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Boundary states for moving D-branes*

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Abstract

We determine the boundary state for both the NS-NS and R-R sectors of superstring theory. We show how they are modified under a boost. The boosted boundary state is then used for computing the interaction of two D-branes moving with constant velocity reproducing with a completely different method a recent calculation by Bachas.

In the early years of string theory it was realized that the open string planar loop could be factorized in the closed string channel in which a closed string is disappearing in the vacuum [1, 2]. The boundary states were originally introduced [3] to write the planar loop for the oriented string with gauge group SU(N) in the following very simple form[1]:

$$(2\pi)^d \, \delta^{(d)}\left(\sum_{i=1}^{M} p_i\right) A_P(p_1 \ldots p_M) = \frac{N}{(2\alpha')^{d/2}} \frac{\langle B_0|D|B_M\rangle}{\pi} \ . \tag{1}$$

The boundary state with open strings attached on it is given by

$$|B_M\rangle = |B\rangle_{\rm gh} \sum_{\lambda,\mu} |\lambda\rangle_a |\hat{\mu}\rangle_{\tilde{a}} |p=0\rangle \frac{\langle \mu, p=0|T_M|\lambda, p=0\rangle}{\langle p=0|p=0\rangle} \ , \tag{2}$$

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[1]We use the conventions of Ref. [4] with $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, \ldots, 1)$ and we denote $|\hat{\lambda}\rangle = (-1)^{\sum_{n=1}^{\infty} a_n^\dagger \cdot a_n} |\lambda\rangle$.

where

$$T_M = \mathrm{Tr}\left(\lambda^{a_1}\dots\lambda^{a_M}\right)\left[2g_S\left(2\alpha'\right)^{\frac{d-2}{4}}\right]^M$$
$$\times(-2\pi\mathrm{i})^M\int_0^1 \mathrm{d}\nu_M\int_0^{\nu_M}\mathrm{d}\nu_{M-1}\dots\int_0^{\nu_3} d\nu_2\prod_{i=1}^M\left(\frac{V_i(p_i,\mathrm{e}^{2\pi\mathrm{i}\nu_i})}{\mathrm{e}^{2\pi\mathrm{i}\nu_i}}\right)\ . \qquad (3)$$

The vertex operators V_i correspond to the emitted open string states. The state $|p=0\rangle$ is the state with zero momentum and $|\lambda\rangle$ and $|\mu\rangle$ are arbitrary states with norm equal to one. Finally $|B\rangle_{\mathrm{gh}}$ is the ghost part of the boundary state that can be found in Ref. [5]. When no open string is attached to the boundary Eq. (2) becomes

$$\langle B_0| = {}_{\mathrm{gh}}\langle B|\langle p=0|\sum_\lambda {}_{\tilde{a}}\langle\hat{\lambda}|_a\langle\lambda| = {}_{\mathrm{gh}}\langle B|\langle p=0|\ {}_{\tilde{a}}\langle 0|_a\langle 0|\mathrm{e}^{-\sum_{n=1}^\infty a_n\cdot\tilde{a}_n}\ . \qquad (4)$$

In the one loop case the ghost part has the only effect of eliminating in the partition function two powers of the 26 ones coming from the contribution of the non-zero modes of the string coordinates and of shifting by one the intercept of the Regge trajectory in the propagator. Therefore for our purposes in this paper we can limit ourselves to the string coordinates provided that we subtract two powers of the 26 ones coming from the non-zero modes of the string coordinates and that we use the following closed string propagator[2]:

$$D = \frac{1}{2\pi}\int_{|z|\le 1}\frac{\mathrm{d}^2 z}{|z|^2} z^{L_0-1}\bar{z}^{\tilde{L}_0-1}\ , \qquad (5)$$

with the integral being done on the unit disk.

The formalism of the boundary state was subsequently and extensively developed to include a slowly varying external gauge field [5, 6], the superstring [7, 8] and Dirichlet boundary conditions [9]. It has been recently used in Refs. [10, 11, 12] to study the properties of D-branes, but the authors of the above references have mostly limited themselves to consider only the massless content of the boundary state. It has also been discussed in the framework of the light-cone quantization [13] and of the D0-brane dynamics [14, 15].

