

Soft Supersymmetry Breaking in Calabi-Yau Orientifolds with D-branes and Fluxes¹

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ABSTRACT

In this paper we compute the $\mathcal{N} = 1$ effective low energy action for a stack of N space-time filling D3-branes in generic type IIB Calabi-Yau orientifolds with non-trivial background fluxes by reducing the Dirac-Born-Infeld and Chern-Simons actions. Specifically, we determine the Kähler potential for the excitations of the D-brane including their couplings to all bulk moduli fields. In the effective theory, $\mathcal{N} = 1$ supergravity is spontaneously broken by the presence of fluxes and we compute the induced soft supersymmetry breaking terms. We find an interesting structure in the resulting soft terms with generically universal soft scalar masses.

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1 Introduction

After the discovery of D-branes as non-perturbative BPS objects in string theory [1], it was realized that they also can serve as a new ingredient in string model building. Recently phenomenologically viable models with chiral fermions in representations of gauge groups similar to the $SU(3) \times SU(2) \times U(1)$ of the Standard Model (SM) were constructed from stacks of space-time filling D-branes [2]. Non-supersymmetric models generically suffer from instabilities and a hierarchy problem while in supersymmetric models a viable mechanism for hierarchical supersymmetry breaking has to be employed. Apart from standard non-perturbative mechanisms for supersymmetry breaking such as gaugino condensation, it has been suggested to use background fluxes in order to stabilize the moduli and break supersymmetry spontaneously [3]–[19].

From a phenomenological point of view spontaneously broken $\mathcal{N} = 1$ theories are of particular interest. Starting from type II string theories in ten space-time dimensions ($D = 10$), one can compactify on Calabi-Yau threefolds to obtain $\mathcal{N} = 2$ theories in $D = 4$. This $\mathcal{N} = 2$ is further broken to $\mathcal{N} = 1$ if in addition background D-branes and/or orientifold planes are present. Turning on background fluxes results in a spontaneously broken $\mathcal{N} = 1$ theory which can be best discussed in terms of a low energy effective action. For Calabi-Yau compactifications of type IIB theories this effective theory was determined in refs. [5, 20, 21, 22] while for Calabi-Yau orientifolds it can be found in refs. [11, 18, 19, 23].

In brane-world models the SM, or rather its supersymmetric extension, arises from the dynamical excitations of the D-branes. Thus it is necessary to determine the low energy effective action of these excitations including their couplings to the gauge neutral bulk moduli fields. This can be done in different ways. In refs. [24] T-duality is used to determine the effective action for a stack of D3-branes starting from the type I action and in refs. [25] supergravity consistency considerations are employed to determine the effective action.² The most direct way is to use the Dirac-Born-Infeld (DBI) and Chern-Simons (CS) action (and their supersymmetric extension) in an appropriate bulk background and perform a Kaluza-Klein reduction [14, 26, 27]. It is the latter approach we will follow in this paper.

The presence of background fluxes breaks supersymmetry, which is communicated to the observable sector, i.e. the charged matter fields on the D-branes, by bulk moduli fields. This results in a set of soft supersymmetry breaking terms which can be computed from the effective low energy action. For a specific class of models this has been carried out in refs. [26, 28].

The purpose of this paper is twofold. First (section 2), we determine the low energy effective action of type IIB string theory compactified on Calabi-Yau orientifolds with a stack of space-time filling D-branes and background fluxes. For the bulk action (section 2.1), we truncate the spectrum as dictated by the orientifold projection, and then reduce the ten-dimensional type IIB supergravity action. For the D-branes (section 2.2), we use the bosonic non-Abelian Dirac-Born-Infeld and Chern-Simons action as proposed in [29] and their supersymmetric completion as obtained in [27] using the κ -symmetric

²This works particularly well for backgrounds with a high degree of supersymmetry, but appears to be less efficient for $\mathcal{N} = 1$ theories.

action of [30].³ We determine the Kähler potential of the charged matter excitations of the D-brane coupled to all Calabi-Yau orientifold bulk moduli including the complex structure deformations. We find that generically it is not of the ‘sequestered form’ [31] in agreement with the discussion of ref. [32]. In the limit of just one Kähler modulus (parameterizing the overall volume) and frozen complex structure moduli we confirm the Kähler potential suggested in refs. [19, 33].

The second purpose of this paper (section 3) is to compute the soft supersymmetry breaking terms arising from turning on three-form flux. After briefly reviewing the generic supergravity analysis of refs. [34, 35] (section 3.1), we determine the resulting supersymmetric (section 3.2) and the soft breaking terms (section 3.3) in the D-brane action. We find that the fermionic masses are generated by a Giudice-Masiero mechanism [36] induced via fluxes into specific couplings in the D-brane Kähler potential. In sections 3.4.1 and 3.4.2 we briefly discuss some phenomenological properties of the resulting soft supersymmetry breaking terms. We find that for (0,3) fluxes a ‘strict no-scale’ breaking [37] occurs in that all soft terms vanish. For (3,0) and (1,2) fluxes on the other hand, we find A -terms which are proportional to the Yukawa couplings, and universal scalar masses.

We also display the consistency of the computed soft terms with the generic supergravity formulas of refs. [34, 35]. This is a highly non-trivial check on the computation performed in section 2. It is important to stress that we do not choose a particular model but our analysis is valid for any Calabi-Yau orientifold compactifications with D3-branes and background fluxes.

When this manuscript was being prepared the paper [38] appeared, which has substantial overlap with our analysis.

2 Effective Actions

2.1 The Bulk: Calabi-Yau orientifolds with three-form flux

In order to set the stage for this paper let us first briefly summarize the results of ref. [23] where the low energy effective action of Calabi-Yau orientifold compactifications of type IIB string theory is derived. The class of orientifolds studied are obtained by modding out type IIB string theory by the world-sheet parity combined with a discrete isometry of the Calabi-Yau manifold Y . Depending on the specific form of the orientifold projection, O3/O7- or O5/O9-orientifold planes are induced. These negative tension objects are needed in brane-world scenarios with compact internal spaces to ensure cancellation of gravitational and electro-magnetic tadpoles. Furthermore, the presence of localized sources requires a deviation from the standard Calabi-Yau compactifications in that a non-trivial warp factor e^{-2A} has to be included into the Ansatz for the metric [11, 39]

$$ds^2 = e^{2A(y)} \tilde{g}_{\mu\nu}(x) dx^\mu dx^\nu + e^{-2A(y)} g_{mn}(y) dy^m dy^n, \quad (2.1)$$

where $\tilde{g}_{\mu\nu}$, $\mu, \nu = 0, \dots, 3$ is a Minkowski metric and g_{mn} , $m, n = 1, \dots, 6$ is the metric on the Calabi-Yau manifold. However, in this paper we perform our analysis in the

³For simplicity we confine our analysis to D3-branes leaving the higher-dimensional D-branes to a separate investigation.

unwarped Calabi-Yau manifold since in the large radius limit the warp factor approaches one and the metrics of the two manifolds coincide [11, 14]. This in turn also implies that the metrics on the moduli space of deformations agree and as a consequence the kinetic terms in the low energy effective actions are the same. The difference appears in the potential when some of the Calabi-Yau zero modes are rendered massive.

In this paper we confine our attention to space-time filling D3-branes and O3/O7 orientifold planes leaving a more general analysis to a separate publication. For this case the orientifold projection acting on the type IIB fields is of the form [41]–[44]

$$\mathcal{O} = (-1)^{F_L} \Omega_p \sigma^*, \quad (2.2)$$

where Ω_p is the world-sheet parity and F_L is the space-time fermion number in the left-moving sector. σ^* is the pull-back of an isometric and holomorphic involution acting on the Calabi-Yau manifold [43, 44]. This involution leaves the Kähler form J of Y automatically invariant but can act non-trivially on the holomorphic three-form Ω . O3/O7-orientifold planes are present when in addition

$$\sigma^* \Omega = -\Omega \quad (2.3)$$

holds.⁴ Thus, we do not have to specify a particular Calabi-Yau manifold but merely need to demand that it admits an isometric and holomorphic involution obeying (2.3). The analysis performed in this paper then holds for all Calabi-Yau manifolds with this property.

2.1.1 The spectrum

The massless spectrum of the effective low energy theory derived from such an orientifold compactification is determined in ref. [44] and here we need to briefly recall the result. Standard Calabi-Yau compactifications of type IIB lead to an effective $\mathcal{N} = 2$ supergravity in $D = 4$ [20] which is further truncated to an $\mathcal{N} = 1$ supergravity by the orientifold projection.⁵ Focusing on the bosonic spectrum one starts from the ten-dimensional massless type IIB fields including the dilaton ϕ , the metric g and a two-form $B^{(2)}$ in the NS-NS sector, and the axion l , a second two-form $C^{(2)}$ and a four-form $C^{(4)}$ with a self-dual field strength in the R-R sector which transform under (2.2) according to

$$\begin{aligned} \mathcal{O} \phi &= \sigma^* \phi, & \mathcal{O} l &= \sigma^* l, \\ \mathcal{O} g &= \sigma^* g, & \mathcal{O} C^{(2)} &= -\sigma^* C^{(2)}, \\ \mathcal{O} B^{(2)} &= -\sigma^* B^{(2)}, & \mathcal{O} C^{(4)} &= \sigma^* C^{(4)}. \end{aligned} \quad (2.4)$$

In the compactified theory these ten-dimensional fields are expanded in terms of harmonic forms on Y and only the invariant states of the projection (2.2) together with (2.3) and (2.4) are kept in the spectrum. The harmonic forms are in one-to-one correspondence with the elements of the cohomology groups $H^{(p,q)}$ which split into two eigenspaces under the action of σ^*

$$H^{(p,q)} = H_+^{(p,q)} \oplus H_-^{(p,q)}. \quad (2.5)$$

⁴The case $\sigma^* \Omega = \Omega$ leads to O5/O9-planes and is analyzed in ref. [23] but plays no role in this paper. Whenever $\sigma^* = id$ the theory has O9-planes and coincides with type I if one introduces D9-branes to cancel tadpoles.

⁵From a supergravity point of view such truncations have been discussed in ref. [45].

$H_+^{(p,q)}$ has dimension $h_+^{(p,q)}$ and denotes the $+1$ eigenspace of σ^* while $H_-^{(p,q)}$ has dimension $h_-^{(p,q)}$ and denotes the -1 eigenspace of σ^* . The Hodge $*$ -operator commutes with σ^* since σ preserves the orientation and the metric of the Calabi-Yau manifold and thus the Hodge numbers obey $h_{\pm}^{(1,1)} = h_{\pm}^{(2,2)}$. Holomorphicity of σ further implies $h_{\pm}^{(3,0)} = h_{\pm}^{(0,3)}$ and $h_{\pm}^{(2,1)} = h_{\pm}^{(1,2)}$. Combining these rules with the transformation properties (2.4) one can systematically determine the massless $D = 4$ (bosonic) spectrum. We will use the following basis for the spaces $H_{\pm}^{(p,q)}$:

$\omega_a \in H_-^{(1,1)}$	$\tilde{\omega}^a \in H_-^{(2,2)}$	$\chi_{\hat{a}} \in H_-^{(2,1)}$	$\bar{\chi}_{\hat{a}} \in H_-^{(1,2)}$	$\Omega \in H_-^{(3,0)}$	$\bar{\Omega} \in H_-^{(0,3)}$
$\omega_{\alpha} \in H_+^{(1,1)}$	$\tilde{\omega}^{\alpha} \in H_+^{(2,2)}$	$\chi_{\hat{\alpha}} \in H_+^{(2,1)}$	$\bar{\chi}_{\hat{\alpha}} \in H_+^{(1,2)}$		

Table 2.1: Basis for $H_{\pm}^{(p,q)}$

The scalar fields arising from the metric are the deformations of the Kähler form $g_{k\bar{j}}$ and the deformations of the complex structure which are proportional to $\delta g_{k\bar{j}}$. Since the Kähler form is left invariant by the orientifold projection, only $h_+^{(1,1)}$ Kähler deformations v^{α} survive and one has

$$g_{k\bar{j}} = -iv^{\alpha}(x) (\omega_{\alpha})_{k\bar{j}}, \quad k, \bar{j} = 1, 2, 3, \quad \alpha = 1, \dots, h_+^{(1,1)}, \quad (2.6)$$

where ω_{α} denotes a basis of $H_+^{(1,1)}$. Due to (2.3) the complex structure deformations of the metric correspond to the elements in $H_-^{(1,2)}$ and one expands

$$\delta g_{k\bar{j}} = \frac{i}{\|\Omega\|^2} \bar{z}^{\hat{a}} (\bar{\chi}_{\hat{a}})_{k\bar{l}} \Omega^{\bar{l}j}, \quad \hat{a} = 1, \dots, h_-^{(1,2)}, \quad (2.7)$$

where $\bar{\chi}_{\hat{a}}$ denotes a basis of $H_-^{(1,2)}$ and we abbreviate $\|\Omega\|^2 \equiv \frac{1}{3!} \Omega_{ijk} \bar{\Omega}^{ijk}$.

Eqs. (2.4) further imply that the two-forms $B^{(2)}, C^{(2)}$ can only be expanded in terms of harmonic forms residing in $H_-^{(1,1)}$ while the four-form $C^{(4)}$ can be expanded in terms of harmonic forms in $H_+^{(1,1)}, H_+^{(2,2)}$ and $H_+^{(3)}$. One thus has

$$B^{(2)} = b^a(x) \omega_a, \quad C^{(2)} = c^a(x) \omega_a, \quad a = 1, \dots, h_-^{(1,1)}, \quad (2.8)$$

$$C^{(4)} = D_{(2)}^{\alpha}(x) \wedge \omega_{\alpha} + V^{\hat{\alpha}}(x) \wedge \alpha_{\hat{\alpha}} + U_{\hat{\alpha}}(x) \wedge \beta^{\hat{\alpha}} + \rho_{\alpha}(x) \tilde{\omega}^{\alpha}, \quad \hat{\alpha} = 1, \dots, h_+^{(1,2)},$$

where $b^a(x), c^a(x)$ and $\rho_{\alpha}(x)$ are space-time scalars, $V^{\hat{\alpha}}(x)$ and $U_{\hat{\alpha}}(x)$ are space-time one-forms and $D_{(2)}^{\alpha}(x)$ is a space-time two-form. ω_a is a basis of $H_-^{(1,1)}$, $\tilde{\omega}^{\alpha}$ is a basis of $H_+^{(2,2)}$ which is dual to ω_{α} , and $(\alpha_{\hat{\alpha}}, \beta^{\hat{\alpha}})$ is a real symplectic basis of $H_+^{(3)} = H_+^{(1,2)} \oplus H_+^{(2,1)}$. Imposing the self-duality on the five-form field strength of $C^{(4)}$ eliminates half of the degrees of freedom in the expansion of $C^{(4)}$ leaving, for example, only ρ_{α} and $V^{\hat{\alpha}}$ in the spectrum.

Let us summarize the low energy spectrum. Starting from the type IIB massless fields and compactifying on a Calabi-Yau manifold one keeps only fields that are invariant under the orientifold projections (2.2), (2.3) and (2.4) and obtains $h_+^{(2,1)}$ vectors $V^{\hat{\alpha}}$,

$h_-^{(2,1)}$ complex scalars $z^{\hat{a}}$ parameterizing the deformations of the complex structure, $2h_+^{(1,1)}$ scalars (v^α, ρ_α) including the Kähler deformations v^α , $2h_-^{(1,1)}$ scalars (b^a, c^a) , the complex dilaton $\tau \equiv l + ie^{-\phi}$ and the space-time metric $g_{\mu\nu}$. Including the fermions, these fields assemble in an $\mathcal{N} = 1$ gravitational multiplet, $h_+^{(2,1)}$ vector multiplets and $h_-^{(2,1)} + h^{(1,1)} + 1$ chiral multiplets (see Table 2.2).

gravity multiplet	1	$g_{\mu\nu}$
vector multiplets	$h_+^{(2,1)}$	$V^{\hat{a}}$
chiral multiplets	$h_-^{(2,1)}$	$z^{\hat{a}}$
	$h_+^{(1,1)}$	(v^α, ρ_α)
	$h_-^{(1,1)}$	(b^a, c^a)
	1	(ϕ, l)

Table 2.2: $\mathcal{N} = 1$ spectrum of moduli.

