

RESEARCH ARTICLE

On Architectural Practice and Arithmetic Abilities in Renaissance Italy

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The article examines the figure of the architect at work in Renaissance Italy, when a major change occurred in the practice of design with the spread of arithmetic. This deep scientific, technical, methodological, and cultural shift involved the image of the architect and his profession, his relationship with the patron, as well as the cultural conception of architecture.

The essay, crossing disciplinary boundaries, analyses some technical aspects of architectural design in early modern Italy only marginally investigated. If proportional systems and architecture's theoretical questions have been amply studied, the practical culture, the daily professional practice and its working tools, such as the operative arithmetic actually known to architects, have been only sporadically analysed.

During the Renaissance, especially in Italy, an important development of mathematics occurred and arithmetic was clarified and simplified so to allow its diffusion, but at the same time those disciplines remained essentially despised by aristocratic and intellectual elites. What was the architects' role in this moment of deep change? Which was the arithmetic usually employed by them in the design process? When did Hindu-Arabic numbers and fractions become familiar in the field of architecture? In the secular battle between geometry and arithmetic, which system was used in what professional cases?

The essay illustrates how architects with different backgrounds responded to this change, through a comparative analysis of all the architectural drawings containing numbers and calculations made by Michelangelo Buonarroti (1475–1564), Baldassarre Peruzzi (1481–1536), and Antonio da Sangallo the Younger (1484–1546).

Introduction

This article examines the figure of the architect at work in Renaissance Italy, when a major change occurred in the practice of design with the spread of arithmetic. This deep scientific, technical, methodological, and cultural shift, during which competing models coexisted over many decades in the fight between tradition and innovation, involved the image of the architect and his profession, his relationship with the patron, and the cultural conception of architecture. As is well known, proportional systems are an essential part of the design process in this period, and have been, along with architecture's theoretical questions, amply studied (see, for instance, Cohen and Delbeke 2014). However, the culture of daily professional practice and its working tools, such as the operative arithmetic actually known to architects, have received only sporadic attention. Therefore, this essay aims to bring to light this technical aspect of architectural design in Early Modern Italy through a systematic examination and a comparative analysis of all those drawings containing numbers and calculations made by three of the most representative Italian architects of the 16th century: Michelangelo Buonarroti

(1475–1564), Baldassarre Peruzzi (1481–1536), and Antonio da Sangallo the Younger (1484–1546). These cases, moreover, are particularly suited to this research because fortunately a very large corpus of drawings survives, many in the form of rough drafts. While presentation- and construction-drawings are rare, rapidly sketched preliminary studies, not intended to be seen by anyone other than the architect, are by far the most numerous among them, allowing us to catch the architect's thought while designing (Ackerman 1954: 8).

Renaissance Arithmetic

In the Renaissance, especially in Italy, there was an important development in the study of mathematics, and algebra was clarified and simplified enough to allow for its wider diffusion (Smith 1958, I: 242–264, 292–305). Compared with the rest of Europe, which remained tied to geometry and the graphic abacus, 16th-century Italy precociously adopted modern numeracy and algorism,¹ making it the centre of advanced speculative mathematical research until the end of the century, when it ceded this position to France (Nenci 2008: 627–640). Essential to the spread of this system was the publication of the *Summa de arithmetica, geometria, proporzioni e proporzionalità* in 1494 by Luca Pacioli – in reality, the result of an already well-defined process, the novelty of which consisted mainly in the systematization and

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completeness of its contents, rather than in its innovation *per se* (Pacioli 1993; Rouse Ball 1960: 173–176; Smith 1958, I: 252–253).² This was followed by many other abacus textbooks, such as *Nuovo lume* by the Siennese Giovanni Sfortunati, published in Venice in 1534 and very widely read, as well as texts by two other Siennese scholars, Pietro Cataneo and Dionigi Gori (Sfortunati 1534; Smith 1958, I: 306; Moscadelli 1991: 209–211; Ulivi 2008: 419–420; Cataneo 1546; Franci and Toti Rigatelli 1982; Gori 1984a; Gori 1984b).

Hindu-Arabic numerals were developed in India, and arrived in the West in the 12th century through Middle Eastern authors, such as the Persian al-Khwarizmi.³ They were introduced into Italy at the beginning of the 13th century by Fibonacci, but remained ignored and unappreciated by the intellectual elite and the university and ecclesiastical schools. However, in the secondary abacus schools that sprung up in commercial cities, where pupils between the ages of about twelve and sixteen, after learning reading and writing at the primary grammar school, received preparation for mercantile practice (abacus schools were also useful for artistic and professional training, and were sometimes attended by nobles, as well), these numerals were quickly adopted, and the Roman and Hindu-Arabic numeral traditions coexisted for a long time (Swetz 1987: 18–24; Grendler 1989: 22–33; Swetz 2002: 393).⁴ Even at the end of the 1400s, a mixture of Roman and Hindu-Arabic numerals was still used, and the transition from one system to the other was very gradual and irregular (Struik 1954: 103–106; Rouse Ball 1960: 155; Moscadelli 1991; Franci 1993: 62–67; Ulivi 2008).⁵

Particularly in Siena, applied science was notably developed by well-known masters and directly funded by the municipality as early as the 13th century (Franci and Toti Rigatelli 1981; Moscadelli 1991; Franci 1998: 125–133; Ulivi 2008: 404–406). While they were especially widespread in Tuscany, abacus schools also existed in Brescia, Genoa, Verona, and Venice. At the beginning of the 16th century, schools taught the Hindu-Arabic numeral system, the four operations – addition, subtraction, multiplication and an extensive study of division – fractions, square-roots, ratios, and the monetary system, in addition to practical solid and plane geometry (Goldthwaite 1972; Grendler 1989: 307–319; Ulivi 2008: 413–414).⁶

Bearing in mind the profound difference between the speculative mathematics of the scientists and the operative arithmetic commonly employed by professionals, did architects have an active role in the development and diffusion of algebra? What degree of confidence did they have in numbers, and how much arithmetic did they know and regularly employ in daily practice? In the century-old battle between geometrical models and numerical values, which system was most frequently adopted, and under what circumstances?⁷

Baldassarre Peruzzi

The systematic analysis of Peruzzi's drawings, examined alongside those of Michelangelo and Sangallo, shows that his are by far the most rich in numbers and calculations: in Sangallo's case, they appear occasionally, while

in Michelangelo's they are practically nonexistent. This observation indicates immediately the varying degree of confidence each architect had in the use of numbers. A drawing, for the plan of San Domenico in Siena (Fig. 1) is a good example of how calculations were an integral part of Peruzzi's design process: while drawing part of the plan, he sketched a small perspective and a bay of the elevation, calculating at the same time the project's dimensions. It is clear that calculation was extremely natural for Peruzzi, that he noted it down as if thinking aloud, and that it was something inseparable from his drawing in the creative process.⁸

Moreover, it is worth noting that he used Roman numerals exclusively in presentation drawings, and a good example of this habit is the well-known folio U 368Ar for Palazzo Massimo alle Colonne.⁹ Roman numerals, much like the use of water-colours and ancient terminology, were used particularly in the public sphere and in the rhetoric of relationships with patrons.¹⁰ A kind of demonstrative-didactic intent, frequently present in his survey drawings of ancient architecture, could also enter into these conventions. To cite just one example, drawing U 549Ar, with the survey of the ancient cornice in Piazza della Minerva in Rome, includes an explanatory note regarding the units of measurement used for the survey itself: 'measured with the Roman palm, divided into 8 inches, and every inch into four grains' ('misurata con palmo romano partito in once 8 e ognj oncja in quattro granj'). Conversely, in drawings done for himself, in which his reasoning about the project is made visible, he invariably adopted the Hindu-Arabic