It can be checked that the boundary state is a physical BRST invariant state. This means that a closed string theory contains also these physical states in addition to the BRST invariant perturbative string states. They can for instance be used to compute open string multiloop diagrams by saturating the N-string closed string vertex with N of them. We will see later on that these BRST invariant states can be generalized to satisfy also Dirichlet boundary conditions and in this case they will be describing D-branes [16].

In this short note we explicitly construct the D-brane boundary state for the bosonic string and for both the NS-NS and the R-R sectors of the superstring. We

[2]When dealing with the superstring, we have $d=10$ and the intercepts in the closed string propagators are $(1/2,1/2)$ for the NS-NS sector and $(0,0)$ for the R-R sector.

then boost these states with a Lorentz transformation obtaining a boundary state describing a D-brane moving with constant velocity. Finally we use these boosted states to compute the vacuum energy obtaining precisely the expression previously found by Bachas [17] using open strings. This alternative looks promising and opens the way to more complicated situations, for example the scattering of more than two branes, higher loop corrections to the potential between two D-branes, D-branes excitations, *etc.*

In the case of a D-brane the boundary state must be suitably modified. The components longitudinal to the D p-brane (that we will sometimes denote as $\|$) and the time component are treated as in the previous case, and we will reserve the indices α and β for them. For the transverse components (denoted by $\perp$, or by the indices i, j and k) we must change the sign of the operators $\tilde{a}$, corresponding to Dirichlet boundary conditions for the open string. With these modifications the boundary state in Eq. (4) becomes [11]

$$|B, y\rangle = (2\pi\sqrt{\alpha'})^{d_\perp}\, \delta^{(d_\perp)}(q^\perp - y^\perp)\, \mathrm{e}^{-\sum_{n=1}^{\infty} a_n^\dagger \cdot \tilde{a}_n^\dagger} |0\rangle_a |0\rangle_{\tilde{a}} |p = 0\rangle \ , \tag{6}$$

where the position y of the D-brane is fixed by a δ-function in $d_\perp = d - p - 1$ dimensions. We omit to write the boundary state for the ghost degrees of freedom since its effect for our calculation will be exactly the same as in the case of the planar loop. As already discussed the non-zero mode part in the previous equation should be understood as follows:

$$a_n^\dagger \cdot \tilde{a}_n^\dagger = \eta_{\alpha\beta} a_n^{\dagger\alpha} a_n^{\dagger\beta} - \delta_{ij} a_n^{\dagger i} a_n^{\dagger j} \ . \tag{7}$$

The boundary state (6) satisfies the conditions [11, 12, 13]

$$\partial_\tau X^\alpha|_{\tau=0}\, |B, y\rangle = \partial_\sigma X^i|_{\tau=0}\, |B, y\rangle = 0 \ . \tag{8}$$

In addition, it is crucial in our analysis to fix the location of the D-brane by imposing an extra condition on the position operator in the transverse direction corresponding to the δ-function in Eq. (6). Thus, the boundary state (6) is characterized by the conditions

$$P^\alpha|_{\tau=0}\, |B, y\rangle = 0 \ , \qquad\qquad (X^i - y^i)|_{\tau=0}\, |B, y\rangle = 0 \tag{9}$$

on its longitudinal momentum and transverse coordinates.

The D-brane can be boosted by applying to the boundary state the boost operator

$$|B, y, \mathbf{v}\rangle = \mathrm{e}^{\mathrm{i} v^j J^0{}_j}\, |B, {}^{(\mathbf{v})}y\rangle \ , \tag{10}$$

where the "velocity" $\mathbf{v}$ is taken transverse to the D-brane, ${}^{(\mathbf{v})}y^i = y^i + v^i(v_j y^j)(\cosh|\mathbf{v}| - 1)/\mathbf{v}^2$ is the boosted position of the D-brane and the generator of the Lorentz transformation is equal to

$$J^{\mu\nu} = q^\mu p^\nu - q^\nu p^\mu - \mathrm{i} \sum_{n=1}^{\infty} \left(a_n^{\dagger\mu} a_n^\nu - a_n^{\dagger\nu} a_n^\mu + \tilde{a}_n^{\dagger\mu} \tilde{a}_n^\nu - \tilde{a}_n^{\dagger\nu} \tilde{a}_n^\mu \right) \ . \tag{11}$$

