Reliability Measures of a Repairable System with Standby Switching Failures and Reboot Delay

Jyh-Bin Ke, Jyh-Wei Chen and Kuo-Hsiung Wang
Department of Applied Mathematics, National Chung-Hsing University, Taichung, Taiwan
(Received March 2009, accepted December 2009)

Abstract: In this paper, we study the reliability measures of a repairable system with $M$ operating units, $W$ warm standby units, and $R$ repairmen in which switching failures and reboot delay are considered. It is assumed that there is a significant failure probability $q$ in the switching process. Failure times of an operating unit and of a standby unit are assumed to be exponentially distributed with parameters $\lambda$ and $\alpha$, respectively. The delay times of reboot are also assumed to be exponentially distributed with parameter $\beta$. The explicit expressions of the reliability characteristics such as the system reliability, $R_Y(t)$, and the mean time to system failure $(MTTF)$ are derived. Several cases are analyzed graphically to investigate the effects of various system parameters on the $R_Y(t)$ and the $MTTF$. We also perform a sensitivity analysis for the reliability characteristics along with changes in specific values of the system parameters.

Keywords: Mean time to failure, reboot delay, reliability, sensitivity, switching failure.

1. Introduction

The reliability of a system with standbys employs an increasingly important issue in power systems, manufacturing systems, and industrial systems. Maintaining a high or required level of reliability and/or availability is often an essential requisite. There always is the possibility of failures during the switching from standby state to operating state. Therefore, we consider the possibility that the switching device will have a failure probability. Reboot delay takes place in this switching process of a standby unit to an operating unit. A standby unit is called a ‘warm-standby’ if its failure rate is non-zero and is less than the failure rate of an operating unit.

Markovian systems are more natural in real life and may be more appropriate in practice. It is important to study a Markovian repairable system with warm standby units and switching failures and reboot delay of units. We can then develop the explicit expressions of the reliability function and the mean time to system failure $(MTTF)$. In this paper, we attempt to study the reliability and sensitivity measures of a repairable system with warm standby units where standby switching failures and reboot delay are considered. We will determine how system reliability can be improved by providing sufficient spares as standbys or by adding repairmen. The failure of a repairable system is defined as when the number of the operating units is less than $M$ or the system is in the reboot states. This paper differs from previous works in that (i) the reliability problem with standby units has distinct characteristics which are different from the machine repair problem with standby units; (ii) it considers the standby switching failures and reboot delay; and (iii) it performs sensitivity analysis for the reliability with different system parameters.

Wang and Sivazlian [9] examined the reliability characteristics of a multiple-server $(M + W)$-unit system with exponential failure and exponential repair time distributions.

The purpose of this paper is to accomplish three objectives. The first is to provide a Laplace transform method for developing the probability that the system has failed on or before time $t$, from which the reliability characteristics such as the reliability function and the MTTF can be obtained. The second is to perform a parametric investigation which presents numerical results to analyze the effects of the various system parameters on the system reliability and on the MTTF. We also determine how reliability can be improved by providing sufficient spares as standbys or increasing the number of repairmen. The third is to perform a sensitivity analysis in the system reliability and the MTTF along with changes in specific values of the system parameters.

### 1.1. Notations

- $M$: number of operating units.
- $W$: number of warm standby units.
- $n$: number of failed units in the system.
- $\lambda$: failure rate of an operating unit.
- $\alpha$: failure rate of a warm standby unit.
- $\mu$: repair rate of a failed unit.
- $1/\beta$: reboot delay of a standby unit to operating unit.
- $\mu_n$: mean repair rate when there are $n$ failed units in the system.
- $P_{i,j}(t)$: probability that at time $t$ there are $i$ operating units and $j$ warm standby units in the system, where $i = M, M-1$ and $j = 0, 1, 2, ..., W$.
- $s$: Laplace transform variable.
- $P_{i,j}(s)$: Laplace transform of $P_{i,j}(t)$.
- $Y$: time to failure of the system.
- $R_s(t)$: reliability function of the system.
- $MTTF$: mean time to system failure.