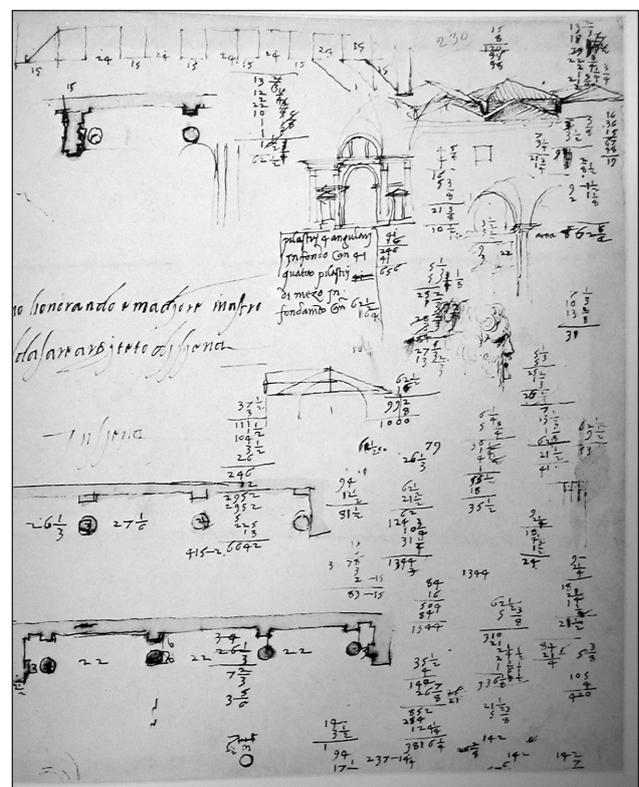


Figure 1: Baldassarre Peruzzi, Plan for the church of San Domenico in Siena, Firenze, Gabinetto dei disegni e stampe degli Uffizi, U 545Ar, detail.

system used by merchants. Working with Roman numerals is extremely difficult: he did not worry about philological virtuosity when dealing with design or other working aspects of his profession.

A more in-depth observation of Peruzzi's calculations allows an evaluation of his arithmetic ability and demonstrates how he moved with agility through long sums and single-digit multiplication. In a typical case for an architect-surveyor, such as calculating the surface of city walls – here, those of Orbetello (**Fig. 2**) – he proceeded to make the survey of the perimeter in an orderly way (A), then the linear sum of the measurements (B), and then the multiplication of the total for the height of the walls (C). There are also sums with fractions, as in folio U 531Ar, with the portal for Palazzo Massimo, which are rather complex operations compared to the decimal system, requiring the calculation of the lowest common denominator and simplifications of fractions. In this respect, one must bear in mind that the decimal system was adopted for the first time in 1585 by the Dutchman Simon Stevin – nearly half a century after Peruzzi's death – while the present system was not introduced until the 1600s. Therefore, none of the architects from the 1500s, from Peruzzi to Palladio, used decimals (Stevin 1965; Smith 1958, II: 235–246).

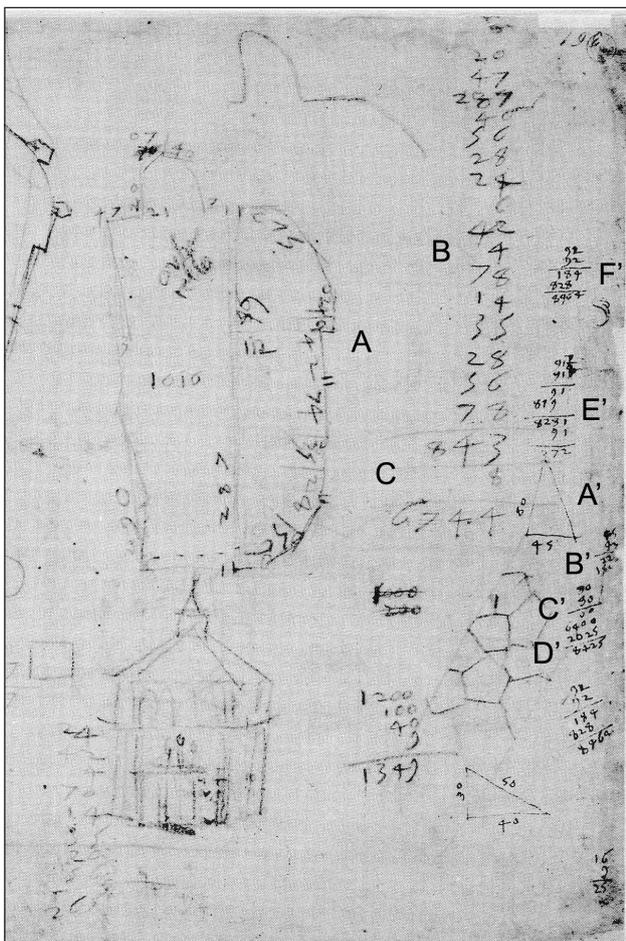


Figure 2: Baldassarre Peruzzi, Survey of the fortifications of Orbetello, Firenze, Gabinetto dei disegni e stampe degli Uffizi, U 361Av, detail.

Together with the abundant calculations in the Peruzzi's drawings, easy operations also appear, such as additions and elementary subtraction like $120 - 6 = 114$ in the aforementioned folio, which seem to indicate a clear preference for written rather than mental calculation.¹¹

More difficult single or double-digit multiplications, solved the same way as we do today, that is, with the placement-system, are found in many drawings, such as U 453Av, containing the plan of a house and various calculations. Here, again, operations that would seem to be automatic, like 100×83 , are solved at length, and there is even an error, perhaps due to distraction or Peruzzi's failing eyesight: in multiplying 4183×100 he mistook the 8 for a 6. The operations get more complicated when he needed to multiply with fractions, but Peruzzi managed to do it, even if some calculations were done with approximations. In **Figure 3**, a drawing for Saint Peter's, he had to multiply $352 \times 10 \frac{1}{3}$. Since the result of $352 \div 3$ is 117, with a remainder of 1, he rounded it up to 118, to then add the partial results and arrive at the total of the multiplication (there is also another possible 'visual' mistake on the same folio: multiplying 428×10 , he confused the 2 for a 9). Other quantities are multiplied by $10 \frac{1}{3}$, but addition continued to be preferred over multiplication. For Peruzzi, it was better to add a number three times than to multiply it by 3.

There is a surprise when one moves on to division. I wondered why I could not find anything that resembled modern long division. The absence of arithmetic symbols, introduced only at the end of the century and not regularly used when making calculations as personal notes, does not help. The reason is that during this period the most commonly used system to illustrate division was either the 'galley' or the 'boat' ('a galera' or 'per battello'), as Sfortunati teaches us (Sfortunati 1534: 20r–21v; Cataneo 1567: 15r; Rouse Ball 1960: 160–161; Smith 1958, II: 136–140).¹² This is the system that Peruzzi used, as one can see in the drawing for the cupola of the Duomo in Siena (**Fig. 4**): to cut a long story short, the dividend was put above and the divisor below, with the remainder under the line and the quotient on the right side (A). Although this is the only division problem I have been able to identify among Peruzzi's papers, it nevertheless demonstrates that he did, indeed, venture into the territory of two-digit division.

The drawing for the fortifications of Orbetello (**Fig. 2**) also shows Peruzzi confronting basic geometrical problems. He wanted to know the length of the hypotenuse of a triangle with sides of 45 and 80 (A'), and he clearly resorted to the Pythagorean theorem; so he proceeded to calculate the squares of the other two sides (B' and C': 45×45 and 80×80), then the sum of those squares, which works out at 8425 (D'). At this point there was the problem of solving the square root. Peruzzi proceeded very simply, by attempting first to square 91 (E') and then 92 (F'), where he stopped, because it seemed an adequate approximation to him. One will shortly see the way Sangallo dealt with an analogous situation.

