Tracking Control of an Underactuated Ship

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***Abstract*—In this paper, we address the tracking problem for an underactuated ship using two controls, namely surge force and yaw moment. A simple state-feedback control law is developed and proved to render the tracking error dynamics globally $\mathcal{K}$-exponentially stable. Experimental results are presented where the controller is implemented on a scale model of an offshore supply vessel.**

***Index Terms*—Cascade control, marine vehicle control, nonlinear systems, tracking.**

I. Introduction

IN THIS paper, we study the underactuated tracking control of a ship. For a conventional ship it is common to consider the motion in *surge* (forward), *sway* (sideways), and *yaw* (heading), see Fig. 1. Often, we have surge and sway control forces and yaw control moment available for steering the ship. However, this assumption is not valid for all ships. For instance, some ships are either equipped with two independent aft thrusters or with one main aft thruster and a rudder, but are without any bow or side thrusters, like, for instance, many supply vessels. As a result, we have no sway control force. In this paper, we consider tracking control for ships having only surge control force and yaw control moment available. Since we need to control three degrees of freedom and have only two inputs available, we are dealing with an underactuated problem.

Since we seek to control the ship motion in the horizontal plane, we neglect the dynamics associated with the motion in heave, roll, and pitch when modeling the ship. Moreover, as a first step toward finding a solution to the underactuated tracking control problem, we do not include the environmental forces due to wind, currents, and waves in the model. Furthermore, we assume that the inertia, added mass and damping matrices are diagonal. In this case, the ship dynamics can be described by (see, e.g., [1]):

$$\begin{aligned}
\dot{u} &= \frac{m_{22}}{m_{11}} vr - \frac{d_{11}}{m_{11}} u + \frac{1}{m_{11}} u_1 \\
\dot{v} &= -\frac{m_{11}}{m_{22}} ur - \frac{d_{22}}{m_{22}} v \\
\dot{r} &= \frac{m_{11} - m_{22}}{m_{33}} uv - \frac{d_{33}}{m_{33}} r + \frac{1}{m_{33}} u_2 \\
\dot{x} &= u \cos\psi - v \sin\psi \\
\dot{y} &= u \sin\psi + v \cos\psi \\
\dot{\psi} &= r
\end{aligned} \tag{1}$$

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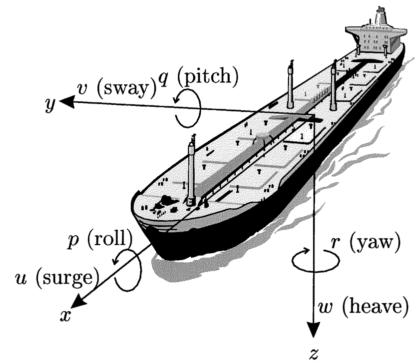

Fig. 1. Definition of state variables in surge, sway, heave, roll, pitch, and yaw for a marine vessel.

where u, v, and r are the velocities in surge, sway, and yaw, respectively, and x, y, ψ denote the position and orientation of the ship in the earth-fixed frame. The parameters $m_{ii} > 0$ are given by the ship inertia and added mass effects. The parameters $d_{ii} > 0$ are given by the hydrodynamic damping. The available controls are the surge force u_1, and the yaw moment u_2.

The ship model (1) is neither static feedback linearizable, nor can it be transformed into chained form. It was shown in [2] that no continuous or discontinuous static state-feedback law exists which makes the origin asymptotically stable. The stabilization problem for an underactuated ship has been studied in [2]–[6].

Tracking control of ships has mainly been based on linear models, giving local results, and steering only two degrees of freedom. In [7] and [8] output-tracking control based on nonlinear ship models has been investigated. Using feedback linearization and Lyapunov theory, respectively, tracking controllers were developed that stabilized the desired trajectories. The trajectories were, however, position trajectories, and the yaw angle was not controlled.

In the case where only the position variables are controlled, the ship may turn around such that the desired position trajectory is followed backward. That is why we focus on state-tracking instead of output-tracking.

The first complete state-tracking controller based on a nonlinear model was developed in [5] and yields global practical stability. Another result yielding global practical stability can be found in [9]. In [10] semi-global asymptotic stability has been achieved by means of backstepping, inspired by the results of [11]. We are not aware of any global tracking results for the tracking control of an underactuated ship in literature.

In this paper, we present a global solution to the tracking problem for an underactuated ship. Based on a result for (time-varying) cascaded systems [12] we divide the tracking error dynamics into a cascade of two linear subsystems which we can stabilize independently of each other.

The organization of this paper is as follows: In Section II, we present some preliminary results. In Section III, we state the problem formulation. In Section IV, a full state feedback control law is developed and proven to globally asymptotically stabilize the tracking-error dynamics. Section V contains experimental results and some conclusions are given in Section VI.

II. Preliminaries

In this section, we recall some results that we need in this paper.

For basic stability concepts, the reader is referred to [13, Sec. 3.4]. A slightly weaker notion than global exponential stability is the following.

Definition 1 (cf. [14]): We call the system

$$\dot{x} = f(t, x), \quad f(t, 0) = 0, \qquad \forall\, t \geq 0 \tag{2}$$

with $x \in \mathbb{R}^n$ and $f(t, x)$ piecewise continuous in t and locally Lipschitz in x, *globally $\mathcal{K}$-exponentially stable* if there exist a $\gamma > 0$ and a class $\mathcal{K}$ function $\kappa(\cdot)$ such that

$$\|x(t)\| \leq \kappa(\|x(t_0)\|) \exp[-\gamma(t - t_0)].$$

Consider a system $\dot{x} = f(t, x)$ that can be written as

$$\dot{x}_1 = f_1(t, x_1) + g(t, x_1, x_2)x_2 \tag{3a}$$
$$\dot{x}_2 = f_2(t, x_2) \tag{3b}$$

where $x_1 \in \mathbb{R}^n$, $x_2 \in \mathbb{R}^m$, $f_1(t, x_1)$ is continuously differentiable in (t, x_1) and $f_2(t, x_2)$, $g(t, x_1, x_2)$ are continuous in their arguments, and locally Lipschitz in x_2 and (x_1, x_2) respectively.

