PSO-based Multiuser Detectors for High-Order Modulation DS/CDMA Systems under Spatial and Multipath Diversities

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Abstract In the present study, the uplink direct-sequence code-division multiple access (DS-CDMA) multiuser detection (MuD) problem is studied from a heuristic perspective called particle swarm optimization (PSO). The motivation to use a heuristic approach is due to the NP-complexity of the wireless multiuser detection optimization problem. The challenge is to solve the associated hard complexity problem in a polynomial time with assurance of suitable data detection performance. Previous studies have indicated that the application of the heuristic search algorithm to several wireless optimization problems can achieve excellent performance-complexity trade-offs. Regarding different system improvements for future technologies, such as high-order modulation and diversity exploitation, a complete parameter optimization procedure for PSO applied to the MuD problem is provided herein, representing the major contribution to this work. Furthermore, the PSO-MuD performance is analyzed through Monte-Carlo simulations. Simulation results show that, after convergence, the performance reached by the PSO-MuD is much better than that shown by the conventional detector (CD), and somewhat closer to that given by the single-user bound (SuB). The flat Rayleigh channel is initially considered, but the results are further extended to the diversity of space-time wireless channels.

Keywords: Heuristic swarm intelligence, PSO algorithm, DS-CDMA, Multiple access communication network, Multiuser detection, Input parameter optimization, Computational complexity.


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1 Introduction

In a DS-CDMA system, a conventional detector may not provide by itself a desirable quality of service, once the system capacity is strongly affected by multiple access interference (MAI). The DS-CDMA system capacity in multipath channels is limited mainly by MAI, self-interference (SI), near-far ratio (NFR), and fading. The conventional Rake receiver explores path diversity in order to reduce the fading effect, but it is not able to mitigate neither the MAI nor the near-far effects Moshavi (1996), Verdu (1998). In this context, multiuser detection is emerged as a solution to overcome the MAI Verdu (1998). The best performance is acquired by the optimum multiuser detection (OMuD) based on the log-likelihood function (LLF) Verdu (1998). In Verdu (1989), it has been demonstrated that the multiuser detection problem appears to be non-deterministic polynomial-time hard (NP-hard). After the Verdu’s revolutionary study, a great variety of sub-optimal approaches have been proposed: from linear multiuser detectors Verdu (1998), Castoldí (2002) to heuristic multiuser detectors Juntti et al. (1997), Ergün and Hacioglu (2000).

Alternatives to the OMuD within the class of linear multiuser detectors include the decorrelator Verdu (1986) and the MMSE detector Poor and Verdu (1997). Besides, the classical non-linear multiuser detectors include the subtractive interference cancellation (IC) MuD Patel and Holtzman (1994) and the zero-forcing decision feedback (ZFDF) Duel-Hallen (1995). In spite of the relatively low complexity, the drawback of (non-)linear, ZFDF, and hybrid cancellation sub-optimal MuDs is the failure in approaching the ML performance in realistic channel and system scenarios.

More recently, heuristic methods have been proposed for solving the MuD problem and obtaining near-ML performance at cost of polynomial computational complexity Ergün and Hacioglu (2000), Abraão et al. (2009). The motivation to use heuristic search algorithms is due to the nature of the NP-complexity that poses the multiuser detection and other optimization problems, frequently found in wireless communication systems. Hence, from a practical engineering point of view, the challenge is to obtain good performances, suitable for solving those hard complexity problems in polynomial time, since the multiuser detection must be performed online, on a symbol-time basis, by supporting multimedia applications (high data rate) under limited computational power resources and aiming at the solution implementation with the use of modern digital signal processing (DSP) platforms.

Furthermore, the heuristic input parameter optimization procedures, based on generalized scenarios, are of paramount importance to fast and on fly applications, such as for the problem treated herein, i.e., this work proposes to develop a methodology for PSO input parameter optimization applied to DS-CDMA multiuser detection problem in generalized system operation and channel fading scenarios.

Previous results have indicated that the application of the heuristic search algorithm to several wireless optimization problems can achieve excellent performance-complexity trade-offs, including the use of genetic algorithm (GA), evolutionary programming (EP), particle swarm optimization (PSO), and local search (LS). In particular, in the context of DS-CDMA multiuser detection optimization problem, the most employed heuris-
tic multiuser detection (HEUR-MuD) methods are as follows: evolutionary programming (EP), genetic algorithm (GA) Ergün and Hacioglu (2000), Ciriaci et al. (2006), particle swarm optimization (PSO) Abraão et al. (2011), Abraão et al. (2010), Khan et al. (2006), Zhao et al. (2006), Oliveira et al. (2006), and ant colony optimization (ACO) Xu et al. (2007), Marinello Filho et al. (2012); sometimes, the deterministic local search (LS) methods Lim and Venkatesh (2003), Oliveira et al. (2009), which have been shown to present a very attractive performance-complexity trade-off for low-order modulations and the number of transmit and receive antennas in multiple-input-multiple output (MIMO) systems Abraão et al. (2009), are also included in this classification.

Nevertheless, there are few works dealing with quasi-optimum multiuser detectors in complex and realistic wireless communication system configurations. Higher-order modulation HEUR-MuD in SISO or MIMO systems have been previously addressed Khan et al. (2006) Zhao et al. (2006), Oliveira et al. (2008). In Oliveira et al. (2008), swarm intelligence was applied to near-optimum asynchronous DS-CDMA multiuser detection problem using 16-QAM modulation and SISO multipath fading channels. The previous reports for the heuristic multiuser detection problem Oliveira et al. (2006), Abraão et al. (2009) have suggested that evolutionary algorithms and particle swarm optimization present similar performance-complexity tradeoffs; furthermore, they have indicated that simple local search heuristic optimization is enough to solve the MuD problem using low-order modulation and a reduced number of antennas Oliveira et al. (2009). However, for higher-order modulation formats, the LS-MuD does not achieve good performances due to a lack of search diversity, whereas the PSO-MuD has been shown to be more efficient for solving the optimization problem under M-QAM modulation Oliveira et al. (2008), Abraão et al. (2010).