Another classic geometry problem, constantly found in architecture, is the calculation of the circumference and, therefore, the value of π . The drawing for the cupola of the

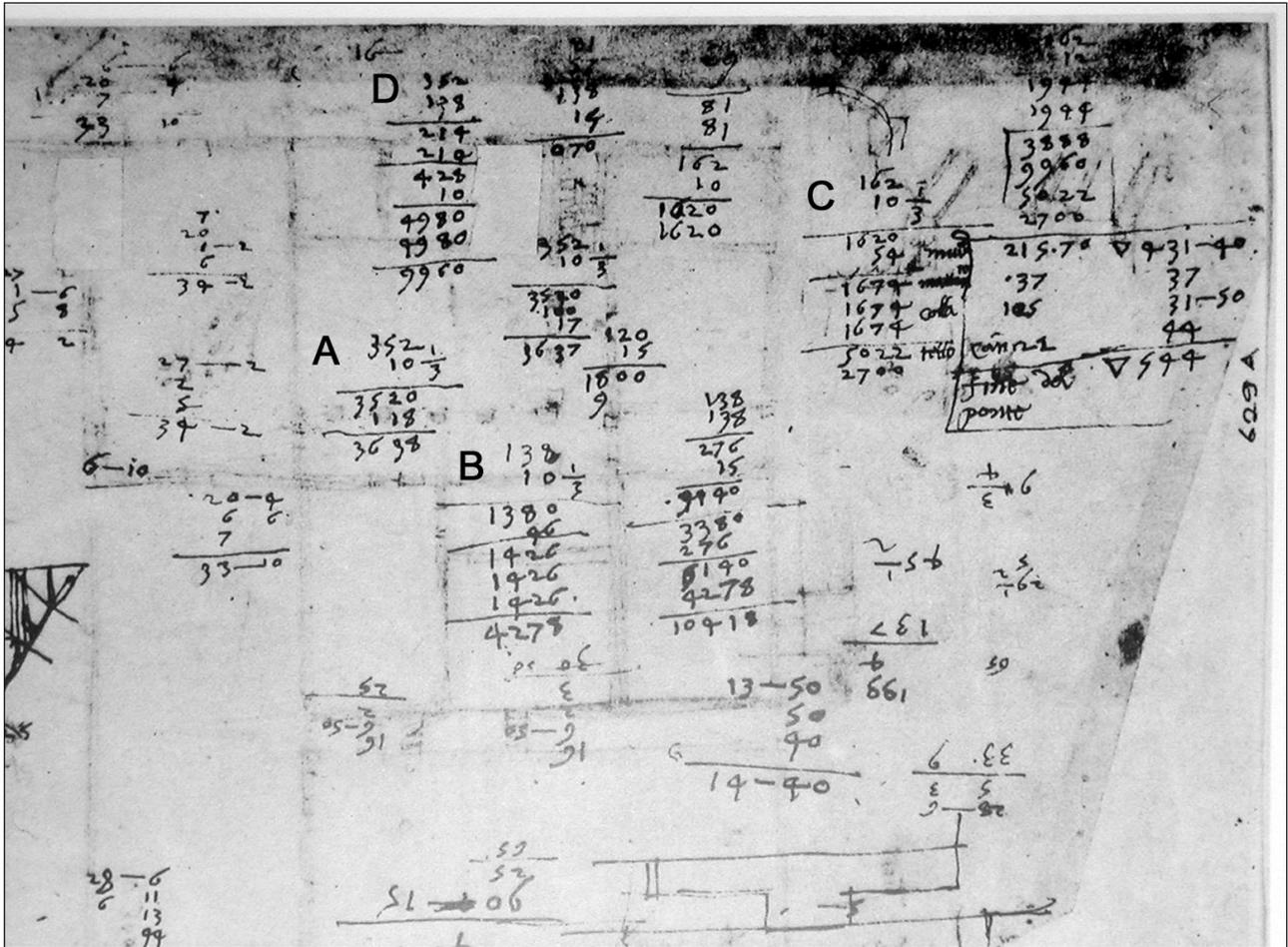


Figure 3: Baldassarre Peruzzi, Studies for Saint Peter's in Rome, Firenze, Gabinetto dei disegni e stampe degli Uffizi, U 629Av, detail.

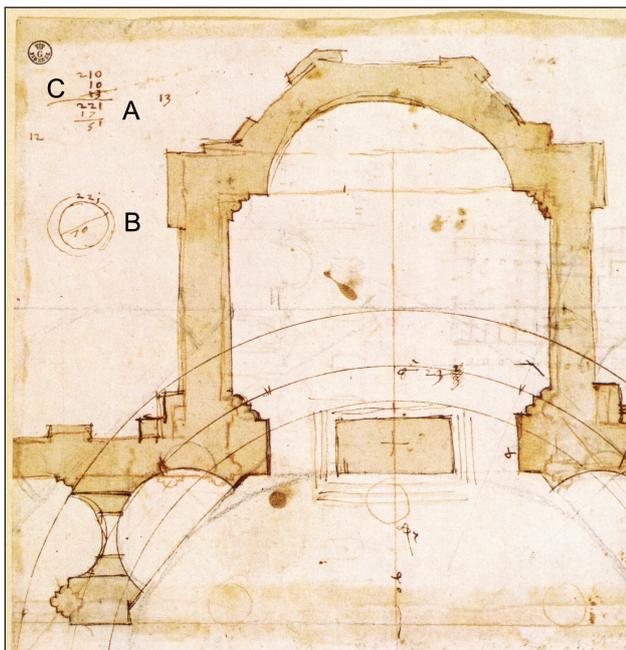


Figure 4: Baldassarre Peruzzi, Project for the dome of Siena's Cathedral, Firenze, Gabinetto dei disegni e stampe degli Uffizi, U 494Ar, detail.

Duomo in Siena (Fig. 4) shows the system Peruzzi adopted. The little note on the margin of the paper reveals that the cupola has a diameter of 70 and a circumference of 221 (B), but how did he get this result? Peruzzi adopted $3 \frac{1}{7}$ as the value of π . The result of $70 \times 3 \frac{1}{7}$ would have been 220, but evidently he rounded up the value to 221, perhaps knowing that the number can never be exact (C). Confirmation that he used $3 \frac{1}{7}$ as the value of π can be found on the verso of a drawing for Palazzo Ricci in Montepulciano (U 355Av), on which he did the same operation for a circumference with a diameter of 25, where that value is clearly shown. Now, this value for π was derived by Archimedes in the 3rd century BCE, but it returned to Europe only in the Middle Ages through the texts of Al-Khwarizmi, and was first cited by Pacioli. One can therefore suppose that it was the value normally used in abacus schools and so also in architectural practice (Pacioli 1993: 31r).¹³ It should be noted that Peruzzi did not consider even for a moment returning to the value proposed by Vitruvius (Vitruvius 1997: 1330–1333 [L. X, chap. 9, 1]), equivalent to $3 \frac{1}{8}$ (with this number, the value of the circumference would have been $218 \frac{3}{4}$), and applied the value commonly used for practical activities. Once more, he did not engage in philological virtuosity when it comes to professional practice.

desired perimeter, as Christof Thoenes says, ‘seeing that things are getting too complicated: Antonio gives up on calculations and changes method’ (‘pare che la cosa stesse diventando troppo complicata: Antonio rinuncia ai calcoli e cambia metodo’, Thoenes 1997: 194; see also Thoenes 2000: 101–103). He moved on to geometry, by which he derived the measurements, working them out directly on the drawing (U 798Ar; Thoenes 2000: 153).

Later, in the text inscribed in U 267Ar (still for the model of Saint Peter’s dome), Sangallo also illustrated the proposal for the cupola’s profile as semielliptical (transcription in Thoenes 2000: 129). Thoenes has amply discussed this drawing and proposed that the method with which the architect constructed this profile was *arithmetical* (Thoenes 1997: 192, 197), marking ‘the passage from the geometrical construction to the new Sangallesque method’ (‘il passaggio dalla costruzione geometrica al nuovo metodo sangallesco’, Thoenes 1997: 198; see also Thoenes 2000: 131 [‘the curve could be constructed by means of an arithmetic calculation’]). However, Sangallo’s written explanation is very close to the method illustrated in Dürer’s treatise *Underweysung der Messung mit dem*

Zirkel und Richtscheyt, explaining to masters and stone-masons how to construct an ellipse from a circumference *geometrically* (Dürer 1525: CIII, Fig. 33; Thoenes 1997: 197): following this method one can easily reach the same result for the dome without using any kind of calculation (Fig. 8). One can suggest that, rather than using a quite complicated arithmetical construction method, Sangallo employed this basic geometrical one instead, which, as Dürer’s treatise shows, was very likely the same that would have been widely used in the world of carpenters and masons in which he was trained.

