_{CL}(s)=\frac{Y(s)}{R(s)}=\frac{G_P(s)G_{CR}(s)}{1+G_P(s)G_{CY}(s)}, \quad (1)$$

the controller is determined in such a way that

$$G_{CL}(0)=1\,. \quad (2)$$

$$\lim_{\omega\to 0}\left[\frac{d^{2k}\left|G_{CL}(j\omega)\right|^2}{d\omega^{2k}}\right]=0\,;\ k=1,2,\cdots,k_{\max} \quad (3)$$

for as many $k$ as possible [6,7].

The first equation (2) is simply fulfilled by using a controller structure containing the integral term (under the condition that the closed-loop response is stable), because the steady-state control error is zero. The number of conditions in (3) that can be satisfied depends on controller order (number of controller parameters).

Let us now calculate the parameters of 2-DOF PI controller, which can be described by the following transfer functions:

$$\begin{aligned} G_{CR}(s)&=bK_P+\frac{K_i}{s} \\ G_{CY}(s)&=K_P+\frac{K_i}{s} \end{aligned}, \quad (4)$$

where $K_P$, $K_i$ and $b$ are proportional gain, integral gain and reference weighting factor, respectively.

The process is given by the following rational transfer function:

$$G_P(s)=\frac{K_{PR}}{s}\frac{1+b_1s+b_2s^2+\cdots+b_ms^m}{1+a_1s+a_2s^2+\cdots+a_ns^n}e^{-sT_{del}}\,. \quad (5)$$

By applying expressions (4) and (5) into expression (1) and by solving equations (3) for $k$=1 and $k$=2 and fixing value $b$=0, the following PI controller parameters are obtained [9]:

$$K_P=\frac{-A_1+\sqrt{A_0A_2}}{A_0A_2-A_1^2}, \quad (6)$$

$$K_i=0.5A_0K_P^2, \quad (7)$$

where symbols $A_0$ to $A_2$ represent the so-called “characteristic areas” of the process [6]:

$$\begin{aligned} A_0&=K_{PR} \\ A_1&=K_{PR}(a_1-b_1+T_{del}) \\ A_2&=K_{PR}\left(b_2-a_2-T_{del}b_1+\frac{T_{del}^2}{2!}\right)+A_1a_1 \end{aligned}\,. \quad (8)$$

Note that “area” $A_0$ equals the steady-state gain of the process.

The name “characteristic areas” is associated with the fact that they can be calculated from a non-parametric process model in time-domain by changing the steady-state of the process and performing multiple integrations of the process input ($u(t)$) and output ($y(t)$) signals [6,7]:

$$\begin{aligned} u_0&=\frac{u(t)-u(0)}{\Delta U} & y_0&=\frac{\dot y(t)-\dot y(0)}{\Delta U} \\ I_{U1}(t)&=\int_0^t u_0(\tau)d\tau & I_{Y1}(t)&=\frac{y(t)-y(0)-\dot y(0)\cdot t}{\Delta U} \\ I_{U2}(t)&=\int_0^t I_{U1}(\tau)d\tau & I_{Y2}(t)&=\int_0^t I_{Y1}(\tau)d\tau \\ &\vdots & &\vdots \end{aligned}, \quad (9)$$

where

$$\Delta U=u(\infty)-u(0)\,. \quad (10)$$

This procedure is relatively easy to perform in practice and does not require explicit identification of the process transfer function parameters (5) [7,8,9]. The areas can be calculated as follows:

$$\begin{aligned} A_0&=y_0(\infty);\ y_1=A_0I_{U1}(t)-I_{Y1}(t) \\ A_1&=y_1(\infty);\ y_2=A_1I_{U1}(t)-A_0I_{U2}(t)+I_{Y2}(t) \\ A_2&=y_2(\infty) \\ y_3&=A_2I_{U1}(t)-A_1I_{U2}(t)+A_0I_{U3}(t)-I_{Y3}(t) \\ A_3&=y_3(\infty) \\ &\vdots \end{aligned}\,. \quad (11)$$

It is assumed that $\dot y(0)=\ddot y(0)=\cdots=0$. Since in practice the integration horizon should be limited, there is no need to wait until $t$=∞. It is enough to integrate until the transient expressions in (10) and (11) die out.

The PI controller tuning proceeds as follows:

- Calculate characteristic areas $A_0$ to $A_2$ from expression (8) or change the steady-state of the process and measure the process input ($u$) and the process output ($y$) signals. The areas $A_0$ to $A_2$ can be calculated from expressions (9) to (11). The initial values of signals can be calculated as mean value of the same signals before the first change of process input signal.
- Calculate the PI controller parameters from expressions (6) and (7). Set reference weighting factor to $b$=0.