Recent works applying PSO to MuD usually assume standard values for PSO input parameters, which are not necessarily optimal in the sense of performance-complexity, such as Soo et al. (2007), or optimized values obtained only for a specific system or channel scenario, such as Oliveira et al. (2006) for a flat Rayleigh channel, Oliveira et al. (2008) for multipath fading channels and higher-order modulation, or Abraão et al. (2009), Abraão et al. (2010) for multicarrier CDMA systems as well.

In the present work, heuristic PSO multiuser detectors (PSO-MuD) with BPSK, QPSK, and 16-QAM modulation schemes are extensively characterized by employing spatial diversity exploration, i.e., single-input single/multiple output (SISO/SIMO) flat Rayleigh channels, as well as frequency selective (multipath) Rayleigh channels. This work is an extension of previous authors’ work Abraão et al. (2010). For this class of problem, a single-objective optimization criterion (SOO) and a discrete version of the PSO algorithm are adopted. Extensive simulations are carried out in order to obtain optimized input parameters for the PSO-MuD algorithm. After convergence, the performance reached by the PSO-MuD with optimized input parameters is much better than that shown by the conventional detector (CD), and somewhat closer to that given by the single user bound (SuB), with the advantage of the computational complexity being substantially lower than the optimum multiuser detection (OMuD) and not excessively higher than the CD complexity.

This paper provides systematic and complete input parameter optimization for the PSO-MuD applied to DS-CDMA systems using Rayleigh fading channels and BPSK, QPSK, and 16-QAM modulation formats, and it is organized in the following way: Section 2 describes the system model, including DS-CDMA, OMuD, and PSO-MuD. The PSO input parameter optimization is presented in Section 3, whereas Section 4 gives representative performance results, obtained through the Monte Carlo simulation (MCS) technique. Finally, Section 5 summarizes the main conclusions of this work.

2 System model

In this Section, a single-cell asynchronous multiple access DS-CDMA system model is described for Rayleigh channels, considering different modulation schemes, such as binary/quadrature phase shift keying (BPSK/QPSK), 16-quadrature amplitude modulation (16-QAM), and single or multiple antennas at the base-station receiver. After describing the conventional detection approach with a maximum ratio combining (MRC) rule, the OMuD and the PSO-MuD are examined. The model is generic enough to allow modeling the effect of additive white Gaussian noise (AWGN) and Rayleigh flat channels, as well as of other modulation formats and single-antenna receivers.

2.1 DS-CDMA

The baseband transmitted signal of the k-th user is described as Proakis (1989)

\[ s_k(t) = \sqrt{\frac{E_k}{T}} \sum_{i=-\infty}^{\infty} d_k^{(i)} g_k(t - iT), \]  

(1)

where \( E_k \) is the symbol energy, and \( T \) is the symbol duration. Each symbol \( d_k^{(i)}, k = 1, \ldots, K \) is taken independently and with an equal probability from the complex alphabet set \( \mathcal{A} \) of cardinality \( M = 2^m \) in a squared constellation, i.e., \( d_k^{(i)} \in \mathcal{A} \subseteq \mathbb{C} \), where \( \mathbb{C} \) is the set of complex numbers. Fig. 1 shows the modulation formats considered, whereas Fig. 2 sketches \( K \) baseband DS-CDMA transmitters.

The normalized spreading sequence for the k-th user is given by

\[ g_k(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} a_k(n)p(t - nT_c), \quad 0 \leq t \leq T, \]  

(2)
where \( a_k(n) \) is a random sequence with \( N \) chips accepting the values \( \{\pm 1\} \), \( p(t) \) is the pulse shaping, assumed to be rectangular with unitary amplitude and duration \( T_c \), with \( T_c \) being the chip interval. The processing gain is given by \( N = T/T_c \).

The equivalent baseband received signal at the \( q \)-th receive antenna, \( q = 1, 2, \ldots, Q \), containing \( I \) symbols for each user in the multipath fading channel, can be expressed by

\[
r_q(t) = \sum_{i=0}^{I-1} \sum_{k=1}^{K} \sum_{\ell=1}^{L} A_k d_i^{(q)} g_k(t - nT - \tau_q, k, \ell) h_q^{(i)}(t) + \eta_q(t),
\]

(3)

with \( A_k = \sqrt{\frac{\xi_k}{T_c}} \), \( L \) being the number of channel paths, admitted as equal for all \( K \) users, \( \tau_q, k, \ell \) is the total delay\(^2\) for the signal of the \( k \)-th user, the \( \ell \)-th path at the \( q \)-th receive antenna, \( c_{\tau_q, k, \ell} \) is the respective received carrier phase; \( \eta_q(t) \) is the additive white Gaussian noise with bilateral power spectral density, equal to \( N_0/2 \), and \( h_q^{(i)}(t) \) is the complex channel coefficient for the \( i \)-th symbol, defined as

\[
h_q^{(i)}(t) = \gamma_q^{(i)}(t) e^{j \phi_q^{(i)}(t)},
\]

(4)

where the gain \( \gamma_q^{(i)}(t) \) is characterized by the Rayleigh distribution, and the phase \( \phi_q^{(i)}(t) \) - by the uniform distribution \([0, 2\pi]\).

At large, a slow frequency-selective channel\(^3\) is assumed. The expression (3) is quite general, and it includes some special and important cases: if \( Q = 1 \), a SISO system is obtained; if \( L = 1 \), the channel becomes non-selective (flat Rayleigh); if \( h_q^{(i)} = 1 \), it results in the AWGN channel; moreover, if \( \tau_q, k, \ell = 0 \), a synchronous DS-CDMA system is characterized.

At the base station, the received signal is submitted to a matched filter bank (CD), with \( D \leq L \) branches (fingers) per antenna for each user. When \( D \geq 1 \), the CD is known as Rake receiver. Assuming perfect carrier phase estimation, after despreading, the resultant signal is given by

\[
y^{(i)}_{q, k, \ell}(t) = \frac{1}{T} \int_{(i+1)T}^{iT} r_q(t) g_k(t - \tau_q, k, \ell) dt
\]

(5)

\[
= A_k h_q^{(i)}(t) d_q^{(i)} + S f_d + r_q^{(i)}(t) + \eta_q^{(i)},
\]

where the MRC weights \( w_q^{(i)} k, \ell \) are

\[
\gamma_q^{(i)}(t) e^{-j \phi_q^{(i)(t)}} + \eta_q^{(i)},
\]

(6)

The first term is the signal of interest, the second corresponds to the self-interference (SI), the third, to the multiple-access interference (MAI), and the last one corresponds to the filtered AWGN.