2.1.2 The effective action without background fluxes

The low energy effective action of the massless modes is determined by a Kaluza-Klein compactification of the ten-dimensional type IIB supergravity. This reduction is carried out in detail in ref. [23] and briefly summarized in appendix A. The resulting four dimensional supergravity theory can be written in the standard $\mathcal{N} = 1$ form

$$\begin{aligned}
S_{\text{Bulk}}^{\text{E}} = & \int_{\mathbb{M}_{3,1}} -\frac{1}{2}R * \mathbf{1} - \hat{K}_{I\bar{J}} dM^I \wedge *d\bar{M}^{\bar{J}} - e^{\hat{K}} \left(\hat{K}^{I\bar{J}} D_I \hat{W} \bar{D}_{\bar{J}} \hat{W} - 3|\hat{W}|^2 \right) \\
& - \frac{1}{2}(\text{Re } f_{\hat{\alpha}\hat{\beta}}) F_{\hat{A}}^{\hat{\alpha}} \wedge *F_{\hat{A}}^{\hat{\beta}} - \frac{1}{2}(\text{Im } f_{\hat{\alpha}\hat{\beta}}) F_{\hat{A}}^{\hat{\alpha}} \wedge F_{\hat{A}}^{\hat{\beta}} , \tag{2.9}
\end{aligned}$$

where $M^I, I = 1, \dots, h_-^{(2,1)} + h^{(1,1)} + 1$ collectively denotes all scalar fields in chiral multiplets and $F_{\hat{A}}^{\hat{\alpha}} = dV^{\hat{\alpha}}$ is the field strength of the $h_+^{(2,1)}$ (Abelian) vector multiplets. $\hat{K}_{I\bar{J}} = \partial_I \bar{\partial}_{\bar{J}} \hat{K}(M^I, \bar{M}^{\bar{J}})$ is the Kähler metric (with Kähler potential \hat{K}) on the moduli space of the compactification while $f(M^I)$ and $\hat{W}(M^I)$ are the holomorphic gauge kinetic function and the holomorphic superpotential, respectively.⁶ The action given in terms of the scalar fields arising in the expansions (2.6)–(2.8) is discussed in appendix A. To obtain the standard form (2.9), a (complicated) field redefinition is necessary. One finds that the Kähler structure of the moduli space is manifest in the complex coordinates $M^I = (\tau, G^a, T_\alpha, z^{\hat{a}})$ defined as

$$\begin{aligned}
\tau &= l + ie^{-\phi} , & G^a &= c^a - \tau b^a , \tag{2.10} \\
T_\alpha &= \frac{3i}{2}\rho_\alpha + \frac{3}{4}\mathcal{K}_\alpha - \frac{3i}{4(\tau - \bar{\tau})} \mathcal{K}_{abc} G^b (G - \bar{G})^c ,
\end{aligned}$$

⁶We use the notation \hat{K}, \hat{W} in order to distinguish from K, W used later on when also matter fields arising from D-branes are included.

where we abbreviate $\mathcal{K}_\alpha \equiv \mathcal{K}_{\alpha\beta\gamma} v^\beta v^\gamma$. $\mathcal{K}_{\alpha\beta\gamma}$ and $\mathcal{K}_{ab\gamma}$ are (constant) intersection numbers defined as

$$\mathcal{K}_{\alpha\beta\gamma} = \int_Y \omega_\alpha \wedge \omega_\beta \wedge \omega_\gamma, \quad \mathcal{K}_{ab\gamma} = \int_Y \omega_a \wedge \omega_b \wedge \omega_\gamma, \quad (2.11)$$

which are the only non-vanishing intersection numbers after the orientifold projection [23]. In terms of the coordinates defined in (2.10), the Kähler potential is given by

$$\hat{K} = -\ln \left[-i \int \Omega(z) \wedge \bar{\Omega}(\bar{z}) \right] - \ln \left[-i(\tau - \bar{\tau}) \right] - 2\ln \left[\frac{1}{6} \mathcal{K}(\tau, T, G) \right], \quad (2.12)$$

where $\mathcal{K} \equiv \mathcal{K}_{\alpha\beta\gamma} v^\alpha v^\beta v^\gamma = 6 \text{Vol}(Y)$ is related to the volume of the Calabi-Yau manifold. \mathcal{K} should be understood as a function of the Kähler coordinates (τ, T, G) which enter by solving (2.10) for v^α in terms of (τ, T, G) . Unfortunately this solution cannot be given explicitly and therefore \mathcal{K} is known only implicitly via $v^\alpha(\tau, T, G)$.⁷ However, for one overall Kähler metric modulus v parametrizing the volume (i.e. for $h_+^{(1,1)} = 1$, $T_\alpha \equiv T$), keeping all $h_-^{(1,1)}$ moduli, eq. (2.10) can be solved for v and one finds

$$-2 \ln \mathcal{K} = -3 \ln \frac{2}{3} \left[T + \bar{T} + \frac{3i}{4(\tau - \bar{\tau})} \mathcal{K}_{1ab} (G - \bar{G})^a (G - \bar{G})^b \right]. \quad (2.13)$$

which has the standard no-scale structure.

The first two terms in (2.12) are the standard Kähler potentials for the complex structure deformations and the dilaton, respectively. \mathcal{K} also depends on τ and therefore the metric mixes τ with T_α and G^a . It is block diagonal in the complex structure deformations which do not mix with the other scalars. Thus, the moduli space has the form

$$\mathcal{M} = \mathcal{M}_{\text{cs}}^{h_-^{(1,2)}} \times \mathcal{M}_{\text{k}}^{h_-^{(1,1)}+1}, \quad (2.14)$$

where each factor is a Kähler manifold.⁸

For completeness let us also give the gauge-kinetic coupling functions $f_{\hat{\alpha}\hat{\beta}}$ which only depend on $z^{\hat{a}}$ but not on any of the other moduli. They are given by

$$f_{\hat{\alpha}\hat{\beta}} = -\frac{i}{2} \mathcal{F}_{\hat{\alpha}\hat{\beta}}, \quad (2.15)$$

where $\mathcal{F}_{\hat{\alpha}\hat{\beta}}$ is a holomorphic function of the complex structure deformations $z^{\hat{a}}$. It is computed as a second derivative from the full $\mathcal{N} = 2$ prepotential $\mathcal{F}^{\mathcal{N}=2}(z)$ of all complex structure deformation via [45]

$$\mathcal{F}_{\hat{\alpha}\hat{\beta}} = \partial_{z^{\hat{\alpha}}} \partial_{z^{\hat{\beta}}} \mathcal{F}^{\mathcal{N}=2}(z^{\hat{\alpha}}, z^{\hat{\beta}})|_{z^{\hat{a}}=0}. \quad (2.16)$$

Since we do not need these couplings in this paper we refer the reader to [23] for further details.

Finally, the superpotential vanishes as long as no background fluxes are turned on and $\mathcal{N} = 1$ supersymmetry remains unbroken. Let us now turn to the situation when non-trivial background fluxes are present.

⁷This is in complete analogy to the situation encountered in compactifications of M-theory on Calabi-Yau fourfolds studied in [46].

⁸In the next section we will see that including matter fields from the D-branes also mixes the complex structure deformations non-trivially with all the other moduli and the product structure is lost.

2.1.3 The effective action with non-trivial background fluxes

We are still in the process of preparing the ground for the next section where we add space-time filling D3-branes into the picture. For consistency, this requires the presence of negative tension objects such as orientifolds and non-trivial five-form flux. Such configurations break $\mathcal{N} = 2$ supersymmetry of the original Calabi-Yau compactification to $\mathcal{N} = 1$. In addition, one can also turn on three-form flux which induces a superpotential. This leads to the stabilization of part of the moduli and allows for the possibility of spontaneous $\mathcal{N} = 1$ supersymmetry breaking.

Let us first discuss the five-form flux. The self-dual field strength of the four-form $C^{(4)}$ is defined (in $D = 10$) as $\tilde{F}^{(5)} = F^{(5)} - \frac{1}{2}C^{(2)} \wedge H^{(3)} + \frac{1}{2}B^{(2)} \wedge F^{(3)}$ where $F^{(5)} = dC^{(4)}$, $H^{(3)} = dB^{(2)}$ and $F^{(3)} = dC^{(2)}$. Since D3-branes and O3-planes are 4D Poincaré invariant sources, the 5-form flux should respect this symmetry. Thus the only possible flux that can be turned on is of the form

$$\tilde{F}^{(5)} = dC^{(4)}, \quad C^{(4)} = \alpha(y) dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3. \quad (2.17)$$

In addition to the five-form flux one can also turn on three-form fluxes $H^{(3)}$ and $F^{(3)}$ on the Calabi-Yau manifold Y . The Bianchi identity combined with the equations of motion imply that both fluxes have to be harmonic three-forms and therefore can be parameterized in terms of the third cohomology group $H^3(Y)$. For the orientifold compactifications under consideration they also have to respect the orientifold projection and remain invariant under the action of \mathcal{O} . This implies that they can only take values in $H^3_-(Y)$.

The five-form flux α is further constrained by the Bianchi identity and the trace of the Einstein equations. Taking the difference between these two equations one obtains [11]

$$\nabla^2(e^{4A} - \alpha) = e^{-6A} |\partial(e^{4A} - \alpha)|^2 + \frac{e^{2A}e^\phi}{6} |*_6G^{(3)} - iG^{(3)}|^2, \quad (2.18)$$

where

$$G^{(3)} \equiv F^{(3)} - \tau H^{(3)}, \quad (2.19)$$

and $*_6$ is the six-dimensional Hodge $*$ -operator on Y .⁹ On a compact space, the LHS of (2.18) integrates to zero, while the RHS is non-negative. This implies a relation between $\alpha(y)$ and the warp factor $e^{-2A(y)}$

$$e^{4A} = \alpha \quad (2.20)$$

and

$$*_6G^{(3)} = iG^{(3)}. \quad (2.21)$$

Fluxes which obey this condition are called imaginary self-dual (ISD). The equation of motion for the dilaton is solved by constant τ and as a consequence $G^{(3)}$ is harmonic since both $F^{(3)}$ and $H^{(3)}$ are harmonic three forms. The Hodge operator $*_6$ acts on such forms according to

$$\begin{aligned} *_6\Omega &= -i\Omega, & *_6\chi_{\hat{a}} &= i\chi_{\hat{a}}, \\ *_6\bar{\Omega} &= i\bar{\Omega}, & *_6\bar{\chi}_{\hat{a}} &= -i\bar{\chi}_{\hat{a}}, \end{aligned} \quad (2.22)$$

⁹Note that for the Bianchi identity of the five-form flux and the Einstein equations there should be source terms. They cancel in the difference $\nabla^2(e^{4A} - \alpha)$ in the RHS of (2.18) for D3-branes and O3-planes [11].

where Ω is the $(3, 0)$ form while $\chi_{\hat{a}}$ are $(2, 1)$ forms introduced in Table 2.1. This implies that the $G^{(3)}$ which obeys (2.21) is a sum of $(2, 1)$ and $(0, 3)$ forms only, and for these cases a consistent supergravity background on a warped compact Calabi-Yau orientifold exists.¹⁰

The supersymmetry transformations in this background were analysed in refs. [39, 40]. It was shown that a primitive $(2, 1)$ piece of the three-form flux preserves the $\mathcal{N} = 1$ supersymmetry while any other three-form flux breaks it. Let us already mention that including D-branes does not change this conclusion as can be seen from eq. (D.6). This implies that when we turn on $(2, 1)$ three-form flux, the CY-orientifold including D3-branes is $\mathcal{N} = 1$ supersymmetric.¹¹ On the other hand $(0, 3)$ flux does break supersymmetry spontaneously with vanishing cosmological constant [11]. This fact is best seen from the effective action including background fluxes.

The Kaluza-Klein reduction briefly reviewed in the previous section can also be performed when the three-form flux $G^{(3)}$ is non-vanishing. The Kähler potential and the gauge kinetic function of the previous section are unchanged¹² but a non-trivial superpotential is induced. One finds¹³

$$\hat{W} = \int \Omega \wedge G^{(3)} , \quad (2.23)$$

which depends on the $h_-^{(1,2)}$ complex structure moduli $z^{\hat{a}}$ through Ω and on τ through the definition of $G^{(3)}$. It vanishes for $(2, 1)$ flux (and also for $(3, 0)$ and $(1, 2)$ flux) but is non-zero for $(0, 3)$ -flux.

The Kähler covariant derivatives $D_I \hat{W} = \partial_I \hat{W} + \hat{W} \partial_I \hat{K}$ are the order parameters for spontaneous supersymmetry breaking. For the case at hand these derivatives are evaluated in (B.8) and one sees that they vanish if

$$\hat{W} = 0 , \quad \mathcal{I} \equiv \int \bar{\Omega} \wedge G^{(3)} = 0 , \quad \mathcal{I}_{\hat{a}} \equiv \int \chi_{\hat{a}} \wedge G^{(3)} = 0 , \quad (2.24)$$

hold. \hat{W} is non-vanishing for $(0, 3)$ flux, \mathcal{I} is non-vanishing for $(3, 0)$ flux and $\mathcal{I}_{\hat{a}}$ is non-vanishing for $(1, 2)$ flux or in other words \hat{W}, \mathcal{I} and $\mathcal{I}_{\hat{a}}$ are integral representations of $(0, 3)$, $(3, 0)$ and $(1, 2)$ flux, respectively. Hence, supersymmetry is unbroken whenever the $(0, 3)$, $(3, 0)$ and $(1, 2)$ pieces of $G^{(3)}$ vanish, while the $(2, 1)$ piece can be arbitrary. In this case also \hat{W} is zero implying the vanishing of the cosmological constant or in other words the existence of a Minkowskian supersymmetric ground state in full agreement

¹⁰Other sources such as $\overline{D3}$ -branes, \overline{O} -planes or exotic orientifold planes do not cancel in the difference (2.18), and can therefore evade this no-go result (we thank E.Dudas and E.Kiritsis for discussions on this point). In this paper we consider only O3-planes and D3-branes, leaving this possibility for future investigation.

¹¹The primitivity condition on the 3-form flux $G^{(3)} \wedge J = 0$ is automatically satisfied on a Calabi-Yau since there are no harmonic 5-forms.

¹²This again assumes a small warp factor. Including the warp consistently is beyond the scope of this paper. The case of one Kähler modulus has been discussed in [19]. The issue is that the harmonic analysis performed in the previous subsection has to be reconsidered and appropriately adjusted to warped compactifications on conformal Calabi-Yau manifolds [9].

¹³This superpotential was first suggested in ref. [47]; for orientifold backgrounds it was rederived in [23].

with the result of ref. [39]. Conversely, any (0,3), (3,0) and (1,2) pieces of $G^{(3)}$ break supersymmetry spontaneously since they lead a non-zero $D_I \hat{W}$.

In order to determine the cosmological constant for these cases, we need to evaluate the scalar potential. It can be computed from (2.9) using (2.12), (2.23), (B.7) and (B.8), or directly from a Kaluza-Klein reduction. Both computations yield

$$\begin{aligned} \hat{V} &= \frac{18ie^\phi}{\mathcal{K}^2 \int \Omega \wedge \bar{\Omega}} \left(\int \Omega \wedge \bar{G}^{(3)} \int \bar{\Omega} \wedge G^{(3)} + \mathcal{G}_w^{\hat{a}\hat{b}} \int \chi_{\hat{a}} \wedge G^{(3)} \int \bar{\chi}_{\hat{b}} \wedge \bar{G}^{(3)} \right) \\ &= e^{\hat{K}} \left(|\mathcal{I}|^2 + \mathcal{G}_w^{\hat{a}\hat{b}} \mathcal{I}_{\hat{a}} \mathcal{I}_{\hat{b}} \right), \end{aligned} \quad (2.25)$$

where $\mathcal{G}_w^{\hat{a}\hat{b}}$ is defined in (A.2). We see that the potential does not depend on \hat{W} , but only on the two other integrals \mathcal{I} and $\mathcal{I}_{\hat{a}}$. Thus, for (0,3) flux the potential vanishes identically which, in fact, is an example of “no-scale” supersymmetry breaking [37, 11]. The (3,0) and (1,2)-pieces on the other hand, contribute positive semi-definite terms to the potential and therefore correspond to spontaneous supersymmetry breaking in a Minkowski or de Sitter background. Due to the overall dilaton and volume dependence generically a ‘run-away’ solution will force the potential to zero for zero string coupling or infinite volume. This instability is just another manifestation of the fact that within our setup, eq. (2.18) does not allow imaginary anti-self dual (IASD) 3-form fluxes, i.e. the (1,2) and (3,0) pieces of $G^{(3)}$.

The potential (2.25) generated by three-form fluxes stabilizes the dilaton and the complex structure deformations, but leaves the Kähler moduli unfixed. Modifying the setup by other localized sources and/or including other non-perturbative effects is beyond the scope of this paper. However, it is still interesting to study the structure of soft supersymmetry breaking which is induced by IASD fluxes under the assumption that the present configuration is rendered stable by other effects. In other words we can continue under the assumption that further terms in the superpotential are generated which stabilize the moduli but otherwise do not interact with the matter fields on the D-branes which we are going to discuss in the next section. In terms of eq. (2.18), this corresponds to its local solution without taking into account the global properties. As we will see in section 3, this is enough to see the structure of the soft terms.

2.2 The Brane: D3-branes coupled to Calabi-Yau orientifolds

In this section we derive the low energy effective action of a stack of N space-time filling D3-branes in type IIB string theory compactified on a Calabi-Yau orientifold as described in the previous section. Our starting point for the bosonic terms is the non-Abelian Dirac-Born-Infeld and Chern-Simons action as proposed in ref. [29]. These action functionals are expanded to fourth order in the fields including the fluctuations of the brane position in the internal space Y . In the effective action, the fluctuations give rise to scalar fields ϕ charged under a non-Abelian gauge group, which in our case will be $U(N)$. The supersymmetric extension of this action leads to appropriate fermionic fields which combine with the ϕ to form $\mathcal{N} = 1$ chiral superfields. They can be viewed as the charged matter multiplets of the theory coupling to the bulk moduli introduced in the previous section. The purpose of this section is to derive the effective action for

the matter fields ϕ (and their superpartners) and their couplings to the moduli. Apart from the supersymmetric terms we also compute moduli-dependent masses and trilinear couplings which arise as a consequence of spontaneous supersymmetry breaking.

2.2.1 Bosonic action

The bosonic part of the action of a single Dp -brane is captured by the Dirac-Born-Infeld action, which reads in string frame

$$S_{\text{DBI}}^{\text{sf}} = -\mu_p \int_{\mathcal{W}} d^{p+1}\xi e^{-\phi} \sqrt{-\det(\varphi^* E_{\mu\nu} + \ell F_{\mu\nu})}, \quad (2.26)$$

and the topological Chern-Simons action

$$S_{\text{CS}} = \mu_p \int_{\mathcal{W}} \varphi^* \left(\sum_q C^{(q)} e^B \right) e^{\ell F}, \quad (2.27)$$

where $\ell = 2\pi\alpha'$. The absolute value of the RR-charge μ_p of the brane is equal to the brane tension for BPS branes. The integrals are taken over the $p+1$ -dimensional world-volume \mathcal{W} of the Dp -brane, which is embedded in the ten dimensional space-time manifold M via the map $\varphi : \mathcal{W} \hookrightarrow M$. For simplicity the combination of the metric g and the B -field

$$E_{\mu\nu} = g_{\mu\nu} + B_{\mu\nu} \quad (2.28)$$

is used. In the brane action the bulk fields E and the bulk RR-fields $C^{(q)}$ are pulled back with φ^* to the world-volume \mathcal{W} of the brane. The dynamics of the Dp -brane is encoded in the pull-back of E .

The Dirac-Born-Infeld action contains a $U(1)$ field strength F , which describes the $U(1)$ gauge theory of the endpoints of open-strings attached to the brane to all orders in $\alpha'F$ [48]. To leading order, the gauge theory reduces to a $U(1)$ Yang-Mills theory on the world-volume \mathcal{W} of the brane.

Since Dp -branes carry RR-charges [1], they couple as extended objects to appropriate RR-forms of the bulk, namely the $p+1$ dimensional world-volume couples naturally to the RR-form $C^{(p+1)}$. Moreover, generically D-branes contain lower dimensional D-brane charges, and hence interact also with lower degree RR-forms [49]. All these couplings to the bulk are implemented in the Chern-Simons action in a way compatible with T-duality. In type IIB string theory we only have even degree RR-forms $C^{(q)}$, and as a consequence we only can have Dp -branes for odd p , in particular D3-branes. Furthermore it implies that in type IIB the sum in (2.27) runs only over even forms. Note that $C^{(0)}$ is the axion which we called l in the previous section.