Michelangelo Buonarroti

The case of Michelangelo Buonarroti is profoundly different from that of the two architects already considered, although the three were roughly contemporaries, and his procedures in arithmetic are even further removed from our current practices than theirs (see Ceriani Sebregondi 2013).

Michelangelo treated architectural drawings essentially like figure drawings, working on the page more or less freehand, without preliminary geometric construction, although sometimes with a few lines drawn with a stylus or lead pencil. He also showed great liberty regarding proportions, which he determined empirically with *ad sensum* adjustments, or by ‘judgment of the eye’ (‘giudizio dell’occhio’), following an essentially pictorial drawing and designing method. He relied almost exclusively on pencil to create rapid sketches, especially in his old age, with ideas and solutions superimposed one on top of the other in innumerable layers. His predominant use of this method, ill-adapted to defining detail, is symptomatic of the way he understood architectural drawing, for which the degree of precision obtainable with pen and ink rarely seemed necessary to him (Elam 2006: 51–54; Brothers 2006: 86–90; Brothers 2008, esp. chap I, II).¹⁵ Consequently, his drawings are not only free of calculations, but almost free of numbers, too, and this applies not just to his figure drawings – as one can imagine – but also to his architectural drawings. The only drawings that do contain measurements are a few pages regarding the dimensions or weights of stone blocks. Moreover, in those cases, the measurements are invariably written out in words rather than numerals: see, for example, the drawings for the Tomb of Julius II (CB 67Ar, 69Ar, 74Ar), those for San Lorenzo (AB, I, 139, 255v; CB 51Ar, 78Ar), and those for the Sagrestia Nuova (AB, I, 82, 225r). The presence of measurements is exclusively connected to the legal value of these drawings, which are contractual forms or acts of control and not part of the design process. This fact justifies the use of dimensions spelt out and not presented in numbers, as we continue to do today in similar official acts of documentation (one thinks of cheques). At the same time, one can be almost certain that these dimensions would not have been there, if they were not documents with such a precise objective.¹⁶

With respect to addition, Michelangelo used a rather laborious method that one can define as ‘accumulation’, whereby he indicated each unit with a sign and then proceeded to do the sum by counting all the signs (Maurer

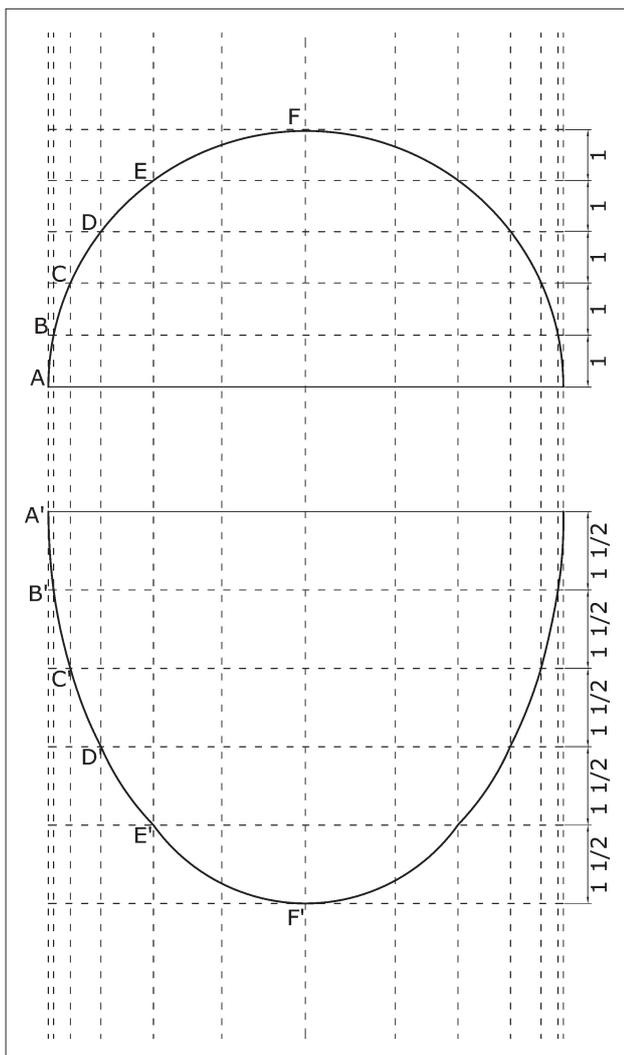


Figure 8: Geometrical construction of a semielliptical profile according to the method illustrated in Dürer’s treatise. Image created by author.

2004: 194–195), as in CB 68Ar; AB, I, 155, 276v (**Fig. 9**); or AB, XIII, 127r. In addition, in **Figure 9** (AB, I, 155, 276v), one can see how he also mixed Roman and Hindu-Arabic numerals: 646, the sum of all the signs on the left where he marked out the hundreds, is written with Roman hundreds and Hindu-Arabic tens. The systematic use of this method is also confirmed by AB, XIII, 127v: here Michelangelo had to add various repeated quantities and, instead of using multiplication and addition (in this case he could have done $2 + 31 \times 3 + 5 \times 2 + 19$), he proceeded to employ his own system of accumulation.

Then, when he had to multiply, Michelangelo resorted to an equally muddled visual-geometric system, constructing a grid with sides of as many units as there were values to multiply and then counting the number of quadrants: he thus transformed multiplication into accumulation. An example can be seen in **Figure 10**, in which he needed to calculate the surfaces areas of various walls: on the left-hand side, he wrote out in letters ‘wall twenty-eight arms long and six high’ (‘parete lunga venti octo braccia e alta sei’) and drew a 28×6 grid; there is also a 24×17 grid, while, on the right-hand side, there is another with 432 units. The result is again written above with the hundreds

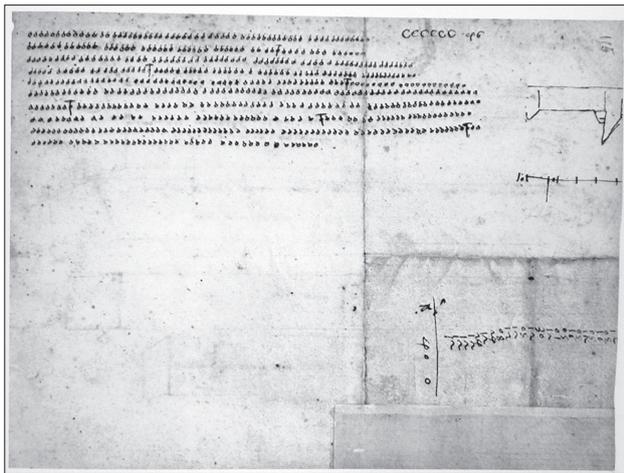


Figure 9: Michelangelo Buonarroti, ‘Accumulations’ and architectural sketches, Firenze, Casa Buonarroti, AB, I, 155, 276v, detail.