Considering a maximum ratio combining (MRC) rule with diversity order, equal to \( DQ \) for each user, the \( M \)-level complex decision variable is given by

\[
\zeta_k^{(i)} = \sum_{q=1}^{Q} \sum_{\ell=1}^{DQ} y_{q, k, \ell}^{(i)} q_{zt}(i), k = 1, \ldots, K
\]

(7)

where the MRC weights \( w_q^{(i)} k, \ell \) are

\[
\gamma_q^{(i)}(t) e^{-j \phi_q^{(i)(t)}} + \eta_q^{(i)},
\]

and \( h_q^{(i)}(t) \) and \( \phi_q^{(i)}(t) \) being the channel amplitude and the phase estimation, respectively.

After that, at each symbol interval, decisions are made on the in-phase and quadrature components\(^4\) of \( \zeta_k^{(i)} \) by scaling them to fit the constellation limits \( \xi_k^{(i)} \) and choosing the complex symbol with the minimum Euclidean distance regarding the scaled decision variable. Alternatively, this procedure can be replaced by separate \( \sqrt{M} \)-level quantizers \( q_{zt} \), acting individually with the in-phase and quadrature terms, as expressed by

\[
\tilde{d}_k^{(i), CD} = q_{zt} \left( \Re \left\{ \xi_k^{(i)} \right\} \right) + j q_{zt} \left( \Im \left\{ \xi_k^{(i)} \right\} \right),
\]

(7)

where \( k = 1, \ldots, K, A_{\text{real}} \) and \( A_{\text{imag}} \) are the real and imaginary value sets, respectively, from the complex alphabet set \( A \), and \( \Re\{\cdot\} \) and \( \Im\{\cdot\} \) represent the real and imaginary operators, respectively. Fig. 3 illustrates the general system structure.

### 2.2 Optimum Detection

The OMuD estimates the symbols for all \( K \) users by choosing the symbol combination, associated with the minimal distance metric, among all possible symbol combinations at \( M = 2^N \) constellation points Verdu (1998). In the asynchronous multipath channel scenario considered in this paper, the one-shot asynchronous channel
approach is adopted, where the configuration with \( K \) asynchronous users, \( I \) symbols, and \( D \) branches is equivalent to a synchronous scenario with \( KID \) virtual users. Furthermore, in order to avoid handling complex-valued variables in high-order squared modulation formats, the alphabet set is henceforth re-arranged as \( \Omega_{\text{real}} = \Omega_{\text{imag}} = \mathcal{Y} \subset \mathbb{Z} \) of cardinality \( \sqrt{M} \), e.g., 16-QAM (\( m = 4 \)); \( d_k^{(i)} \in \mathcal{Y} = \{ \pm 1, \pm 3 \} \).

The OMuD is based on the maximum-likelihood criterion that chooses the vector of symbols \( \mathbf{d}_k \), formally defined in (12), which maximizes the metric

\[
\mathbf{d}_k^{\text{opt}} = \arg \max_{\mathbf{d}_k \in \mathcal{Y}_{KID}} \{ \Omega(\mathbf{d}_k) \},
\]

where, in the SIMO channel, the single-objective function is generally written as a combination of the LLFs from all the receive antennas, given by

\[
\Omega(\mathbf{d}_k) = \sum_{q=1}^{Q} \Omega_q(\mathbf{d}_k).
\]

In the more general case considered here, i.e., \( K \) asynchronous users in the SIMO multipath Rayleigh channel with diversity \( D \leq L \), the LLF can be defined as a decoupled optimization problem with only real-valued variables, as expressed by

\[
\Omega(\mathbf{d}_k) = \sum_{q=1}^{Q} \Omega_q(\mathbf{d}_k) = 2d_k^{\text{opt}} \mathbf{W}_q^T \mathbf{y}_q - d_k^{\text{opt}} \mathbf{W}_q^T \mathbf{R} \mathbf{W}_q d_k^{\text{opt}},
\]

with the following definitions

\[
\mathbf{y}_q := \begin{bmatrix} \Re(\mathbf{y}_q) \\ \Im(\mathbf{y}_q) \end{bmatrix}, \quad \mathbf{W}_q := \begin{bmatrix} \Re(\mathbf{A} \mathbf{H}) - \Im(\mathbf{A} \mathbf{H}) \\ \Im(\mathbf{A} \mathbf{H}) + \Re(\mathbf{A} \mathbf{H}) \end{bmatrix},
\]

\[
\mathbf{d}_k := \begin{bmatrix} \Re(\mathbf{d}_k) \\ \Im(\mathbf{d}_k) \end{bmatrix}, \quad \mathbf{R} := \begin{bmatrix} \mathbf{R}[0] & \mathbf{R}[1] \\ \mathbf{R}[1] & \mathbf{R}[0] \end{bmatrix},
\]

where \( \mathbf{y}_q \in \mathbb{C}^{KID \times 1} \), \( \mathbf{W}_q \in \mathbb{C}^{KID \times 2KID} \), \( \mathbf{d}_k \in \mathbb{C}^{2KID \times 1} \) and \( \mathbf{R} \in \mathbb{C}^{2KID \times 2KID} \).