In order to describe a stack of Dp -branes the above action has to be generalized. The endpoints of open strings are now labeled by the brane they are attached to. Therefore, the modified world-volume action must be a non-Abelian gauge theory of these labels, which are called Chan-Paton factors. In type IIB compactifications with orientifolds, the possible gauge groups turn out to be $U(N)$, $SO(N)$ and $Sp(N)$. For branes which coincide with orientifold planes the gauge groups are either $SO(N)$ or $Sp(N)$ depending on the type of orientifold plane[44]. Branes that are not invariant under the orientifold

projection have $U(N)$ as a gauge group. We limit our analysis to the latter case and from now on consider a stack of N space-time filling D3-branes.

For the non-Abelian Dirac-Born-Infeld action we use the form[29]

$$S_{\text{DBI}}^{\text{sf}} = -\mu_3 \int_{\mathcal{W}} d^4\xi \text{Tr} e^{-\phi} \sqrt{-\det \left(\varphi^* (E_{\mu\nu} + E_{\mu n} (Q^{-1} - \delta)^{nm} E_{m\nu}) + \ell F_{\mu\nu} \right) \det Q_m^n}, \quad (2.29)$$

where

$$Q_m^n = \delta_m^n + i\ell [\phi^n, \phi^k] E_{km}, \quad n, m = 1, \dots, 6. \quad (2.30)$$

Now the function $\varphi : \mathcal{W} \hookrightarrow M$ describes the embedding of the stack of branes in the space-time manifold M . The six ϕ^n parameterize the fluctuations of the branes and are in the adjoint representation of $U(N)$. For the Chern-Simons action we use [29]

$$\mathcal{S}_{\text{CS}} = \mu_3 \int_{\mathcal{W}} \text{Tr} \left(\varphi^* (e^{i\hat{i}_\phi} \sum_{q \text{ even}} C^{(q)} e^B) e^{\ell F} \right). \quad (2.31)$$

i_ϕ denotes the interior multiplication of a form with ϕ^n , which yields for a local q -form

$$i_\phi C^{(q)} = \frac{1}{q!} \sum_{k=1}^q (-1)^{k+1} \phi^n C_{\nu_1 \dots \nu_{k-1} n \nu_{k+1} \dots \nu_q}^{(q)} dx^{\nu_1} \wedge \dots \widehat{dx^{\nu_k}} \dots \wedge dx^{\nu_q}, \quad (2.32)$$

where the differential with the hat $\widehat{}$ is omitted. Note that in equations (2.29) and (2.31) the symmetrized average [50] of the trace has to be taken with respect to the non-Abelian expressions $F_{\mu\nu}$, $D_\mu \phi^m$ and $[\phi^m, \phi^n]$.

Already in the Abelian case one expects additional corrections in α' involving derivative terms beyond second order. However, the Abelian Dirac-Born-Infeld action is expected to capture all α' corrections in F for “slowly-varying” F (i.e. all derivative-independent terms in F). In the non-Abelian case the distinction between the field strength and its covariant derivative is ambiguous since $[D_\mu, D_\nu] F_{\lambda\rho} = [F_{\mu\nu}, F_{\lambda\rho}]$. The symmetrized trace proposal of ref. [50] treats the matrices F (as well as $D_\mu \phi^m$ and $[\phi^m, \phi^n]$) as if they were commuting, leaving out all commutators among these. This proposal was shown to be reliable only up to fourth order in F [51], but this is enough for our purpose.

In order to expand the Dirac-Born-Infeld action (2.29) we first have to expand the square root of the determinant using the standard formula

$$\sqrt{\det(\mathbf{1} + M)} = 1 + \frac{1}{2} \text{Tr} M - \frac{1}{4} \text{Tr} M^2 + \frac{1}{8} (\text{Tr} M)^2 + \dots \quad (2.33)$$

Second, we need to evaluate the pull-back of the metric which is carefully derived in appendix C. In the Abelian case we obtain from eq. (C.9) for the warped metric

$$\varphi^*(g)_{\mu\nu} = e^{2A(y_0)} \tilde{g}_{\mu\nu} + e^{-2A(y_0)} \ell^2 g_{mn} D_\mu \phi^m D_\nu \phi^n + e^{2A(y_0)} \ell^2 \tilde{g}_{\mu\tau} R_n{}^\tau{}_{\nu m} \phi^n \phi^m, \quad (2.34)$$

where y_0 denotes the locus of the D3-branes in the internal space.¹⁴ D_μ is the covariant derivative with respect to the gauge group, and moreover it contains a connection of the

¹⁴We choose y_0 not to be a fixed point of the orientifold involution so as to get a $U(N)$ gauge theory on the world-volume of the branes.

normal bundle of the D3-brane [52]. The latter connection is, however, trivial in the limit of vanishing warp factor due to the product ansatz of the ten dimensional metric (2.1).

The non-Abelian nature of the ϕ are taken into account by using a non-Abelian Taylor expansion of the background fields [29, 53, 54]. On a generic background field T this expansion yields

$$T = \exp [\ell \phi^n \partial_n] \varphi^* T|_{y_0} = \sum_{k=0}^{\infty} \frac{\ell^k}{k!} \phi^{n_1} \cdot \dots \cdot \phi^{n_k} \partial_{n_1} \dots \partial_{n_k} \left(\varphi^* T|_{y_0} \right) . \quad (2.35)$$

Applying this non-Abelian Taylor expansion to the determinant of (2.30) we obtain

$$\sqrt{\det Q_j^i} = 1 + \frac{i\ell^2}{2} [\phi^m, \phi^n] \phi^k \partial_k B_{nm} + \frac{\ell^2}{4} g_{mn} g_{op} [\phi^o, \phi^m] [\phi^n, \phi^p] + \dots , \quad (2.36)$$

where \dots denotes terms which vanish after taking the trace in the Lagrangian. Assembling all terms together we arrive at the following action in the 4D-Einstein frame [26]

$$\begin{aligned} S_{\text{DBI}}^{\text{E}} = & -\mu_3 \int_{\mathcal{W}} d^4 \xi \sqrt{-g_4} \text{Tr} \left(\frac{36 e^{4A}}{\mathcal{K}_w^2} (1 + \ell^2 R_n{}^\tau{}_{\tau m} \phi^n \phi^m) + \frac{\ell^2}{4} e^{-\phi} F^{\mu\nu} F_{\nu\mu} \right. \\ & + \frac{3\ell^2}{\mathcal{K}_w} g_{mn} D_\mu \phi^m D^\mu \phi^n + \frac{9\ell^2 e^{4A}}{\mathcal{K}_w^2} e^\phi (G^{(3)} - \bar{G}^{(3)})_{lmn} \phi^l \phi^m \phi^n \\ & \left. + \frac{9\ell^2}{\mathcal{K}_w^2} e^{\hat{\phi}} g_{qp} g_{mn} [\phi^q, \phi^m] [\phi^n, \phi^p] \right) , \end{aligned} \quad (2.37)$$

where we used (2.19). Note that the last two terms vanish in the Abelian limit.

g_4 is the determinant of the 4D-Einstein frame metric, which is related to the metric $\tilde{g}_{\mu\nu}$ defined in (2.1) by [19]

$$g_{\mu\nu} = \frac{1}{6} \mathcal{K}_w \tilde{g}_{\mu\nu} , \quad \mathcal{K}_w = 6 \int_Y d^6 y \sqrt{\det g_{mn}} e^{-4A} . \quad (2.38)$$

The factor of e^{-4A} results from the reduction of the ten-dimensional curvature scalar of the warped metric. In the reduction of the brane action we include the warp factor taking the large volume limit only when we combine it with the bulk action.

Since the ϕ^m are scalar components of chiral superfields they have to combine into complex variables. Therefore we have to rewrite the action (2.37) in terms of complex fields or in other words we have to find a complex structure compatible with $\mathcal{N} = 1$ supersymmetry. From the action (2.37) we see that the σ -model metric of the ϕ^m coincides with the Calabi-Yau metric g_{mn} and thus a natural guess is to choose the complex structure J of Y also as the complex structure of the low energy effective action.¹⁵ For fixed complex structure we just rewrite all equations in terms of complex indices, i.e. we choose a basis which is compatible with the complex structure J . With respect to this basis J takes block diagonal form

$$J = \begin{pmatrix} +i\mathbf{1} & \\ & -i\mathbf{1} \end{pmatrix} . \quad (2.39)$$

¹⁵By abuse of notation we use the same symbol J as we used for the Kähler form in section 2.1.

Including the complex structure deformations to lowest order we have to perturb J according to [55]

$$\tilde{J} = J + \delta J = \begin{pmatrix} +i\mathbf{1} & z^{\hat{a}}\chi_{\hat{a}} \\ \bar{z}^{\hat{a}}\bar{\chi}_{\hat{a}} & -i\mathbf{1} \end{pmatrix}, \quad (2.40)$$

where $\chi_{\hat{a}}$ is an element of $H^{(0,1)}(Y, T^{(1,0)})$ related to the basis of $H_-^{(2,1)}$ defined in Table 2.1 via

$$(\chi_{\hat{a}})_j^i = \frac{1}{\|\Omega\|^2} \bar{\Omega}^{ilk} (\chi_{\hat{a}})_{lk\bar{j}}. \quad (2.41)$$

As we perturb the complex structure J to \tilde{J} , the eigenvectors of J are also modified. To first order the perturbed eigenvectors read

$$\begin{pmatrix} \phi \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} \phi \\ -\frac{i}{2}\bar{z}^{\hat{a}}\bar{\chi}_{\hat{a}}\phi \end{pmatrix}, \quad \begin{pmatrix} 0 \\ \bar{\phi} \end{pmatrix} \rightarrow \begin{pmatrix} \frac{i}{2}z^{\hat{a}}\chi_{\hat{a}}\bar{\phi} \\ \bar{\phi} \end{pmatrix}, \quad (2.42)$$

with ϕ a vector of $T^{(1,0)}$ and $\bar{\phi}$ a vector of $T^{(0,1)}$ with respect to the fixed complex structure J . Furthermore $\chi_{\hat{a}}$ maps a tangent vector of type $(0, 1)$ to a tangent vector of type $(1, 0)$, and $\bar{\chi}_{\hat{a}}$ vice versa. Hence the complex structure deformations act (up to first order) on the total vector ϕ^n in component notation as

$$\phi^i \rightarrow \phi^i + \frac{i}{2}z^{\hat{a}}(\chi_{\hat{a}})_l^i \bar{\phi}^{\bar{l}}, \quad \bar{\phi}^{\bar{j}} \rightarrow \bar{\phi}^{\bar{j}} - \frac{i}{2}\bar{z}^{\hat{a}}(\bar{\chi}_{\hat{a}})_{\bar{l}}^{\bar{j}} \phi^l. \quad (2.43)$$

Thus in the kinetic term in the action (2.37) we have to replace

$$g_{mn} \rightarrow g_{i\bar{j}} + g_{\bar{i}j} + \delta g_{ij}(\bar{z}^a) + \delta g_{\bar{i}\bar{j}}(z^a), \quad (2.44)$$

and simultaneously

$$\begin{aligned} \mathcal{D}_\mu \phi^i &\rightarrow \mathcal{D}_\mu \phi^i + \frac{i}{2}z^{\hat{a}}(\chi_{\hat{a}})_l^i \mathcal{D}_\mu \bar{\phi}^{\bar{l}}, \\ \mathcal{D}_\mu \bar{\phi}^{\bar{j}} &\rightarrow \mathcal{D}_\mu \bar{\phi}^{\bar{j}} - \frac{i}{2}\bar{z}^{\hat{a}}(\bar{\chi}_{\hat{a}})_{\bar{l}}^{\bar{j}} \mathcal{D}_\mu \phi^l, \end{aligned} \quad (2.45)$$

where δg_{ij} and $\delta g_{\bar{i}\bar{j}}$ are defined in (2.7), and the covariant derivatives read

$$\begin{aligned} \mathcal{D}_\mu \phi^i &= \mathcal{D}_\mu \phi^i + \frac{i}{2}\partial_\mu z^{\hat{a}}(\chi_{\hat{a}})_l^i \bar{\phi}^{\bar{l}}, \\ \mathcal{D}_\mu \bar{\phi}^{\bar{j}} &= \mathcal{D}_\mu \bar{\phi}^{\bar{j}} - \frac{i}{2}\partial_\mu \bar{z}^{\hat{a}}(\bar{\chi}_{\hat{a}})_{\bar{l}}^{\bar{j}} \phi^l. \end{aligned} \quad (2.46)$$

Note that the definition of the covariant derivative \mathcal{D} contains a connection of the normal bundle of the D3-brane, a connection of the gauge group $U(N)$, and finally the newly added connection to include complex structure fluctuations. Inserting (2.44) and (2.45) into the kinetic term of (2.37) results in a cancellation of the terms proportional to z leaving only terms proportional to $\partial_\mu z$ inside the covariant derivatives. We obtain up to linear order in z ¹⁶

$$\mathcal{L}_{\text{kin}} = -\frac{6\mu_3}{\mathcal{K}_w} \text{Tr } g_{i\bar{j}} \mathcal{D}_\mu \phi^i \mathcal{D}^\mu \bar{\phi}^{\bar{j}} = \frac{6i\mu_3}{\mathcal{K}_w} \text{Tr } v^\alpha (\omega_\alpha)_{i\bar{j}} \mathcal{D}_\mu \phi^i \mathcal{D}^\mu \bar{\phi}^{\bar{j}}, \quad (2.47)$$

¹⁶In the covariant derivatives we dropped the term quadratic in z .

where we also used (2.6). Thus by going to a complex basis as is required by supersymmetry, additional (derivative) couplings between the complex structure deformations z and the matter fields ϕ arise. As we will see in section 3 these couplings also induce additional couplings in the Kähler potential which destroy the product structure of the moduli space as indicated in (2.14).

Similarly, we rewrite the quartic non-Abelian term according to

$$g_{op}g_{mn}[\phi^o, \phi^m][\phi^n, \phi^p] \rightarrow 2g_{i\bar{j}}g_{k\bar{l}}[\phi^i, \phi^k][\bar{\phi}^{\bar{l}}, \bar{\phi}^{\bar{j}}] + 2g_{i\bar{j}}g_{k\bar{l}}[\phi^k, \bar{\phi}^{\bar{j}}][\phi^i, \bar{\phi}^{\bar{l}}]. \quad (2.48)$$

Note that there is no complex structure dependence in these terms because it is of higher order (5th order) in the fields ϕ and z . Substituting (2.47) and (2.48) into the action (2.37), we arrive at

$$\begin{aligned} S_{\text{DBI}}^{\text{E}} = & -\mu_3 \int_{\mathcal{W}} d^4\xi \sqrt{-g_4} \text{Tr} \left(\frac{36 e^{4A}}{\mathcal{K}_w^2} (1 + \ell^2 R_n{}^\tau{}_{\tau m} \phi^n \phi^m) + \frac{\ell^2}{4} e^{-\phi} F^{\mu\nu} F_{\mu\nu} \right. \\ & - \frac{6i\ell^2}{\mathcal{K}_w} v^\alpha (\omega_\alpha)_{i\bar{j}} \mathcal{D}_\mu \phi^i \mathcal{D}^\mu \bar{\phi}^{\bar{j}} + \frac{9\ell^2 e^{4A}}{\mathcal{K}_w^2} e^\phi (G^{(3)} - \bar{G}^{(3)})_{lmn} \phi^l \phi^m \phi^n \\ & \left. + \frac{18\ell^2}{\mathcal{K}_w^2} e^\phi \left(g_{i\bar{j}}g_{k\bar{l}}[\phi^i, \phi^k][\bar{\phi}^{\bar{l}}, \bar{\phi}^{\bar{j}}] + g_{i\bar{j}}g_{k\bar{l}}[\phi^k, \bar{\phi}^{\bar{j}}][\phi^i, \bar{\phi}^{\bar{l}}] \right) \right). \quad (2.49) \end{aligned}$$

Our next task is to expand the Chern-Simons action (2.31). It contains interior multiplications with ϕ acting on forms which in local coordinates is given by (2.32).¹⁷ If the ϕ were commuting quantities, the interior multiplication of a form with $i_\phi i_\phi$ would always yield zero.¹⁸ This, however, is not the case as the ϕ are non-Abelian. From a physics point of view the reduction of the gauge group $U(N)$ to $U(1)$, i.e. the transition from non-Abelian ϕ to Abelian ϕ , corresponds to reducing the non-Abelian Chern-Simons action (2.31) to the Abelian Chern-Simons action. Therefore all terms involving $i_\phi i_\phi$ in (2.31) must vanish for the gauge group $U(1)$, because there are no such terms in the Abelian Chern-Simons action. There is, however, a remnant of the disappearing $i_\phi i_\phi$ -terms in the non-Abelian case, because these non-Abelian terms appear in the trace and if ϕ commutes with the form to be multiplied with, then we can use the cyclic property of the trace to rotate the order of the ϕ . The result differs for an odd and even number of i_ϕ operating on forms. In the former case the cyclic property generates an even permutation of ϕ and the interior multiplication yields a possibly non-vanishing term, however, in the latter case the rotation of ϕ corresponds to an odd permutation and due to the skew-symmetry of forms the expression vanishes. In summary we find for all integers k

$$\text{Tr} (i_\phi^{2k} C^{(q)}) = 0 \quad \text{for} \quad [\phi, C^{(q)}] = 0. \quad (2.50)$$

In the reduction we expand the four-form $C^{(4)}$ around its background value which is determined by eqs. (2.17) and (2.20). However, we do not necessarily want to insert this background immediately but rather allow small deviations. This is related to the fact that we introduced a potential in the bulk action which determines the allowed background

¹⁷This makes sense as the ϕ are really elements of the normal space, namely for D3-branes, of the tangent space of the Calabi-Yau at the brane location y_0 .

¹⁸The interior multiplication is an anti-derivation of degree -1 .

values for the fluxes and the background values for the fields only after minimization. In this way we will be able to also include IASD fluxes into our analysis. One way to include small deviations from its background value into the reduction of the D-brane action is to expand the function α in the four-form potential according to

$$\alpha = e^{4A} - h(y) . \quad (2.51)$$

We expand the pull-back of the function h to quadratic order in ϕ

$$\varphi^*(h) = h(y_0) + \ell \nabla_n h|_{y_0} \phi^n + \frac{1}{2} \ell^2 \nabla_m \nabla_n h|_{y_0} \phi^n \phi^m = \frac{1}{2} \ell^2 h_{nm} \phi^n \phi^m , \quad (2.52)$$

where we use

$$h(y_0) = 0 , \quad \nabla_n h|_{y_0} = 0 , \quad h_{nm} \equiv \nabla_m \nabla_n h|_{y_0} . \quad (2.53)$$

This expresses the fact that we are expanding around the solution $\alpha = e^{4A}$. Furthermore, using a complex basis one has $\varphi^*(h) = \frac{1}{2} \ell^2 (h_{i\bar{j}} \phi^i \bar{\phi}^{\bar{j}} + h_{ij} \phi^i \phi^j + \text{h.c.})$.