### 1.2. Description of the System

We consider a repairable system with $M$ identical units operating simultaneously in parallel, with $W$ warm standbys, $R$ repairmen. In a real life situation, the switching failures and reboot delay of standby units are considered.

It is assumed that each of the operating units fails independently of the state of the others and has an exponential time-to-failure distribution with parameter $\lambda$. If an operating
unit fails, it is immediately replaced by a standby unit if one is available. Let us assume that each of the available standby units fails independently of the state of all the others and has an exponential time-to-failure distribution with parameter $\alpha (0 < \alpha < \lambda)$. Note that the case of $\alpha = 0$ corresponds to cold standby unit and the case of $0 < \alpha < \lambda$ corresponds to warm standby unit. Suppose that the switching time from failure to repair, or from repair to standby state (or operating state if the system is short) is instantaneous. We also assume that there is always the failure possibility $q$ during the switching process from standby state to operating state. After the switching, reboot delay takes place with mean time $1/\beta$ for a standby unit which is exponentially distributed. This kind of system can be modeled a power system with some base generators (e.g. nuclear or fossil turbines) and some spare generators (e.g. hydraulic or gas turbines). Comparing with the base generators, the spare generators are operated with less reboot time (i.e. $\beta >> \lambda, \alpha, \mu$) and lower maintaining cost. In the system, we also assume that no other event can take place during a reboot. (i.e. $\beta >> \lambda, \alpha, \mu$)

Suppose that when a standby unit switches to an operating state successfully, its failure characteristics will be that of an operating unit. After the failed units are repaired, they are treated as standbys. Whenever an operating unit or a standby unit fails, it is immediately sent to $R$ repairmen where it is repaired in the order of breakdowns, with a time-to-repair which is exponentially distributed with parameter $\mu$. Furthermore, the succession of failure times and the succession of repair times are independently distributed random variables. If one operating unit or standby unit is in repair, then arriving failed units have to wait in the queue until the repairman is available. Let us assume that failed units arriving at the repairmen form a single waiting line and are repaired in the order of their breakdowns; i.e., according to the first-come, first-served discipline. Suppose that the repairmen can repair only one failed unit at a time, and that the repair is independent of the failure of the units. Once a unit is repaired, it is as good as new. System reliability is investigated according to the assumptions that system is safe when all $M$ operating units are working.

2. Reliability Analysis of the System

At time $t = 0$, the system has just started operation with no failed units. The reliability function under the exponential failure time and exponential repair time distributions can then be developed through the birth and death process. Let

\[ P_{M,i}(t) = \text{probability of safe states that at time } t \text{ there are } M \text{ operating units and } i \text{ warm standby units in the system, where } i = W, W - 1, \ldots, 0; \]

\[ P_{M-1,i}(t) = \text{probability of reboot states and failure state that at time } t \text{ there are } M - 1 \text{ operating units and } i \text{ warm standby units in the system, where } i = W, W - 1, \ldots, 0. \]

The mean repair rate $\mu_n$ is given by:

\[ \mu_n = \begin{cases} 
  n\mu & \text{if } 1 \leq n \leq \min\{R, W\}; \\
  R\mu & \text{if } R \leq n \leq W; \\
  0 & \text{otherwise.} 
\end{cases} \]

The Laplace transforms of $P_{i,j}(t)$ are defined as:

\[ P_{i,j}^*(s) = \int_0^\infty e^{-st} P_{i,j}(t)dt, \quad i = M, M - 1, \quad j = 0, 1, \ldots, W. \]