The matrix \( \mathbf{H} \) and \( \mathbf{A} \) are the coefficient and amplitude diagonal matrices, and \( \mathbf{R} \) represents the block-tridiagonal, block-Toeplitz cross-correlation matrix, composed by the sub-matrices \( \mathbf{R}[1] \) and \( \mathbf{R}[0] \), as expressed by Verdú (1998)

\[
\mathbf{R} = \begin{bmatrix} \mathbf{R}[0] & \mathbf{R}[1] \\ \mathbf{R}[1] & \mathbf{R}[0] \end{bmatrix},
\]

with \( \mathbf{R}[0] \) and \( \mathbf{R}[1] \) being \( KD \) matrices with the elements

\[
\rho_{u,\ell}^{(i)} = \begin{cases} 1, & \text{if (} k = u \text{) and (} \ell = l \text{)} \\ \rho_{k,l,u,l}^{(i)}, & \text{if (} k < u \text{) or (} k = u, \ell < l \text{)} \\ \rho_{u,l,k,\ell}^{(i)}, & \text{if (} k > u \text{) or (} k = u, \ell > l \text{)} \end{cases}
\]

where

Figure 3: DS-CDMA uplink baseband system model with conventional receiver: \( K \)-user transmitters, SIMO channel, and conventional (Rake) receiver with \( Q \) multiple receive antennas.
\[ p_{u,k}[1] = \begin{cases} 0, & \text{if } k > u \\ \rho_{u,l,k,t}^1, & \text{if } k < u \end{cases}, \] 

where \( a = (k-1)D + \ell \), \( b = (u-1)D + l \) and \( k, u = 1, 2, \ldots, K; \ell, l = 1, 2, \ldots, D \); the cross-correlation vector element between the \( k \)-th user, the \( \ell \)-th path, the \( u \)-th user, and the \( d \)-th path, at the \( q \)-th receive antenna, \( \rho_{k,\ell,u,d}^q \), is

\[ \rho_{k,\ell,u,d}^q = \frac{1}{T} \int_0^T g_k(t - \tau_{q,k,\ell}) g_u(t - \tau_{q,u,d}) dt. \] 

The evaluation for (8) can be extended along the entire message, where all the symbols of the transmitted vector for all \( K \) users are jointly detected (vector ML approach), or the decisions can be taken regarding the optimal single symbol detection of all \( K \) symbols, \( k \)-th symbol, \( \ell \)-th path, the \( u \)-th user, and the \( d \)-th path, at the \( q \)-th receive antenna, \( \rho_{k,\ell,u,d}^q \), is

\[ \rho_{k,\ell,u,d}^q = \frac{1}{T} \int_0^T g_k(t - \tau_{q,k,\ell}) g_u(t - \tau_{q,u,d}) dt. \] 

The vector \( d_i \) in (11) is based on a discrete set of size depending on \( M, K, I, J \), and \( D \). Hence, the optimization problem, posed by (8), can be solved directly using a \( m \)-dimensional (\( m = \log_2 M \)) search method. Therefore, the associated combinatorial problem strictly requires an exhaustive search for \( 2^{KI}D \) possibilities of \( d_i \), or equivalently, an exhaustive search for \( 2^{2KI}D \) possibilities of \( d_i \) for the decoupled optimization problem with only real-valued variables. As a result, the maximum-likelihood detector, coupled with the optimization problem of (10) and the matched filter outputs, \( y_q \), Fig. 3, has a complexity that increases exponentially with the modulation order, number of users, symbols, and branches, becoming prohibitive even for moderate product values \( m D =\), i.e., for a BPSK modulation format, medium system loading \( KI/N \), and a small number of symbols \( I \) and Rake fingers \( D \).

Next, the sub-optimum PSO-MuD algorithm is developed in order to solve efficiently the multiuser detection problem in polynomial time, as an alternative to the maximum-likelihood detector. The PSO-MuD is then coupled with the matched filter outputs (before the MRC blocks, Fig. 3) at the base-station multiuser receiver. It is worth noting that the overall DS-CDMA uplink receiver, including the conventional detector, plus the PSO-MuD, must provide a symbol data decision on a time-symbol basis, \( T \). Hence, the computational resource, necessary to process those signals associated to \( K \) users, \( Q \) receive antennas, and \( D \) branches, suggests that the quasi-optimal heuristic detection approach is properly implemented in a centralized fashion on the base-station side.

### 2.3 Discrete Swarm Optimization Algorithm

Discrete or, in several cases, binary PSO Kennedy and Eberhart (1997) is considered in this paper. Such scheme is suitable to deal with digital information detection/decoding. Hence, the binary PSO is adopted herein.

The particle selection for evolving is based on the highest fitness values, obtained through (10) and (9).

Accordingly, each candidate vector, defined as \( d_i \), has its binary representation, \( b_{p,t} \), of size \( mKI \), used for velocity calculation, and the \( p \)-th PSO-MuD particle position at instant (iteration) \( t \) is represented by \( mKI \times 1 \) binary vector

\[ b_{p}[t] = [b_{p,1}^1 \ b_{p,2}^2 \ \cdots \ b_{p,m}^m]; \] 

\[ b_{p}^i = [b_{p,1}^i \ b_{p,2}^i \ \cdots \ b_{p,m}^i]; \quad b_{p,i}^i \in \{0, 1\}, \] 

where each binary vector \( b_{p}^i \) is associated with one \( d_{(i)}^e \) symbol in Eq. (12). Each particle has a velocity, which is calculated and updated according to

\[ v_p[t + 1] = \omega \cdot v_p[t] + \phi_1 \cdot U_{p_1}[t](b_{p}^{\text{best}}[t] - b_p[t]) + \phi_2 \cdot U_{p_2}[t](b_{g}^{\text{best}}[t] - b_p[t]), \] 

where \( \omega \) is the inertial weight; \( U_{p_1}[t] \) and \( U_{p_2}[t] \) are diagonal matrices of dimension \( mKI \); the elements of which are random variables with the uniform distribution \( U \in [0, 1] \); \( b_{p}^{\text{best}}[t] \) and \( b_{g}^{\text{best}}[t] \) are the best global and the best local positions found until the \( t \)-th iteration, respectively; \( \phi_1 \) and \( \phi_2 \) are the weight factors (acceleration coefficients) regarding the influences of the best individual and the best global positions on the velocity update, respectively.

For the MuD optimization with binary representation, each element \( b_{p,t} \) (in 18) just assumes “0” or “1” values. Hence, a discrete model regarding the position choice is carried out by inserting a probabilistic decision step, based on threshold, depending on the velocity. Several functions have this characteristic, such as the sigmoid function Kennedy and Eberhart (1997)

\[ S(v_{p,i}^t) = \frac{1}{1 + e^{-v_{p,i}^t}}. \] 

where \( v_{p,i}^t \) is the \( r \)-th element of the \( p \)-th particle velocity vector, \( v_p^t = [v_{p,1}^t \ • \ • \ v_{p,R}^t] \), and the selection of the future particle position is obtained through the statement

\[ \begin{cases} b_{p,i}^t[t + 1] = 1, \text{ if } u_{p,i}^t[t] < S(v_{p,i}^t) \\ b_{p,i}^t[t + 1] = 0, \quad \text{otherwise} \end{cases} \] 

where \( b_{p,i}^t \) is the element of \( b_{p} \) in Eq. (18), and \( u_{p,i}^t \) is a random variable with the uniform distribution \( U \in [0, 1] \).