Notice that if $x_2 = 0$ (3a) reduces to

$$\dot{x}_1 = f_1(t, x_1).$$

Therefore, we can view (3a) as the system

$$\Sigma_1: \quad \dot{x}_1 = f_1(t, x_1) \tag{4}$$

that is perturbed by the output of the system

$$\Sigma_2: \quad \dot{x}_2 = f_2(t, x_2). \tag{5}$$

Assume that the systems Σ_1 and Σ_2 are asymptotically stable, then for (4) we know that $\lim_{t\to\infty} x_1(t) = 0$ and for (5) we have $\lim_{t\to\infty} x_2(t) = 0$. It is obvious that then also for (3b) $x_2(t)$ tends to zero. In that case, the dynamics (3a) reduces to the dynamics (4). It seems plausible also, that, therefore, (3a) and as a result the cascaded system (3) become asymptotically stable.

Unfortunately, this is not true in general as can be seen from the system

$$\begin{aligned} \dot{x}_1 &= -x_1 + x_1^2 x_2 \qquad && x_1(0)x_2(0) > 2 \\ \dot{x}_2 &= -\gamma x_2 && \gamma > 0 \end{aligned}$$

which has a finite escape time $t_{\text{esc}} = (1/2\gamma)\ln(x_1(0) x_2(0)/(x_1(0)x_2(0) - 2))$. However, under certain conditions it is possible to conclude asymptotic stability of (3) when both Σ_1 and Σ_2 are asymptotically stable. In [12, Lemma 2], it was mentioned that if the systems (4) and (5) are globally uniformly asymptotically stable and solutions of the cascaded system (3) are globally uniformly bounded, then the system (3) is globally uniformly asymptotically stable. The question that remains then, is when solutions of (3) are globally uniformly bounded. An answer to that question can also be found in [12]. For this paper, a corollary suffices.

Corollary 1 (cf. [15]): Assume that both (4) and (5) are globally $\mathcal{K}$-exponentially stable and that continuous functions k_1: $\mathbb{R}_+ \to \mathbb{R}$ and k_2: $\mathbb{R}_+ \to \mathbb{R}$ exist such that

$$\|g(t, x_1, x_2)\| \leq k_1(\|x_2\|) + k_2(\|x_2\|)\|x_1\|. \tag{6}$$

Then the cascaded system (3) is globally $\mathcal{K}$-exponentially stable.

A third ingredient we need for this paper is a result from linear systems theory. For basic concepts, the reader is referred to [16]. The result we need in this paper is a corollary of [17, Th. 2].

Corollary 2: Consider the time-varying linear system

$$\dot{x} = A(\phi(t))x + Bu \qquad y = Cx \tag{7}$$

where $A(\phi)$ is continuous, $A(0) = 0$, ϕ: $\mathbb{R} \to \mathbb{R}$ continuous. Assume that for all $s \neq 0$ the pair $(A(s), B)$ is controllable. If $\phi(t)$ is bounded, Lipschitz in t, and constants $\delta_c > 0$ and $\epsilon > 0$ exist such that

$$\forall\, t \geq 0,\ \exists\, s\text{: } t - \delta_c \leq s \leq t \qquad \text{such that } |\phi(s)| \geq \epsilon$$

then the system (7) is uniformly completely controllable.

The condition imposed on $\phi(t)$ in Corollary 2 plays an important role, not only in this paper, but also in identification and adaptive control systems. It is known as the “persistence of excitation condition.”

Definition 2: A continuous function ϕ: $\mathbb{R}_+ \to \mathbb{R}$ is said to be *persistently exciting* if all of the following conditions hold:

- constant $K > 0$ exists such that $|\phi(t)| \leq K$ for all $t \geq 0$;
- constant $L > 0$ exists such that $|\phi(t) - \phi(t')| \leq L|t - t'|$ for all $t, t' \geq 0$;
- constants $\delta_c > 0$ and $\epsilon > 0$ exist such that

$$\forall\, t \geq 0,\ \exists\, s\text{: } t - \delta_c \leq s \leq t \quad \text{such that } |\phi(s)| \geq \epsilon.$$

Remark 1: Notice that the third condition on $\phi(t)$ in Definition 2 can be interpreted as follows: assume that we plot the graph of $|\phi(t)|$ and look at this plot through a window of width $\delta_c > 0$. Then, no matter where we put this window on the graph, always a time instant s exists where $|\phi(s)|$ is at least $\epsilon > 0$.

III. Problem Formulation

Consider the system (1). Assume that a feasible reference trajectory $(u_r, v_r, r_r, x_r, y_r, \psi_r, u_{1,r}, u_{2,r})^T$ is given, i.e., a trajectory satisfying

$$\begin{aligned} \dot{u}_r &= \frac{m_{22}}{m_{11}} v_r r_r - \frac{d_{11}}{m_{11}} u_r + \frac{1}{m_{11}} u_{1,r} \\ \dot{v}_r &= -\frac{m_{11}}{m_{22}} u_r r_r - \frac{d_{22}}{m_{22}} v_r \\ \dot{r}_r &= \frac{m_{11} - m_{22}}{m_{33}} u_r v_r - \frac{d_{33}}{m_{33}} r_r + \frac{1}{m_{33}} u_{2,r} \\ \dot{x}_r &= u_r \cos\psi_r - v_r \sin\psi_r \\ \dot{y}_r &= u_r \sin\psi_r + v_r \cos\psi_r \\ \dot{\psi}_r &= r_r. \end{aligned} \tag{8}$$

Notice that a drawback exists in considering the error coordinates $x - x_r$ and $y - y_r$, since these position errors depend on the choice of the inertial frame. This problem is solved by defining the change of coordinates as proposed in [2] which boils down to considering the dynamics in a frame with an earth-fixed origin having the x- and y-axis always oriented along the ship surge- and sway-axis

$$\begin{aligned} z_1 &= x\cos\psi + y\sin\psi \\ z_2 &= -x\sin\psi + y\cos\psi \\ z_3 &= \psi. \end{aligned}$$

The reference variables $z_{1,r}$, $z_{2,r}$, and $z_{3,r}$ are defined correspondingly. Next, we define the tracking errors

$$\begin{aligned} u_e &= u - u_r \\ v_e &= v - v_r \\ r_e &= r - r_r \\ z_{1,e} &= z_1 - z_{1,r} \\ z_{2,e} &= z_2 - z_{2,r} \\ z_{3,e} &= z_3 - z_{3,r}. \end{aligned}$$