Performing the boost on the non-zero modes we find

$$\mathrm{e}^{\mathrm{i}v^j J^0{}_j} a_n^\dagger \cdot \tilde{a}_n^\dagger \mathrm{e}^{-\mathrm{i}v^j J^0{}_j} = a_n^{\dagger\|} \cdot \tilde{a}_n^{\dagger\|} - \sum_{k\neq j} a_n^{\dagger k}\tilde{a}_n^{\dagger k} - \begin{pmatrix} a_n^{\dagger 0} & a_n^{\dagger j} \end{pmatrix} M(\mathbf{v})^2 \begin{pmatrix} \tilde{a}_n^{\dagger 0} \\ \tilde{a}_n^{\dagger j} \end{pmatrix} , \quad (12)$$

where

$$M(\mathbf{v}) = \begin{pmatrix} \cosh v & n^k \sinh v \\ n^j \sinh v & \delta^{jk} + n^j n^k (\cosh v - 1) \end{pmatrix} , \quad (13)$$

with $v \equiv |\mathbf{v}|$ and $n^j \equiv v^j/|\mathbf{v}|$. The action of the boost on the zero modes is also easily computed and one gets

$$\mathrm{e}^{\mathrm{i}v^j(q^0 p_j - q_j p^0)} \mathrm{e}^{\mathrm{i}q^\perp \cdot Q^\perp} \mathrm{e}^{-\mathrm{i}v^j(q^0 p_j - q_j p^0)} = \mathrm{e}^{\mathrm{i}Q_j \left\{ q^j + n^j q^0 \sinh v + n^j (n\cdot q)(\cosh v - 1)\right\}} . \quad (14)$$

Remembering that the physical velocity V of the brane is related to v through the relation $V = \tanh v$, the previous expression becomes

$$\mathrm{e}^{\mathrm{i}Q_j \left\{ q^j + n^j q^0 \sqrt{\frac{V^2}{1-V^2}} + n^j (n\cdot q)\left[\frac{1}{\sqrt{1-V^2}} - 1\right]\right\}} . \quad (15)$$

We choose now to boost in one of the transverse directions, that we call i, and we fix $n^i = -1$ and $n^j = 0$ for $j \neq i$. Using Eq. (15) the δ-function in the boundary state gets modified as follows:

$$\delta^{(d_\perp)}(q^\perp - y^\perp) \to \sqrt{1-V^2}\, \delta(q^i - q^0 V - y^i) \prod_{j\neq i} \delta(q^j - y^j) . \quad (16)$$

Notice that the boost of the zero mode part induces a normalization factor containing the physical velocity of the D-brane and a modification of the δ-function $\delta(q^i - y^i)$. Thus the boosted boundary state becomes

$$|B, y, v\rangle = (2\pi\sqrt{\alpha'})^{d_\perp} \sqrt{1-V^2}\, \delta(q^i - q^0 V - y^i) \prod_{j\neq i} \delta(q^j - y^j)$$
$$\times \prod_{n=1}^{\infty} \mathrm{e}^{-\left\{ a_n^{\dagger\|}\cdot\tilde{a}_n^{\dagger\|} - \sum_{j\neq i} a_n^{\dagger j}\tilde{a}_n^{\dagger j} - \begin{pmatrix} a_n^{\dagger 0} & a_n^{\dagger i}\end{pmatrix} M(v)^2 \begin{pmatrix} \tilde{a}_n^{\dagger 0} \\ \tilde{a}_n^{\dagger i}\end{pmatrix}\right\}} |p=0\rangle |0\rangle_a |0\rangle_{\tilde{a}} , \quad (17)$$

where $M(v)$ is the matrix in Eq. (13) with the above choice for n^j. The non-zero mode structure of the vertex agrees with the expression found in Ref. [11]. The Born-Infeld factor $\sqrt{1-V^2}$ is the Lorentz contraction factor taken out of the correctly boosted δ-function. We will see that this boosted δ-function is essential to reproduce the result of Ref. [17].