The additional piece of information we need in order to reduce the Chern-Simons action is the pull-back formula (C.10) and as before also the non-Abelian Taylor expansion formula (2.35). Furthermore, the trace in (2.31) has to be taken as a symmetrized trace over appropriate non-commuting quantities. Altogether this yields the expanded Chern-Simons action in the Einstein frame

$$\begin{aligned} S_{\text{CS}}^{\text{E}} &= \mu_3 \int_{\mathcal{W}} d^4 \xi \sqrt{-g_4} \text{Tr} \left(\frac{36 e^{4A}}{\mathcal{K}_w^2} (1 + \ell^2 R_n{}^\tau{}_{\tau m} \phi^n \phi^m) - \frac{18 \ell^2}{\mathcal{K}_w^2} (h_{i\bar{j}} \phi^i \bar{\phi}^{\bar{j}} + h_{ij} \phi^i \phi^j + \text{h.c.}) \right) \\ &\quad - \mu_3 \int_{\mathcal{W}} d^4 \xi \sqrt{-g_4} \text{Tr} \left(\frac{9i \ell^2 e^{4A}}{\mathcal{K}_w^2} e^\phi (*_6 G^{(3)} + *_6 \bar{G}^{(3)})_{nmp} \phi^n \phi^m \phi^p \right) \\ &\quad + \frac{\mu_3 \ell^2}{4} \int_{\mathcal{W}} \text{Tr} \left(\phi^i \mathcal{D}_\mu \bar{\phi}^{\bar{j}} - \bar{\phi}^{\bar{j}} \mathcal{D}_\mu \phi^i \right) (\omega_\alpha)_{i\bar{j}} dx^\mu \wedge dD_{(2)}^\alpha + 2l \text{Tr} (F \wedge F) , \end{aligned} \quad (2.54)$$

where we have integrated by parts the pull-back of the fluctuations of the 4-form field in the expansion (2.8).

Now we can combine the Dirac-Born Infeld action (2.49) and the Chern-Simons action (2.54) to arrive at

$$\begin{aligned} S_{\text{bos}}^{\text{E}} &= - \frac{\mu_3 \ell^2}{4} \int_{\mathcal{W}} d^4 \xi \sqrt{-g_4} \text{Tr} e^{-\phi} F^{\mu\nu} F_{\mu\nu} + \frac{\mu_3 \ell^2}{2} \int_{\mathcal{W}} l \text{Tr} (F \wedge F) \\ &\quad + \mu_3 \ell^2 \int_{\mathcal{W}} d^4 \xi \sqrt{-g_4} \text{Tr} \left(\frac{6i}{\mathcal{K}_w} v^\alpha (\omega_\alpha)_{i\bar{j}} \mathcal{D}_\mu \phi^i \mathcal{D}^\mu \bar{\phi}^{\bar{j}} - \frac{18}{\mathcal{K}_w^2} (h_{i\bar{j}} \phi^i \bar{\phi}^{\bar{j}} + h_{ij} \phi^i \phi^j + \text{h.c.}) \right) \\ &\quad + \frac{\mu_3 \ell^2}{4} \int_{\mathcal{W}} \text{Tr} \left(\phi^i \mathcal{D}_\mu \bar{\phi}^{\bar{j}} - \bar{\phi}^{\bar{j}} \mathcal{D}_\mu \phi^i \right) (\omega_\alpha)_{i\bar{j}} dx^\mu \wedge dD_{(2)}^\alpha \\ &\quad - \mu_3 \ell^2 \int_{\mathcal{W}} d^4 \xi \sqrt{-g_4} \text{Tr} \left(\frac{9i e^{4A}}{\mathcal{K}_w^2} e^\phi (*_6 G^{(3)} - i G^{(3)})_{nmp} \phi^n \phi^m \phi^p + \text{h.c.} \right. \\ &\quad \left. + \frac{18}{\mathcal{K}_w^2} e^\phi \left(g_{i\bar{j}} g_{k\bar{l}} [\phi^i, \phi^k] [\bar{\phi}^{\bar{l}}, \bar{\phi}^{\bar{j}}] + g_{i\bar{j}} g_{k\bar{l}} [\phi^k, \bar{\phi}^{\bar{j}}] [\phi^i, \bar{\phi}^{\bar{l}}] \right) \right) . \end{aligned} \quad (2.55)$$

Note that the the first terms in (2.49) and (2.54) exactly canceled each other. This is a consequence of the fact that we are expanding around a consistent background determined by (2.20). The deviation from this background is captured by the mass terms $h_{nm}\phi^n\phi^m$. The term trilinear in the ϕ^n vanishes in the Abelian limit and also for ISD-fluxes.

This action has to be added to the bulk action (A.1) in appendix A and the self duality of the (modified) five-form field strength has to be imposed. This eliminates the two forms $D_{(2)}^\alpha$ in favour of the scalars ρ_α , leading to a modification of the derivatives of $\partial_\mu\rho_\alpha$ according to

$$\partial_\mu\rho_\alpha \rightarrow \partial_\mu\rho_\alpha + \ell^2(\omega_\alpha)_{ij} \text{Tr} \left(\bar{\phi}^{\bar{j}}\mathcal{D}_\mu\phi^i - \phi^i\mathcal{D}_\mu\bar{\phi}^{\bar{j}} \right). \quad (2.56)$$

This covariant derivative was also introduced in [14], where it was argued to come from a modified five-form Bianchi identity due to the source term of the charged D3-branes. In our analysis, it appears naturally through the expansion of the Chern-Simons action, which describes the couplings of the RR-charges of the branes to the bulk fields. Furthermore, in our case the covariant derivative also includes couplings to the complex structure deformations.

The final chore is to rewrite the trilinear terms in terms of complex variables. In order to do so, we use the decomposition [19]¹⁹

$$e^{4A}(*_6G_3 - iG_3) = \frac{2i}{w} \left(\mathcal{I}\Omega + \mathcal{G}_{ww}^{\hat{a}\hat{b}} \bar{\chi}_{\hat{b}}\mathcal{I}_{\hat{a}} \right), \quad (2.57)$$

where $\mathcal{I}, \mathcal{I}_{\hat{a}}$ are defined in eq. (2.24) and $\mathcal{G}_{ww}^{\hat{a}\hat{b}}$ is the warped version of the metric defined in (A.2). i.e.

$$\mathcal{G}_{w\hat{a}\hat{b}} \equiv -\frac{1}{w} \int e^{-4A} \chi_{\hat{a}} \wedge \bar{\chi}_{\hat{b}}, \quad w \equiv \int e^{-4A} \Omega \wedge \bar{\Omega}. \quad (2.58)$$

We should apply the substitution rule (2.43) and, in addition, we need to take into account the complex structure dependence of Ω and $\chi_{\hat{a}}$ in (2.57). According to [56]²⁰

$$\frac{\partial\Omega}{\partial z^{\hat{a}}} = k_{\hat{a}}\Omega + i\chi_{\hat{a}}, \quad \frac{\partial\chi_{\hat{a}}}{\partial z^{\hat{b}}} = k_{\hat{b}}\chi_{\hat{a}} + \kappa_{\hat{a}\hat{b}}^{\hat{c}}\bar{\chi}_{\hat{c}}, \quad (2.59)$$

where $\kappa_{\hat{a}\hat{b}}^{\hat{c}}$ is defined in [56] but here we do not need its precise form. With this in mind, we get the following substitution formula for the (3, 0) piece of the flux

$$\begin{aligned} \frac{1}{3!}\Omega_{ijk}\phi^i\phi^j\phi^k &\rightarrow \frac{1}{3!}(1 + k_{\hat{a}}z^{\hat{a}})\Omega_{ijk}(\phi + z^{\hat{a}}\chi_{\hat{a}}\bar{\phi})^i(\phi + z^{\hat{a}}\chi_{\hat{a}}\bar{\phi})^j(\phi + z^{\hat{a}}\chi_{\hat{a}}\bar{\phi})^k \\ &+ \frac{i}{2!}z^{\hat{a}}(\chi_{\hat{a}})_{ij\bar{k}}(\phi + z^{\hat{a}}\chi_{\hat{a}}\bar{\phi})^i(\phi + z^{\hat{a}}\chi_{\hat{a}}\bar{\phi})^j(\bar{\phi} + \bar{z}^{\hat{a}}\bar{\chi}_{\hat{a}}\phi)^{\bar{k}}, \end{aligned} \quad (2.60)$$

which simplifies to linear order in z to

$$\frac{1}{3!}\Omega_{ijk}\phi^i\phi^j\phi^k \rightarrow \frac{1}{3!}(1 + k_{\hat{a}}z^{\hat{a}})\Omega_{ijk}\phi^i\phi^j\phi^k. \quad (2.61)$$

¹⁹The combination $e^{4A} *_6 G_3 - i\alpha G_3$ is closed when τ is constant. Up to to second order in ϕ we can set $\alpha = e^{4A}$, which makes $e^{4A}(*_6G_3 - iG_3)$ closed and consequently coclosed. Therefore, it can be expanded in harmonic three-forms of the warped Calabi-Yau manifold. On a six-dimensional manifold they coincide with the harmonic three-forms of the unwarped Calabi-Yau.

²⁰The expression differs from our conventions by an i compared to ref. [56].

Analogously we obtain linearly in z for the (2, 1) piece of the flux

$$\frac{1}{2!}(\chi_{\hat{a}})_{ijk}\phi^i\phi^j\bar{\phi}^k \rightarrow \frac{1}{2!}(1+k_{\hat{b}}z^{\hat{b}})(\chi_{\hat{a}})_{ijk}\phi^i\phi^j\bar{\phi}^k + \frac{1}{2!}\eta_{\hat{a}\hat{b}}^{\hat{c}}z^{\hat{b}}(\bar{\chi}_{\hat{c}})_{\bar{i}\bar{j}\bar{k}}\bar{\phi}^{\bar{i}}\bar{\phi}^{\bar{j}}\phi^k + \frac{1}{3!}\zeta_{\hat{a}\hat{b}}z^{\hat{b}}\Omega_{ijk}\phi^i\phi^j\phi^k, \quad (2.62)$$

where $\eta_{\hat{a}\hat{b}}^{\hat{c}}$ and $\zeta_{\hat{a}\hat{b}}$ are combinations of χ and κ appropriately contracted but again the precise form is not relevant in the following.

Thus we obtain for the trilinear coupling to first order in z

$$\begin{aligned} \mathcal{L}_{\phi^3} &\sim \mathcal{I}(1+k_{\hat{a}}z^{\hat{a}})\Omega_{ijk}\phi^i\phi^j\phi^k \\ &+ \mathcal{G}_w^{\hat{a}\hat{b}}\mathcal{I}_{\hat{a}}\left(3(1+\bar{k}_{\hat{c}}z^{\hat{c}})\bar{\chi}_{\hat{b}\bar{i}\bar{j}\bar{k}}\bar{\phi}^{\bar{i}}\bar{\phi}^{\bar{j}}\phi^k + 3\bar{\eta}_{\hat{a}\hat{b}}^{\hat{c}}z^{\hat{b}}\chi_{\hat{c}ijk}\phi^i\phi^j\bar{\phi}^k + \bar{\zeta}_{\hat{b}\hat{c}}z^{\hat{c}}\bar{\Omega}_{\bar{i}\bar{j}\bar{k}}\bar{\phi}^{\bar{i}}\bar{\phi}^{\bar{j}}\bar{\phi}^{\bar{k}}\right) + \text{h.c.} \end{aligned} \quad (2.63)$$

where we used (2.57). Finally, we insert (2.63) into (2.55) to obtain

$$\begin{aligned} S_{\text{D3}}^{\text{E}} &= -\frac{\mu_3\ell^2}{4}\int_{\mathcal{W}}d^4\xi\sqrt{-g_4}\text{Tr}e^{-\phi}F^{\mu\nu}F_{\mu\nu} + \frac{\mu_3\ell^2}{2}\int_{\mathcal{W}}l\text{Tr}(F\wedge F) \\ &+ \mu_3\ell^2\int_{\mathcal{W}}d^4\xi\sqrt{-g_4}\text{Tr}\left(\frac{6i}{\mathcal{K}_w}v^\alpha(\omega_\alpha)_{i\bar{j}}\mathcal{D}_\mu\phi^i\mathcal{D}^\mu\bar{\phi}^{\bar{j}} - \frac{18}{\mathcal{K}_w^2}(h_{i\bar{j}}\phi^i\bar{\phi}^{\bar{j}} + h_{ij}\phi^i\phi^j + \text{h.c.})\right) \\ &+ \frac{\mu_3\ell^2}{4}\int_{\mathcal{W}}\text{Tr}(\phi^i\mathcal{D}_\mu\bar{\phi}^{\bar{j}} - \bar{\phi}^{\bar{j}}\mathcal{D}_\mu\phi^i)(\omega_\alpha)_{i\bar{j}}dx^\mu\wedge dD_{(2)}^\alpha \\ &- \mu_3\ell^2\int_{\mathcal{W}}d^4\xi\sqrt{-g_4}\text{Tr}\left[\frac{18}{\mathcal{K}_w^2}e^\phi\left(g_{i\bar{j}}g_{k\bar{l}}[\phi^i,\phi^k][\bar{\phi}^{\bar{l}},\bar{\phi}^{\bar{j}}] + g_{i\bar{j}}g_{k\bar{l}}[\phi^k,\bar{\phi}^{\bar{j}}][\phi^i,\bar{\phi}^{\bar{l}}]\right) \right. \\ &\quad \left. - \frac{18e^\phi}{w\mathcal{K}_w^2}\left(\mathcal{I}(1+k_{\hat{a}}z^{\hat{a}})\Omega_{ijk}\phi^i\phi^j\phi^k + 3\mathcal{G}_w^{\hat{a}\hat{b}}\mathcal{I}_{\hat{a}}\left((1+\bar{k}_{\hat{c}}z^{\hat{c}})(\bar{\chi}_{\hat{b}})_{\bar{i}\bar{j}\bar{k}}\bar{\phi}^{\bar{i}}\bar{\phi}^{\bar{j}}\phi^k \right. \right. \right. \\ &\quad \left. \left. \left. + 3i\bar{\eta}_{\hat{a}\hat{b}}^{\hat{c}}z^{\hat{b}}(\chi_{\hat{c}})_{ijk}\phi^i\phi^j\bar{\phi}^k + \bar{\zeta}_{\hat{b}\hat{c}}z^{\hat{c}}\bar{\Omega}_{\bar{i}\bar{j}\bar{k}}\bar{\phi}^{\bar{i}}\bar{\phi}^{\bar{j}}\bar{\phi}^{\bar{k}}\right) + \text{h.c.}\right)\right]. \end{aligned} \quad (2.64)$$

(Note that $\mu_3\ell^2 = \frac{1}{2\pi}$ holds for D3-branes.)

This concludes our derivation of the bosonic action of a stack of N space-time filling D3-branes in a Calabi-Yau orientifold compactification. We now turn to its fermionic counterpart.

2.2.2 The fermionic action

The supersymmetrized version of the Abelian DBI-CS action has been proposed by different groups [30]. The supersymmetric action looks exactly like the bosonic one, but in the former case the brane lives in superspace, and every space-time field is promoted to a superfield. The explicit form of the action is given in terms of a background supervielbein and super NS-NS and R-R gauge fields, that have to be expanded in terms of component fields. The action has a fermionic kappa-symmetry, which allows to project out half of the fermions by a gauge choice and is obviously reparametrization invariant. When choosing static gauge to fix this latter symmetry, the space-time fermions become world-volume fermions, and these are the fermionic world-volume degrees of freedom.

The non-Abelian version of the supersymmetric action is not known yet. We expect it to contain a trace over the gauge indices of the corresponding Abelian expression plus eventually extra terms. Any extra term must have dimension greater than three, so up to dimension three, which is all we need to compute fermionic supersymmetric and gaugino soft masses, there should not be any additional intrinsically non-Abelian terms.

The terms in the Abelian D3-brane action containing fermion bilinears were found in [27]. We write this action in a fixed gauge for the kappa-symmetry, adding a trace over the gauge indices. In the four-dimensional Einstein frame the Lagrangian is²¹

$$\mathcal{L}_{\text{ferm}}^{\text{E}} = \frac{36\mu_3}{\mathcal{K}_w^2} \text{Tr} \left(-\frac{i\mathcal{K}_w^{1/2}e^{3A}}{2\sqrt{6}} \bar{\theta}\Gamma^\mu D_\mu\theta + \frac{e^{\phi/2}e^{4A}}{48} \text{Re}[(*_6G^{(3)} - iG^{(3)})_{pqr}]\bar{\theta}\Gamma^{pqr}\theta \right) \quad (2.65)$$

where the covariant derivative D_μ contains a spin connection and, in the non-Abelian generalization of the fermionic action, it should also contain the connection of the gauge group $U(N)$.

The spinor θ is a $D = 10$ Majorana-Weyl fermion of negative chirality, i.e. it is in the $\mathbf{16}$ of $SO(9, 1)$. Under $SO(9, 1) \rightarrow SO(3, 1) \times SO(6)$, it decomposes into $(\mathbf{2}, \mathbf{4}) \oplus (\mathbf{2}, \bar{\mathbf{4}})$. This implies that from the four-dimensional point of view, there are four fermions: the gaugino λ , which together with A_μ forms an $\mathcal{N} = 1$ vector multiplet, and three fermions ψ^i , which are the superpartners of the scalars ϕ^i , resulting in three $\mathcal{N} = 1$ chiral multiplets. Ten-dimensional gamma matrices also decompose as

$$\Gamma^\mu = \gamma^\mu \otimes 1, \quad \Gamma^m = \gamma_5 \otimes \gamma^m \quad (2.66)$$

where γ^μ and γ^m are Dirac gamma matrices of $SO(3, 1)$ (in four-dimensional Einstein frame) and $SO(6)$ respectively.