In this way, we obtain the tracking error dynamics

$$\dot{u}_e = \frac{m_{22}}{m_{11}}(v_e r_e + v_e r_r(t) + v_r r_e) - \frac{d_{11}}{m_{11}}u_e + \frac{1}{m_{11}}(u_1 - u_{1,r}) \tag{9a}$$

$$\dot{v}_e = -\frac{m_{11}}{m_{22}}(u_e r_e + u_e r_r(t) + u_r r_e) - \frac{d_{22}}{m_{22}}v_e \tag{9b}$$

$$\dot{r}_e = \frac{m_{11} - m_{22}}{m_{33}}(u_e v_e + u_e v_r + u_r v_e) - \frac{d_{33}}{m_{33}}r_e + \frac{1}{m_{33}}(u_2 - u_{2,r}) \tag{9c}$$

$$\dot{z}_{1,e} = u_e + z_{2,e}r_e + z_{2,e}r_r(t) + z_{2,r}r_e \tag{9d}$$

$$\dot{z}_{2,e} = v_e - z_{1,e}r_e - z_{1,e}r_r(t) - z_{1,r}r_e \tag{9e}$$

$$\dot{z}_{3,e} = r_e. \tag{9f}$$

As in [10], we study the problem of stabilizing the tracking error dynamics (9).

Problem: Find appropriate state feedback laws u_1 and u_2 of the form

$$\begin{aligned} u_1 &= u_1(u, v, r, x, y, \psi, u_r, v_r, r_r, x_r, y_r, \psi_r) \\ u_2 &= u_2(u, v, r, x, y, \psi, u_r, v_r, r_r, x_r, y_r, \psi_r) \end{aligned} \tag{10}$$

such that the closed-loop trajectories of (9) and (10) are globally uniformly asymptotically stable.

IV. Controller Design

Our controller design aims at arriving at a closed-loop error dynamics of the form (3). To start with, we look for a way to obtain in closed loop a subsystem Σ_2, i.e., a subsystem (3b). In that light, it is good to remark that we can use one input for stabilization of a subsystem of the control system (9).

By defining the preliminary feedback

$$u_2 = u_{2,r} - (m_{11} - m_{22})(uv - u_r v_r) + d_{33}r_e + m_{33}\nu \tag{11}$$

where ν is a new input, the subsystem (9c) and (9f) reduces to the linear system

$$\dot{r}_e = \nu \qquad \dot{z}_{3,e} = r_e \tag{12}$$

which can easily be stabilized by choosing a suitable control law for ν, for example

$$\nu = -c_1 r_e - c_2 z_{3,e} \qquad c_1, c_2 > 0. \tag{13}$$

As a result, the subsystem (9c) and (9f) is rendered globally exponentially stable. In the closed-loop system this stabilized subsystem can be considered as the system Σ_2, i.e., the system (3b). Now one input is left that should be chosen such that the overall closed-loop system is rendered asymptotically stable.

We aim for a closed-loop system of the form (3). Besides, for asymptotic stability of the system (3) it is necessary that the part

$$\dot{z}_1 = f_1(t, z_1) \tag{14}$$

is asymptotically stable. This is something that should be guaranteed by the controller design. From Corollary 1 we furthermore know that it might be sufficient too! As a result, we can conclude that it might suffice in the controller design for the remaining input to render the part (14) asymptotically stable and "forget" about the $g(t, z_1, z_2)z_2$ part.

Notice that it is fairly easy to arrive from (3a) at (14), simply by substituting $z_2 \equiv 0$. This is also the way to proceed in the controller design. In the first step we designed a control law for one of the two inputs in such a way that in closed loop a subsystem was stabilized. Before we proceed with the controller design we assume that the stabilization of this subsystem worked out, i.e., we substitute $r_e \equiv 0$ and $z_{3,e} \equiv 0$ in (9a), (9b), (9d), (9e). This results in

$$\begin{aligned} \dot{u}_e &= \frac{m_{22}}{m_{11}}v_e r_r(t) - \frac{d_{11}}{m_{11}}u_e + \frac{1}{m_{11}}(u_1 - u_{1,r}) \\ \dot{v}_e &= -\frac{m_{11}}{m_{22}}u_e r_r(t) - \frac{d_{22}}{m_{22}}v_e \\ \dot{z}_{1,e} &= u_e + z_{2,e}r_r(t) \\ \dot{z}_{2,e} &= v_e - z_{1,e}r_r(t) \end{aligned}$$

which is a linear time-varying system

$$\begin{bmatrix} \dot{u}_e \\ \dot{v}_e \\ \dot{z}_{1,e} \\ \dot{z}_{2,e} \end{bmatrix} = \begin{bmatrix} -\frac{d_{11}}{m_{11}} & \frac{m_{22}}{m_{11}}r_r(t) & 0 & 0 \\ -\frac{m_{11}}{m_{22}}r_r(t) & -\frac{d_{22}}{m_{22}} & 0 & 0 \\ 1 & 0 & 0 & r_r(t) \\ 0 & 1 & -r_r(t) & 0 \end{bmatrix} \cdot \begin{bmatrix} u_e \\ v_e \\ z_{1,e} \\ z_{2,e} \end{bmatrix} + \begin{bmatrix} \frac{1}{m_{11}} \\ 0 \\ 0 \\ 0 \end{bmatrix} [u_1 - u_{1,r}]. \tag{15}$$

All that remains to be done, is to find a feedback controller for u_1 that stabilizes the system (15). It follows from Corollary 2 that the system (15) is uniformly completely controllable (UCC) if the reference yaw velocity $r_r(t)$ is persistently exciting. As a result, if the reference yaw velocity $r_r(t)$ is persistently exciting, we can use any of the control laws available in literature for stabilizing linear time-varying systems. In addition to these results, we propose the following control law.

Proposition 1: Consider the system (15) in closed loop with the control law

$$u_1 = u_{1,\,r} - k_1 u_e + k_2 r_r(t) v_e - k_3 z_{1,\,e} + k_4 r_r(t) z_{2,\,e} \quad (16)$$

where k_i ($i = 1, \ldots, 4$) satisfy

$$\begin{aligned} k_1 &> d_{22} - d_{11} \\ k_2 &= \frac{m_{22} k_4 (k_4 + k_1 + d_{11} - d_{22})}{d_{22} k_4 + m_{11} k_3} \\ 0 < k_3 &< (k_1 + d_{11} - d_{22}) \frac{d_{22}}{m_{11}} \\ k_4 &> 0. \end{aligned} \quad (17)$$

If $r_r(t)$ is persistently exciting then the closed-loop system (15) and (16) is globally exponentially stable.