The next step is to compute the matrix element $\langle B, y_1, v_1 | D | B, y_2, v_2\rangle$, where the closed string propagator is given in Eq. (5) and the "velocities" v_1 and v_2 are both taken in the common direction i. The contribution of the non-zero modes and the ghosts is equal to

$$\prod_{n=1}^{\infty} \left[(1-(z\bar{z})^n)^{-(d-2)} \frac{(1-(z\bar{z})^n)^2}{[1-\mathrm{e}^{2\pi\epsilon}(z\bar{z})^n][1-\mathrm{e}^{-2\pi\epsilon}(z\bar{z})^n]}\right] , \quad (18)$$

with $\pi\epsilon \equiv |v_1 - v_2|$. The contribution of the zero modes is equal instead to

$$(2\pi\sqrt{\alpha'})^{2d_\perp} \frac{V_p}{(2\pi)^p} \frac{1}{(2\pi)^{2d_\perp}} \frac{\sqrt{1-V_1^2}\sqrt{1-V_2^2}}{|V_1 - V_2|} \, \mathrm{e}^{\frac{b^2}{\alpha' \log r^2}} \left[-\frac{\alpha'}{4\pi} \log r^2 \right]^{-\frac{d_\perp - 1}{2}} , \tag{19}$$

with $r \equiv |z|$. We have chosen the normalization

$$\langle p = 0 | \mathrm{e}^{\mathrm{i}q\cdot Q} | p = 0 \rangle = \delta^{(d)}(Q) \, , \qquad\qquad \delta^{(d)}(0) = \frac{V_d}{(2\pi)^d} \tag{20}$$

and we have introduced $b^2 \equiv \sum_{j\neq i}(y_1^j - y_2^j)^2$. It is easy to check that

$$\frac{\sqrt{1-V_1^2}\sqrt{1-V_2^2}}{|V_1 - V_2|} = \frac{1}{\sinh \pi\epsilon} \, . \tag{21}$$

In conclusion we get the following expression for the matrix element:

$$\begin{aligned} \langle B, y_1, v_1 | D | B, y_2, v_2 \rangle = & \frac{V_p}{(2\pi)^p} \frac{(\alpha')^{d_\perp}}{(\alpha'/2)^{(d_\perp - 1)/2}} \int_0^1 \frac{\mathrm{d}r}{r} r^{-2} \left[\prod_{n=1}^{\infty} \left(1 - r^{2n} \right) \right]^{-(d-2)} \\ & \times \mathrm{e}^{\frac{b^2}{\alpha' \log r^2}} \left[-\frac{1}{2\pi} \log r^2 \right]^{-\frac{d_\perp - 1}{2}} \frac{1}{\sinh \pi\epsilon} \prod_{n=1}^{\infty} \frac{(1 - r^{2n})^2}{[1 - \mathrm{e}^{2\pi\epsilon} r^{2n}][1 - \mathrm{e}^{-2\pi\epsilon} r^{2n}]} \, . \end{aligned} \tag{22}$$

In order to get the correctly normalized vacuum energy F one must multiply Eq. (22) by a normalization factor:

$$F = \frac{1}{\pi} (2\alpha')^{-d/2} \langle B, y_1, v_1 | D | B, y_2, v_2 \rangle \, . \tag{23}$$

Notice that this normalization factor is the same as in Eq. (1) and does not depend on the dimension of the D-branes. After the change of variable $r = \mathrm{e}^{-\pi\tau}$ we can write the vacuum energy in the final form:

$$\begin{aligned} F \;=\; & V_p \left(8\pi^2 \alpha' \right)^{-p/2} \int_0^\infty \mathrm{d}\tau \, \tau^{-(d-2)/2 + p/2} \, \mathrm{e}^{-\frac{b^2}{2\pi\alpha'\tau}} \mathrm{e}^{2\pi\tau} \left[\prod_{n=1}^{\infty} \left(1 - \mathrm{e}^{-2\pi\tau n} \right) \right]^{-d+2} \\ & \times \frac{1}{2\sinh \pi\epsilon} \prod_{n=1}^{\infty} \frac{(1 - \mathrm{e}^{-2\pi\tau n})^2}{[1 - \mathrm{e}^{2\pi\epsilon} \mathrm{e}^{-2\pi\tau n}][1 - \mathrm{e}^{-2\pi\epsilon} \mathrm{e}^{-2\pi\tau n}]} \, , \end{aligned} \tag{24}$$

that reproduces Bachas' calculation for the bosonic string [17]. Remember that in this case $d = 26$.