We want to write the action in terms of the four-dimensional gaugino λ and the fermions ψ^i , so we need to see how these fermions are “hidden” in θ . This decomposition is carried out in detail in Appendix D. Here we quote the result:

$$\theta = \ell e^{-3A/2} \left(\left(\frac{\mathcal{K}_w}{6} \right)^{1/4} \frac{1}{2\|\Omega\|} \Omega_{ijk} \psi^i \otimes \gamma^{jk} \chi + \left(\frac{\mathcal{K}_w}{6} \right)^{3/4} e^{-\phi/2} \lambda \otimes \chi \right) + \text{h.c.} \quad (2.67)$$

Inserting this decomposition in the action we get the following kinetic terms²²

$$\mathcal{L}_{\text{kin}} = -i\mu_3\ell^2 \text{Tr} \left(\frac{6}{\mathcal{K}_w} \bar{\psi}^i \gamma^\mu D_\mu \psi^l g_{il} + e^{-\phi} \bar{\lambda} \gamma^\mu D_\mu \lambda \right) \quad (2.68)$$

This is the correct supersymmetrization of the kinetic terms in the bosonic action (2.64).

The interaction term in (2.65) contains the combination $e^{4A}(*_6G^{(3)} - iG^{(3)})$ that appeared already in the bosonic action, and contains only the IASD pieces of 3-form flux. Inserting (2.57) in the second term of (2.65), we get the following interaction Lagrangian²³

$$\mathcal{L}_{\text{int}} = \frac{3i\mu_3 e^{\phi/2}}{4\mathcal{K}_w^2 w} \left[\mathcal{I} \Omega_{ijk} \bar{\theta} \Gamma^{ijk} \theta + 3 \mathcal{G}_w^{\hat{a}\hat{b}} \mathcal{I}_{\hat{a}}(\bar{\chi}_{\hat{b}})_{i\bar{j}\bar{k}} \bar{\theta} \Gamma^{i\bar{j}\bar{k}} \theta \right] + \text{h.c.} \quad (2.69)$$

²¹Our conventions differ from those in [26] by a factor of i in the kinetic term.

²²We also get terms of the form $\bar{\lambda} \gamma^\mu \lambda \partial_\mu \mathcal{K}_w$, for example, but these are the same order as terms we have already neglected in the action, so we also neglect them here.

²³We should also insert here the expansion of Ω and $\chi_{\hat{a}}$ to first order in the complex structure deformations, as done in the bosonic action, but this will give additional terms of the same order of those we have neglected in the fermionic action.

The first contraction in this equation is

$$\Omega_{ijk}\bar{\theta}\Gamma^{ijk}\theta = \frac{1}{6\sqrt{6}}\ell^2\mathcal{K}_w^{3/2}e^{-\phi}\lambda\lambda\Omega_{ijk}\chi^T\gamma^{ijk}\chi = \frac{1}{\sqrt{6}}\ell^2\|\Omega\|\mathcal{K}_w^{3/2}e^{-\phi}\lambda\lambda \quad (2.70)$$

where in the last equality we have used (D.11). Similarly, the second contraction gives

$$(\bar{\chi}_b)_{i\bar{j}\bar{k}}\bar{\theta}\Gamma^{i\bar{j}\bar{k}}\theta = -3\sqrt{6}\ell^2\mathcal{K}_w^{1/2}\frac{1}{\|\Omega\|}(\bar{\chi}_b)_{i\bar{k}\bar{l}}\Omega^{\bar{s}\bar{k}\bar{l}}g_{j\bar{s}}\psi^i\psi^j = -3\sqrt{6}\ell^2\mathcal{K}_w^{1/2}\|\Omega\|(\chi_b)_{i\bar{s}}g_{j\bar{s}}\psi^i\psi^j \quad (2.71)$$

So we get for the interaction Lagrangian

$$\mathcal{L}_{\text{int}} = -\frac{1}{4}\mu_3\ell^2e^{\hat{K}/2}\left[\frac{e^{-\phi}}{2}\mathcal{I}\lambda\lambda - \frac{27}{\mathcal{K}_w}\mathcal{I}_{\hat{a}}\mathcal{G}_w^{\hat{a}\hat{b}}(\bar{\chi}_b)_{i\bar{s}}g_{j\bar{s}}\psi^i\psi^j\right] + \text{h.c.}, \quad (2.72)$$

where we use

$$\|\Omega\| = \sqrt{\frac{6iw}{\mathcal{K}_w}}, \quad e^{\hat{K}/2} = \frac{\sqrt{18ie^{\hat{\phi}/2}}}{\mathcal{K}_w\sqrt{w}}. \quad (2.73)$$

This completes our computation of the fermionic kinetic and mass terms. Their consistency with the bosonic Lagrangian we are going to check in the next section.

3 Soft Supersymmetry Breaking Terms

So far we computed the low energy effective action of type IIB Calabi-Yau orientifold compactification with background D-branes and fluxes. Our next task is to rewrite this action in a standard supergravity form. Since supersymmetry is spontaneously broken by the three-form fluxes it is convenient to use instead of the action (2.9), one that is already adopted to this situation. The case of F -term supersymmetry breaking by moduli fields has been analyzed generically in refs. [34, 35]. In the limit $M_{\text{Pl}} \rightarrow \infty$ with the gravitino mass $m_{3/2}$ fixed, the resulting effective action corresponds to a softly broken, globally supersymmetric theory which is characterized by a Kähler potential K , a superpotential $W^{(\text{eff})}$, a gauge kinetic function f and a set of soft supersymmetry breaking terms. After briefly recalling the notation and results of [34, 35] in section 3.1, we determine in sections 3.2–3.4.2 the supersymmetric couplings and the soft supersymmetry breaking terms by comparing the generic action of [34, 35] with the action computed for D3-branes in a Calabi-Yau orientifold bulk in the previous section. The internal consistency of the derived formulas is highly non-trivial check on the results of the previous section.

3.1 Soft Supersymmetry Breaking from Supergravity

In $\mathcal{N} = 1$ supergravities arising from compactification of string theory it is convenient to distinguish between the chiral charged matter fields ϕ^i and the gauge neutral moduli M^I . (Here ϕ^i and M^I denote the bosonic (lowest) components of chiral multiplets.) In terms of the fields introduced in the previous section this corresponds to the identification $M^I = (T_\alpha, z^{\hat{a}}, G^a, \tau)$ while ϕ^i are the same charged fields as introduced in section 2.2. As long as the gauge symmetry is unbroken, the vacuum expectation value (VEV) of the ϕ^i vanishes and therefore it is convenient²⁴ to expand the Kähler potential $K(M^I, \bar{M}^I, \phi^i, \bar{\phi}^i)$

²⁴Here we follow the analysis of refs. [34, 35] where the notation is adjusted to our situation. We also choose to set $M_{\text{Pl}} = 1$.

and the superpotential $W(M^I, \phi^i)$ in a power series in ϕ^i

$$\begin{aligned} K(M, \bar{M}, \phi, \bar{\phi}) &= \hat{K}(M, \bar{M}) + Z_{i\bar{j}}(M, \bar{M}) \phi^i \bar{\phi}^{\bar{j}} + \left(\frac{1}{2} H_{ij}(M, \bar{M}) \phi^i \phi^j + \text{h.c.} \right) + \dots , \\ W(M, \phi) &= \hat{W}(M) + \frac{1}{2} \tilde{\mu}_{ij}(M) \phi^i \phi^j + \frac{1}{3} \tilde{Y}_{ijk}(M) \phi^i \phi^j \phi^k + \dots . \end{aligned} \quad (3.1)$$

The gauge couplings g are only computed at tree level and therefore obey

$$g^{-2} = \text{Re}f(M) , \quad (3.2)$$

where f is the holomorphic gauge kinetic function.

Spontaneous supersymmetry breaking occurs if a D -term or a F -term has a non-vanishing VEV. As we already noted in section 2.1.3, the three-form fluxes $G^{(3)}$ only lead to non-vanishing F -terms induced by the moduli dependent superpotential $\hat{W}(M)$ given in (2.23). Since the ϕ^i vanish in the ground state they do not contribute to the F -terms and one has

$$\bar{F}^{\bar{I}} = e^{\hat{K}/2} \hat{K}^{\bar{I}J} \left(\partial_J \hat{W} + \hat{W} \partial_J \hat{K} \right) , \quad (3.3)$$

where $\hat{K}^{\bar{I}J} = (\hat{K}_{\bar{I}J})^{-1}$. For vanishing cosmological constant the gravitino mass

$$m_{3/2} = e^{\hat{K}/2} \hat{W} \quad (3.4)$$

is an alternative measure for the supersymmetry breaking.

Without going through the analysis let us just state the resulting effective low energy supergravity potential in the standard limit where one sends $M_{\text{Pl}} \rightarrow \infty$ with $m_{3/2}$ fixed. In this limit one finds [34, 35]

$$\begin{aligned} V^{(\text{eff})} &= \frac{1}{2} D^2 + Z^{i\bar{j}} (\partial_i W^{(\text{eff})}) (\partial_{\bar{j}} \bar{W}^{(\text{eff})}) + m_{i\bar{j}, \text{soft}}^2 \phi^i \bar{\phi}^{\bar{j}} \\ &\quad + \frac{1}{3} A_{ijk} \phi^i \phi^j \phi^k + \frac{1}{2} B_{ij} \phi^i \phi^j + \text{h.c.} , \end{aligned} \quad (3.5)$$

where the ‘supersymmetric’ terms are given by

$$\begin{aligned} D &= -g \bar{\phi}^{\bar{i}} Z_{i\bar{j}} \phi^j , \\ W^{(\text{eff})} &= \frac{1}{2} \mu_{ij} \phi^i \phi^j + \frac{1}{3} Y_{ijk} \phi^i \phi^j \phi^k , \\ \mu_{ij} &= e^{\hat{K}/2} \tilde{\mu}_{ij} + m_{3/2} H_{ij} - F^{\bar{I}} \bar{\partial}_{\bar{I}} H_{ij} , \\ Y_{ijk} &= e^{\hat{K}/2} \tilde{Y}_{ijk} . \end{aligned} \quad (3.6)$$

$W^{(\text{eff})}$ also determines the fermionic masses via

$$m_{ij}^f = \partial_i \partial_j W^{(\text{eff})} = \mu_{ij} . \quad (3.7)$$

The soft supersymmetry breaking terms read

$$\begin{aligned} m_{i\bar{j}, \text{soft}}^2 &= (|m_{3/2}|^2 + V_0) Z_{i\bar{j}} - F^I F^{\bar{J}} R_{I\bar{J}i\bar{j}} , \\ A_{ijk} &= F^I D_I Y_{ijk} , \\ B_{ij} &= (2|m_{3/2}|^2 + V_0) H_{ij} - \bar{m}_{3/2} \bar{F}^{\bar{J}} \bar{\partial}_{\bar{J}} H_{ij} + m_{3/2} F^I D_I H_{ij} \\ &\quad - F^I \bar{F}^{\bar{J}} D_I \bar{\partial}_{\bar{J}} H_{ij} - e^{K/2} \tilde{\mu}_{ij} \bar{m}_{3/2} + e^{K/2} F^I D_I \tilde{\mu}_{ij} , \end{aligned} \quad (3.8)$$

where

$$\begin{aligned}
R_{I\bar{J}i\bar{j}} &= \partial_I \partial_J Z_{i\bar{j}} - \Gamma_{Ii}^k Z_{k\bar{l}} \Gamma_{\bar{J}\bar{j}}^{\bar{l}}, & \Gamma_{Ii}^l &= Z^{\bar{l}j} \partial_I Z_{\bar{j}i}, \\
D_I Y_{ijk} &= \partial_I Y_{ijk} + \frac{1}{2} \hat{K}_I Y_{ijk} - 3 \Gamma_{Ii}^l Y_{ljk}, \\
D_I \tilde{\mu}_{ij} &= \partial_I \tilde{\mu}_{ij} + \frac{1}{2} \hat{K}_I \tilde{\mu}_{ij} - 2 \Gamma_{Ii}^l \tilde{\mu}_{lj}.
\end{aligned} \tag{3.9}$$

This form of the soft terms differs from the ones given in [34] in three respects. First of all, it includes the possibility of a non-vanishing cosmological constant $V_0 = \langle V \rangle$. Secondly, the B -term given in [34] reads $B_{ij} = F^I D_I \mu_{ij} - \bar{m}_{3/2} \mu_{ij}$ (for vanishing cosmological constant). This equation can be obtained from the B -term given in (3.8) by using the condition for the ground state $\partial_I V = 0$ and a vanishing cosmological constant $V_0 = 0$. However, for the purpose of this paper, the form of (3.8) is more convenient in that we do want to allow for a cosmological constant but more importantly later on we compare the terms computed from the D3-brane action with the supergravity formulas just given. In this comparison it is inconvenient to impose $\partial_I V = 0$. Thirdly, in [34] it was assumed that Yukawa couplings Y_{ijk} and the supersymmetric mass terms μ_{ij} are symmetric in its indices which led to slightly different formulas. Here we allow for arbitrary symmetries and changed the covariant derivatives accordingly.

Finally, for the gaugino masses one finds

$$m_g = F^I \partial_I \ln(\text{Re}f). \tag{3.10}$$

The next step is to read off the couplings and masses of this softly broken supersymmetric theory from the actions (2.9), (2.64), (2.68) and (2.72).

3.2 The supersymmetric terms

In this section we extract the supersymmetric couplings, that is $Z_{ij}, H_{ij}, W^{(\text{eff})}$ and f , by comparing with the action computed in (2.9), (2.64), (2.68) and (2.72). This analysis is then completed in the following two sections where the soft supersymmetry breaking terms $m_{i\bar{j}, \text{soft}}, B_{ij}, A_{ijk}, m_g$ are determined.

Let us start with the gauge kinetic function $f(M)$ which can be straightforwardly determined from the first two terms of (2.64) to be

$$f = -i\mu_3 \ell^2 \tau = -\frac{i}{2\pi} \tau, \tag{3.11}$$

where we use $\mu_3 \ell^2 = \frac{1}{2\pi}$.

Let us continue with the kinetic terms and determine the Kähler potential or rather $Z_{i\bar{j}}$ and H_{ij} . $\hat{K}(M, \bar{M})$ was already determined in (2.12) and we should read off Z and H from (2.64). Surprisingly it is possible and in fact easier to compute the full $K(M, \bar{M}, \phi, \bar{\phi})$ and then do the expansion as in (3.1) rather than just reading off $Z_{i\bar{j}}$ and H_{ij} from (2.64).²⁵ The reason is the somewhat complicated field redefinition (2.10) which

²⁵In fact it is straightforward to read off $Z_{i\bar{j}} \sim g_{i\bar{j}}$ from (2.64) but H_{ij} is more involved.

was necessary in order to transform the action obtained from the Kaluza-Klein reduction to the standard $\mathcal{N} = 1$ form (2.9). The same is true if one includes the kinetic terms of the matter fields ϕ^i computed in (2.64). One finds that the Kähler potential takes the form

$$K(\tau, T, G, z, \phi) = -\ln \left[-i \int \Omega \wedge \bar{\Omega} \right] - \ln [-i(\tau - \bar{\tau})] - 2 \ln \left[\frac{1}{6} \mathcal{K}(\tau, T, G, z, \phi) \right], \quad (3.12)$$

which strongly resembles the \hat{K} given in (2.12). As before \mathcal{K} is proportional to the volume of the Calabi-Yau manifold, i.e. $\mathcal{K} \equiv \mathcal{K}_{\alpha\beta\gamma} v^\alpha v^\beta v^\gamma$ still holds, but the dependence of the v^α on the Kähler coordinates is modified in that they are now also a function of the complex structure moduli $z^{\hat{a}}$ and the brane fluctuations ϕ . More precisely, the Kähler coordinates (τ, T, G, z, ϕ) are defined in close analogy with (2.10), and read²⁶

$$\begin{aligned} \tau &= l + ie^{-\phi}, & G^a &= c^a - \tau b^a, & (3.13) \\ T_\alpha &= \frac{3i}{2} \rho_\alpha + \frac{3}{4} \mathcal{K}_\alpha - \frac{3i}{4(\tau - \bar{\tau})} \kappa_{abc} G^b (G - \bar{G})^c + \frac{3}{2} i \mu_3 \ell^2 (\omega_\alpha)_{i\bar{j}} \text{Tr} \phi^i \left(\bar{\phi}^{\bar{j}} - \frac{i}{2} \bar{z}^{\hat{a}} (\bar{\chi}_{\hat{a}})_{\bar{l}}^{\bar{j}} \phi^l \right). \end{aligned}$$

Note that exactly as in (2.10), the functional dependence of \mathcal{K} in terms of the Kähler coordinates can only be stated implicitly by solving (3.13) for v^α in terms of (τ, T, G, z, ϕ) and inserting into \mathcal{K} . Note also that now all moduli fields mix non-trivially in the Kähler potential (3.13), and thus the Kähler metric is no longer block diagonal. In addition, \mathcal{K} is also not of the sequestered form suggested in [31] since \mathcal{K} generically does not split into a sum $\mathcal{K} \neq \mathcal{K}_{\text{hid}}(\tau, T, G, z) + \mathcal{K}_{\text{obs}}(\phi)$.

For a single (radial) Kähler modulus v , (3.13) can be solved explicitly and one obtains

$$\begin{aligned} -2 \ln \mathcal{K} &= -3 \ln \frac{2}{3} \left[T + \bar{T} + \frac{3i}{4(\tau - \bar{\tau})} \mathcal{K}_{1ab} (G - \bar{G})^a (G - \bar{G})^b \right. & (3.14) \\ &\quad \left. + 3i \mu_3 \ell^2 (\omega_1)_{i\bar{j}} \text{Tr}(\phi^i \bar{\phi}^{\bar{j}}) + \frac{3}{4} \mu_3 \ell^2 ((\omega_1)_{i\bar{j}} \bar{z}^{\hat{a}} (\bar{\chi}_{\hat{a}})_{\bar{l}}^{\bar{j}} \text{Tr}(\phi^i \phi^l) + h.c.) \right]. \end{aligned}$$

Note that in this case the Kähler potential is of the sequestered form if we also freeze the complex structure moduli $z^{\hat{a}} = 0$. For $G^a = z^{\hat{a}} = 0$, a similar form of the Kähler potential was suggested in [19, 33]. Here we derived it from a reduction of the DBI-action and also included the couplings to G^a and the complex structure deformations $z^{\hat{a}}$.