Referring to the state-transition-rate diagram in Figure 1 for the reliability system with standby switching failures and reboot delay, it leads to the following Laplace transform
expressions for $P_{i,j}^*(s)$:

$$\begin{align*}
(M \lambda + W \alpha + s)P_{M,w}^*(s) - \mu_1 P_{M,w-1}^*(s) &= P_{M,w}(0), \\
[M \lambda + (W - n)\alpha + \mu_n + s]P_{M,w-n}^*(s) - (W - n + 1)\alpha P_{M,w-n+1}^*(s) &= P_{M,w-n}(0), \quad 1 \leq n \leq W - 1, \\
-\mu_{n+1} P_{M,w-n-1}^*(s) - \beta P_{M-1,w-n+1}^*(s) &= P_{M-1,w-n}(0), \quad 1 \leq n \leq W - 1,
\end{align*}$$

$$\begin{align*}
(M \lambda + \mu_w + s)P_{M,0}^*(s) - \alpha P_{M,1}^*(s) - \beta P_{M-1,1}^*(s) &= P_{M,0}(0), \\
-M \lambda (1-q)P_{M,w}^*(s) + (s+\beta)P_{M-1,w}^*(s) &= P_{M-1,w}(0), \\
-\sum_{k=0}^{n} M \lambda q^{n-k}(1-q) P_{M,w-k}^*(s) + (s+\beta)P_{M-1,w-n}^*(s) &= P_{M-1,w-n}(0), \quad 1 \leq n \leq W - 1,
\end{align*}$$

Equations (1)-(6) can be written in the following matrix form

$$D(s) \cdot Q^*(s) = Q(0),$$

where $D(s)$ is the matrix with dimension $(2W+2) \times (2W+2)$; $Q^*(s)$ is a column vector containing the set of elements $[P_{M,w}^*(s), P_{M,w-1}^*(s), ..., P_{M-1,w}^*(s), P_{M-1,w-1}^*(s), ..., P_{M-1,0}^*(s)]^T$, and $Q(0)$ is a column vector of initial states containing the set of elements $[P_{M,w}(0), P_{M,w-1}(0), ..., P_{M-1,w}(0), P_{M-1,w-1}(0), ..., P_{M-1,0}(0)]^T$, where the symbol $T$ denotes the transpose. At time $t = 0$, we set $Q(0) = [1, 0, 0, ..., 0]^T$. 

![State-transition-rate diagram for reliability system with switching failures and reboot delay](image-url)
Solving Equation (7) in accordance with Cramer's rule, we obtain the expressions for $P^*_{M,L}(s)$

$$P^*_{M,L}(s) = \frac{\det[N_{W-L+1}(s)]}{\det[D(s)]}, \quad L = W, W-1, W-2, \ldots, 0,$$

(8)

where $\det[D(s)]$ denotes the determinant of matrix $D(s)$, $\det[N_{W-L+1}(s)]$ denotes the determinant obtained by replacing the $(W-L+1)$th column in matrix $D(s)$ by the initial vector $Q(0) = [1, 0, 0, \ldots, 0]^T$.

It is too complex to derive the explicit solutions $P^*_{M,L}(s)$ of Equation (8). Therefore, an efficient MAPLE computer program is used to evaluate the solutions $P^*_{M,L}(s)$. We first consider the denominator $\det[D(s)]$ in Equation (8). It is easy to find that $s = 0$ is a root of $\det[D(s)] = 0$. Let $s = -r$ ($r = \text{unknown values}$), then we have $D(-r) = A - rl$, where $A = D(0)$ is an $(2W + 2)\times(2W + 2)$ matrix and $I$ is the identity matrix. Thus Equation (8) becomes

$$(A - rl) \cdot Q^*(s) = Q(0).$$

We set the determinant of the matrix $A - rl = 0$ and find the corresponding distinct eigenvalues $r_j (j \neq 0)$ and $l = 1, 2, \ldots, 2W + 1$ which may be real or complex. Suppose that there are $j$ real distinct eigenvalues (excluding zero), say $r_1, r_2, \ldots, r_j$ and $k$ pairs distinct conjugate complex eigenvalues, say $r_{j+1}, \bar{r}_{j+1}, r_{j+2}, \bar{r}_{j+2}, \ldots, r_{j+k}, \bar{r}_{j+k}$ where $j$ and $k$ satisfy $j + 2k = 2W + 1$. It is noted that $j = 0$ denotes all eigenvalues (excluding 0) are complex, and $k = 0$ represents all eigenvalues are real.