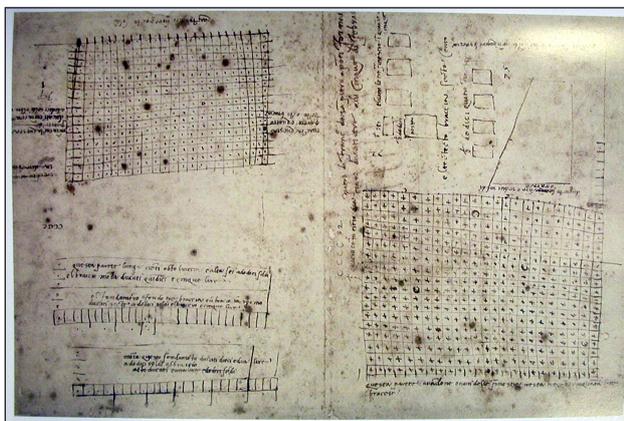


Figure 10: Michelangelo Buonarroti, ‘Accumulations’ and architectural studies, Firenze, Casa Buonarroti, CB 75Av, detail.

in Roman numerals and the tens in Hindu-Arabic numerals. The same thing also takes place with very elementary multiplication, such as $10 \times 7 = 70$ (AB, X, 627v). This was Michelangelo’s habitual method of calculating large numbers, found in many other drawings, and seems to indicate that he was not capable of doing multiplication arithmetically.¹⁷

There is no general consensus among scholars on this hypothesis.¹⁸ A few drawings show arithmetic calculations solved with a mastery of the Hindu-Arabic system, among which is **Figure 11**, for the façade of San Lorenzo. However, because of the handwriting, doubts exist about the author of these calculations. On the folio’s right-hand side, in fact, there is a note in Michelangelo’s own handwriting that indicates the measurements for a pillar, ‘fourteen arms high’, ‘two arms wide’, and ‘one and a half thick’ (‘alto braccia quattordici’, ‘largo braccia dua’ e ‘uno emezzo grosso’), all written strictly in letters. Underneath, one finds the grid system for the calculation $14 \times 2 \times 1 \frac{1}{2}$, which equals 14×3 , and, in fact, Michelangelo drew a 14×3 grid. It seems unlikely that the person who used this laborious system was the same one who, on the other half of the sheet, performed the rapid calculations that can also be seen on the back of the same page. One can confirm that assumption by comparing the handwriting with that on another documents, definitively attributed to Michelangelo, such as **Figure 12**. First of all, the Hindu-Arabic numerals in **Figure 11** seem to be written by a single hand (compare the 8, 3, and 7 in the pen and red pencil sections). Secondly, comparing these numbers with those in **Figure 12**, which is incontestably Michelangelo’s, clear differences appear, especially in the 2, 4, and 7. Michelangelo’s 2 and 7 have a very sharp, well-defined angle, while in **Figure 11** they are traced with a single, curved line, and the 2 is rotated horizontally; Michelangelo’s 4 is made with one stroke with a loop, while in **Figure 11** it is made with two separate strokes and without a loop.

In sum, the hand that had performed the calculation in **Figure 11** differs from that of Michelangelo, who had performed the accumulation on the same sheet. The same first hand of the person who was able to do arithmetical operations can also be recognized in other drawings. The accumulation system, universally employed by Michelangelo – I could not find any evidence of a calculation with Michelangelo’s handwriting in any of his drawings – is so laborious that it appears to be not a question of preference but of inability. A person who needed to make a grid of seventy cells and employed the accumulation method to operate the very easy multiplication $10 \times 7 = 70$ was unlikely to be able to solve the same problem with the algorism system.

Further proof that Michelangelo usually did not use Hindu-Arabic numerals can be found in AB, I, 127, 240r–241r, in which blocks of marble are drawn in red pencil with measurements in numbers. If the only unmistakable difference in the handwriting of the numbers is the round and horizontally rotated 2, the handwriting of the letters is still completely different from Michelangelo’s unique and easily distinguishable hand. Suffice it to note the presence of an ‘l’ with the loop and the ‘b’ with two

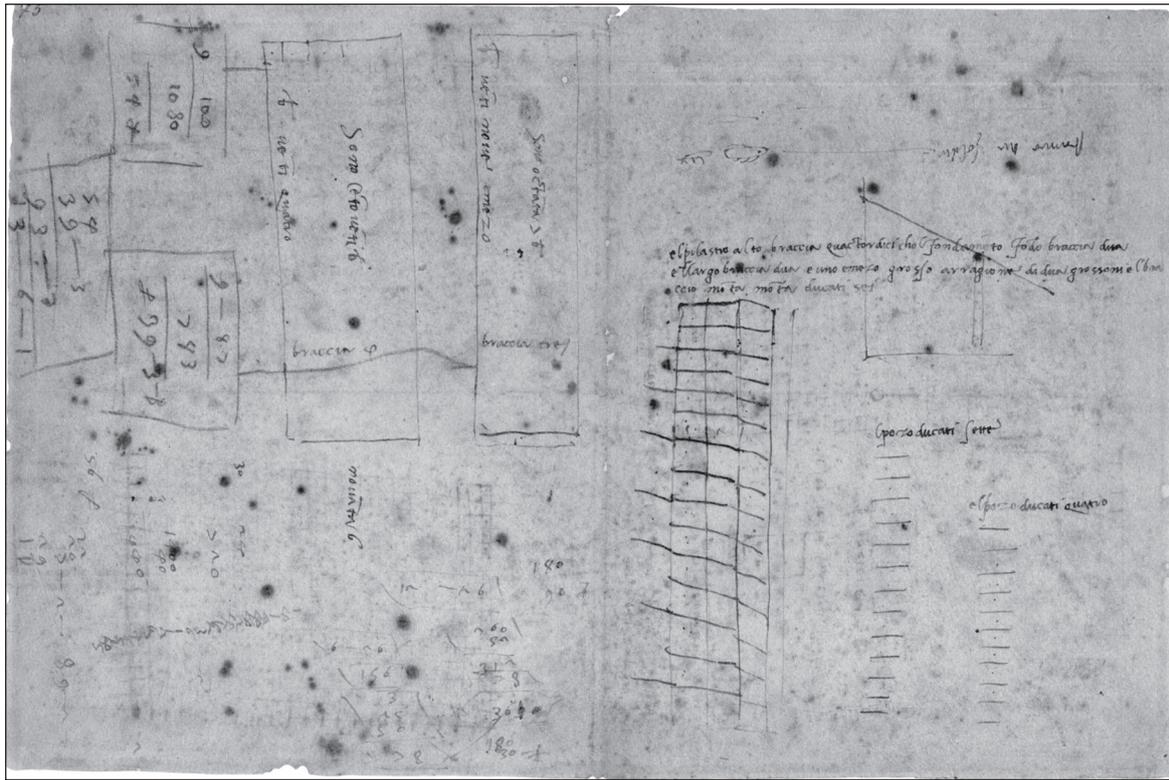


Figure 11: Michelangelo Buonarroti and a second hand, Architectural studies and calculations, Firenze, Casa Buonarroti, CB 75Ar, detail.