The full $K(M, \bar{M}, \phi, \bar{\phi})$ can be expanded for small ϕ according to (3.1) which yields

$$Z_{i\bar{j}} = \frac{6\mu_3 \ell^2}{\mathcal{K}} g_{i\bar{j}} = -i \frac{6\mu_3 \ell^2}{\mathcal{K}} v^\alpha (w_\alpha)_{i\bar{j}}, \quad (3.15)$$

and

$$H_{ij} = \frac{3\mu_3 \ell^2}{\mathcal{K} \|\Omega\|^2} v^\alpha (w_\alpha)_{(j|\bar{s}} \Omega^{\bar{s}k\bar{l}} (\bar{\chi}_{\hat{a}})_{|i)k\bar{l}} \bar{z}^{\hat{a}} = \frac{3\mu_3 \ell^2}{\mathcal{K}} v^\alpha (w_\alpha)_{(j|\bar{s}} (\bar{\chi}_{\hat{a}})_{|i) \bar{s}}^{\bar{j}} \bar{z}^{\hat{a}}. \quad (3.16)$$

Next let us determine $W^{(\text{eff})}$. The Yukawa couplings Y_{ijk} are best determined from the quartic couplings in the potential while the μ -terms are best determined from the fermionic masses. We start with quartic couplings which we read off from (2.64)

$$V_{\text{quartic}}^{\text{D3}} = \frac{18 \ell^2 \mu_3 e^\phi}{\mathcal{K}^2} \text{Tr} \left(g_{i\bar{j}} g_{k\bar{l}} [\phi^i, \phi^k] [\bar{\phi}^{\bar{j}}, \bar{\phi}^{\bar{l}}] + g_{i\bar{j}} g_{k\bar{l}} [\phi^k, \bar{\phi}^{\bar{j}}] [\phi^i, \bar{\phi}^{\bar{l}}] \right). \quad (3.17)$$

²⁶In this coordinate transformation we are only working to leading (linear) order in $z^{\hat{a}}$.

Using the Jacobi identity the two terms can be identified with the sum of a D -term and a term derived from a superpotential. Using eqs. (3.5) and (3.15) we find²⁷

$$Y_{ijk} = 3\mu_3\ell^2 e^{\hat{K}/2} \Omega_{ijk} , \quad \tilde{Y}_{ijk} = 3\mu_3\ell^2 \Omega_{ijk} = \frac{3}{2\pi} \Omega_{ijk} . \quad (3.18)$$

The supersymmetric μ -terms can be determined either from the fermionic mass terms of (2.72) for the ψ^i (which are the superpartners of the ϕ^i) or from eq. (3.6). As we are going to show both determinations agree up to numerical factors and we find²⁸

$$\mu_{ij}^{(D3)} = -\frac{\mu_3\ell^2}{2w} e^{\hat{K}/2} \mathcal{G}^{\hat{a}\hat{b}} \Omega^{\bar{s}\bar{k}\bar{l}} (\bar{\chi}_{\hat{b}})_{(i|\bar{k}\bar{l}} g_{|j)\bar{s}} \mathcal{I}_{\hat{a}} . \quad (3.19)$$

We see that they are proportional to IASD-fluxes (through $\mathcal{I}_{\hat{a}}$) and thus vanish in the supersymmetric limit (or more precisely for ISD-fluxes). $\mu_{ij}^{(D3)}$ has to be compared with the μ_{ij} of (3.6) which has two distinct pieces. The first $\tilde{\mu}_{ij}$ term survives in the supersymmetric limit while the second and third term are proportional to $m_{3/2}$ and F , respectively, which vanish in the supersymmetric limit. Thus, the $\mu_{ij}^{(D3)}$ given in (3.19) has to correspond to these latter terms or in other words is induced by a Giudice-Masiero mechanism [36] and we conclude

$$\tilde{\mu}_{ij} = 0 . \quad (3.20)$$

This is consistent with the fact that for vanishing fluxes all matter fields should be exactly massless. In order to check the consistency of (3.19) with the supergravity expression (3.6), we need to compute

$$\mu_{ij}^{(\text{eff})} = m_{3/2} H_{ij} - F^{\bar{I}} \bar{\partial}_{\bar{I}} H_{ij} , \quad (3.21)$$

and show that it coincides with $\mu_{ij}^{(D3)}$ of (3.19). From eq. (3.16) we learn that H depends explicitly on the complex structure moduli $z^{\hat{a}}$, and through v^α and \mathcal{K}_w also implicitly on the other moduli. Using the formulas given in appendix B, we find

$$\bar{F}^{\bar{I}} \bar{\partial}_{\bar{I}} H_{ij} = m_{3/2} H_{ij} - \bar{F}^{\bar{z}^{\hat{a}}} \partial_{\bar{z}^{\hat{a}}} H_{ij} . \quad (3.22)$$

Inserted into (3.21) yields

$$\mu_{ij}^{(\text{eff})} = -\bar{F}^{\bar{z}^{\hat{a}}} \partial_{\bar{z}^{\hat{a}}} H_{ij} . \quad (3.23)$$

From (3.16) we obtain

$$\partial_{\bar{z}^{\hat{a}}} H_{ij} = \frac{3\mu_3\ell^2}{\mathcal{K}} v^\alpha (w_\alpha)_{(j|\bar{s}} (\bar{\chi}_{\hat{b}})_{|i)}^{\bar{s}} = \frac{\mu_3\ell^2}{2w} g_{(j|\bar{s}} \Omega^{\bar{s}\bar{k}\bar{l}} (\bar{\chi}_{\hat{b}})_{|i)\bar{k}\bar{l}} , \quad (3.24)$$

which, when inserted into (3.23) shows (up to numerical factors) $\mu_{ij}^{(\text{eff})} = \mu_{ij}^{(D3)}$.

²⁷Since we have restricted our analysis to fourth in the fields we do not see the holomorphic $z^{\hat{a}}$ dependence in the quartic term (3.17). Including such terms would result in $\Omega \rightarrow \Omega(1 + k_{\hat{a}} z^{\hat{a}})$ where at linear order in $z^{\hat{a}}$ the function $k_{\hat{a}}$ is constant.

²⁸Here we have taken the numerical factor from the bosonic computation since it is under better control.

Let us close this subsection with the observation that due to the presence of both μ_{ij} and Y_{ijk} a cubic mixed term arises from the $Z^{i\bar{j}}(\partial_i W^{(\text{eff})})(\partial_{\bar{j}} \bar{W}^{(\text{eff})})$ of (3.5). Inserting μ_{ij} of eq. (3.19) and the Yukawa couplings Y_{ijk} of (3.18), we get

$$\begin{aligned}
V_{\text{cubic, susy}}^{(\text{eff})} &= \frac{\mathcal{K}}{9\mu_3\ell^2} \mu_{li} \bar{Y}_{\bar{j}\bar{k}\bar{l}} g^{l\bar{j}} \phi^i \phi^{\bar{k}} \phi^{\bar{l}} + \text{h.c.} \\
&= -\frac{\mu_3\ell^2}{6} e^{\hat{K}} \mathcal{G}^{\hat{a}\hat{b}} \mathcal{I}_{\hat{a}} \frac{1}{w} g_{(l|\bar{s}} \bar{\Omega}^{\bar{s}\bar{t}\bar{u}} (\bar{\chi}_{\hat{b}})_{|i)\bar{t}\bar{u}} \bar{\Omega}_{\bar{j}\bar{k}\bar{l}} g^{l\bar{j}} \phi^i \phi^{\bar{k}} \phi^{\bar{l}} + \text{h.c.} \\
&= -3i\mu_3\ell^2 e^{\hat{K}} \mathcal{G}^{\hat{a}\hat{b}} \mathcal{I}_{\hat{a}} (\bar{\chi}_{\hat{b}})_{i\bar{k}\bar{l}} \phi^i \phi^{\bar{k}} \phi^{\bar{l}} + \text{h.c.}
\end{aligned} \tag{3.25}$$

which is indeed equal to the trilinear mixed term in (2.64).²⁹

This completes the determination of the supersymmetric terms and we now turn to the soft terms.

3.3 Soft supersymmetry breaking terms from supergravity

In this section we compute the soft supersymmetric terms from eqs. (3.8). Let us start with the soft mass terms. Using (B.11) we derive

$$F^I F^{\bar{J}} R_{I\bar{J}i\bar{j}} = |m_{3/2}|^2 Z_{i\bar{j}} . \tag{3.26}$$

Inserted into (3.8) we arrive at

$$m_{i\bar{j}, \text{soft}}^2 = V_0 Z_{i\bar{j}} , \tag{3.27}$$

which corresponds to universal soft scalar masses. For the B -terms we first compute using (B.11) and (B.14)

$$\left(\bar{m}_{3/2} \bar{F}^{\bar{I}} \partial_{\bar{I}} + m_{3/2} F^I D_I - F^I \bar{F}^{\bar{J}} D_I \partial_{\bar{J}} \right) H_{ij} = -2|m_{3/2}|^2 H_{ij} . \tag{3.28}$$

Inserted into (3.8) and using $\tilde{\mu} = 0$ we obtain

$$B_{ij} = V_0 H_{ij} . \tag{3.29}$$

Note that this B -term is not proportional to the μ -term (3.19). For the gaugino masses we insert (3.11) and (B.11) into (3.10) to arrive at³⁰

$$m_g^{(\text{eff})} = -\frac{i}{2} e^\phi F^\tau = -e^{\hat{K}/2} \mathcal{I} . \tag{3.30}$$

For the trilinear A -terms we first compute using (B.12) and (B.14)

$$F^I \left(\hat{K}_I \tilde{Y}_{ijk} - 3\Gamma_{Ii}^l \tilde{Y}_{ljk} \right) = 3\mu_3\ell^2 e^{\hat{K}/2} \left(\mathcal{I} - k_{\hat{a}} G^{\hat{b}\hat{a}} \bar{\mathcal{I}}_{\hat{b}} \right) \Omega_{ijk} , \tag{3.31}$$

²⁹One might have worried that these mixed cubic terms are hard breaking terms since they are proportional to the fluxes yet at the same time they cannot be holomorphic A -terms. This ‘puzzle’ is resolved by the fact that they arise as mixed supersymmetric terms of a Giudice-Masiero μ -term with Yukawa-terms.

³⁰The masses for the bulk gauginos can be computed using (2.15).

and

$$F^I \partial_I \tilde{Y}_{ijk} = F^{z^{\hat{a}}} \partial_{z^{\hat{a}}} \tilde{Y}_{ijk} = 3\mu_3 \ell^2 k_{\hat{a}} \Omega_{ijk} , \quad (3.32)$$

where we needed to include the linear $z^{\hat{a}}$ dependence of the Yukawa couplings discussed earlier. When inserted into (3.8), we arrive at

$$A_{ijk}^{(\text{eff})} = 3\mu_3 \ell^2 e^{\hat{K}} \mathcal{I} \Omega_{ijk} = e^{\hat{K}/2} \mathcal{I} Y_{ijk} . \quad (3.33)$$

We see that the A -terms are proportional to the Yukawa couplings.

This completes our computation of the soft terms from the supergravity formula (3.8). We now compare these results to the terms computed in (2.64), (2.72) and find agreement.

3.4 Comparison with the D-brane action

3.4.1 ISD fluxes

Let us first concentrate on the simplest situation where only imaginary self dual (ISD) fluxes, which obey (2.21), are turned on. These are the (2,1) and (0,3) fluxes and for this case one has

$$\mathcal{I} = \mathcal{I}_{\hat{a}} = V_0 = h_{ij} = h_{i\bar{j}} = 0 . \quad (3.34)$$

For \mathcal{I} and $\mathcal{I}_{\hat{a}}$ this can be seen directly from the definition (2.24) while $V_0 = 0$ for ISD fluxes was shown in (2.25). h vanishes since for ISD fluxes eq. (2.20) holds, implying $h = 0$ in (2.51). Inspecting the D-brane action (2.64), (2.72) we see that for (3.34) all soft terms vanish and apart from the kinetic terms only the quartic couplings survive which result in the Yukawa couplings (3.18). Inserting (3.34) into the soft terms computed in the previous subsection we also find

$$m_{i\bar{j},\text{soft}}^2{}^{(\text{eff})} = B_{ij} = m_g^{(\text{eff})} = A_{ijk}^{(\text{eff})} = 0 . \quad (3.35)$$

The supersymmetric μ -terms, which we already compared in section 3.2, also vanish for ISD fluxes. The vanishing of all soft terms for (2,1) fluxes is a direct consequence of the unbroken supersymmetry while for (0,3) fluxes we have instead a ‘strict’ no-scale supersymmetry breaking [37] in that supersymmetry is broken but it is not communicated to the observable sector. Higher order α' and also loop corrections will induce soft terms and it would be interesting to study their phenomenological properties.

3.4.2 IASD fluxes

As we stated in section 2, ISD fluxes are well understood but not very interesting phenomenologically, as all masses and soft terms vanish when we turn on these fluxes only. So in this subsection we discuss the situation where IASD fluxes are turned on and compare the soft terms computed from the D-brane action with the supergravity formulas.

Let us start with the (normalized) gaugino mass, which can be read off from (2.72) to be

$$m^{(\text{D3})} \sim e^{\hat{K}/2} \mathcal{I} , \quad (3.36)$$

which agrees with (3.30) up to numerical factors.

For the cubic terms in the D-brane action (2.64) we already accounted for the mixed $\phi^i \phi^j \bar{\phi}^{\bar{k}}$ (and their complex conjugate) as arising from the effective superpotential $W^{(\text{eff})}$ as a mixed μ -Yukawa term (c.f.(3.25)). This leaves the holomorphic trilinear terms which should be identified as A -terms. Thus we have

$$A_{ijk}^{(D3)} = 3\mu_3 \ell^2 e^{\hat{K}} \mathcal{I} \Omega_{ijk} (1 + k_{\hat{a}} z^{\hat{a}}) , \quad (3.37)$$

which agrees with (3.33) for $z^{\hat{a}} = 0$. The term linear $z^{\hat{a}}$ could not be computed rigorously in (3.33) since one would need to know \tilde{Y}_{ijk} in (3.32) to quadratic order.

For the mass terms we obtain from (2.64)

$$\begin{aligned} m_{ij}^{2(D3)} &= \frac{36\mu_3 \ell^2}{\mathcal{K}_w^2} h_{ij} , \\ m_{i\bar{j}}^{2(D3)} &= \frac{36\mu_3 \ell^2}{\mathcal{K}_w^2} h_{i\bar{j}} , \end{aligned} \quad (3.38)$$

which should obey

$$\begin{aligned} m_{ij}^{2(D3)} &= B_{ij} , \\ m_{i\bar{j}}^{2(D3)} &= m_{i\bar{j}, \text{susy}}^2 + m_{i\bar{j}, \text{soft}}^2 = Z^{i\bar{k}} \mu_{i\bar{l}}^{(\text{eff})} \bar{\mu}_{\bar{j}\bar{k}}^{(\text{eff})} + m_{i\bar{j}, \text{soft}}^2 \\ &= 6\mu_3 \ell^2 \frac{e^{\hat{K}}}{\mathcal{K}_w} \left[-i \frac{\mathcal{K}_w}{w} \mathcal{G}_w^{\hat{a}\hat{b}} \mathcal{G}_w^{\hat{c}\hat{d}} (\bar{\chi}_{\hat{b}})_{i\bar{k}\bar{l}} (\chi_{\hat{c}})_{\bar{j}}^{\tilde{k}\tilde{l}} \mathcal{I}_{\hat{a}} \bar{\mathcal{I}}_{\hat{d}} + \left(\mathcal{I} \bar{\mathcal{I}} + \mathcal{G}_w^{\hat{a}\hat{b}} \mathcal{I}_{\hat{a}} \bar{\mathcal{I}}_{\hat{b}} \right) g_{i\bar{j}} \right] , \end{aligned} \quad (3.39)$$

where we used (3.19) and (3.27). It would be nice to confirm (3.39) more directly from the D-brane action. Here we have determined them indirectly via a supergravity analysis using the available input from the action. The equations of motion, in particular the one given in (2.18), relate the trace of the mass matrix to the IASD fluxes. For (3,0) fluxes we find agreement (up to numerical factors) while for (1,2) fluxes the first and the second term in (3.39) agree but the third term is missing. We suspect that this is related to the problem of not satisfying global tadpole cancellation conditions when turning on IASD fluxes as also noticed in ref. [38].

Let us close with a few phenomenological observations. We already observed that the B -terms are not proportional to the μ -terms as it is sometimes assumed in phenomenological models. On the other hand the A -terms are proportional to the Yukawa couplings. However, most importantly the soft scalar masses (3.27) are generically universal as a consequence of (3.26) or in other words as a consequence of the Kähler potential (3.12).

4 Conclusions

In this paper we determined the low-energy effective action of type IIB string theory compactified on Calabi-Yau orientifolds with a stack of space-time filling D3-branes, in the presence of background fluxes. Our analysis is quite general in the sense that we did not choose any particular Calabi-Yau manifold but only demanded that it admits

an isometric and holomorphic involution. Furthermore, we considered all $h_-^{2,1} + h^{1,1} + 1$ bulk moduli surviving the orientifold projection. Reducing the appropriate bulk and brane actions we computed the Kähler potential of the four-dimensional effective theory containing the bulk moduli and the charged matter excitations of the brane. In particular we determined the couplings of the complex structure deformations to the matter fields on the brane and showed that all fields mix non-trivially in the Kähler potential. As a consequence it is generically not of the sequestered form. In the limit of just one overall Kähler modulus and frozen complex structure deformations, we recovered the Kähler potential suggested in [19, 33]. In this case the Kähler potential is of the sequestered form but turning on the complex structure deformations already destroys this property.

The presence of the three-form fluxes generically break supersymmetry spontaneously. The couplings of the bulk moduli to the matter fields on the brane communicate this breaking to the observable sector. Via a Giudice-Masiero mechanism ‘supersymmetric’ fermionic masses of the matter fields are induced. We computed them both directly from the D-brane action but also from the appropriate term in the Kähler potential using a supergravity analysis. Similarly, soft A -terms arise which can be computed from the D-brane action but also via a supergravity relation from the Yukawa couplings. The resulting agreement is a strong consistency check on our computation. For the soft scalar masses and the B -terms we can only use the supergravity analysis and compute these terms from the couplings in the Kähler potential.

The phenomenological properties of the resulting soft terms depend on which components of three-form flux is turned on. For ISD fluxes one finds either no supersymmetry breaking at all (for (2,1) fluxes) or a strict no-scale supersymmetry breaking with all soft terms vanishing (for (0,3) fluxes). For IASD fluxes on the other hand we find universal soft masses using the supergravity analysis.

It is important to note that all soft terms get contributions from the VEVs of the auxiliary fields in the dilaton and complex structure multiplets only. Including all Kähler moduli does not affect the result compared to the situation where only one Kähler modulus, the overall volume, is taken into account. This can be traced directly to its “no-scale” property and can be seen from (2.25).