An efficient MAPLE computer program is used to evaluate $\det[N_{W-L+1}(s)]$. Thus, substituting $\det[D(s)]$ and $\det[N_{W-L+1}(s)]$ into Equation (8) yields

$$P^*_{M,L}(s) = \frac{b_{L,1}}{s + r_1} + \ldots + \frac{b_{L,j}}{s + r_j} + \frac{c_{L,1} \cdot s + d_{L,1}}{s^2 + (r_{j+1} + \bar{r}_{j+1}) \cdot s + r_{j+1} \cdot \bar{r}_{j+1}} + \ldots + \frac{c_{L,k} \cdot s + d_{L,k}}{s^2 + (r_{j+k} + \bar{r}_{j+k}) \cdot s + r_{j+k} \cdot \bar{r}_{j+k}}, \quad L = 0, 1, 2, \ldots, W,$$

(10)

where $b_{L,1}, \ldots, b_{L,j}, c_{L,1}, d_{L,1}, \ldots, c_{L,k}, d_{L,k}$ are unknown real numbers.

Let $u_l$ and $v_l$ represent the real part and the imaginary part of complex eigenvalue $r_{j+1}$, respectively. Inverting the Laplace transform in Equation (10), we get the explicit expressions of $P_{M,L}(t)$ given by

$$P_{M,L}(t) = \sum_{l=1}^{j} b_{L,l} \cdot e^{-rt} + \sum_{l=1}^{k} c_{L,l} \cdot e^{-\eta t} \cos(v_l t) + \frac{d_{L,l} - c_{L,l} \cdot u_l}{v_l} e^{-\eta t} \sin(v_l t), \quad L = 0, 1, 2, \ldots, W.$$

(11)

2.1. The Reliability Function $R_Y(t)$

Let $Y$ be the random variable representing the time to failure of the system. Since $P_{M,L}(t)$ are the probabilities that the system is safe on or before time $t$, we have the reliability function given by

$$R_Y(t) = \sum_{L=0}^{W} P_{M,L}(t), \quad t \geq 0.$$

(12)
2.2. The Mean Time to System Failure MTTF

The mean time to system failure MTTF, which is always finite, is defined as

$$MTTF = \lim_{s \to 0} \left[ \int_0^\infty R_T(t) \cdot e^{-st} \, dt \right] = \lim_{s \to 0} \left[ \frac{W}{s} \sum_{L=0}^\infty P^*_{M,L}(s) \right].$$

(13)

Substituting Equation (10) into Equation (13) and setting $s = 0$, we can have

$$MTTF = \sum_{L=0}^W \left[ \sum_{i=1}^L \frac{b_{L,i}}{r_i} + \sum_{i=1}^k \frac{d_{L,i}}{r_{L+i} \cdot \mathcal{F}_{L+i}} \right].$$

(14)

2.3. Sensitivity Analysis for $R_T(t)$ and MTTF

We first perform a sensitivity analysis for changes in the $R_T(t)$ resulting from changes in system parameters $\lambda, \alpha, \mu, q$ and $\beta$. Differentiating Equation (7) with respect to $\lambda$, we obtain

$$\frac{\partial D(s)}{\partial \lambda} \cdot Q^*(s) + D(s) \cdot \frac{\partial Q^*(s)}{\partial \lambda} = 0,$$

or equivalently

$$\frac{\partial Q^*(s)}{\partial \lambda} = -D(s)^{-1} \cdot \frac{\partial D(s)}{\partial \lambda} \cdot Q^*(s).$$

(15)

After inverting the Laplace transform of vector $\frac{\partial Q^*(s)}{\partial \lambda}$, we get $\frac{\partial q(t)}{\partial \lambda}$. Differentiating Equation (12) with respect to $\lambda$ yields

$$\frac{\partial R_T(t)}{\partial \lambda} = \sum_{L=0}^W \frac{\partial P^*_{M,L}(t)}{\partial \lambda},$$

(16)

where $\frac{\partial P^*_{M,L}(t)}{\partial \lambda}$ are some elements of vector $\frac{\partial Q(t)}{\partial \lambda}$. Using the same procedure described above, we can get $\frac{\partial R_T(t)}{\partial \alpha}, \frac{\partial R_T(t)}{\partial \mu}, \frac{\partial R_T(t)}{\partial \beta}$ and $\frac{\partial R_T(t)}{\partial q}$.