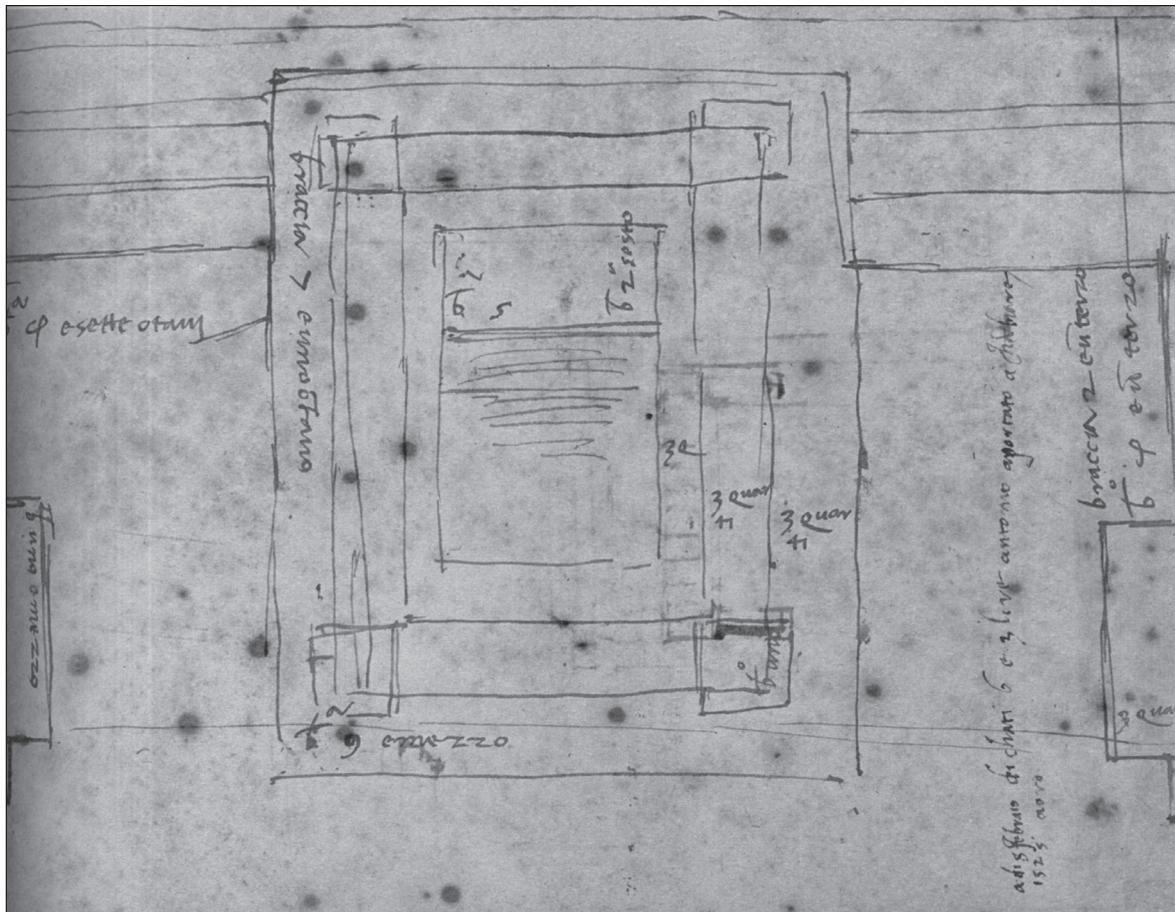


Figure 12: Michelangelo Buonarroti, Architectural studies, Firenze, Casa Buonarroti, CB 76Ar, detail.

loops and a flourish, elements that are completely absent from the master's handwriting. Also, the columnar addition in **Figure 9** does not seem to be in Michelangelo's hand: above all, the 2 has the same characteristics noted above. The same collaborator could have executed all of the above-mentioned examples of algorism. Conversely, in CB 50Av, where there is again the 'graphic calculation' system, with a 30×50 grid, one finds numbers with the same characteristics as Michelangelo's: the 2 with a sharp corner and the 4 with a loop. Further examples can be found in AB, I, 151, 269r–v, and CB 12Av, where there are columnar additions of money, aligned in the way typical of mercantile practice, indicated by Tolnay as 'not Michelangelo's calculations' (Tolnay 1978-80: 547r-v, 565v).¹⁹

Therefore, one can conclude that Hindu-Arabic numerals and algorism remained completely unfamiliar to Michelangelo, and something he used with insecurity, if at all, preferring instead to transform the problems into a visually controllable system. What is sometimes called Michelangelo's 'lack of interest' in numbers and measurements – in line with the idea that he approached architecture as a figure artist, concentrated on the perfection of the 'gesture' more than the process (Brothers 2006: 86–89; Brothers 2008: 4–6; Thoenes 2009: 26, 30, 36) – might actually be due to his insufficient acquaintance with arithmetic.

Michelangelo's avoidance of preliminary line and square constructions and proportional procedures, the usual practice in all Renaissance architecture, can also be read as a way of reinforcing his image as an artist, trying to distance himself as much as possible from the figure of the artisan, tied to practical, utilitarian activity. Michelangelo pursued this idea with increased stubbornness over the years. One of the most renowned and significant episodes in this respect dates to 1547, when he wrote to his nephew: 'I would like you to send Giovan Francesco [Ughi?] to measure the height of the cupola of Santa Maria del Fiore, from where the lantern starts down to the ground, and the height of the whole lantern, and send them to me; and mark on the letter the length of a third of the Florentine arm' ('vorrei che per mezzo di messer Giovan Francesco [Ughi?] tu avel' altezza della cupola di Santa Maria del Fiore, da dove comincia la lanterna insino a terra, e poi l'altezza di tuca la lanterna, e mandassimela; e mandami segnato in su la lettera un terzo del braccio fiorentino').²⁰ About a month later, he responded, disdainful and offended: 'you sent me a brass arm, as if I were a mason or carpenter who had to carry it around with him. I am ashamed of having it in my house and I gave it away!' ('tu mi mandasti un braccio d'octone, come se io fussi muratore o legnaiuolo che l'abbi a portare meco. Mi vergognai d'averlo in casa e dectilo via').²¹ He repeated the same idea in 1548, when he explicitly claimed, 'I was never a painter or sculptor like those who make a business out of it. I have always been careful to avoid that for the honour of my father and brothers' ('che io non fu' mai pictore né scultore come chi ne fa boctega. Sempre me ne sono guardato per l'onore di mie padre e de' mia frategli').²² Fundamentally, he wanted his activity to appear as if it were a nobleman's amusement. For example, in Condivi's account (much of

which should be considered as Michelangelo's autobiography) of the early commission of the statues for San Petronio, San Procolo, and an angel holding a candelabra in Bologna, Michelangelo appeared like 'a kind of gentleman artist who receives the assignment, by pure chance, thanks to the intervention of another sophisticated gentleman' (Zöllner 2008b: 22; Bellini 2011, I: 47).

Throughout his life, Michelangelo tried to reinforce his noble status in innumerable letters, especially in his later years and in correspondence with his hard-pressed nephew Lionardo, in which he underlined his membership in the aristocracy. In July 1540, for example, he wrote, 'I received three shirts with your letter, and I was amazed that you had sent them to me, because they are so large, and there is no peasant here' ('I' ò ricevuto con la tua lettera tre camice, e sonmi molto maravigliato me l'abbiate mandate, perché son sì grosse che qua non è contadino nessuno'). On April 14, 1543 he extended this theme: 'When you write to me, don't write: 'Michelangelo Simoni' or 'sculptor.' It is enough to say 'Michelagnol Buonarroti,' as I am known here' ('quando mi scrivi, non far nella sopra scripta: 'Michelangelo Simoni', né 'scultore'. Basta dir: 'Michelagnol Buonarroti', ché così son conosciuto qua'). And finally, on February 1, 1549: 'It is well known that we are ancient Florentine citizens and nobles as much as any other house' ('gli è noto che noi siano antichi cittadini fiorentini e nobili quante ogni altra casa', Buonarroti 1965–83, IV, 971: 108; 1009: 166; 1119: 310).²³

This strategy was put in practice, first, from the material point of view, as Michelangelo became enormously rich,²⁴ acquiring a palace for his family in Florence,²⁵ and advising his nephew on the type of marriage to contract (Buonarroti 1965–83, IV, 1091: 211); and second, from the point of view of Michelangelo's behaviour, emphasizing such aristocratic traits as his handwriting by transforming it from the 1400s mercantile script learned at school²⁶ into an elegant humanistic cursive – certainly by 1506–7, and probably as early as his arrival in Rome in 1502. He deliberately worked on acquiring a modern *all'antica* script that was sufficiently personal to be uniquely and unmistakably his.²⁷

I propose that his avoidance of learning and using Hindu-Arabic numerals and mercantile arithmetic, which were already ignored and even scorned by the intellectual elite and aristocratic culture of the time, was part of Michelangelo's broader strategy to align himself with the nobility. During his years of training, in fact, first as an apprentice painter with Ghirlandaio and then as a sculptor under the guidance of Bertoldo di Giovanni in the Giardino di San Marco, it seems likely that he did not attend an abacus school, which was normally frequented after primary education for learning reading and writing, education he received at Settignano among marble and stone cutters.²⁸ Both Vasari and Condivi confirm this hypothesis, the first saying that 'Lodovico [Michelangelo's father] had many children, and being badly-off and with a low income, he placed his sons in the wool and silk crafts, and Michelangelo, who was already grown up, was placed at Maestro Francesco da Urbino's grammar school, and, because his genius drew him to dabble in drawing, all the

time he could secretly snatch was taken up with drawing' ('crebbe col tempo in figliuoli assai Lodovico, et essendo male agiato e con poche entrate, andò accomodando all'arte della lana e seta i figliuoli; e Michelangelo, che era già cresciuto, fu posto con Maestro Francesco da Urbino alla scuola di grammatica; e perché l'ingegno suo lo tirava a dilettersi del disegno, tutto il tempo che poteva mettere di nascosto lo consumava nel disegnare'); the second sums it up with: 'He completely abandoned his studies' ('in tutto abbandonò le lettere') (Vasari 1962, I: 5–6; Condivi 2009: 13).²⁹ But from what has been discussed above, one can be sufficiently certain that subsequently he would not have wanted to learn this type of arithmetic either.