Proof: See the Appendix. ■

Remark 2: Notice that the condition that $r_r(t)$ has to be persistently exciting appeared in the literature before. Not only in the paper [10] on tracking control of an underactuated ship, but also in the literature on tracking control of a mobile robot [15], [16], [19]. In these papers, the reference angular velocity had to be persistently exciting.

Remark 3: Notice that determining gains k_i ($i = 1, \ldots, 4$) which meet (17) is feasible. The gains k_1 and k_4 could for instance be chosen first. The condition on k_1 guarantees that a gain k_3 can be chosen. After choosing the gains k_1, k_3, and k_4, the required value for k_2 can be determined.

Combining the controllers (11), (13), and (16) we are now able to formulate the cascaded systems based solution to the tracking control problem:

Proposition 2: Consider the ship tracking error dynamics (9) in closed loop with the control law

$$u_1 = u_{1,r} - k_1 u_e + k_2 r_r(t) v_e - k_3 z_{1,e} + k_4 r_r(t) z_{2,e} \quad \text{(18a)}$$

$$\begin{aligned} u_2 = &\, u_{2,r} - (m_{11} - m_{22})(u_e v_e + v_r u_e + u_r v_e) \\ &- k_5 r_e - k_6 z_{3,e} \end{aligned} \quad \text{(18b)}$$

where

$$\begin{aligned} k_1 &> d_{22} - d_{11} \\ k_2 &= \frac{m_{22} k_4 (k_4 + k_1 + d_{11} - d_{22})}{d_{22} k_4 + m_{11} k_3} \\ 0 < k_3 &< (k_1 + d_{11} - d_{22}) \frac{d_{22}}{m_{11}} \\ k_4 &> 0 \\ k_5 &> -d_{33} \\ k_6 &> 0. \end{aligned}$$

If u_r, v_r, $z_{1,\,r}$ and $z_{2,\,r}$ are bounded and $r_r(t)$ is persistently exciting, then the closed-loop system (9) and (18) is globally $\mathcal{K}$-exponentially stable.

Proof: Due to the design, the closed-loop system (9) and (18) has a cascaded structure as shown in the equation at the bottom of the page. From Proposition 1 we know that the system $\dot{z}_1 = f_1(t, z_1)$ is globally exponentially stable and from standard linear control that the system $\dot{z}_2 = f_2(t, z_2)$ is globally exponentially stable. Furthermore, due to the fact that u_r, v_r, $z_{1,\,r}$, and $z_{2,\,r}$ are bounded, $g(t, z_1, z_2)$ satisfies (6). Applying Corollary 1 provides the desired result. ■

Remark 4: Notice that the only property of the system $\dot{z}_1 = f_1(t, z_1)$ that we need in this proof, is the fact that it is globally exponentially stable. Under the assumption that $r_r(t)$ is persistently exciting (which yields uniform complete controllability according to Corollary 2), more control laws for u_1 are available in literature that also guarantee global exponential stability of the system (15). In case we replace u_1 with any of these, the proof still holds. Therefore, several other choices for u_1 can be made. For instance, one might consider the following:

- a "standard" linear control law [16] which involves using the state-transition matrix of the system (15);
- a less complicated control law [which also needs the state-transition matrix of the system (15)] as presented by [20];
- a pole-placement based control law, like for instance the one presented by [21] [which requires the signals $\dot{r}_r(t)$, $\ddot{r}_r(t)$, and $(d^3 r_r/dt^3)(t)$ to be continuous and available], or any other control law one prefers that guarantees global exponential stability of the system (15).

$$\begin{bmatrix} \dot{u}_e \\ \dot{v}_e \\ \dot{z}_{1,e} \\ \dot{z}_{2,e} \end{bmatrix} = \underbrace{\begin{bmatrix} -\frac{k_1 + d_{11}}{m_{11}} & \frac{k_2 + m_{22}}{m_{11}} r_r(t) & -\frac{k_3}{m_{11}} & \frac{k_4}{m_{11}} r_r(t) \\ -\frac{m_{11}}{m_{22}} r_r(t) & -\frac{d_{22}}{m_{22}} & 0 & 0 \\ 1 & 0 & 0 & r_r(t) \\ 0 & 1 & -r_r(t) & 0 \end{bmatrix} \begin{bmatrix} u_e \\ v_e \\ z_{1,e} \\ z_{2,e} \end{bmatrix}}_{f_1(t,z_1)} + \underbrace{\begin{bmatrix} \frac{m_{22}}{m_{11}} (v_e + v_r) & 0 \\ -\frac{m_{11}}{m_{22}} (u_e + u_r) & 0 \\ z_{2,e} + z_{2,r} & 0 \\ -(z_{1,e} + z_{1,r}) & 0 \end{bmatrix}}_{g(t,z_1,z_2)} \begin{bmatrix} r_e \\ z_{3,e} \end{bmatrix}$$

$$\begin{bmatrix} \dot{r}_e \\ \dot{z}_{3,e} \end{bmatrix} = \underbrace{\begin{bmatrix} -\frac{d_{33} + k_5}{m_{33}} & -\frac{k_6}{m_{33}} \\ 1 & 0 \end{bmatrix} \begin{bmatrix} r_e \\ z_{3,e} \end{bmatrix}}_{f_2(t,z_2)}.$$

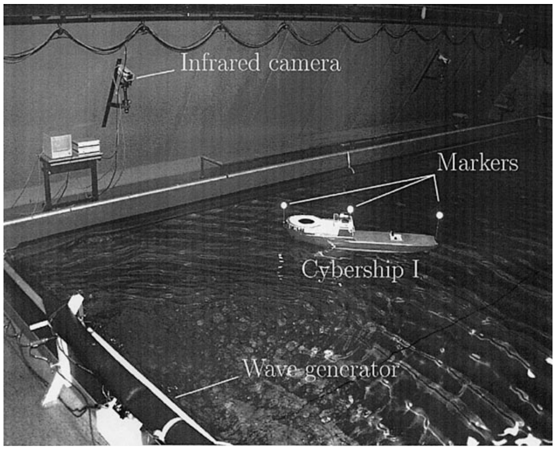

Fig. 2. Guidance, navigation, and control laboratory.