Next, we perform a sensitivity analysis of changes in MTTF for a specified system with respect to system parameters $\lambda, \alpha, \mu, q$ and $\beta$. Differentiating Equation (13) with respect to $\lambda$, we obtain

$$\frac{\partial}{\partial \lambda} MTTF = \lim_{s \to 0} \left[ \int_0^\infty \frac{\partial}{\partial \lambda} (R_T(t)) \cdot e^{-st} \, dt \right] = \lim_{s \to 0} \left[ \frac{W}{s} \sum_{L=0}^\infty \frac{\partial}{\partial \lambda} P^*_{M,L}(s) \right].$$

(17)

Using the same procedure, $\frac{\partial MTTF}{\partial \alpha}, \frac{\partial MTTF}{\partial \mu}, \frac{\partial MTTF}{\partial q}$ and $\frac{\partial MTTF}{\partial \beta}$ can be obtained.

3. Numerical Results

In this section we perform numerical experiments to investigate the effect of various parameters on the $R_T(t)$ and the MTTF. In real life situations, the failure rate of an operating unit (or a standby unit) always is relatively small compared to the repair rate. We choose $M = 6$ and fix $\lambda = 0.01, \alpha = 0.005, \mu = 1.0$. The following cases are first analyzed graphically to study the effect of various parameters on the $R_T(t)$.
2.3. Sensitivity Analysis for $\text{MTTF}_R(t)$

The mean time to system failure $\text{MTTF}_R(t)$ can be obtained.

Substituting Equation (10) into Equation (13) and setting $\text{MTTF}_R(t) = \lim \left[ (\text{MTTF}_R(t)) \right]$.

Next, we perform a sensitivity analysis for changes in $r$. We first perform a sensitivity analysis for changes in the $\beta$.

$\text{MTTF}_R(t) = \lim \left[ (\text{MTTF}_R(t)) \right]$. To further investigate the effect of reboot delay rate $\beta$ on the $\text{MTTF}_R(t)$. Figure 3 reveals that $\text{MTTF}_R(t)$ rarely changes for $\beta$, that is, the corresponding reliability curves for four various values of $\beta$ are almost identical. Intuitively, the reboot delay has very little effect on the system reliability. It appears from Figure 4 that $\text{MTTF}_R(t)$ is increased by increasing the number of repairmen $R$. From Figure 5 we can easily observe that $\text{MTTF}_R(t)$ improves dramatically as $W$ increases from 1 to 3.

Next, we study the cross effect of various parameters on the $\text{MTTF}$.

To further investigate the impact of switching failure probability $q$, we vary the parameters $\lambda$ and $q$ simultaneously. As presented in Table 1, we find that the $\text{MTTF}$ with $\lambda = 0.01$ drops from 217,076 to 6,174 as $q$ increases from 0 to 0.1. It should be noted that the effect of $q$ on the $\text{MTTF}$ becomes more significant when $\lambda$ is smaller. Table 2 describes the cross effect of $q$ and the number of operating units $M$ on the $\text{MTTF}$. From Table 2, we observe that the effect of $q$ becomes more significant as $M$ decreases. We note that the results for $q = 0$ are verified by the measures of the classical model of the machine repair problems. Finally, we investigate the cross effect of $q$ and the number of repairmen $R$ on the $\text{MTTF}$. Table 3 shows that the effect of $q$ becomes more significant as $R$ increases. The corresponding reliability curves of systems with various values of $R$ are demonstrated in Figure 4. It is clear that the performance of the system can be enhanced several times with the addition of one repairman when $q = 0$; but this effect decreases when $q$ increases.