Conclusion

In answering the questions raised at the beginning, I have tried to demonstrate that Peruzzi and Sangallo most likely went to an abacus school in their youth, while Michelangelo did not. Even if Italian architects, particularly those who attended an abacus school, were at the forefront of Europe, and 16th-century Italy was the mathematical centre of excellence in general, one cannot claim that Italian architects had an active role in developing the discipline of mathematics; rather, they simply used their operative arithmetic knowledge well. Peruzzi, for example, was a learned architect and came from Siena, celebrated for its tradition of engineering and applied mathematics, but he did not show any theoretical interest in the field, except when it applied to problems faced in the daily design process. The same seems true for Sangallo the Younger: the idea that in the design process both followed a 'doctrine conceived in strictly technical terms, without any reference to philological-antiquarian culture' (Thoenes 1997: 197) seems to be amply confirmed, as I believe the case of the value of π demonstrates. There are, however, a few differences between these two figures. Although they had almost the same knowledge in algorism, Sangallo seems considerably more bound to the thousand-year-old geometric tradition, possibly because he came from the world of the building site and was trained as a carpenter,³⁰ while Peruzzi, perhaps due to the influence of local Siennese culture, seems more strongly oriented toward numbers and arithmetic, closely resembling the modern figure of the architect. With Peruzzi in particular, we can detect the presence of both a private system, which may come from his training at the abacus school and which he used for professional activity, and a public system for presentation and communication with patrons, tied to theoretical elaborations and to the didactic aspect, in which he continued to use Roman numerals and the proportional system.

The degree of confidence with numbers and operative arithmetic demonstrated by these two architects, though, seems quite high, apart from the issue of division, which appears to have remained mysterious to them.³¹ At the same time, however, the glaring case of Michelangelo shows that arithmetic knowledge was not yet completely established and its diffusion was very gradual and uneven, even if one could have expected general diffusion of Hindu-Arabic numerals and algorism given their introduction by Fibonacci as early as the 13th century.³²

It is not unlikely that Michelangelo's initial ignorance and then refusal of algorism displayed a certain level of snobbishness, which made him, also on this matter, closer to the nobility to which he so much wanted to belong.³³ In other words, he did not want to practice arithmetic, because he did not know how to do it, but also, and especially, because he did not want to know how to do it. Vasari's well-known story, according to which the artist said that 'one must have the compass in the eyes and not in the hand' ('bisognava avere le seste negli occhi e non in mano', Vasari 1568, VI: 109), illustrates Michelangelo's drawing method well and can be read in a new light to fit well with this other aspect of the question: no numbers for someone who judges everything with the eyes. So, one may ask: did Michelangelo, too, make a virtue of necessity?

Author's Note

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Notes

- ¹ This term indicates the practical, basic arithmetic, or art of computing. See Smith (1958, II: 9): 'Arithmetic based on the Hindu-Arabic numerals, more especially those that made use of the zero, came to be called algorism as distinct from the theoretical work with numbers that was still called arithmetic'.
- ² To evaluate the role this text played among other abacus treatises, see Ulivi (2008: 419) and Nenci (2008: 631–634).
- ³ Al-Khwarizmi lived at the beginning of the 9th century and the term 'algorithm' comes from his name.
- ⁴ For an overview on schooling in the rest of Europe, see Grendler (1990).
- ⁵ Struik (1954: 105, n. 3) underlines that it was only in 1494 that the Hindu-Arabic numerals completely substituted the Roman ones in the Medici accounts books. Berggren (2002: 361) confirms that 'not until the seventeenth century did ordinary people in Europe become familiar with the Hindu-Arabic system of numeration'. On 'suspicion' and 'reluctance' to employ the new numbers in the Middle Ages, see also Swetz (2002: 402–403).
- ⁶ A Florentine contract of 1519 for an abacus teacher, including the syllabus, is illustrated in Goldthwaite (1972), a pioneering study.
- ⁷ Some of these problems have been pointed out in Carpo (2003: 463). This author has investigated the shift from geometry to numeracy in Early Modern architectural treatises. I began working on the subject of the present article by analysing Baldassarre Peruzzi's drawings for the final presentation for the seminar 'Drawings and Numbers: Five Centuries of Digital

Design' led by Mario Carpo in 2002 at the Massachusetts Institute of Technology, Cambridge, MA.

Swetz (2002: 391) explains how 'the division between theoretical arithmetic and practical or applied arithmetic was conceived in the Ancient Greece world and perpetuated, modified and further institutionalized during the European Middle Ages'.

⁸ For an overview of his life and activity, see Ceriani Sebregondi (2015, forthcoming). For his activity as an architect in Siena in particular, see Ceriani Sebregondi (2014; 2011; 2008; 2007; 2007b).

⁹ The only other surviving drawings with Roman numerals are U 352Ar, for Palazzo Lambertini in Bologna, and U 355Ar, U 356Ar, U 357Ar, U 358Ar for Palazzo Ricci in Montepulciano.

¹⁰ Elam (2006: 57–58) also hints at this 'protocol' as an integral part of the relations between architects and Renaissance patrons.

¹¹ Wallace (1995: 103), too, notes this approach in Antonio da Sangallo the Younger's drawings, observing that mathematical operations that we would compute in our head are instead visualised on paper. For his comparison with Michelangelo, see note 18, below.

¹² Smith (1958, II: 136) confirms that 'by far the most common plan in use before 1600 is known as the galley, *batello*'. See Sfortunati (1534: 14v–21v) on the different methods of performing division. The 'a danda' method began to replace the galley method at the beginning of the 17th century (Sfortunati 1534: 17v–21v; Cataneo 1567: 15r–16v; Smith 1958, II: 141–144, esp. 141–142). On the history of other systems of processing divisions, see Smith (1958, II: 128–143), and, more generally, for the history of the terminology and the process of the four fundamental operations, see Smith (1958, II: 88–143).

¹³ This figure was still the value commonly used in professional practice in the 1600s – Francesco Righi, Francesco Borromini's assistant, uses it for the estimates for Sant'Agnes Church in Rome (Bellini 2004: 30).

¹⁴ Pagliara and Veronese (1994) and Veronese (1994), in their entries on Antonio da Sangallo the Younger's drawings with mathematical calculations and geometrical constructions, confirm it, and record only the presence of calculations involving addition, subtraction, and multiplication, noting the absence of division.

¹⁵ In any case, Bramante, Raphael, and Peruzzi also used pencil: the difference is mostly in Michelangelo's extensive and prevailing use of it.

¹⁶ On these drawings and their function for the San Lorenzo building site, see Wallace (1992) and Wallace (1994: 41–43). The drawings consist of two notebooks preserved at the Archivio Buonarroti, referring to the façade of San Lorenzo, with the measurements of the three dimensions of every block of marble in 'arms', mentioned in Michelangelo's letters as 'books' or 'memoranda'. The first one, drafted after January 1517 Lex Hermans 28 mei 2015 23:30, according to Wallace (1992: 126), was the method for ordering blocks from the supervisor of the quarry, and it survived because it was never sent, while the second one, drafted after

the beginning of 1518 according to Wallace (1992: 126), was probably either Michelangelo's personal copy or the rough copy of one later sent to the quarries. For our purposes, it is significant that in the latter only dimensions in letters appear, with sporadic use of single Hindu–Arabic numbers. Other similar drawings can be found in AB, I, 82, 223–233, related to the Sagrestia Nuova building site. This notebook, drafted in July 1521 according to Wallace (1992: 129), was a formal notarized deed, validated on the first page and on every sheet by the notary Galvano di ser Niccolò da Carrara. Therefore, it was not about blocks to order, but about blocks already quarried and delivered, with Michelangelo's sign and other sculpted trademarks, the three-dimensional measurements, as well as notations about irregularities or damage (226r, 230r for example). So it was one of Michelangelo's official memoranda, in which the measurements were therefore spelt out in letters without exception.