Let us consider now the fermionic string. The fermionic part of the boundary states must obey the boundary conditions

$$\begin{aligned} (\psi^\alpha - \mathrm{i}\eta\tilde{\psi}^\alpha)|_{\tau=0} \, |\mathcal{B}, \eta\rangle &= 0 \, , \\ (\psi^i + \mathrm{i}\eta\tilde{\psi}^i)|_{\tau=0} \, |\mathcal{B}, \eta\rangle &= 0 \, , \end{aligned} \tag{25}$$

where $\eta = \pm 1$. By consistency, the fields ψ^μ and $\tilde{\psi}^\mu$ must have the same periodicity, and therefore only the NS-NS and R-R sectors are to be considered.

In the NS-NS case, Eqs. (25) are solved by

$$|\mathcal{B}, \eta\rangle_{\rm NS} = {\rm e}^{{\rm i}\eta \sum_{n=1}^{\infty} b^\dagger_{n-1/2}\cdot\tilde{b}^\dagger_{n-1/2}}\, |0\rangle_b |0\rangle_{\tilde{b}}\, |\mathcal{B}\rangle_{\rm gh}\ . \tag{26}$$

The dot in the exponent should be understood as in Eq. (7); we do not write the ghost contribution that is explicitly given in Refs. [7, 8]. It is easy to check that the corresponding GSO projected state is

$$|\mathcal{B}\rangle_{\rm NS} \equiv \frac{1-(-)^{\tilde{F}}}{2}\,\frac{1-(-)^{F}}{2}\,|\mathcal{B},+\rangle_{\rm NS} = \frac{1}{2}\left(|\mathcal{B},+\rangle_{\rm NS} - |\mathcal{B},-\rangle_{\rm NS}\right)\ , \tag{27}$$

with F being the fermion number operator: $F = \sum_{n=1}^{\infty} b^\dagger_{n-1/2}\cdot b_{n-1/2}$.

In the R-R sector, we have

$$|\mathcal{B}, \eta\rangle_{\rm R} = {\rm e}^{{\rm i}\eta \sum_{n=1}^{\infty} d^\dagger_{n}\cdot\tilde{d}^\dagger_{n}}\, |0\rangle_d |0\rangle_{\tilde{d}}\, |\eta\rangle_{\rm R}\, |\mathcal{B}\rangle_{\rm gh}\ , \tag{28}$$

where $|\eta\rangle_{\rm R}$ is the fermionic zero mode boundary state. The correct normalization of the R-R contribution to the boundary state requires some care in the definition of $|\eta\rangle_{\rm R}$. The Clifford algebras formed by the d_0^μ's and the one formed by the $\tilde{d}_0^\mu$'s are conveniently represented by gamma matrices acting on normalized sixteen-dimensional spinor states $|a\rangle$ and $|\tilde{a}\rangle$ $[a, \tilde{a} = 1, \ldots, 16]$. Since moreover d_0^μ and $\tilde{d}_0^\nu$ anticommute, we need to introduce an extra normalized two-dimensional spinor state $|m\rangle$ $[m = 1, 2]$ on which d_0^μ and $\tilde{d}_0^\nu$ act as Pauli matrices:

$$\begin{aligned} d_0^\mu\ |a\rangle|\tilde{a}\rangle|m\rangle &= \frac{1}{\sqrt{2}}\,{\sigma_1}^m{}_n {\Gamma^{\mu a}}_b\ |b\rangle|\tilde{a}\rangle|n\rangle\ , & (29)\\ \tilde{d}_0^\mu\ |a\rangle|\tilde{a}\rangle|m\rangle &= \frac{1}{\sqrt{2}}\,{\sigma_2}^m{}_n {\Gamma^{\mu\tilde{b}}}_{\tilde{a}}\ |a\rangle|\tilde{b}\rangle|n\rangle\ . & (30)\end{aligned}$$