Case 1: We fix $W = 3, R = 2, \beta = 2.4$ and choose $q = 0.0, 0.02, 0.04, 0.06, 0.08, 0.1$.

Case 2: We fix $W = 3, R = 2, q = 0.01$ and choose $\beta = 2.4, 24, 240, 2400$.

Case 3: We fix $M = 6, \beta = 2.4, \lambda = 0.01, q = 0.01$ and choose $R = 1, 2, 3$.

Case 4: We fix $R = 2, \beta = 2.4, q = 0.01$ and choose $W = 1, 2, 3$.

The effect of various parameters on $\text{MTTF}_R(t)$ is shown in Figures 2-5. We first examine the effect of switching failure probability $q$ on $\text{MTTF}_R(t)$. It is seen from Figure 2 that $\text{MTTF}_R(t)$ decreases as $q$ increases. Next, we investigate the effect of reboot delay rate $\beta$ on the $\text{MTTF}_R(t)$. Figure 3 reveals that $\text{MTTF}_R(t)$ rarely changes for $\beta$, that is, the corresponding reliability curves for four various values of $\beta$ are almost identical. Intuitively, the reboot delay has very little effect on the system reliability. It appears from Figure 4 that $\text{MTTF}_R(t)$ is increased by increasing the number of repairmen $R$. From Figure 5 we can easily observe that $\text{MTTF}_R(t)$ improves dramatically as $W$ increases from 1 to 3.

Next, we study the cross effect of various parameters on the $\text{MTTF}$. To further investigate the impact of switching failure probability $q$, we vary the parameters $\lambda$ and $q$ simultaneously. As presented in Table 1, we find that the $\text{MTTF}$ with $\lambda = 0.01$ drops from 217,076 to 6,174 as $q$ increases from 0 to 0.1. It should be noted that the effect of $q$ on the $\text{MTTF}$ becomes more significant when $\lambda$ is smaller. Table 2 describes the cross effect of $q$ and the number of operating units $M$ on the $\text{MTTF}$. From Table 2, we observe that the effect of $q$ becomes more significant as $M$ decreases. We note that the results for $q = 0$ are verified by the measures of the classical model of the machine repair problems. Finally, we investigate the cross effect of $q$ and the number of repairmen $R$ on the $\text{MTTF}$. Table 3 shows that the effect of $q$ becomes more significant as $R$ increases. The corresponding reliability curves of systems with various values of $R$ are demonstrated in Figure 4. It is clear that the performance of the system can be enhanced several times with the addition of one repairman when $q = 0$; but this effect decreases when $q$ increases.

![Figure 2. System reliability for different switching failure probabilities.](image-url)
Finally, the sensitivities of the system reliability ($R_t$) with respect to system parameters are shown in Figure 6. We can easily observe from Figure 6 that the biggest impact almost happened at the same time for all system parameters. Moreover, we find that $R$ is the most prominent parameter, $D$ and $q$ are the second and the third in magnitude, and the effect of $P$ is trivial. The sensitivities of reboot delay rate $E$ on the $R_t$ are almost equal to zero. The sensitivities of various values of $q$ on the $R_t$ are shown in Figure 7. We observe that the influence of $q$ on reliability increases as $q$ decreases and the time with maximum sensitivity delays. Moreover, Table 4 shows that the gross sensitivity of various values of $q$ on the MTTF decreases rapidly from 6,536,000 to 233,000 as $q$ increases from 0.01 to 0.08. Figure 8 reveals that the sensitivities of various values of $E$ on the $R_t$ which reverse the sign from positive to negative nearly at the same certain time for four cases. We conclude that a faster reboot increases the system reliability in the interval of $t$, but decreases the system reliability after that time. The gross sensitivity of various values of $E$ on the MTTF as presented in Table 5. We find amazingly that the total effect of various reboot rates on the MTTF is all equal to zeros.

$$R_t(t) \times 10^5$$

$$R_t(t) \times 10^5$$

$$R_t(t) \times 10^4$$
Table 1. The MTTF for different $q$ and $\lambda$. ($M = 6, W = 3, R = 2, \alpha = 0.005, \mu = 1.0, \beta = 2.4$).