¹⁷ See also Wallace (1992: 122). Other examples can be found in CB 51Ar, 75Ar, 50Av; AB, I, 29, 72r. Carpo (2003: 469, n. 54) accordingly describes Michelangelo as 'almost completely innumerate', taking the same drawing 75Av as an example.

¹⁸ Tolnay, for instance, defines the Hindu–Arabic numerals in CB 17Av (Tolnay 579v) as 'calculations in Michelangelo's hand,' but, although they are certainly by his hand, they are actually just a list of numbers, not a calculation. Maurer (2004: 203) suggests that Michelangelo was in fact capable of solving problems like double-digit multiplication and that he did not use it because he found the 'visual' method more practical. Elam (2006: 55) cites a few drawings with calculations of the masses of walls and costs, implying that they are by Michelangelo. Wallace (1995: 103) states that he has found a combination of 'mathematical sophistication and literalism' in Michelangelo's drawings (in my view actually generated by two hands, as I try to demonstrate above).

¹⁹ Estimates, accounts, and expenses were, in fact, overseen by others in big building programs most of the time. See Wallace (1994: 87, 137–138) for the case of Michelangelo's work at San Lorenzo; pages 106–107, in the Medici Chapel; and pages 138–139, 178, at the Laurentian Library. In the hundreds of drawings by Sangallo for Saint Peter's, none contain references to costs or estimates of material, while in several of Peruzzi's drawings one can find cost calculations: further evidence of the high level of his numeracy that allowed him to consider economic factors during the design process itself (U 338Ar, U 339Ar, U 339Av, U 342Ar, U 343r, U 344Ar, U 344Av, U 345Ar, U 457Ar, U 545Ar for San Domenico in Siena; U 16Ar, U 18Ar for Saint Peter's; U 107Ar with a study for a church crossing; and U 629Ar, U 629Av with studies for a palace, Saint Peter's, and calculations).

²⁰ 30.7.1547, Michelangelo to Lionardo (Buonarroti 1965–83, IV, 1086: 271–272).

²¹ 3.9.1547, Michelangelo to Lionardo (Buonarroti 1965–83, IV, 1086: 274–275). Elam (1996) also reports the episode.

- ²² 2.5.1548, Michelangelo to Lionardo (Buonarroti 1965–83, IV, 1109: 299).
- ²³ Also in October 1542, he wrote to an unknown monsignor: 'I am a noble Florentine citizen, and the son of a respectable man' ('sono cittadino fiorentino, nobile, e figliolo d'omo dabbene', Buonarroti 1965–83, IV, 1001: 150–155). In any case, at least until 1523, he himself signed as 'sculptor': 1522–23, letters to Giovan Francesco Fattucci to and from Florence, signed 'Your very faithful sculptor in Via Moza' ('Vostro fedelissimo schultore in Via Moza', Bardeschi Ciulich 1989: 34).
- ²⁴ Michelangelo's visceral avarice is notorious by now, aimed at assuring the social rise of the Buonarroti family, often achieved with personal privations and by conducting a life of extreme poverty (Hatfield 2002). To 'reestablish' the honour and family dignity, and 'renew and expand the family line' ('rifare et acrescere la casa': 20.4.1545, Luigi del Riccio and Michelangelo to Lionardo, Buonarroti 1965–83, IV, 1041: 210–211), however, he could only count on his reluctant nephew Lionardo, his only heir.
- ²⁵ This question became one of his main preoccupations, especially in the 1540s and 50s, and his correspondence shows constant concern with Florentine real estate: the Buonarroti lineage was Florentine, and therefore, he had to concentrate his efforts there. See, for example, the letter to Lionardo of December 4, 1546: 'You must have received the letter I wrote you about buying a respectable house for one thousand five hundred or two thousand ecus, which should be in our Neighborhood, if possible. I say these things because *a respectable house in the city does us great honour, because it is more noticeable than owning land, and because we are also citizens of very noble descent*' ('Tu debbi aver ricevuta la lettera che ti scrissi del comperare una casa onorevole di mille cinquecento o duemila scudi e che sia nel Quartier nostro, se si può. Io dico questo, perché una casa onorevole nella città fa onore assai, perché si vede più che non fanno le possessione, e perché noi siam pure cittadini discesi di nobilissima stirpe', Buonarroti 1965–83, IV, 1070: 249–259; italics mines.)
- ²⁶ See the letter of July 1, 1497 to his father from Rome, in which he signed himself 'Michelagnuolo, sculptor in Rome' ('Michelagnuolo scultore in Roma', Bardeschi Ciulich 1989: 19).
- ²⁷ In his letters from Bologna from 1506–07, one can recognize the second phase of Michelangelo's youthful handwriting, in which the well-known Michelangelesque characters appear, including the 'ch' and 'c', elongated below the line; the 'q' cut off below the line; and the 'g', with its restrained flourish (Bardeschi Ciulich 1989: 20–22). Elam (2006: 63) also notes these peculiarities. In particular, the letters to Tommaso Cavalieri from 1523 to 1533 are distinguished by the very tidy, beautiful handwriting, with the same distance between the letters as in classical humanistic script, analogous to the later letters to Vittoria Colonna (Bardeschi Ciulich 1989: 48).
- ²⁸ On June 28, 1487, at only 12 years of age, he was already documented as Domenico Ghirlandaio's shop assis-

- tant, and April 1, 1488 he was cited as an apprentice painter, while between approximately 1490 and 1496 he was at the Giardino di San Marco at the expense of Lorenzo de' Medici, becoming a member of the family (Zöllner 2008a: 14, 17; Elam 1992: 159–170).
- ²⁹ Bardeschi Ciulich (1989: 13) recognizes from Michelangelo's way of writing that his knowledge of Latin was 'quite modest'.
- ³⁰ On the education and training of Antonio, see Bruschi (1983), Frommel (1994: 10–11), and Frommel (2000: 1). Carpo (2003: 468, n. 54) affirms that from the published papers Sangallo is evidently a talented mathematician, and very advanced in all the arts of algorism, although his use of numbers has so far received little attention. In the present essay I have instead tried to demonstrate that while he had the basic knowledge acquired in the abacus schools, he preferred to turn to geometry when the operations became too complicated for him, as Christof Thoenes also states.
- ³¹ Rouse Ball (1960 (1908): 159) comments that 'if multiplication was considered difficult, division was at first regarded as a feat which could be performed only by skilled mathematicians'; Smith (1958, II: 132) confirms that 'even in the 15th century it was commonly looked upon in the commercial training of the Italian boy as a hard matter. Pacioli (1494) remarked that 'if a man can divide well, everything else is easy, for all the rest is involved therein'. He consoles the learner, however, by a homily on 'the benefits of hard work'.
- ³² It seems that only from the end of the 17th century were architects across Europe comfortable with the use of numbers (Carpo 2003: 460).
- ³³ Grendler (1989: 311) explains that 'Latin schools almost always omitted mathematics and rejected *abbaco* completely. The Latin schools it ignored because it added nothing to the social status and career goals of their students. The Latin schools sought to train society's leaders', and Swetz (2002: 393) confirms this, reminding us how 'strong was this association of the new European mathematics with merchants' and that 'calculations with the new numerals now become the "mercantile art", *l'arte della merchadantia*. It was through this commercial use of numbers that the very concept of numbers itself changed'.

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