As already noted in [38], one nice feature of supersymmetry breaking by fluxes is that the supersymmetry breaking scale can be much smaller than the string scale. Since 3-form fluxes are quantized in units of $\ell = 2\pi\alpha'$, the supersymmetry breaking scale is of order $M_{susy} = \alpha'/R^3$, where R is an average radius of compactification. In the large volume limit ($R \gg \sqrt{\alpha'}$), this is much lower than the string scale, $M_{string} = 1/\sqrt{\alpha'}$. Furthermore, in the large radius limit, the supersymmetry breaking scale is also much lower than the masses of Kaluza-Klein tower of states, $M_{KK} = 1/R$. Finally, in compactifications with a volume of order R^6 , the 4-dimensional Planck mass is $M_P = R^3/\alpha'^2$, so it is indeed much larger than the rest of the scales in the large volume limit. In summary, spontaneous supersymmetry breaking by 3-form flux in large volume compactifications leads to $M_{susy} \ll M_{KK} \ll M_{string} \ll M_P$.

Appendix

A Kaluza-Klein reduction of Calabi-Yau orientifolds

In this appendix we briefly summarize the calculation of the $\mathcal{N} = 1$ effective action obtained by compactifying type IIB supergravity on Calabi-Yau orientifolds allowing O3/O7 planes [23].³¹ The spectrum of the low-energy theory which is invariant under the orientifold projection (2.2) is discussed in section 2.1.1. Inserting the expansions (2.6)–(2.8) into the ten-dimensional type II B action one obtains (after Weyl rescaling)

$$\begin{aligned}
S_{\text{Bulk}}^{\text{E}} &= \int_{\mathbb{M}_{3,1}} -\frac{1}{2}R * \mathbf{1} - \mathcal{G}_{\hat{a}\hat{b}} dz^{\hat{a}} \wedge *d\bar{z}^{\hat{b}} - G_{\alpha\beta} dv^{\alpha} \wedge *dv^{\beta} - \frac{1}{4}d \ln \mathcal{K} \wedge *d \ln \mathcal{K} \\
&\quad - \frac{1}{4}d\phi \wedge *d\phi - \frac{\mathcal{K}^2}{72}G_{\alpha\beta} dD_{(2)}^{\alpha} \wedge *dD_{(2)}^{\beta} - \frac{1}{8}\mathcal{K}_{\alpha ab} dD_{(2)}^{\alpha} \wedge (c^a db^b - b^a dc^b) \\
&\quad - \frac{1}{4}e^{2\phi}dl \wedge *dl - e^{-\phi}G_{ab} db^a \wedge *db^b - e^{\phi}G_{ab} (dc^a - ldb^a) \wedge * (dc^b - ldb^b) \\
&\quad - \frac{9}{8\mathcal{K}^2}G^{\alpha\beta} \left(d\rho_{\alpha} - \frac{1}{2}\mathcal{K}_{\alpha ab}(c^a db^b - b^a dc^b) \right) \wedge * \left(d\rho_{\beta} - \frac{1}{2}\mathcal{K}_{\beta cd}(c^c db^d - b^c dc^d) \right) \\
&\quad + \frac{1}{8}(\text{Im } \mathcal{M})^{-1} \hat{\alpha}\hat{\beta} (dU_{\hat{\alpha}} - (\mathcal{M}dV)_{\hat{\alpha}}) \wedge * (dU_{\hat{\alpha}} - (\bar{\mathcal{M}}dV)_{\hat{\alpha}}) . \tag{A.1}
\end{aligned}$$

In (A.1) three metrics appear: $G_{\alpha\beta}(v)$ is the metric on $H_+^{(1,1)}$ and thus the metric for the Kähler deformations v^{α} . $G_{ab}(v)$ also depends on the Kähler deformations but is the metric on $H_-^{(1,1)}$, i.e. the -1 -eigenspace of σ^* . $\mathcal{G}_{\hat{a}\hat{b}}(z)$ is the metric on the space of complex structure deformations, i.e. a metric on $H_-^{(2,1)}$. These three metrics are defined as

$$\begin{aligned}
G_{\alpha\beta} &\equiv \frac{3}{2\mathcal{K}} \int \omega_{\alpha} \wedge *\omega_{\beta} = -\frac{3}{2} \left(\frac{\mathcal{K}_{\alpha\beta}}{\mathcal{K}} - \frac{3}{2} \frac{\mathcal{K}_{\alpha}\mathcal{K}_{\beta}}{\mathcal{K}^2} \right) , \\
G_{ab} &\equiv \frac{3}{2\mathcal{K}} \int \omega_a \wedge *\omega_b = -\frac{3}{2} \frac{\mathcal{K}_{ab}}{\mathcal{K}} , \\
\mathcal{G}_{\hat{a}\hat{b}} &\equiv - \frac{\int \chi_{\hat{a}} \wedge \bar{\chi}_{\hat{b}}}{\int \Omega \wedge \bar{\Omega}} ,
\end{aligned} \tag{A.2}$$

where we abbreviated

$$\mathcal{K}_{\alpha\beta} \equiv \mathcal{K}_{\alpha\beta\gamma}v^{\gamma} , \quad \mathcal{K}_{\alpha} \equiv \mathcal{K}_{\alpha\beta\gamma}v^{\beta}v^{\gamma} , \quad \mathcal{K}_{ab} \equiv \mathcal{K}_{ab\alpha}v^{\alpha} , \tag{A.3}$$

³¹This computation is performed for large Calabi-Yau manifolds, i.e. neglecting the warp factor.

and $\mathcal{K}_{\alpha\beta\gamma}, \mathcal{K}_{ab\alpha}$ are the intersection numbers defined in (2.11). The complex matrix \mathcal{M} is defined by using the symplectic basis $\alpha_{\hat{\alpha}}, \beta^{\hat{\beta}}$ of H_+^3 to be

$$\begin{aligned}\int \alpha_{\hat{\alpha}} \wedge * \alpha_{\hat{\beta}} &= -(\text{Im } \mathcal{M} + (\text{Re } \mathcal{M})(\text{Im } \mathcal{M})^{-1}(\text{Re } \mathcal{M}))_{\hat{\alpha}\hat{\beta}}, \\ \int \beta^{\hat{\alpha}} \wedge * \beta^{\hat{\beta}} &= -(\text{Im } \mathcal{M})^{-1}{}^{\hat{\alpha}\hat{\beta}}, \\ \int \alpha_{\hat{\alpha}} \wedge * \beta^{\hat{\beta}} &= -((\text{Re } \mathcal{M})(\text{Im } \mathcal{M})^{-1})_{\hat{\alpha}}^{\hat{\beta}}.\end{aligned}\tag{A.4}$$

The self-duality $*\tilde{F}^{(5)} = \tilde{F}^{(5)}$ is ensured by adding a term $\delta S^E = \frac{1}{4}dV^{\hat{\alpha}} \wedge dU_{\hat{\alpha}} + \frac{1}{4}dD_{(2)}^{\alpha} \wedge d\rho^{\alpha}$ to this action. The equation of motion for $dU_{\hat{\alpha}}$ and $dD_{(2)}^{\alpha}$ coincides with the constraints obtained from the self-duality condition for $F^{(5)}$. Eliminating $dU_{\hat{\alpha}}$ and $dD_{(2)}^{\alpha}$ from the action and one obtains the action (2.9) for the coordinates (2.10) and the Kähler potential (2.12).

The same procedure can be repeated when also D-brane action (2.64) is included. In this case one finds after imposing the self-duality of the modified five-form field strength for $S^E = S_{\text{Bulk}}^E + S_{D3}^E$

$$\begin{aligned}S^E &= \int -\frac{1}{2}R * \mathbf{1} - \mathcal{G}_{\hat{a}\hat{b}} dz^{\hat{a}} \wedge * dz^{\hat{b}} - G_{\alpha\beta} dv^{\alpha} \wedge * dv^{\beta} - \frac{1}{4}d \ln \mathcal{K} \wedge * d \ln \mathcal{K} - \frac{1}{4}d\phi \wedge * d\phi \\ &\quad - \frac{1}{4}e^{2\phi} dl \wedge * dl - e^{-\phi} G_{ab} db^a \wedge * db^b - e^{\phi} G_{ab} (dc^a - l db^a) \wedge * (dc^b - l db^b) \\ &\quad - \frac{9}{4\mathcal{K}^2} G^{\alpha\beta} \left(d\rho_{\alpha} - \frac{1}{2} \mathcal{K}_{\alpha ab} (c^a db^b - b^a dc^b) - \mu_3 \ell^2 \text{Tr} (\phi^i \mathcal{D}_{\mu} \bar{\phi}^{\bar{j}} - \bar{\phi}^{\bar{j}} \mathcal{D}_{\mu} \phi^i) (\omega_{\alpha})_{ij} dx^{\mu} \right) \\ &\quad \wedge * \left(d\rho_{\beta} - \frac{1}{2} \mathcal{K}_{\beta cd} (c^c db^d - b^c dc^d) - \mu_3 \ell^2 \text{Tr} (\phi^i \mathcal{D}_{\mu} \bar{\phi}^{\bar{j}} - \bar{\phi}^{\bar{j}} \mathcal{D}_{\mu} \phi^i) (\omega_{\alpha})_{i\bar{j}} dx^{\mu} \right) \\ &\quad + \frac{1}{4} \text{Im } \mathcal{M}_{\hat{\alpha}\hat{\beta}} dV^{\hat{\alpha}} \wedge * dV^{\hat{\beta}} + \frac{1}{4} \text{Re } \mathcal{M}_{\hat{\alpha}\hat{\beta}} dV^{\hat{\alpha}} \wedge dV^{\hat{\beta}} \\ &\quad + \frac{\mu_3 \ell^2}{2} \left(-e^{-\phi} \text{Tr } F \wedge * F + l \text{Tr} (F \wedge F) \right) \\ &\quad + \mu_3 \text{Tr} \left(\frac{6i\ell^2}{\mathcal{K}} v^{\alpha} (\omega_{\alpha})_{i\bar{j}} \mathcal{D}_{\mu} \phi^i \mathcal{D}^{\mu} \bar{\phi}^{\bar{j}} + \frac{18\ell^2}{\mathcal{K}^2} (h_{i\bar{j}} \phi^i \bar{\phi}^{\bar{j}} + h_{ij} \phi^i \phi^j + \text{h.c.}) \right) * \mathbf{1} \\ &\quad - \mu_3 \text{Tr} \left[\frac{18\ell^2}{\mathcal{K}^2} e^{\phi} \left(g_{ij} g_{k\bar{l}} [\phi^i, \phi^k] [\bar{\phi}^{\bar{l}}, \bar{\phi}^{\bar{j}}] + g_{i\bar{j}} g_{k\bar{l}} [\phi^k, \bar{\phi}^{\bar{j}}] [\phi^i, \bar{\phi}^{\bar{l}}] \right) \right. \\ &\quad \left. - \frac{18\ell^2 e^{\phi}}{w\mathcal{K}^2} \left(\mathcal{I} (1 + k_{\hat{a}} z^{\hat{a}}) \Omega_{ijk} \phi^i \phi^j \phi^k + 3\mathcal{G}^{\hat{a}\hat{b}} \mathcal{I}_{\hat{a}} ((1 + \bar{k}_{\hat{c}} \bar{z}^{\hat{c}}) (\bar{\chi}_{\hat{b}})_{\bar{i}\bar{j}\bar{k}} \bar{\phi}^{\bar{i}} \bar{\phi}^{\bar{j}} \bar{\phi}^{\bar{k}} \right. \right. \\ &\quad \left. \left. + 3i\bar{\eta}_{\hat{a}\hat{b}}^{\hat{c}} \bar{z}^{\hat{b}} (\chi_{\hat{c}})_{ij\bar{k}} \phi^i \phi^j \bar{\phi}^{\bar{k}} + \bar{\zeta}_{\hat{b}\hat{c}}^{\hat{c}} \bar{\Omega}_{\bar{i}\bar{j}\bar{k}} \bar{\phi}^{\bar{i}} \bar{\phi}^{\bar{j}} \bar{\phi}^{\bar{k}} \right) + \text{h.c.} \right] * \mathbf{1}.\end{aligned}\tag{A.5}$$

Using the coordinates given in (3.13) one shows that the kinetic terms of (A.5) combine into a Kähler metric with the Kähler potential (3.12).

B Computing F -terms

In section 3 we need the F -terms derived from the Kähler potential \hat{K} given in (2.12) and the superpotential \hat{W} given in (2.23). For convenience let us repeat these formulæ here

$$\hat{W} = \int \Omega \wedge G^{(3)} \quad (\text{B.1})$$

$$\hat{K}(z, T, G, \tau) = -\ln \left[-i \int \Omega(z) \wedge \bar{\Omega}(\bar{z}) \right] - \ln \left[-i(\tau - \bar{\tau}) \right] - 2\ln \left[\frac{1}{6} \mathcal{K}(T, G, \tau) \right],$$

where $\mathcal{K} \equiv \mathcal{K}_{\alpha\beta\gamma} v^\alpha v^\beta v^\gamma$. The v^α depend implicitly via the equation

$$T_\alpha = \frac{3i}{2} \rho_\alpha + \frac{3}{4} \mathcal{K}_\alpha - \frac{3i}{4(\tau - \bar{\tau})} \mathcal{K}_{abc} G^b (G - \bar{G})^c, \quad (\text{B.2})$$

on the Kähler coordinates T, G and τ defined as

$$\tau = l + i e^{-\phi}, \quad G^a = c^a - \tau b^a, \quad (\text{B.3})$$

and we abbreviated $\mathcal{K}_\alpha \equiv \mathcal{K}_{\alpha\beta\gamma} v^\beta v^\gamma$. The derivatives of the Kähler potential $K_I \equiv \partial_I K$ are

$$\hat{K}_\tau = \frac{i}{2} e^\phi + i G_{ab} b^a b^b, \quad \hat{K}_{T_\alpha} = -2 \frac{v^\alpha}{\mathcal{K}}, \quad \hat{K}_{G^a} = 2i G_{ab} b^b, \quad (\text{B.4})$$

where G_{ab} is defined in (A.2). For the complex structure one has (c.f. eq. (2.59)) [56]

$$\hat{K}_{z^{\hat{a}}} = -k_{\hat{a}}, \quad \partial_{\hat{a}} \Omega = k_{\hat{a}} \Omega + i \chi_{\hat{a}}. \quad (\text{B.5})$$

The Kähler metric is derived by taking one more (antiholomorphic) derivative leading to

$$\begin{aligned} \hat{K}_{T_\alpha \bar{T}_\beta} &= \frac{G^{\alpha\beta}}{\mathcal{K}^2}, \\ \hat{K}_{T_\alpha \bar{G}^a} &= -\frac{3i}{4} \frac{G^{\alpha\beta}}{\mathcal{K}^2} \mathcal{K}_{\beta ab} b^b, \\ \hat{K}_{T_\alpha \bar{\tau}} &= -\frac{3i}{4} \frac{G^{\alpha\beta}}{\mathcal{K}^2} \mathcal{K}_{\beta ab} b^a b^b, \\ \hat{K}_{G^a \bar{G}^b} &= e^\phi G_{ab} + \frac{9}{4} \frac{G^{\alpha\beta}}{\mathcal{K}^2} \mathcal{K}_{\beta ac} b^c \mathcal{K}_{\beta bd} b^d, \\ \hat{K}_{G^a \bar{\tau}} &= e^\phi G_{ab} b^b + \frac{9}{8} \frac{G^{\alpha\beta}}{\mathcal{K}^2} \mathcal{K}_{\beta ac} b^c \mathcal{K}_{\beta bd} b^b b^d, \\ \hat{K}_{\tau \bar{\tau}} &= \frac{1}{4} e^{2\phi} + e^\phi G_{ab} b^a b^b + \frac{9}{16} \frac{G^{\alpha\beta}}{\mathcal{K}^2} \mathcal{K}_{\beta ac} b^a b^c \mathcal{K}_{\beta bd} b^b b^d, \\ \hat{K}_{z^{\hat{a}} \bar{z}^{\hat{b}}} &= \mathcal{G}_{\hat{a}\hat{b}}. \end{aligned} \quad (\text{B.6})$$

Its inverse is found to be

$$\begin{aligned}
\hat{K}^{T_\alpha \bar{T}_\beta} &= \mathcal{K}^2 G_{\alpha\beta} - \frac{9}{4} e^{-\phi} G^{ab} \mathcal{K}_{\beta ac} b^c \mathcal{K}_{\beta bd} b^d + \frac{9}{4} e^{-2\phi} \mathcal{K}_{\beta ac} b^a b^c \mathcal{K}_{\beta bd} b^b b^d, \\
\hat{K}^{T_\alpha \bar{G}^a} &= -\frac{3i}{2} e^{-\phi} G^{ab} \mathcal{K}_{abc} b^c - 3i e^{-2\phi} \mathcal{K}_{abc} b^b b^c b^a, \\
\hat{K}^{T_\alpha \bar{\tau}} &= 3i e^{-2\phi} \mathcal{K}_{abc} b^c, \\
\hat{K}^{G^a \bar{G}^b} &= e^{-\phi} G^{ab} + 4e^{-2\phi} b^a b^b, \\
\hat{K}^{G^a \bar{\tau}} &= -4e^{-2\phi} b^a \\
\hat{K}^{\tau \bar{\tau}} &= 4e^{-2\phi}, \\
\hat{K}^{z^{\hat{a}} \bar{z}^{\hat{b}}} &= \mathcal{G}^{\hat{a}\hat{b}}.
\end{aligned} \tag{B.7}$$

Using (B.4) and (B.1) we can determine the covariant derivatives $D_I W \equiv (\partial_I + K_I)W$ to be

$$\begin{aligned}
D_\tau \hat{W} &= \frac{i}{2} e^{\phi} \bar{\mathcal{I}} + i G_{ab} b^a b^b \hat{W}, & D_{T_\alpha} \hat{W} &= -2 \frac{v^\alpha}{\mathcal{K}} \hat{W}, \\
D_{G^a} \hat{W} &= 2i G_{ab} b^b \hat{W}, & D_{z^{\hat{a}}} \hat{W} &= \mathcal{I}_{\hat{a}},
\end{aligned} \tag{B.8}$$

where

$$\mathcal{I} \equiv \int \bar{\Omega} \wedge G^{(3)}, \quad \mathcal{I}_{\hat{a}} \equiv \int \chi_{\hat{a}} \wedge G^{(3)}. \tag{B.9}$$