Similarly, any control law that renders the system (12) globally uniformly asymptotically stable can be used for ν.

Remark 5: In case $r_r(t)$ is constant (but not equal to zero), then the system (15) becomes a standard time-invariant linear system which can be stabilized by means of standard linear control theory.

Remark 6: As pointed out by [22], it is possible to normalize the system's equations in terms of the advancement velocity $|r_r(t)|$, in order to replace time by the distance gone by the reference vehicle. This "time normalization" makes the solutions "geometrically" unaffected by velocity changes, yielding convergence in terms of this distance, instead of time. In practice, this has the advantage that the damping rate does not change with different values of $r_r(t)$.

V. Experimental Results

To support our claims, we performed some experiments at the Guidance, Navigation and Control Laboratory located at the Department of Engineering Cybernetics, Norwegian University of Science and Technology, Trondheim, Norway, shown in Fig. 2. In the experiments we used Cybership I, which is a 1 : 70 scale model of an offshore supply vessel. The model ship has a length of 1.19 m, and a mass of 17.6 kg. The maximum surge force is approximately 0.9 N and the maximum yaw moment is approximately 0.9 Nm. The vessel moves in a 10-by-6 m pool with a depth of about 0.25 m.

Three spheres mounted on the model ship can be identified by infrared cameras. Three infrared cameras are mounted in such a way that (almost always) one or two cameras can see the boat. From each camera the positions of the spheres are transmitted via a serial line to a dSPACE signal processor (DSP). From these positions the ship position and orientation can be calculated. A nonlinear passive observer of [23] is used to estimate the unmeasured states. The estimates for position and velocities generated by this observer are used for feedback in the control law. No theoretical guarantee for a stable controller observer combination can be given (yet), as for nonlinear systems no general separation principle exists. However, in the experiments it turned out to work satisfactorily.

The control law and position estimates are implemented on a Pentium 166 MHz PC which is connected with the DSP via a dSPACE bus. By using Simulink blocks, the software is compiled and then downloaded into the DSP. The DSP sends the thruster commands to the ship via a radio-transmitter. The sampling frequency used in the experiments was 50 Hz.

The reference trajectory to be tracked was similar to that in [10], namely a circle with a radius of 1 m that should be tracked at a constant surge velocity of 0.05 m/s. From the initial reference state

$$\begin{aligned} u_r(0) &= 0.05 \text{ m/s} \\ v_r(0) &= 0 \text{ m/s} \\ r_r(0) &= 0.05 \text{ rad/s} \\ x_r(0) &= 4.75 \text{ m} \\ y_r(0) &= 3.5 \text{ m} \\ \psi_r(0) &= \pi \text{ rad} \end{aligned}$$

and the requirement

$$\begin{aligned} u_r(t) &= 0.05 \text{ m/s} \qquad \forall\, t \geq 0 \\ r_r(t) &= 0.05 \text{ rad/s} \qquad \forall\, t \geq 0 \end{aligned}$$

the reference trajectory $[u_r,\, v_r,\, r_r,\, x_r,\, y_r,\, \psi_r]^T$ can be generated, since it has to satisfy (8).

As in [10], we chose in the experiments not to cancel or compensate for the damping terms (i.e., assume $d_{11} = d_{33} = 0$), since these are restoring terms, and due to possible parameter uncertainties cancellations could result in destabilizing terms.

For tuning the gains of (18) we considered the two linear subsystems (12) and (15) that resulted from the cascaded analysis. Both can be expressed as a standard linear time-invariant system of the form $\dot{x} = Ax + Bu$. We used optimal control to arrive at the control law $u = -Kx$ for which the costs

$$\int_0^\infty \left(x(t)^T Q x(t) + u(t)^T R u(t)\right)\, dt$$

are minimized. For Q we chose a diagonal matrix with entries $q_{ii} = (1/\Delta x_i)$ $(i = 1, \ldots, 4)$, where Δx_i is the maximum error we would tolerate in x_i. For R we took the inverse of maximum allowed input. This resulted in the choice

$$Q = \begin{bmatrix} 10 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 20 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix} \qquad R = 1.1$$

for the system (15) and

$$Q = \begin{bmatrix} 50 & 0 \\ 0 & 50 \end{bmatrix} \qquad R = 1.1$$

for the system (12). In this way, we obtained the following gains for the control law:

$$u_1 = u_{1,r} - 10.28 u_e + 9.2 v_e - 4.44 z_{1,e} + 2.74 z_{2,e} \tag{19a}$$

$$\begin{aligned} u_2 = {} & u_{2,r} - (m_{11} - m_{22})(u_e v_e + v_r u_e + u_r v_e) \\ & - 9.02 r_e - 6.74 z_{3,e}. \end{aligned} \tag{19b}$$

The resulting performance of this controller is shown in Fig. 3. In the first two graphs, we compare the actual position of the ship with its desired position. The third graph contains the error