The Ramond zero mode state is then defined in terms of the $(p+1)$ chiral matrix $\Gamma_{p+2} = -{\rm i}^{p(p+1)/2+1}\Gamma^0\Gamma^1\cdots\Gamma^p$ (satisfying $\Gamma_{p+2}^2 = 1$ and $\Gamma_{p+2}^\dagger = \Gamma_{p+2}$) [13, 18]:

$$\begin{aligned} |+\rangle_{\rm R} &= \frac{\rm i}{2}\left(1+(-)^p\sigma_3\right)^m{}_n \left(\Gamma_{p+2}\right)^{\tilde{a}}{}_a\ |a\rangle|\tilde{a}\rangle|n\rangle\ , & (31)\\ |-\rangle_{\rm R} &= \frac{\rm i}{2}\left(1+(-)^p\sigma_3\right)^m{}_n \left(\Gamma_{p+2}\Gamma_{11}\right)^{\tilde{a}}{}_a\ |a\rangle|\tilde{a}\rangle|n\rangle\ , & (32)\end{aligned}$$

with $m = 1$ for p even and $m = 2$ for p odd. It obeys the zero mode part of the conditions (25) and verifies the following properties:

$$\begin{aligned} &32\, d_0^{11}\ |\eta\rangle_{\rm R} \equiv 32\, d_0^0 d_0^1\cdots d_0^9\ |\eta\rangle_{\rm R} = |-\eta\rangle_{\rm R} = (-)^p\, 32\, \tilde{d}_0^{11}\ |\eta\rangle_{\rm R}\ , & (33)\\ &{}_{\rm R}\langle\eta|\eta'\rangle_{\rm R} = -16\ \delta_{\eta\eta'}\ . & (34)\end{aligned}$$

The crucial minus sign in the inner product comes from the exchange in the ordering of the spinor states in the conjugate state. The fermion number operator in the R sector has a zero mode part: $(-)^F = 32\, d_0^{11} (-)^{\sum d_n^\dagger d_n}$, and analogously for $(-)^{\tilde{F}}$. In the above explicit representation, the GSO projected R-R state becomes

$$|\mathcal{B}\rangle_{\rm R} \equiv \frac{1+(-)^p(-)^{\tilde{F}}}{2}\,\frac{1+(-)^F}{2}\,|\mathcal{B},+\rangle_{\rm R} = \frac{1}{2}\left(|\mathcal{B},+\rangle_{\rm R} + |\mathcal{B},-\rangle_{\rm R}\right) \tag{35}$$

with the sign $(-)^p$ taken in order to get a non-vanishing result. The GSO projection is thus of type IIB for p odd, and of type IIA for p even, in accordance with the Ramond-Ramond charge that is carried by the Dirichlet p-brane [19].

As in the case of the bosonic oscillators we can boost the fermionic boundary states with a Lorentz transformation:

$$|\mathcal{B},\eta,v_i\rangle = {\rm e}^{{\rm i}v^i J^0{}_i}\;|\mathcal{B},\eta\rangle\ , \tag{36}$$

where the generator of the Lorentz transformation is given by

$$J^{\mu\nu} = -{\rm i}\sum_{n=1}^{\infty}\left(b^{\dagger\mu}_{n-1/2}b^{\nu}_{n-1/2} - b^{\dagger\nu}_{n-1/2}b^{\mu}_{n-1/2} + \tilde{b}^{\dagger\mu}_{n-1/2}\tilde{b}^{\nu}_{n-1/2} - \tilde{b}^{\dagger\nu}_{n-1/2}\tilde{b}^{\mu}_{n-1/2}\right) \tag{37}$$