Using (B.6) and (B.8) one computes the F-terms needed in the main text. They are defined as $F^I = e^{\hat{K}/2} K^{I\bar{J}} D_{\bar{J}} \hat{W}$ and read

$$\begin{aligned}
F^\tau &= -2i e^{\hat{K}/2} e^{-\phi} \mathcal{I}, \\
F^{T_\alpha} &= e^{\hat{K}/2} \left(-\frac{3}{2} \mathcal{K}_\alpha \hat{W} + \frac{3}{2} e^{-\hat{\phi}} \mathcal{K}_{\alpha ab} b^a b^b \mathcal{I} \right), \\
F^{G^a} &= e^{\hat{K}/2} 2i e^{-\phi} b^a \mathcal{I}, \\
F^{z^{\hat{a}}} &= e^{\hat{K}/2} \mathcal{G}^{\hat{a}\hat{b}} \bar{\mathcal{I}}_{\hat{b}}.
\end{aligned} \tag{B.10}$$

Using (B.10) and (B.4) one obtains the following useful identities

$$\begin{aligned}
F^I \hat{K}_I &= e^{\hat{K}/2} \left(\mathcal{I} - k_{\hat{a}} \mathcal{G}^{\hat{a}\hat{b}} \bar{\mathcal{I}}_{\hat{b}} + 3 \hat{W} \right) \\
F^I \partial_I v^\alpha &= -\frac{1}{2} e^{\hat{K}/2} v^\alpha \hat{W}, & F^I \partial_I \mathcal{K} &= -\frac{3}{2} e^{\hat{K}/2} \mathcal{K} \hat{W}, \\
F^I F^{\bar{J}} \partial_I \partial_{\bar{J}} v^\alpha &= -\frac{1}{4} e^{\hat{K}} |\hat{W}|^2 v^\alpha, & F^I F^{\bar{J}} \partial_I \partial_{\bar{J}} \mathcal{K} &= -\frac{3}{4} e^{\hat{K}} |\hat{W}|^2 \mathcal{K}.
\end{aligned} \tag{B.11}$$

The other quantities we need in the main text are the the Christoffel following symbols defined as $\Gamma_{i\bar{j}}^k = Z^{\bar{k}j} \partial_I Z_{\bar{j}i}$ for the matter metric

$$Z_{i\bar{j}} = -i \frac{6\mu_3 \ell^2}{\mathcal{K}} v^\alpha (\omega_\alpha)_{i\bar{j}}. \tag{B.12}$$

One finds

$$\begin{aligned}
\Gamma_{i\tau}^k &= \frac{3\mu\ell^2}{2\mathcal{K}}\kappa^{\alpha\beta}\mathcal{K}_{\beta ab}b^ab^b(\omega_\alpha)_{\bar{j}i}Z^{k\bar{j}} - \frac{i}{2}G_{ab}b^ab^b\delta_k^i, \\
\Gamma_{iG^\alpha}^k &= \frac{3\mu_3\ell^2}{\mathcal{K}}\kappa^{\alpha\beta}\kappa_{\beta ab}b^b(\omega_\alpha)_{\bar{j}i}Z^{k\bar{j}} - iG_{ab}b^b\delta_k^i, \\
\Gamma_{iT^\alpha}^k &= -\frac{2i\mu_3\ell^2}{\mathcal{K}}\kappa^{\alpha\beta}(\omega_\beta)_{\bar{j}i}Z^{k\bar{j}} + \frac{1}{\mathcal{K}}v^\alpha\delta_k^i,
\end{aligned} \tag{B.13}$$

obeying the identity

$$F^I\Gamma_{Ii}^l = e^{\hat{K}/2}\hat{W}\delta_i^l. \tag{B.14}$$

C Normal coordinates expansion

In the Dirac-Born-Infeld action (2.29) and the Chern-Simons action (2.31) of the Dp -brane, there appear various contributions which have to be pulled back from the space-time manifold M to the world-volume \mathcal{W} of the brane via $\varphi : \mathcal{W} \hookrightarrow M$.

The pull-back of the D-brane action contains the whole dynamics of the brane in the following way: The pull-back is not just performed with a rigid map φ , but we allow for small fluctuations normal to the embedded world-volume $\mathcal{W} \subset M$. Hence we must describe these fluctuations in an appropriate way.

The fluctuations of the embedded world-volume are described by considering displacements of the embedding in the normal direction of the world-volume as in [57, 58]. These displacements are encoded in sections of the normal bundle $N\mathcal{W}$ of a fixed world-volume. Let us take the section ξ of the normal bundle to represent a fluctuation. The section ξ gives rise to a map $\hat{\varphi} : \mathcal{W} \times I \hookrightarrow M$, $(y, t) \mapsto \hat{\varphi}(y, t)$ with the following properties:

$$\hat{\varphi}(y, 0) = \varphi(y), \tag{C.1}$$

and for fixed y the function $\hat{\varphi}(y, t)$ parameterizes a geodesic in the direction $\xi|_y$ in such a way that the geodesic from $t = 0$ to $t = 1$ has arc-length $\|\xi\|$. In mathematical terms the function $\hat{\varphi}$ is given by the exponential map of ξ . Thus

$$\frac{d}{dt}\hat{\varphi}(y, 0) = \xi|_y, \tag{C.2}$$

and we extend ξ to $\mathcal{W} \times I$ by parallel transport along t . As there exists a tubular neighborhood of \mathcal{W} , we always have this construction for sufficiently small fluctuations ξ .

Having expressed the fluctuations in terms of the map $\hat{\varphi}$, we are now able to pull-back a tensor T of M and expand using

$$\begin{aligned}
(\hat{\varphi}^*T)|_{\hat{\varphi}(y,t)} &= \hat{\varphi}^*(e^{\nabla^t\xi}T)|_{\hat{\varphi}(y,0)} \\
&= \hat{\varphi}^*(T)|_{\hat{\varphi}(y,0)} + t\hat{\varphi}^*(\nabla_\xi T)|_{\hat{\varphi}(y,0)} + \frac{1}{2}t^2\hat{\varphi}^*(\nabla_\xi\nabla_\xi T)|_{\hat{\varphi}(y,0)} + \dots \tag{C.3}
\end{aligned}$$

Recall that the Riemann tensor and the torsion of a (Pseudo-) Riemannian manifold is given by

$$R(X, Y)Z = \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X, Y]} Z \quad (\text{C.4})$$

$$T(X, Y) = \nabla_X Y - \nabla_Y X - [X, Y] = 0, \quad (\text{C.5})$$

where the Levi-Cevita connection ∇ is metric and torsion-free.

In local coordinates (x^μ, t) of $U \times I \subset \mathcal{W} \times I$ where $\mu = 1, \dots, \dim \mathcal{W}$, we have the vector fields $(\partial_\mu, \partial_t)$. Since the Lie-bracket $[\partial_t, \partial_\mu]$ vanishes and we get (C.5)

$$\nabla_\xi \partial_\mu = \nabla_{\partial_\mu} \xi, \quad (\text{C.6})$$

and hence with (C.4) and $\nabla_\xi \xi = 0$

$$R(\xi, \partial_\mu) \xi = \nabla_\xi \nabla_{\partial_\mu} \xi = \nabla_\xi \nabla_\xi \partial_\mu. \quad (\text{C.7})$$

The developed tools we readily apply to the metric and obtain up to second order in the parameter t

$$\begin{aligned} (\hat{\varphi}^* g(\partial_\mu, \partial_\nu))|_{\hat{\varphi}(y, t)} &= \hat{\varphi}^* (g(\partial_\mu, \partial_\nu))|_{\hat{\varphi}(y, 0)} + \\ &+ t \hat{\varphi}^* (g(\nabla_{\partial_\mu} \xi, \partial_\nu))|_{\hat{\varphi}(y, 0)} + t \hat{\varphi}^* (g(\partial_\mu, \nabla_{\partial_\nu} \xi))|_{\hat{\varphi}(y, 0)} \\ &+ t^2 \hat{\varphi}^* (g(\nabla_{\partial_\mu} \xi, \nabla_{\partial_\nu} \xi))|_{\hat{\varphi}(y, 0)} + t^2 \hat{\varphi}^* (g(R(\xi, \partial_\mu) \xi, \partial_\nu))|_{\hat{\varphi}(y, 0)} + \dots \end{aligned} \quad (\text{C.8})$$

This index free notation translates in a slightly abusive way into the component expression (for $t = 1$)

$$\begin{aligned} \varphi^*(g)_{\mu\nu} &= g_{\mu\nu} + \ell g_{\mu n} D_\nu \phi^n + \ell g_{\nu n} D_\mu \phi^n \\ &+ \ell^2 g_{nm} D_\mu \phi^n D_\nu \phi^m + \ell^2 g_{\mu\tau} R_n{}^\tau{}_{\nu m} \phi^n \phi^m + \dots, \end{aligned} \quad (\text{C.9})$$

where the $\ell\phi$ s denote the fluctuations of the brane in the normal direction and D is a covariant derivative of the normal bundle. Greek indices are used for the world-volume coordinates of the brane and Latin indices for the normal directions of the world-volume. As we consider a space-time filling D3-brane, the Greek indices correspond also to the non-compact space-time manifold and the Latin indices to the internal (Calabi-Yau) manifold. In the same way one can derive the pull-back of various forms appearing in (2.29) and (2.31). For a p -form we obtain in local coordinates up to second order (for $t = 1$)

$$\begin{aligned} (\hat{\varphi}^* C^{(p)})|_{\hat{\varphi}(y, t)} &= \left(\frac{1}{p!} C_{\nu_1 \dots \nu_p}^{(p)} + \frac{\ell}{p!} \phi^n \partial_n (C_{\nu_1 \dots \nu_p}^{(p)}) - \frac{\ell}{(p-1)!} D_{\nu_1} \phi^n C_{n\nu_2 \dots \nu_p}^{(p)} \right. \\ &+ \frac{\ell^2}{2p!} \phi^n \partial_n (\phi^m \partial_m (C_{\nu_1 \dots \nu_p}^{(p)})) - \frac{\ell^2}{(p-1)!} D_{\nu_1} \phi^n \cdot \phi^m \partial_m (C_{n\nu_2 \dots \nu_p}^{(p)}) \\ &+ \frac{\ell^2}{2(p-2)!} D_{\nu_1} \phi^n D_{\nu_2} \phi^m C_{nm\nu_3 \dots \nu_p}^{(p)} \\ &\left. + \frac{p-2}{2p!} \ell^2 R_n{}^\tau{}_{\nu_1 m} \phi^n \phi^m C_{\tau\nu_2 \dots \nu_p}^{(p)} \right) dx^{\nu_1} \wedge \dots \wedge dx^{\nu_p}. \end{aligned} \quad (\text{C.10})$$

D Decomposition of the 10D spinor.

The spinor θ is a 10d Majorana-Weyl fermion of negative chirality, i.e. in the **16** of $SO(9,1)$. Under $SO(9,1) \rightarrow SO(3,1) \times SO(6)$, it decomposes into $(\bar{\mathbf{2}}, \mathbf{4}) \oplus (\mathbf{2}, \bar{\mathbf{4}})$. From the 4-dimensional point of view there are four fermions, the gaugino λ , superpartner of the gauge field A_μ , and 3 fermions ψ^i , superpartners of the scalars ϕ^i . The fermions ψ^i and λ should then be contained in θ , in the following way

$$\theta = a \psi^i \otimes \chi_i + b \lambda \otimes \chi_4 + \text{h.c.} , \quad (\text{D.1})$$

where $i = 1, 2, 3$ and a and b could be functions of internal space. In order to find these functions, and the precise form of χ_i and χ_4 , let us perform a world-volume supersymmetry transformation of the world-volume fields ϕ^i, A_μ , and compare it to the standard $\mathcal{N} = 1$ supersymmetry transformation of the scalar in a chiral and the vector in a vector multiplet. World-volume fields transform according to

$$\delta_\epsilon \phi^i = \bar{\epsilon} \tilde{\Gamma}^i \theta , \quad (\text{D.2})$$

$$\delta_\epsilon A_\mu = \bar{\epsilon} \Gamma_\mu \theta , \quad (\text{D.3})$$

where $\tilde{\Gamma}^i$ is a gamma-matrix for the warped metric. On the other hand, the usual $\mathcal{N} = 1$ supersymmetry transformations of the scalar in a chiral multiplet and the vector in the vector multiplet are

$$\delta_\epsilon \phi^i = \epsilon \psi^i , \quad (\text{D.4})$$

$$\delta_\epsilon A_\mu = \bar{\epsilon} \gamma_\mu \lambda . \quad (\text{D.5})$$

We want to perform a supersymmetry transformation that leaves the supergravity background invariant. With a background metric of the warped form, eq.(2.1), the invariant spinor is [39]

$$\epsilon = e^{A/2} (\epsilon_4 \otimes \chi + \epsilon_4^* \otimes \chi^*) , \quad (\text{D.6})$$

where ϵ_4 is a constant spinor in 4 dimensions with positive chirality, $\gamma_5 \epsilon_4 = \epsilon_4$, and χ is a nowhere vanishing covariantly constant with respect to the unwarped metric g_{mn} , i.e., $D_m \chi = 0$, and has negative 6d chirality. We can take χ to be normalized to one, i.e. $\chi^\dagger \chi = 1$. By doing an $SO(6)$ rotation, we can also choose χ to be annihilated by all lowering operators

$$\gamma^{\bar{i}} \chi = 0. \quad (\text{D.7})$$

Using the formula for the world-volume supersymmetry transformations (D.3), for a supersymmetry parameter given by (D.6), and inserting the decomposition (D.1) we get

$$\delta_\epsilon A_\mu = b e^{A/2} \bar{\epsilon}_4 \gamma_\mu \lambda \chi^\dagger \chi_4 = b e^{3A/2} \bar{\epsilon}_4 \tilde{\gamma}_\mu \lambda \chi^\dagger \chi_4. \quad (\text{D.8})$$

Then, taking $b = e^{-3A/2}$; $\chi_4 = \chi$, we recover the global $\mathcal{N} = 1$ supersymmetry transformation (D.5). Repeating the process for ϕ^i , we get

$$\delta_\epsilon \phi^i = a e^{3A/2} \epsilon_4 \psi^j \chi^T \gamma^i \chi_j . \quad (\text{D.9})$$

We recover the usual $\mathcal{N} = 1$ supersymmetry transformation for the scalar in the chiral multiplet if we set

$$a \chi_i = \frac{1}{3! \|\Omega\|} e^{-3A/2} \Omega_{ijk} \gamma^{jk} \chi \quad (\text{D.10})$$

where the γ s are taken with respect to the unwarped metric $g_{mn}(y)$, Ω is the holomorphic (3,0)-form of the Calabi-Yau, $\|\Omega\|^2 = \frac{1}{3!}\Omega_{ijk}\bar{\Omega}^{ijk}$ and we have used

$$\chi^T \gamma^{ijk} \chi = \frac{1}{\|\Omega\|} \bar{\Omega}^{ijk}. \quad (\text{D.11})$$

Combining both results, the spinor θ is

$$\theta = e^{-3A/2} \left(\frac{1}{6\|\Omega\|} \Omega_{ijk} \psi^i \otimes \gamma^{jk} \chi + \lambda \otimes \chi \right) + h.c. \quad (\text{D.12})$$

When inserting this spinor in the fermionic Lagrangian (2.65), we get a kinetic term of the form

$$\mathcal{L}_{kin} = -i\mu_3 \frac{\sqrt{6}}{\mathcal{K}_w^{3/2}} Tr \left(\frac{2}{3} \bar{\psi}^i \gamma^\mu D_\mu \psi^l \tilde{g}_{il} + 6\bar{\lambda} \gamma^\mu D_\mu \lambda \right). \quad (\text{D.13})$$

From supersymmetry, the fermionic and bosonic kinetic terms should have a common Kähler metric, i.e.

$$\begin{aligned} \mathcal{L}_{kin,susy} = & - K_{\bar{i}l} D_\mu \bar{\phi}^{\bar{i}} D^\mu \phi^l - i K_{\bar{i}l} \bar{\psi}^{\bar{i}} \gamma^\mu D_\mu \psi^l \\ & - Re \left[\frac{1}{4} f_{ab} (F^a + i *_4 F^a)_{\mu\nu} (F^b + i *_4 F^b)^{\mu\nu} + i f_{ab} \bar{\lambda}^a \gamma^\mu D_\mu \lambda^b \right] \end{aligned} \quad (\text{D.14})$$

where $K_{\bar{i}l}$ is the derivative of the Kähler potential.

From the bosonic DBI action (2.49), we see that $K_{\bar{i}l} = \frac{6\ell^2}{\mathcal{K}_w} g_{\bar{i}l}$. Then, from susy, we should have the same metric in the kinetic term for the fermions in the chiral multiplet, which is different from the $\mathcal{K}_w^{-3/2} g_{\bar{i}l}$ that we have in (D.13)³². This means that we need to renormalize the fermions ψ^i , i.e. the actual susy partners of the bosons ϕ^i are the fermions ψ'^i , defined as

$$\psi'^i = \frac{1}{\ell} \left(\frac{2}{27\mathcal{K}_w} \right)^{1/4} \psi^i. \quad (\text{D.15})$$

Besides, we see from the bosonic DBI plus CS action (2.64) that

$$f_{ab} = -i\mu_3 \ell^2 \tau \delta_{ab} \quad (\text{D.16})$$

which means that we also need to renormalize the gaugino, such that its kinetic term follows the form (D.14), namely

$$\lambda' = \frac{1}{\ell} \left(\frac{6}{\mathcal{K}_w} \right)^{3/4} \lambda. \quad (\text{D.17})$$

Then, the decomposition of θ in terms of these spinors is

$$\theta = \ell e^{-3A/2} \left(\left(\frac{\mathcal{K}_w}{6} \right)^{1/4} \frac{1}{2\|\Omega\|} \Omega_{ijk} \psi'^i \otimes \gamma^{jk} \chi + \left(\frac{\mathcal{K}_w}{6} \right)^{3/4} e^{-\phi/2} \lambda' \otimes \chi \right) + h.c. \quad (\text{D.18})$$

and kinetic term for the fermions is of the form (D.14). In the main sections of the paper we have dropped the primes for the fermions.

³²If we had found the complete action (bosonic and fermionic) from the supersymmetric DBI-CS, supersymmetry would be automatic. But since we need bosonic terms that appear only in the non-Abelian action, and we do not have its supersymmetric version, we are forced to compute bosonic and fermionic terms separately.

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