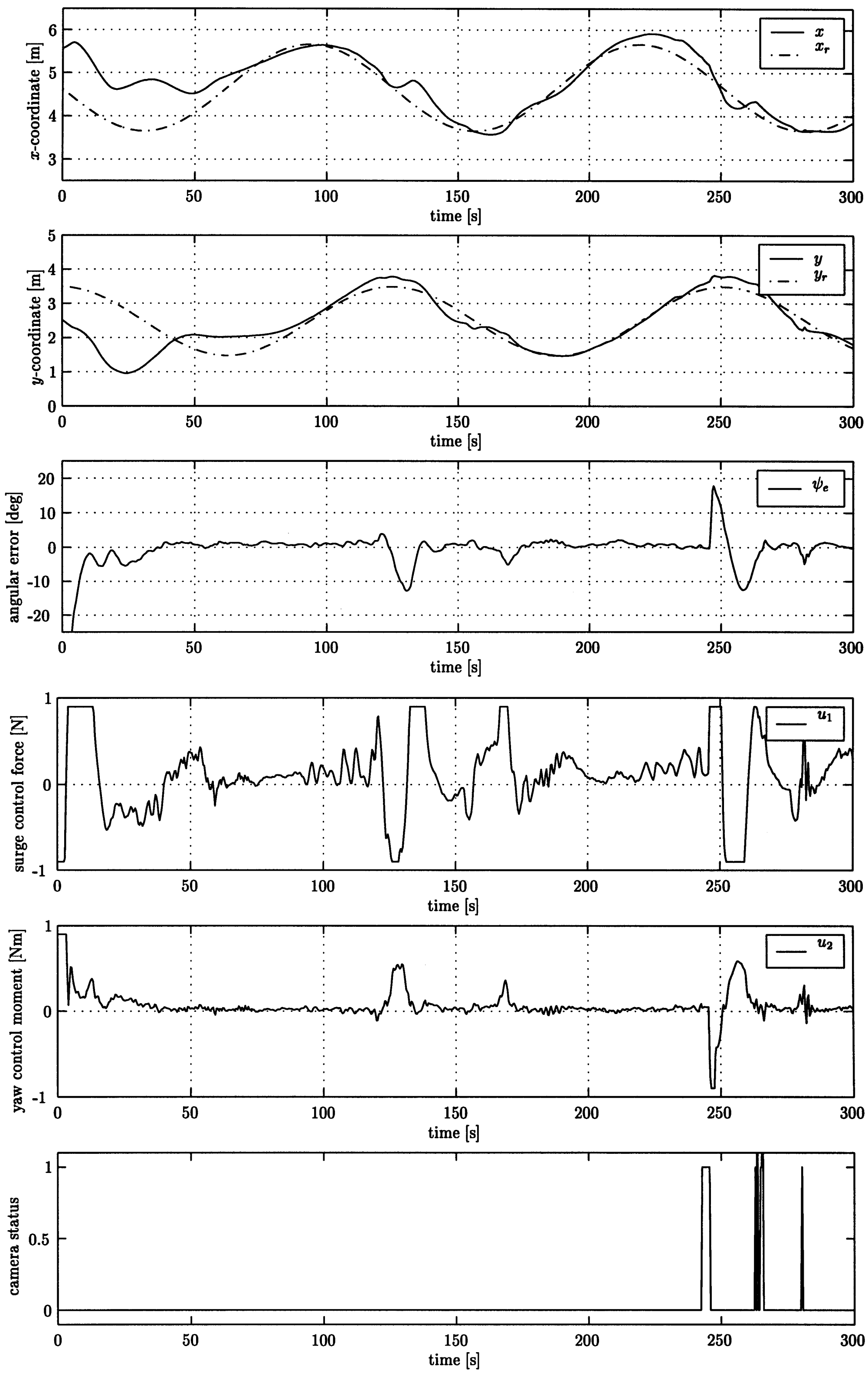

Fig. 3. Cascade controller (19) with gains based on optimal control.

in orientation. The fourth and fifth graph depict the controls applied to the ship. The bottom graph depicts the camera status. The reason for showing this is that the infrared cameras from time to time loose track of the ship. As long as the camera status equals zero we have position measurements from the camera-system, but as soon as the camera status is nonzero we no longer get correct position measurements. In Fig. 3, we can see that for instance after about 240 s we had a temporary failure of the camera-system. This explains the sudden change in the orientation error ψ_e and in the controls u_1 and u_2. Note, however, that