for the Neveu-Schwarz sector and by

$$J^{\mu\nu} = -\frac{{\rm i}}{2}[d_0^\mu, d_0^\nu] - \frac{{\rm i}}{2}[\tilde{d}_0^\mu,\tilde{d}_0^\nu] - {\rm i}\sum_{n=1}^{\infty}\left(d_n^{\dagger\mu}d_n^\nu - d_n^{\dagger\nu}d_n^\mu + \tilde{d}_n^{\dagger\mu}\tilde{d}_n^\nu - \tilde{d}_n^{\dagger\nu}\tilde{d}_n^\mu\right) \tag{38}$$

for the Ramond sector. For the part containing only non zero modes we obtain for both sectors an expression very similar to the one obtained in Eq. (12) for the non-zero mode of the bosonic coordinates:

$${\rm e}^{{\rm i}v^i J^0{}_i} b_r^\dagger\cdot\tilde{b}_r^\dagger {\rm e}^{-{\rm i}v^i J^0{}_i} = b_r^{\dagger;\parallel}\cdot\tilde{b}_r^{\dagger;\parallel} - \sum_{j\neq i} b^\dagger_{r;j}\tilde{b}^\dagger_{r;j} - \begin{pmatrix} b_r^{0\dagger} & b_r^{i\dagger}\end{pmatrix} M(v)^2 \begin{pmatrix}\tilde{b}_r^{0\dagger}\\ \tilde{b}_r^{i\dagger}\end{pmatrix}\ , \tag{39}$$

where r is a positive half-integer for the Neveu-Schwarz sector. The same holds true with b_r replaced by d_n [n a positive integer] for the Ramond sector. The zero mode part of the boost operator is conveniently expressed as

$${\rm e}^{{\rm i}v^i J^0{}_i} = {\rm e}^{v^i\left[d^{0\,(+)}d_i^{(-)} + d^{0\,(-)}d_i^{(+)}\right]} \tag{40}$$

in terms of the combinations $d^{\mu\,(\pm)} = \frac{1}{\sqrt{2}}\left(d_0^\mu \pm {\rm i}\tilde{d}_0^\mu\right)$, whose only non-zero anticommutators are $\{d^{\mu\,(+)}, d^{\nu\,(-)}\} = \eta^{\mu\nu}$. This yields

$$|\eta, v\rangle_{\rm R} \equiv {\rm e}^{{\rm i}v^i J_i^0}\,|\eta\rangle_{\rm R} = \left[\cosh v + \sinh v\, d^{0\,(+\eta)} d_i^{(-\eta)}\right]|\eta\rangle_{\rm R} \tag{41}$$

and

$${}_{\rm R}\langle\eta, v_2|\eta, v_1\rangle_{\rm R} = -16\cosh(v_1 - v_2) = -16\cosh\pi\epsilon\ , \tag{42}$$

where we have taken into account that $d_0^{\alpha(-\eta)}|\eta\rangle_{\rm R} = d_0^{i(\eta)}|\eta\rangle_{\rm R} = 0$ and the normalization in Eq. (34).

We now compute the matrix element of the closed string propagator between two boundary states. With our conventions, the conjugate of a fermionic boundary state defined as in Eq. (25) satisfies $0 = \langle\mathcal{B},\eta|\,(\psi^\alpha + \mathrm{i}\eta\tilde{\psi}^\alpha)|_{\tau=0} = \langle\mathcal{B},\eta|\,(\psi^i - \mathrm{i}\eta\tilde{\psi}^i)|_{\tau=0}$.

For the NS-NS part, we get

$$
{}_{\mathrm{NS}}\langle\mathcal{B},v_1|z^{L_0-1/2}\,\bar{z}^{\bar{L}_0-1/2}|\mathcal{B},v_2\rangle_{\mathrm{NS}} = \frac{1}{2r}\left[Z_+(\epsilon) - Z_-(\epsilon)\right]\;, \tag{43}
$$

where the functions

$$
Z_\pm(\epsilon) = \prod_{n=1}^\infty \left[1 \pm r^{2n-1}\right]^8 \prod_{n=1}^\infty \frac{\left[1 \pm \mathrm{e}^{2\pi\epsilon} r^{2n-1}\right]\left[1 \pm \mathrm{e}^{-2\pi\epsilon} r^{2n-1}\right]}{(1 \pm r^{2n-1})^2}\;. \tag{44}
$$

are related to the Jacobi's theta functions.