After obtaining the new particle position \( b_{p,t} \), it is mapped back into its correspondent symbol vector \( d_p[t+1] \), and further into the real form \( d_{(i)}^e[t+1] \), for the objective function evaluation in (9).

In order to obtain further diversity for the search universe, the \( V_{\text{max}} \) factor is added to the PSO model, Eq. (18), being responsible for limiting the velocity in the range \([±V_{\text{max}}]\). The insertion of this factor to the velocity calculation enables the algorithm to escape from possible local optima. The likelihood of a bit change increases
The population size $P$ is typically in the range of 10 to 40 Eberhart and Shi (2001). However, based on Oliveira et al. (2006), it is set to $P = 10 \left[ 0.3454 \left( \sqrt{\pi (mK\tau - 1)} + 2 \right) \right]$. (21)

Algorithm 1 describes the pseudo-code for the PSO implementation.

**Algorithm 1** PSO Algorithm for the MuD Problem

**Input:** $d_{\text{CD}}$, $P$, $G$, $\omega$, $\phi_1$, $\phi_2$, $V_{\text{max}}$. **Output:** $d_{\text{PSO}}$

begin
1. initialize the first population: $t = 0$;
   $B[0] = B_{\text{CD}} \cup \tilde{B}$, where $\tilde{B}$ contains $(P - 1)$ particles randomly generated;
   $b_p^{\text{best}}[0] = b_p[0]$ and $b_p^{\text{best}}[0] = B_{\text{CD}}$;
   $v_p[0] = 0$: null initial velocity;
2. while $t \leq G$
   a. calculate $\Omega(d_p[t])$, $\forall b_p[t] \in B[t]$ using (9);
   b. update the velocity $v_p[t]$, $p = 1, \ldots, P$, through (18);
   c. update the best positions:
      for $p = 1, \ldots, P$
      if $\Omega(d_p[t]) > \Omega(d_p^{\text{best}}[t])$, $b_p^{\text{best}}[t + 1] \leftarrow b_p[t]$
         else $b_p^{\text{best}}[t + 1] \leftarrow b_p^{\text{best}}[t]$
      end
      if $\exists b_p[t]$ such as $[\Omega(d_p[t]) > \Omega(d_p^{\text{best}}[t])] \land
      [\Omega(d_j[t]) \geq \Omega(d_j[t]), j \neq p]$
      $b_j^{\text{best}}[t + 1] \leftarrow b_j[t]$
      else $b_j^{\text{best}}[t + 1] \leftarrow b_j^{\text{best}}[t]$
   d. Evolve to a new swarm population $B[t + 1]$, using (20);
3. $b_{\text{PSO}} = b_{g^{\text{best}}[t]}$, $b_{\text{PSO}} \xrightarrow{\text{map}} d_{\text{PSO}}$.
end $d_{\text{CD}}$: CD output.

$P$: Population size.

$G$: number of swarm iterations.

For each $d_p[t]$, $b_p[t]$ is associated.

### 3 PSO-MuD Parameter Optimization

In this section, the PSO-MuD parameter optimization is carried out using Monte Carlo simulation (MCS) technique. Further details for the MCS technique can be found in the Appendices A and B. Such optimization is directly related to the complexity-performance trade-off of the algorithm. Herein, this performance analysis is carried out in terms of bit or symbol error-rate (BER or SER, respectively) versus signal-to-noise ratio (SNR), system loading ($L = \frac{K}{T}$) and iterations (convergence), considering BPSK, QPSK, and 16-QAM modulation schemes with diversity exploration.

The first analysis of the PSO parameters gives rise to the following behaviors: $\omega$ is responsible for creating the inertia of the particles, inducing them to keep the movement towards the last directions of their velocities; $\phi_1$ aims at guiding the particles to each individual best position, inserting diversification into the search; $\phi_2$ leads all the particles towards the best global position, hence intensifying the search and reducing the convergence time; $V_{\text{max}}$ inserts perturbation limits into the movement of the particles, allowing more or less diversification in the algorithm.

The optimization process for the initial velocity of the particles achieves similar results for three different conditions: null, random, and CD output as initial velocity. Hence, the null initial velocity, i.e., $v[0] = 0$, is adopted here for simplicity.

In Oliveira et al. (2006), the best performance-complexity trade-off for the BPSK PSO-MuD algorithm was obtained by setting $V_{\text{max}} = 4$. Herein, the simulations were carried out by varying $V_{\text{max}}$ for different modulations and diversity exploration, accomplish this value as a good trade-off. This optimization process is quite similar for the systems using QPSK and 16-QAM modulation formats.

#### 3.1 $\omega$ Optimization

It is worth noting that a relatively larger value for $\omega$ is helpful for a global optimum and less influenced by the best global and local positions, whereas a relatively smaller value for $\omega$ is helpful for a course convergence, i.e., a smaller inertial weight encourages the local exploration Eberhart and Shi (2001); Shi and Eberhart (1998) as the particles are more attracted towards $b_p^{\text{best}}[t]$ and $b_j^{\text{best}}[t]$.

Fig. 4 shows the convergence of the PSO scheme for different values of $\omega$ considering the BPSK modulation and the flat channel. It is evident that the best performance-complexity trade-off is achieved with $\omega = 1$.

Many research papers have proposed new strategies for the principle of PSO in order to improve its performance and reduce its complexity. For instance, in Chatterjee and Siarry (2006), the adaptive non-linear inertia weight has been applied in order to improve the PSO convergence. However, the current analysis indicates that no further specialized strategy is necessary, since the conventional PSO works well to solve the MuD DS-CDMA problem in several practical scenarios.