Matlab files, which performs tuning of PI controller parameters from the process model or from the process time-responses, are available on-line [8].

Note that only the process gain and two areas are required for the calculation of PI controller parameters. However, the MO criterion results in relatively fast and stable closed-loop responses for different types of process models [9].

## III. Comparison to some other methods

The new method has been compared to some other existing PI controller tuning methods for integrating processes on four process models.

Case 1

The following slightly delayed first-order integrating process is chosen:

$$G_{P1}(s)=\frac{e^{-0.05s}}{s(1+s)}. \tag{12}$$

The characteristic areas are calculated from expression (8):

$$\begin{aligned} A_0 &= 1 \\ A_1 &= 1.05 \\ A_2 &= 1.0513 \end{aligned}. \tag{13}$$

The PI controller parameters are then calculated from expressions (6) and (7):

$$\begin{aligned} K_P &= 0.482 \\ K_I &= 0.116 \\ b &= 0 \end{aligned}. \tag{14}$$

The proposed method has been compared to Åström's tuning method [1] and Taguchi and Araki's method for integrating processes [5]. The Åström's method is based on fixing maximum sensitivity value to either Ms=1.4 (in subsequent text the method will be denoted as *Ms*14) or Ms=2.0 (in subsequent text it will be denoted as *Ms*20). Method proposed by Taguchi and Araki is based on the goal that the overshoot should be less than 20% and that settling time should be the same or less to Chien-Hrones-Reswick method [5]. The tuning rules depend on chosen process model. In subsequent text, the method will be denoted as *TA*.

The calculated PI controller parameters for Ms14 method are:

$$\begin{aligned} K_P &= 0.386 \\ K_I &= 0.06 \\ b &= 0.37 \end{aligned}, \tag{15}$$

and the PI controller parameters for Ms20 are:

$$\begin{aligned} K_P &= 0.733 \\ K_I &= 0.203 \\ b &= 0.71 \end{aligned}. \tag{16}$$

TA method defines the following PI controller parameters:

$$\begin{aligned} K_P &= 5.67 \\ K_I &= 1.26 \\ b &= 0.34 \end{aligned}. \tag{17}$$

The closed-loop responses for reference change and for process input disturbance ($d$=0.1 at t=50s), when using 2-DOF PI controllers tuned by applying mentioned tuning rules, are shown in Figure 3.

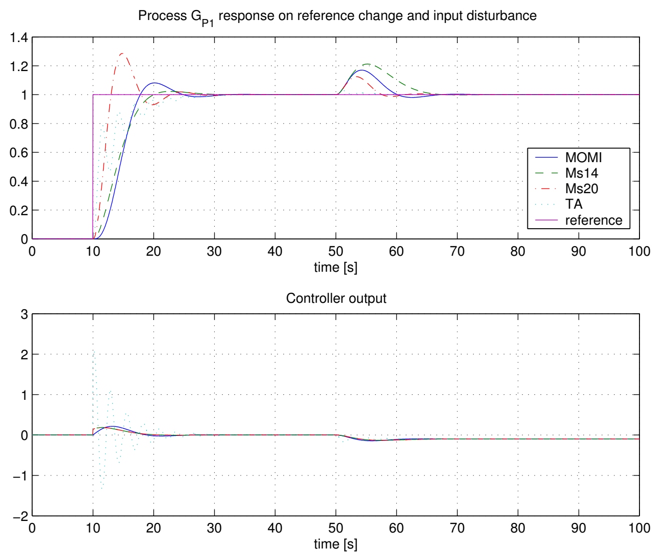


Fig. 3. Closed-loop response of the process $G_{P1}$ with PI controllers tuned by MOMI, Ms14, Ms20 and TA method.

Settling times (within 5% of the final value) for tracking are quite similar for all the tested methods. However, Ms20 results in a relatively high overshoot, while TA method exhibits oscillatory response. Disturbance rejection performance is the best when using TA method (although response is still oscillatory), the MOMI response is somewhere between Ms20 (faster) and Ms14 (slower).