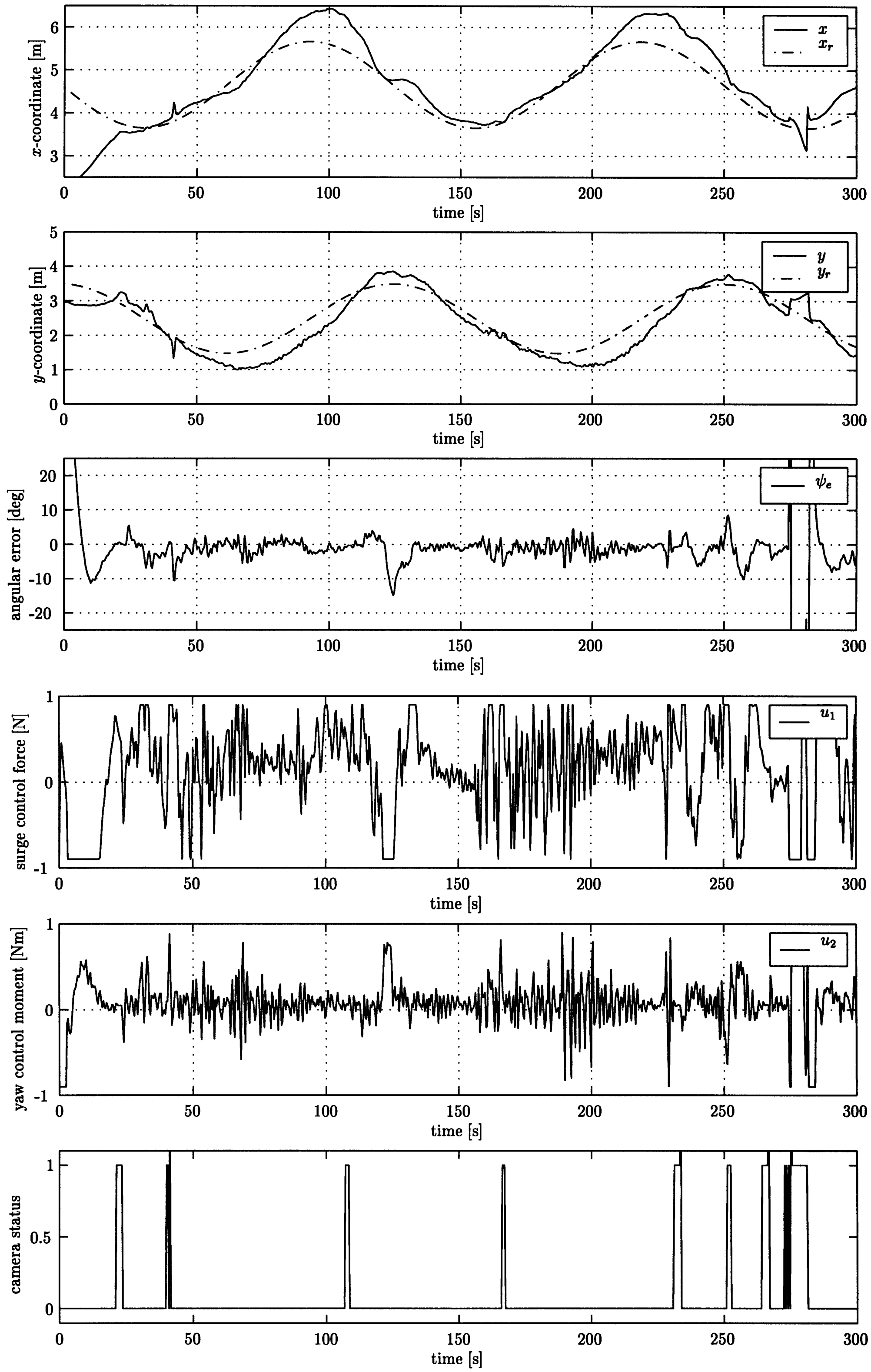

Fig. 4. Cascade controller (19) under disturbance of author walking through the pool.

the peaks in the error time evolution corresponding to camera failures mostly were due to observer estimation errors and not reflecting the actual ship behavior.

From the fact that the presented controller can be applied successfully in experiments, we might conclude that it possesses some robustness with respect to modeling errors and with respect to disturbances due to currents and wave drift forces.

To illustrate this robustness even more, we performed one experiment in which the first author was wearing boots and walking through the pool, trying to create as much waves as possible and disturbing the ship as much as he could. The results are depicted in Fig. 4. It can be noticed that, due to the heavy waves, the camera system had much more difficulties in keeping track of the ship. Nevertheless, a reasonable tracking performance was achieved.

VI. Concluding Remarks

In this paper, we studied the complete state-tracking problem for an underactuated ship that has only surge control force and yaw control moment, which is a common situation for many supply vessels.

By means of a cascaded approach we developed a global tracking controller for this tracking problem. The resulting control law has a very simple structure and guarantees global $\mathcal{K}$-exponential stability of the tracking error dynamics. The cascaded approach reduced the problem of stabilizing the nonlinear tracking error dynamics to two separate problems of stabilizing linear systems. This insight simplified the gain-tuning a lot, since optimal control could be used to arrive at suitable gains.

A disadvantage of both the cascade controller and the backstepping-based controller presented in [10] is the demand that the reference angular velocity does not tend to zero. Solutions to the tracking of a straight line are presented in [10] and [24]–[26].

The controller presented in this paper also proved to work reasonably well in experiments. This implies a certain robustness against modeling errors and disturbances due to currents and wave drift forces. In an attempt to get better robustness results, the cascaded approach might be helpful, as well, since robustness results from linear theory can be used.

Appendix

Before we prove Proposition 1, we first prove the following lemma.

Lemma 1: Let the following conditions be given:

$$k_1 > d_{22} - d_{11} \tag{20a}$$

$$k_2 = \frac{m_{22}k_4(k_4 + k_1 + d_{11} - d_{22})}{d_{22}k_4 + m_{11}k_3} \tag{20b}$$

$$0 < k_3 < (k_1 + d_{11} - d_{22})\,\frac{d_{22}}{m_{11}} \tag{20c}$$

$$k_4 > 0. \tag{20d}$$

Define λ and μ ($\lambda < \mu$) by means of

$$\lambda + \mu = \frac{k_1 + d_{11}}{m_{11}} \tag{21a}$$

$$\lambda\mu = \frac{k_3}{m_{11}} \tag{21b}$$

which is similar to saying that λ and μ are the roots of the polynomial

$$p(x) = m_{11}x^2 - (k_1 + d_{11})x + k_3. \tag{22}$$

Then λ and μ are well defined, and furthermore

$$0 < \mu - \lambda \tag{23a}$$

$$0 < d_{22} - m_{11}\lambda \tag{23b}$$

$$0 < m_{11}\mu - d_{22} \tag{23c}$$

$$0 < m_{11}^2\lambda\mu + d_{22}k_4 \tag{23d}$$

$$0 < k_4 + m_{11}\lambda \tag{23e}$$

$$0 < k_4 + m_{11}\mu. \tag{23f}$$

Proof: First, we remark that from (20a) and the fact that $d_{22} > 0$, $m_{11} > 0$ we have

$$0 < \frac{d_{22}}{m_{11}} < \frac{k_1 + d_{11}}{m_{11}}.$$

Consider the polynomial (22). Then, obviously

$$p(0) = p\left(\frac{k_1 + d_{11}}{m_{11}}\right) = k_3 > 0$$

and

$$\begin{aligned} p\left(\frac{d_{22}}{m_{11}}\right) &= m_{11}\left(\frac{d_{22}}{m_{11}}\right)^2 - (k_1 + d_{11})\,\frac{d_{22}}{m_{11}} + k_3 \\ &= (d_{22} - k_1 - d_{11})\,\frac{d_{22}}{m_{11}} + k_3 < 0. \end{aligned}$$

Therefore, from the intermediate value theorem we know that a constant λ exists, $0 < \lambda < d_{22}/m_{11}$, such that $p(\lambda) = 0$ and also a μ, $d_{22}/m_{11} < \mu < (k_1 + d_{11})/m_{11}$, such that $p(\mu) = 0$. As a result we obtain that λ and μ are well defined by means of (21). From (20) and

$$0 < \lambda < \frac{d_{22}}{m_{11}} < \mu$$

we can conclude that the inequalities (23) hold true. ■

Proof [Proof of Proposition 1]: The closed-loop system (15) and (16) is given by

$$\begin{bmatrix} \dot{u}_e \\ \dot{v}_e \\ \dot{z}_{1,e} \\ \dot{z}_{2,e} \end{bmatrix} = \begin{bmatrix} -\dfrac{k_1 + d_{11}}{m_{11}} & \dfrac{k_2 + m_{22}}{m_{11}}\,r_r(t) & -\dfrac{k_3}{m_{11}} & \dfrac{k_4}{m_{11}}\,r_r(t) \\ -\dfrac{m_{11}}{m_{22}}\,r_r(t) & -\dfrac{d_{22}}{m_{22}} & 0 & 0 \\ 1 & 0 & 0 & r_r(t) \\ 0 & 1 & -r_r(t) & 0 \end{bmatrix} \cdot \begin{bmatrix} u_e \\ v_e \\ z_{1,e} \\ z_{2,e} \end{bmatrix}. \tag{24}$$

If we define λ and μ as in (21) and use (20b), the closed-loop dynamics (24) can be expressed as shown in the equation at the top of the next page.