The optimization of the inertial weight, $\omega$, achieves analogous results for the QPSK and 16-QAM modulation schemes, where adopting $\omega = 1$ the PSO-MuD obtains the best performance-complexity trade-off as well (the numerical results not shown herein). A special attention is given for $\phi_1$ and $\phi_2$ optimization, presented in the next section, since their values impact deeply the PSO performance, also varying for each modulation.

<table>
<thead>
<tr>
<th>$V_{\text{max}}$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 - S(V_{\text{max}})$</td>
<td>0.269</td>
<td>0.119</td>
<td>0.047</td>
<td>0.018</td>
<td>0.007</td>
</tr>
</tbody>
</table>

as the particle velocity crosses the limits established by $[\pm V_{\text{max}}]$, as shown in Tab. 1.
3.2 φ1 and φ2 Optimization

3.2.1 BPSK Modulation

For Rayleigh channels, the performance improvement, expected by the φ1 increment, is not evident, and its value can be reduced without performance losses, as could be seen in Fig. 5. Therefore, a good choice seems to be φ1 = 2 that achieves a reasonable convergence rate.

3.2.2 QPSK Modulation

Different results for input parameter optimization are achieved for high order modulation formats. It is worth noting (Fig. 7) for QPSK modulation that adopting low values for φ2 and the high values for φ1 delay the convergence; the inverse results are found if the lack of diversity occurs. Hence, the best performance × complexity is achieved with φ1 = φ2 = 4.

3.2.3 16-QAM Modulation

With higher order modulation format (16-QAM), the PSO-MuD requires more intensification, once the search becomes more complex due to each symbol mapped to a 4-bits sequence. Fig. 8 shows the convergence curves for different values of φ1 and φ2, from which it is clear that the performance gap is more evident with an increasing Eb/N0 value; similar tendency was verified with an increasing in the number of users (not shown here). Analyzing these results, the chosen values were φ1 = 6 and φ2 = 2.
3.3 Diversity Exploration

The best range for the acceleration coefficients in the case of the resolvable multipath channels \((L \geq 2)\) for the MuD SISO DS-CDMA problem appears to be \(\phi_1 = 2\) and \(\phi_2 \in [12; 15]\), as indicated by the simulation results shown in Fig. 9. For the medium system loading and SNR, Fig. 9 indicates that the best acceleration coefficient values are \(\phi_1 = 2\) and \(\phi_2 = 15\), allowing the combination of fast convergence and near-optimum performance achievement.

3.4 Optimized parameters for PSO-MuD

As previously mentioned, the optimized input parameters for the PSO-MuD vary regarding the system and channel scenario conditions. The Monte-Carlo simulations, exhibited in Section 4, adopt the values presented in Tab. 2 as the optimized PSO input parameters. The system loading range \(L\) indicates the boundaries \(K\), for which the PSO input parameter optimization is carried out. For the system operation, characterized by the spatial diversity \((Q > 1\) receive antennas), the PSO-MuD behavior, in terms of convergence speed and quality of solution, is very similar to that presented for the multipath diversity.

<table>
<thead>
<tr>
<th>Channel &amp; Mod.</th>
<th>(L) range</th>
<th>(\omega)</th>
<th>(\phi_1)</th>
<th>(\phi_2)</th>
<th>(V_{\text{max}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flat BPSK</td>
<td>([0.16; 1.00])</td>
<td>1</td>
<td>2</td>
<td>10</td>
<td>4</td>
</tr>
<tr>
<td>Flat QPSK</td>
<td>([0.16; 1.00])</td>
<td>1</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Flat 16-QAM</td>
<td>([0.03; 0.50])</td>
<td>1</td>
<td>6</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>Diversity BPSK</td>
<td>([0.03; 0.50])</td>
<td>1</td>
<td>2</td>
<td>15</td>
<td>4</td>
</tr>
</tbody>
</table>
4 NUMERICAL RESULTS WITH OPTIMIZED PARAMETERS

In this section, the numerical results for the PSO-MuD performance are obtained by simulations using Monte-Carlo methodology. The results are compared with the theoretical single-user bound (SuB), according to Appendix A, since the OMuD computational complexity results are prohibitive. Furthermore, Appendix B describes the basic setup of the Monte-Carlo simulation technique employed in the PSO-MuD BER (or SER) performance evaluation. The adopted PSO-MuD parameters, as well as the system and channel conditions, employed in the Monte Carlo simulations, are summarized in Tab. 3.

Fig. 10 presents the performance as a function of received $E_b/N_0$ for two different near-far ratio scenarios with the flat Rayleigh channel. Fig. 10(a) has been obtained for perfect power control, whereas Fig. 10(b) has been generated considering half of the users with near-far ratio $NFR = +6$ dB. Here, the BER$_{Avg}$ performance is calculated only for the weaker users. It is worth noting that the PSO-MuD performance is almost constant despite of $NFR = +6$ dB for half of the users, illustrating the robustness of the PSO-MuD against the unbalanced received signal powers in the flat fading channels.

Table 3 System, channel and PSO-MuD parameters for fading channel performance analysis.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Adopted Values</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>DS-CDMA System</strong></td>
<td></td>
</tr>
<tr>
<td>Number of Rx antennas</td>
<td>$Q = 1, 2, 3$</td>
</tr>
<tr>
<td>Spreading Sequences</td>
<td>Random, $N = 31$</td>
</tr>
<tr>
<td>Modulation</td>
<td>BPSK, QPSK and 16-QAM</td>
</tr>
<tr>
<td>Number of mobile users</td>
<td>$K \in [5; 31]$</td>
</tr>
<tr>
<td>Received SNR</td>
<td>$E_b/N_0 \in [0; 30]$ dB</td>
</tr>
<tr>
<td><strong>PSO-MuD Parameters</strong></td>
<td></td>
</tr>
<tr>
<td>Population size, $P$</td>
<td>Eq. (21)</td>
</tr>
<tr>
<td>Acceleration coefficients</td>
<td>$\phi_1 = 2, 6; \phi_2 = 1, 10$</td>
</tr>
<tr>
<td>Inertia weight</td>
<td>$\omega = 1$</td>
</tr>
<tr>
<td>Maximal velocity</td>
<td>$V_{max} = 4$</td>
</tr>
<tr>
<td><strong>Rayleigh Channel</strong></td>
<td></td>
</tr>
<tr>
<td>Channel state info. (CSI)</td>
<td>perfectly known at Rx</td>
</tr>
<tr>
<td>Number of paths</td>
<td>$L = 1, 2, 3$</td>
</tr>
</tbody>
</table>