Case 2

The following integrating process with pure time-delay is chosen:

$$G_{P2}(s)=\frac{e^{-s}}{s}. \tag{18}$$

The characteristic areas are calculated from expression (8):

$$\begin{aligned} A_0 &= 1 \\ A_1 &= 1 \quad . \\ A_2 &= 0.5 \end{aligned} \tag{19}$$

The PI controller parameters are calculated from expressions (6) and (7):

$$\begin{aligned} K_P &= 0.586 \\ K_I &= 0.172 \ . \\ b &= 0 \end{aligned} \tag{20}$$

Again the proposed (MOMI) method has been compared to Ms14, Ms20 and TA tuning methods. The calculated PI controller parameters for Ms14 are:

$$\begin{aligned} K_P &= 0.332 \\ K_I &= 0.021 \ , \\ b &= 0.60 \end{aligned} \tag{21}$$

and the PI controller parameters for Ms20 are:

$$\begin{aligned} K_P &= 0.577 \\ K_I &= 0.129 \ . \\ b &= 0.39 \end{aligned} \tag{22}$$

The PI controller parameters, according to TA method, are:

$$\begin{aligned} K_P &= 0.766 \\ K_I &= 0.187 \ . \\ b &= 0.32 \end{aligned} \tag{23}$$

The closed-loop responses for reference change and for input disturbance (*d*=0.1 at t=30s) are shown in Figure 4.

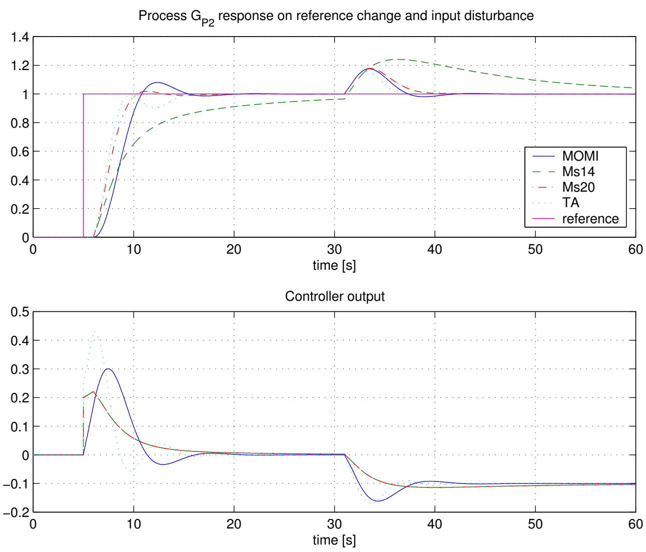


Fig. 4. Closed-loop response of the process $G_{P2}$ with PI controllers tuned by MOMI, Ms14, Ms20 and TA method.

The fastest tracking response is obtained with TA (undershoot) and Ms20 (more stable response). The MOMI method is slightly slower, while Ms14 is very slow with characteristic “long-tail” response. The MOMI method gives similar disturbance rejection performance to TA and Ms20. The slowest response is again obtained with Ms14.

Case 3

The following delayed second-order integrating process is chosen:

$$G_{P3}(s) = \frac{e^{-s}}{s(1+s)^2} \ . \tag{24}$$

The characteristic areas are the following:

$$\begin{aligned} A_0 &= 1 \\ A_1 &= 3 \quad , \\ A_2 &= 5.5 \end{aligned} \tag{25}$$

while the PI controller parameters for MOMI method are:

$$\begin{aligned} K_P &= 0.187 \\ K_I &= 0.018 \ . \\ b &= 0 \end{aligned} \tag{26}$$

The calculated PI controller parameters for Ms14 are:

$$\begin{aligned} K_P &= 0.095 \\ K_I &= 0.0026 \ , \\ b &= 0.60 \end{aligned} \tag{27}$$

and the PI controller parameters for Ms20 are:

$$\begin{aligned} K_P &= 0.154 \\ K_I &= 0.0104 \ . \\ b &= 0.48 \end{aligned} \tag{28}$$

The PI controller parameters for TA method are:

$$\begin{aligned} K_P &= 0.291 \\ K_I &= 0.023 \ . \\ b &= 0.32 \end{aligned} \tag{29}$$

The closed-loop responses for reference change and for input disturbance (*d*=0.1 at 70s) are shown in Figure 5.

The fastest tracking responses are obtained with Ms20, TA and MOMI. The fastest disturbance-rejection performance is obtained with TA method, followed by MOMI and Ms20 method. The slowest response is again obtained with Ms14.