The R-R part reads

$$
\begin{aligned}
{}_{\mathrm{R}}\langle\mathcal{B},v_1|z^{L_0}\,\bar{z}^{\bar{L}_0}|\mathcal{B},v_2\rangle_{\mathrm{R}} &= -8\,\cosh\pi\epsilon \\
&\quad\times \prod_{n=1}^\infty \left[1 + r^{2n}\right]^8 \prod_{n=1}^\infty \frac{\left[1 + \mathrm{e}^{2\pi\epsilon} r^{2n}\right]\left[1 + \mathrm{e}^{-2\pi\epsilon} r^{2n}\right]}{(1 + r^{2n})^2}\;,
\end{aligned} \tag{45}
$$

where the factor $-8\cosh\pi\epsilon$ comes from the boost of the zero modes and the infinite product comes from the non-zero modes.

The sum of the NS-NS contribution, Eq. (43), and of the R-R contribution, Eq. (45), can be recast in the following form:

$$
\frac{1}{2r}\prod_{n=1}^\infty \left(1 - r^{2n}\right)^{-4} \sum_{\alpha=2,3,4} e_\alpha \Theta_\alpha(-\mathrm{i}\epsilon|\mathrm{i}\tau)\left[\Theta_\alpha(0|\mathrm{i}\tau)\right]^3\;, \tag{46}
$$

with $e_3 = 1 = -e_2 = -e_4$.

If we also include the bosonic contribution with the same normalization as in the bosonic case, we arrive at the final result for the vacuum energy F in the case of superstring:

$$
\begin{aligned}
F &= V_p\left(8\pi^2\alpha'\right)^{-p/2}\int_0^\infty \mathrm{d}\tau\,\tau^{-4+p/2}\,\mathrm{e}^{-\frac{b^2}{2\pi\alpha'\tau}}\,\frac{1}{2}\sum_{\alpha=2,3,4} e_\alpha\Theta_\alpha(-\mathrm{i}\epsilon|\mathrm{i}\tau)\left[\Theta_\alpha(0|\mathrm{i}\tau)\right]^3 \\
&\quad\times\left[\mathrm{e}^{-\pi\tau/12}\prod_{n=1}^\infty\left(1 - \mathrm{e}^{-2\pi\tau n}\right)\right]^{-12}\frac{1}{2\pi\mathrm{i}}\frac{\Theta_1'(0|\mathrm{i}\tau)}{\Theta_1(-\mathrm{i}\epsilon|\mathrm{i}\tau)}\;,
\end{aligned} \tag{47}
$$

that exactly reproduces Bachas' calculation: $F = -\delta_{\mathrm{Bachas}}$ [17]. An alternative form is [13]

$$
\begin{aligned}
F &= V_p\left(8\pi^2\alpha'\right)^{-p/2}\int_0^\infty \mathrm{d}\tau\,\tau^{-4+p/2}\,\mathrm{e}^{-\frac{b^2}{2\pi\alpha'\tau}} \\
&\quad\times\left[\mathrm{e}^{-\pi\tau/12}\prod_{n=1}^\infty\left(1 - \mathrm{e}^{-2\pi\tau n}\right)\right]^{-9}\frac{1}{\mathrm{i}}\frac{\left[\Theta_1(-\mathrm{i}\epsilon/2|\mathrm{i}\tau)\right]^4}{\Theta_1(-\mathrm{i}\epsilon|\mathrm{i}\tau)}\;.
\end{aligned} \tag{48}
$$

To compute the vacuum energy of a D-brane anti-D-brane system amounts simply to change the sign of the R-R contribution [20].

In this paper we have shown how the formalism based on the boundary states considerably simplifies the calculation of the interaction between two D-branes moving with constant velocity. The boundary state can also be extended to include an arbitrary number of both open and closed strings attached to it and in fact constitutes the basic element for computing the interaction between any number of D-branes and closed and open strings in pretty much the same way as one can compute multiloops in open string theory starting from the multi-string vertex for closed strings and saturating the external legs with boundary states describing the closed-string vacuum transition after the insertion of a closed string propagator. Work is in progress along these lines.[3]

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