4.1 Diversity

For the results presented here, two assumptions are considered when there are more than one antenna at the receiver (spatial diversity): first, the average received power is equal for all the antennas; and second, the SNR at the receiver input is defined as the received SNR per antenna. Therefore, there is a power gain of 3 dB when adopted $Q = 2$, or 6 dB with $Q = 3$ and so forth. The effect of increasing the number of receive antennas on the convergence curves is shown in Fig. 11, where the PSO-MuD works with systems having $Q = 1, 2,$ and 3.
PSO-based Multiuser Detectors for High-Order Modulation DS/CDMA Systems under Spatial and Antennas. A delay in the PSO-MuD convergence is observed when more antennas are added to the receiver, and it is caused by the larger gap that it has to overpass. Furthermore, the PSO-MuD achieves the optimum ML performance for all the three cases.

![Figure 11](image)

**Figure 11** Convergence performance of PSO-MuD, with $K = 15$, $E_b/N_0 = 15$, BPSK modulation, (a) asynchronous multipath slow Rayleigh channels, $I = 3$, $L = 1, 2, 3$ paths; and (b) synchronous flat Rayleigh channel, $I = 1, Q = 1, 2, 3$ antennas.

The exploitation of the path diversity also improves the system capacity. Fig. 11 shows the BER_{Avg} convergence of PSO-MuD for different number of paths, $L = 1, 2, 3$, when the detector fully explores the path diversity, i.e., the number of fingers of conventional detector is equal to the number of copies of the received signal, $D = L$. The power delay profile considered is a classical decreasing exponential profile, with the mean path energy, as shown in Tab. 4 Proakis (1989). It is worth noting that the mean received energy is equal for the three conditions, i.e., the resultant improvement with an increasing number of paths is due to the diversity gain only.

**Table 4** Three power delay profiles for different Rayleigh fading channels used in Monte-Carlo simulations.

<table>
<thead>
<tr>
<th>Param.</th>
<th>PD-1</th>
<th>PD-2</th>
<th>PD-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Path, $\ell$</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>$\tau_\ell$</td>
<td>0</td>
<td>0</td>
<td>$T_c$</td>
</tr>
<tr>
<td>$L$</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>$E[\gamma_\ell]$</td>
<td>1.0000</td>
<td>.8320</td>
<td>.1680</td>
</tr>
<tr>
<td>$\gamma_\ell$</td>
<td>.8047</td>
<td>.1625</td>
<td>.0328</td>
</tr>
</tbody>
</table>

It is worth noting that there is a performance gain with exploration of such diversity, verified for both the Rake and the PSO-MuD receivers. The performance obtained by PSO-MuD is close to that presented by the SuB in all the cases, exhibiting its capability of exploring the path diversity and dealing with the self-interference (SI) as well. In addition, the convergence aspects are kept for all the conditions.

The PSO-MuD is also evaluated under the condition of channel error estimates, which are modeled through the continuous uniform distributions $U[1 \pm \epsilon]$, centralized on the true values of the coefficients, resulting in

$$\tilde{\gamma}_k^{(i)} = U[1 \pm \epsilon_\gamma] \times \gamma_k^{(i)}, \quad \tilde{\theta}_k^{(i)} = U[1 \pm \epsilon_\theta] \times \theta_k^{(i)}.$$

![Figure 12](image)

**Figure 12** Performance of PSO-MuD with $K = 15$, BPSK modulation, and channel estimation error, for (a) path diversity and (b) spatial diversity.
where $\epsilon_\gamma$ and $\epsilon_\theta$ are the maximum module and phase normalized errors for the channel coefficients, respectively. For a low-moderate SNR and medium system loading ($L = 15/31$), Fig. 12 shows the performance degradation of the PSO-MuD considering BPSK modulation, $L = 1$ and $L = 2$ paths or $Q = 1$ and $Q = 2$ antennas, with estimation errors of 10%- or 25%-order, i.e., $\epsilon_\gamma = \epsilon_\theta = 0.10$ or $\epsilon_\gamma = \epsilon_\theta = 0.25$, respectively. It is worth noting that the PSO-MuD reaches the SuB under both conditions with perfect channel estimation, and the improvement is more evident when the diversity gain increases. However, bearing in mind that, with the spatial diversity, the gain is higher, since the mean energy is equally distributed among the antennas, whereas for the path diversity, a realistic exponential power-delay profile is considered. Although there is a general performance degradation when the error in the channel coefficient estimation increases, the PSO-MuD still achieves much better performance than the conventional detector (CD) under perfect channel estimation, being more evident for a larger number of antennas.

Fig. 13 shows the performance as a function of the number of users $K$. It is evident that the performance of the PSO-MuD is much better than that of the CD scheme. Besides, Fig. 13 indicates that PSO-MuD presents robustness against increasing multiple access interference (system loading, $L$) under path and spatial diversities exploitation scenarios.

### 4.2 QPSK and 16-QAM Modulations

Fig. 14 shows the convergence comparison for the three different modulations: (a) BPSK, (b) QPSK, and (c) 16-QAM. It is worth mentioning, as presented in Tab. 2, that the PSO-MuD optimized parameters are specific for each modulation.

Further and corroborative simulation results were obtained for $\text{SER} \times E_b/N_0$ performance curves with BPSK, QPSK and 16-QAM modulation (not shown here). Nevertheless, Fig. 15 shows that for the 16-QAM modulation with $\phi_1 = 6$ and $\phi_2 = 1$, the PSO-MuD performance degradation is quite slight in the low and medium system loading range ($0 < L \leq 0.5$). However, the performance is substantially degraded in medium-high system loading scenarios ($L > 0.7$), indicating that the heuristic PSO-MuD strategy is not completely robust to intensive multiple access interference, mainly in hard scenarios, e.g., with high order modulation ($M \geq 16$) combined to high or even overloaded system loading ($L \geq 0.8$).