Case 4

The following integrating non-minimum phase process is chosen:

$$G_{P4}(s) = \frac{(1-2s)}{s(1+0.5s)^2}. \tag{30}$$

The characteristic areas are the following:

$$\begin{aligned} A_0 &= 1 \\ A_1 &= 3 \\ A_2 &= 2.75 \end{aligned} . \tag{31}$$

The PI controller parameters for MOMI method are:

$$\begin{aligned} K_P &= 0.215 \\ K_I &= 0.023 \\ b &= 0 \end{aligned} . \tag{32}$$

The calculated PI controller parameters for Ms14 are:

$$\begin{aligned} K_P &= 0.186 \\ K_I &= 0.0074 \\ b &= 0.75 \end{aligned} , \tag{33}$$

And the PI controller parameters for Ms20 are:

$$\begin{aligned} K_P &= 0.287 \\ K_I &= 0.0368 \\ b &= 0.37 \end{aligned} . \tag{34}$$

The TA method is not defined for such process models. The closed-loop responses for reference change and for input disturbance ($d$=0.1 at t=40s) are shown in Figure 6.

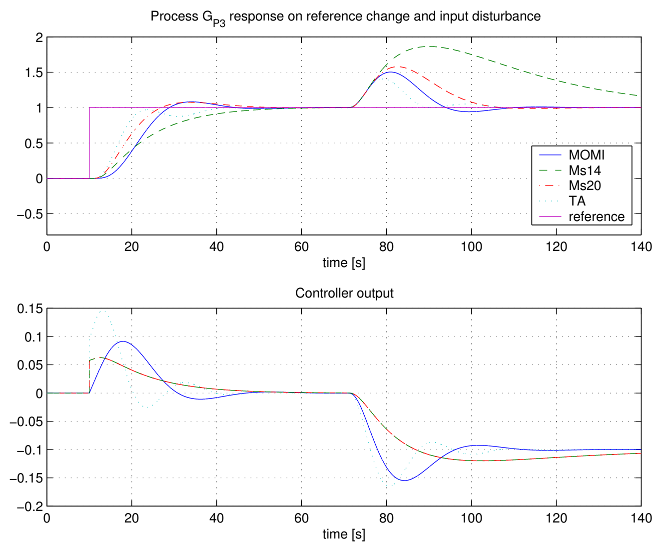


Fig. 5. Closed-loop response of the process $G_{P3}$ with PI controllers tuned by MOMI, Ms14, Ms20 and TA method.

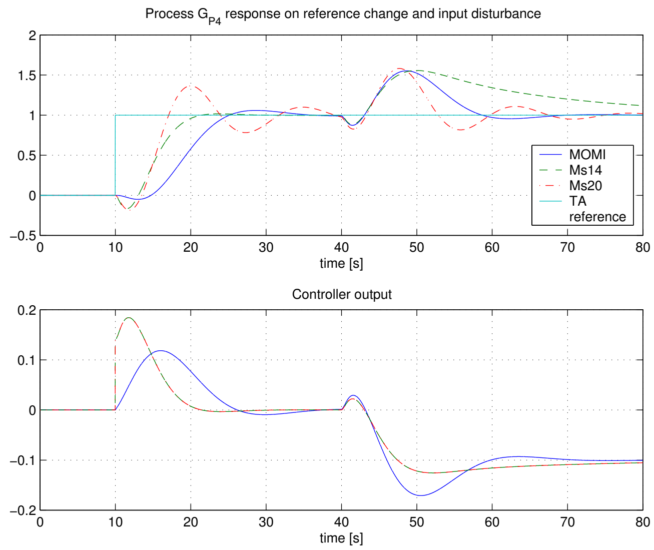


Fig. 6. Closed-loop response of the process $G_{P4}$ with PI controllers tuned by MOMI, Ms14 and Ms20 method.

The fastest tracking response without oscillations is obtained with Ms14, while Ms20 results in a quite oscillatory response. The MOMI method gives quite good disturbance rejection response, while Ms20 results in oscillatory response and Ms14 in a very slow response.

According to results of experiments, it can be concluded that the proposed method gives acceptable closed-loop responses for broad range of process models. The tracking and disturbance rejection responses are stable, relatively fast and with relatively small overshoots.

## IV. Conclusion

The novel MOMI method for integrating processes has been compared to some other existing PI controller methods. It has been shown that the proposed method results in a quite good tracking and disturbance rejection response for different types of process models. On the other hand other compared methods exhibit some variations in performance.

Another advantage of the MOMI method is that it does not require process model. The controller parameters can be derived directly form process time response without any identification procedure. Therefore the proposed method can be considered as very useful tuning method for integrating processes.

Our future research will be concentrated on testing the MOMI method on different process models and reference-weighting parameter $b$.

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