<table>
<thead>
<tr>
<th>$q$</th>
<th>$\lambda = 0.01$</th>
<th>$\lambda = 0.02$</th>
<th>$\lambda = 0.03$</th>
<th>$\lambda = 0.05$</th>
<th>$\lambda = 0.1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>217076</td>
<td>18502</td>
<td>4331</td>
<td>722</td>
<td>76</td>
</tr>
<tr>
<td>0.02</td>
<td>70156</td>
<td>9500</td>
<td>2700</td>
<td>537</td>
<td>66</td>
</tr>
<tr>
<td>0.04</td>
<td>30982</td>
<td>5528</td>
<td>1801</td>
<td>411</td>
<td>57</td>
</tr>
<tr>
<td>0.06</td>
<td>16334</td>
<td>3503</td>
<td>1264</td>
<td>322</td>
<td>50</td>
</tr>
<tr>
<td>0.08</td>
<td>9651</td>
<td>2362</td>
<td>922</td>
<td>258</td>
<td>44</td>
</tr>
<tr>
<td>0.10</td>
<td>6174</td>
<td>1670</td>
<td>695</td>
<td>210</td>
<td>39</td>
</tr>
</tbody>
</table>

Table 2. The MTTF for different $q$ and $M (W = 3, R = 2, \lambda = 0.01, \alpha = 0.005, \mu = 1.0, \beta = 2.4)$.

<table>
<thead>
<tr>
<th>$q$</th>
<th>$M = 6$</th>
<th>$M = 5$</th>
<th>$M = 4$</th>
<th>$M = 3$</th>
<th>$M = 2$</th>
<th>$M = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>217076</td>
<td>408187</td>
<td>871136</td>
<td>2247384</td>
<td>7969676</td>
<td>54955590</td>
</tr>
<tr>
<td>0.02</td>
<td>70156</td>
<td>113609</td>
<td>199160</td>
<td>391367</td>
<td>925047</td>
<td>3208194</td>
</tr>
<tr>
<td>0.04</td>
<td>30982</td>
<td>46464</td>
<td>74182</td>
<td>129706</td>
<td>263625</td>
<td>744974</td>
</tr>
<tr>
<td>0.06</td>
<td>16334</td>
<td>23379</td>
<td>35350</td>
<td>57952</td>
<td>108976</td>
<td>279713</td>
</tr>
<tr>
<td>0.08</td>
<td>9651</td>
<td>13386</td>
<td>19530</td>
<td>30736</td>
<td>55120</td>
<td>133788</td>
</tr>
<tr>
<td>0.10</td>
<td>6174</td>
<td>8371</td>
<td>11909</td>
<td>18219</td>
<td>31644</td>
<td>74049</td>
</tr>
</tbody>
</table>

Table 3. The MTTF for different $q$ and $R (M = 6, W = 3, \lambda = 0.01, \alpha = 0.005, \mu = 1.0, \beta = 2.4)$.

<table>
<thead>
<tr>
<th>$q$</th>
<th>$R = 1$</th>
<th>$R = 2$</th>
<th>$R = 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>56160</td>
<td>217076</td>
<td>322338</td>
</tr>
<tr>
<td>0.02</td>
<td>27437</td>
<td>70156</td>
<td>87254</td>
</tr>
<tr>
<td>0.04</td>
<td>15248</td>
<td>30982</td>
<td>35908</td>
</tr>
<tr>
<td>0.06</td>
<td>9293</td>
<td>16334</td>
<td>18234</td>
</tr>
<tr>
<td>0.08</td>
<td>6064</td>
<td>9651</td>
<td>10527</td>
</tr>
<tr>
<td>0.10</td>
<td>4170</td>
<td>6174</td>
<td>6629</td>
</tr>
</tbody>
</table>