### 5 CONCLUSIONS

This paper provides an analysis of the PSO scheme applied to the multiuser DS-CDMA system, focusing on the algorithm parameters optimization. It has been shown that $\omega = 1$ represents a good choice for the considered detection problem under wide system and channel configuration scenarios.
Regarding the acceleration coefficients ($\phi_1$ and $\phi_2$) for the flat Rayleigh channels, it has been demonstrated that their choices depend on the modulation order. The BPSK PSO-MuD with $\phi_1 = 2$ and $\phi_2 = 10$ represents a good choice. The QPSK PSO-MuD with $\phi_1 = 4$ and $\phi_2 = 4$ shows a good complexity-performance trade-off, whereas for the 16-QAM PSO-MuD, it has been observed that $\phi_1 = 6$ and $\phi_2 = 1$ provide suitable performance results. However, in the latter case, the performance is not optimal for the high system loading. Besides, the PSO-MuD under BPSK modulation and Rayleigh diversity channels, it has been pointed out that $\phi_1 = 2$ and $\phi_2 \in [12; 15]$ provide a good convergence speed and BER performance.

The PSO algorithm is shown to be efficient for the SISO/SIMO MuD asynchronous DS-CDMA problem, when the input parameters are properly chosen. In a variety of simulated and analyzed realistic scenarios, the performance achieved by the PSO-MuD, except for the high-order modulation under the high system loading condition, appears to be near-optimal. In the presence of channel errors, the PSO-MuD is much more efficient than the conventional receiver with perfect channel estimates. Under all the system conditions evaluated, the PSO-MuD results in the small degradation performance if those errors are confined to 10% of the actual instantaneous values.

This work was supported in part by the National Council for Scientific and Technological Development (CNPq) under Grant 303426/2009-8, and the Araucaria Foundation, PR, Brazil under Grant 045/2007.

APPENDIX

A Minimal Number of Trials and Single-User Performance

The minimal number of trials ($T_R$), evaluated at each Monte-Carlo simulated point (SNR), has been ob-
tained basing on the single-user bound (SuB) performance. Considering a confidence interval, and admitting that non-spreading and spreading systems have the same equivalent bandwidth \( BW \approx \frac{1}{T} = BW_{\text{spread}} \approx \frac{1}{T} \), and thus, equivalently, both systems have the same channel response (delay spread, diversity order, etc), the SuB performance in both systems will be equivalent. So, the average symbol error rate for the single-user bound, regarding the \( M \)-QAM DS-CDMA system with \( L \) Rayleigh fading path channels with exponential power-delay profile and maximum ratio combining reception, is found as (Simon and Alouini, 2005, Eq. (9.26)):

\[
\text{SER}_{\text{SU}} = 2\alpha \sum_{\ell=1}^{L} \frac{p_{\ell}}{\pi} \beta_{\ell} \tan^{-1}\left(\frac{1}{\beta_{\ell}}\right) - \sum_{\ell=1}^{L} p_{\ell}
\]

where:

\[
p_{\ell} = \left(\prod_{k=1, k \neq \ell}^{L} \left(1 - \frac{1}{1 + g_{k}}\right)\right)^{-1}, \quad \alpha = \left(1 - \frac{1}{\sqrt{M}}\right), \quad \beta_{\ell} = \sqrt{\frac{\gamma_{\ell} g_{\text{QAM}}}{1 + \gamma_{\ell} g_{\text{QAM}}}}, \quad g_{\text{QAM}} = \frac{3}{2(M-1)},
\]

and \( \gamma_{\ell} = \tau_{\ell} \log_{2} M = m\tau_{\ell}^{*} \) denotes the average received signal-to-noise ratio per symbol for the \( \ell \)-th path, with \( \tau_{\ell}^{*} \) being the correspondent SNR per bit per path.

Once the lower bound is defined, the minimal number of trials can be defined as

\[
\text{TR} = \frac{n_{\text{errors}}}{\text{SER}_{\text{SU}}},
\]

where \( n_{\text{errors}} \) is the number of corrupted received bit (or symbol) from a total number of transmitted bits (equal to TR); the higher \( n_{\text{errors}} \) value, the more reliable will be the SER estimate obtained through the MCS technique Jeruchim et al. (1992). In this work, the minimum adopted is \( n_{\text{errors}} = 100 \), and considering a reliable interval of 95%, it is assured that the estimated symbol error rate lies on the interval \( \text{SER} \subset [0.823; 1.215] \) SER.

### B Monte Carlo Simulation Setup

Monte Carlo simulation is a method for iteratively evaluating a deterministic model using sets of random numbers as inputs and modeling processes. This method is often used when the model is complex, non-linear, or involves more than just a couple of uncertain parameters, where the underlying probabilities are known but the results are more difficult to determine.

The simplified diagram of the adopted Monte Carlo simulation setup is shown in Fig. 16. In the random data generator block, the transmitted data are randomly generated, and all information symbols have an equal probability of being selected. The transmitter block implements Eq. (1), with a set of input variables \( (A, I, K, N, M, \text{ and spreading sequence type}) \) adjusted according to the choices stated in Section 4.

![Figure 16 Monte Carlo simulation diagram.](image)

The channel block adds other stochastic characteristics to the model. Here, the complex additive white Gaussian noise, with bilateral power spectral density equal to \( N_{0}/2 \), corrupts the received signal of all the users. The power delay profile for the Rayleigh fading channel can be adjusted by each user, admitting random delay and normalized power \( \sum_{\ell=1}^{L} \gamma_{\ell} = 1 \). Then, the average SNR at the receiver input is given by

\[
\bar{E} = \sum_{\ell=1}^{L} \gamma_{\ell}, \quad \text{with} \quad \gamma_{\ell} = \text{SNR} \cdot \text{E}[\gamma_{\ell}^2].
\]

For instance, admitting an exponential power-delay profile with 2 paths such that adopted in Section 4, \( \text{E}[\gamma_{\ell}^2] = 0.8320 \) and \( \text{E}[\gamma_{\ell}^2] = 0.1680 \) are assumed, with the respective delays uniformly distributed on the interval \( \tau_{h, \ell} \in [0; N - 1] \).

The symbols, estimated at the conventional receiver stage, are used as start points for the heuristic MuD. Finally, all numerical results have been obtained using mathematical simulation platform Matlab\textsuperscript{\textregistered} v.7.3, The MathWorks, Inc.

### References