Using the change of coordinates

$$\begin{bmatrix} u_e \\ v_e \\ z_{1,e} \\ z_{2,e} \end{bmatrix} = \begin{bmatrix} 0 & 0 & \dfrac{\mu}{\mu - \lambda} & -\dfrac{\lambda}{\mu - \lambda} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \dfrac{-1}{\mu - \lambda} & \dfrac{1}{\mu - \lambda} \\ \dfrac{m_{22}}{d_{22}} & -\dfrac{m_{22}}{d_{22}} & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$\begin{bmatrix} \dot{u}_e \\ \dot{v}_e \\ \dot{z}_{1,e} \\ \dot{z}_{2,e} \end{bmatrix} = \begin{bmatrix} -(\lambda+\mu) & \dfrac{m_{22}(k_4+m_{11}\lambda)(k_4+m_{11}\mu)}{m_{11}(m_{11}^2\lambda\mu+d_{22}k_4)} r_r(t) & -\lambda\mu & \dfrac{k_4}{m_{11}} r_r(t) \\ -\dfrac{m_{11}}{m_{22}} r_r(t) & -\dfrac{d_{22}}{m_{22}} & 0 & 0 \\ 1 & 0 & 0 & r_r(t) \\ 0 & 1 & -r_r(t) & 0 \end{bmatrix} \begin{bmatrix} u_e \\ v_e \\ z_{1,e} \\ z_{2,e} \end{bmatrix}.$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -\dfrac{m_{11}\mu-d_{22}}{m_{22}(\mu-\lambda)} r_r & -\dfrac{d_{22}-m_{11}\lambda}{m_{22}(\mu-\lambda)} r_r \\ 0 & -\dfrac{d_{22}}{m_{22}} & -\dfrac{m_{11}\mu}{m_{22}(\mu-\lambda)} r_r & \dfrac{m_{11}\lambda}{m_{22}(\mu-\lambda)} r_r \\ \dfrac{m_{22}(k_4+m_{11}\lambda)}{m_{11}d_{22}} r_r & \dfrac{m_{22}(k_4+m_{11}\lambda)(d_{22}-m_{11}\lambda)\mu}{d_{22}(m_{11}^2\lambda\mu+d_{22}k_4)} r_r & -\mu & 0 \\ \dfrac{m_{22}(k_4+m_{11}\mu)}{m_{11}d_{22}} r_r & -\dfrac{m_{22}(k_4+m_{11}\mu)(m_{11}\mu-d_{22})\lambda}{d_{22}(m_{11}^2\lambda\mu+d_{22}k_4)} r_r & 0 & -\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}. \tag{25}$$

$$A(t) = \begin{bmatrix} 0 & 0 & -\dfrac{m_{11}\mu-d_{22}}{m_{22}(\mu-\lambda)} r_r & -\dfrac{d_{22}-m_{11}\lambda}{m_{22}(\mu-\lambda)} r_r \\ 0 & -\dfrac{d_{22}}{m_{22}} & -\dfrac{m_{11}\mu}{m_{22}(\mu-\lambda)} r_r & \dfrac{m_{11}\lambda}{m_{22}(\mu-\lambda)} r_r \\ \dfrac{m_{22}(k_4+m_{11}\lambda)}{m_{11}d_{22}} r_r & \dfrac{m_{22}(k_4+m_{11}\lambda)(d_{22}-m_{11}\lambda)\mu}{d_{22}(m_{11}^2\lambda\mu+d_{22}k_4)} r_r & -\mu & 0 \\ \dfrac{m_{22}(k_4+m_{11}\mu)}{m_{11}d_{22}} r_r & -\dfrac{m_{22}(k_4+m_{11}\mu)(m_{11}\mu-d_{22})\lambda}{d_{22}(m_{11}^2\lambda\mu+d_{22}k_4)} r_r & 0 & -\lambda \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & \sqrt{\dfrac{(m_{11}\mu-d_{22})(d_{22}-m_{11}\lambda)m_{22}}{m_{11}(m_{11}^2\lambda\mu+d_{22}k_4)}} & \sqrt{\dfrac{\mu(m_{11}\mu-d_{22})}{(\mu-\lambda)(k_4+\lambda)}} & \sqrt{\dfrac{\lambda(d_{22}-m_{11}\lambda)}{(\mu-\lambda)(k_4+m_{11}\mu)}} \end{bmatrix}$$

which, due to (23a), is well defined, we obtain (25) as shown at the top of the page.

Differentiating the positive definite [cf. (23)] Lyapunov-function candidate

$$V = \frac{m_{22}^2}{2m_{11}d_{22}} x_1^2 + \frac{m_{22}^2(d_{22}-m_{11}\lambda)(m_{11}\mu-d_{22})}{2m_{11}d_{22}(m_{11}^2\lambda\mu+d_{22}k_4)} x_2^2 + \frac{(m_{11}\mu-d_{22})}{2(k_4+m_{11}\lambda)(\mu-\lambda)} x_3^2 + \frac{(d_{22}-m_{11}\lambda)}{2(k_4+m_{11}\mu)(\mu-\lambda)} x_4^2$$

along solutions of (25) yields

$$\dot{V} = -\frac{(m_{11}\mu-d_{22})(d_{22}-m_{11}\lambda)m_{22}}{m_{11}(m_{11}^2\lambda\mu+d_{22}k_4)} x_2^2 - \frac{\mu(m_{11}\mu-d_{22})}{(\mu-\lambda)(k_4+\lambda)} x_3^2 - \frac{\lambda(d_{22}-m_{11}\lambda)}{(\mu-\lambda)(k_4+m_{11}\mu)} x_4^2$$

which is negative semidefinite [cf. (23)].

It is well known [13] that the origin of the system (25) is globally exponentially stable if the pair $(A(t), C)$, as shown in the equation at the top of the page, is uniformly completely observable (UCO). If $r_r(t)$ is persistently exciting, it follows from Corollary 2 that the pair $(A(t), C)$ is UCO, which completes the proof. ∎

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