Finally, the sensitivities of the system reliability $R_s(t)$ with respect to system parameters $\lambda, \alpha, \mu, q$ and $\beta$ are shown in Figure 6. We can easily observe from Figure 6 that the biggest impact almost happened at the same time for all system parameters. Moreover, we find that $\lambda$ is the most prominent parameter, $\alpha$ and $q$ are the second and the third in magnitude, and the effect of $\mu$ is trivial. The sensitivities of reboot delay rate $\beta$ on the $R_s(t)$ are almost equal to zero. The sensitivities of various values of $q$ on the $R_s(t)$ are shown in Figure 7. We observe that the influence of $q$ on reliability $R_s(t)$ increases as $q$ decreases and the time with maximum sensitivity delays. Moreover, Table 4 shows that the gross sensitivity of various values of $q$ on the MTTF decreases rapidly from $-6,536,000$ to $-233,000$ as $q$ increases from 0.01 to 0.08. Figure 8 reveals that the sensitivities of various values of $\beta$ on the $R_s(t)$ which reverse the sign from positive to negative nearly at the same certain time $t = 120,000$ for four cases. We conclude that a faster reboot increases the system reliability in the interval of $t \geq t_1$, but decreases the system reliability after that time. The gross sensitivity of various values of $\beta$ on the MTTF as presented in Table 5. We find amazingly that the total effect of various reboot rates on the MTTF is all equal to zeros.
Figure 6. Sensitivity of system reliability with respect to system parameters.

Figure 7. Sensitivity of system reliability with respect to $q$.

Table 4. Sensitivity analysis for MTTF with respect to $q$ ($q_{0.01}, 0.02, 0.04, 0.06, 0.08$).

<table>
<thead>
<tr>
<th>$q$</th>
<th>$\frac{\partial MTTF}{\partial q}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>$-6,536,000$</td>
</tr>
<tr>
<td>0.02</td>
<td>$-3,300,000$</td>
</tr>
<tr>
<td>0.04</td>
<td>$-1,107,000$</td>
</tr>
<tr>
<td>0.06</td>
<td>$-470,000$</td>
</tr>
<tr>
<td>0.08</td>
<td>$-233,000$</td>
</tr>
</tbody>
</table>
Figure 8. Sensitivity of system reliability with respect to $\beta$.

Table 5. Sensitivity analysis for $MTTF$ with respect to $\beta$ ($\beta = 2.4, 24, 240, 2400$).

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$\frac{\partial MTTF}{\partial \beta}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.4</td>
<td>0</td>
</tr>
<tr>
<td>3.6</td>
<td>0</td>
</tr>
<tr>
<td>4.8</td>
<td>0</td>
</tr>
<tr>
<td>6.0</td>
<td>0</td>
</tr>
</tbody>
</table>

4. Conclusions

In this paper, we study the reliability and sensitivity measures of a repairable system with standby switching failures and reboot delay. The numerical results indicate that the performances of this system are quite different from those of a system without considering switching failure and reboot delay. We provide the expressions for the system reliability and the $MTTF$. Traditionally, increasing the number of repairmen $R$ and number of warm standby units $W$ can greatly improve the system performance but this effect decays a lot when switching failure are considered. Sensitivity analysis of the system parameters on the system reliability are also studied. It is noted that the impacts of the system parameters are different in magnitude compared to the case when switching failure and reboot delay are not considered. Especially, we find that the reboot delay parameter $\beta$ only affect the reliability but not $MTTF$ of the system.

References


Authors' Biographies:

Jyh-Bin Ke is an Associate Professor of Applied Mathematics at National Chung-Hsing University, Taiwan. He received his MS and Ph.D. in Civil Engineering from the University of California, Berkeley, USA. His areas of research include queuing theory, reliability and stochastic modeling. His publications appeared in Computers and Operational Research, Applied Mathematical Modeling, Mathematical Methods of Operations Research, Applied Mathematics and Computation, International Journal of Advanced manufacturing Technology, Physica A and others.

Jyh-Wei Chen received his MS degree from the department of Applied Mathematic at national Chung-Hsing University Taiwan. His research interests are in statistics and queuing theory.