

# **OPERATIONAL CHARACTERISTICS OF BRUSHLESS DC MOTORS**

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## **Abstract**

A brushless dc motor can be described as an inverted brush dc motor with its magnet being the rotor and its stationary windings forming the stator. This design provides many advantages over the brush dc motor. The operational characteristic of a motor is important for its control, modeling and deriving optimum performance. However, a review of the literature did not present motor operation based on the various phenomena occurring in the motor. Smooth and efficient operation of the brushless dc motor relies on a knowledge of the energization sequence of the windings and a technique for its determination is presented. The application of the vector method of analysis to the

brushless dc motor together with developed energization sequence, produced continuous electromagnetic torque utilizing the cross product of the peak flux linkage vector due to the rotor magnet and the stator current vector and classifies this motor as an ac synchronous motor.

*Keywords:* Operational; characteristics; bldcm.

## 1. Introduction

Brushless dc motors are rapidly gaining popularity in the appliance, automotive, aerospace, consumer, medical and industrial automation industries. As a result of the absence of mechanical commutators and brushes and the permanent magnet rotor, brushless dc motors have many advantages over the brush dc and induction motor [1-6]. Some of the advantages of brushless dc motors are:

- (1) High power density, low inertia and high torque to inertia ratio and high dynamic response due to the small size, low weight and high flux density neodymium-iron-boron permanent magnet rotor.
- (2) High efficiency due to the low rotor losses as a result of the absence of current carrying conductors on the rotor and reduced friction and windage losses in the rotor.
- (3) Long operating life and high reliability due to the absence of brushes and metallic commutators.
- (4) Clean operation due to the absence of brushes, resulting in no brush dust during operation and allowing for clean room applications.
- (5) Low audible noise operation due to the absence of brushes, commutators and smooth low air resistance rotor.
- (6) High speed operation in excess of 80,000 rpm is possible, since these motors are electronically commutated and are not subjected to the limitations of conventional commutations.
- (7) Low thermal resistance since most of the machine losses occur in the stationary stator, thereby allowing heat dissipation by the process of direct conduction. In addition, since the rotor losses are small, heat transfer to machine tools and work pieces when these motors are utilized in machine tools is minimal, thereby reducing the effects of heat on the machining operation.
- (8) Low EMI/RFI due to the absence of brushes and metallic commutators.

As a result of the above features, the brushless dc motor has been replacing other motors in many industries. The household appliance industry has been one of the fastest growing end product market for adjustable speed drives [7]. Brushless dc motors are now being used in refrigeration compressors, washing machines, fans, food processing equipment and vacuum cleaners in the household appliance industry. In the automotive industry, brushless dc motors are being used in fuel pumps, air-condition blowers and engine cooling fans.

The exceptional features of brushless dc motors described above are responsible for their widespread use in many industries, however, a review of the literature did not provide motor operational characteristics based on the various phenomena occurring in the motor. Since the operational characteristic of a motor is important for its control, modeling and deriving optimum performance, this paper is focused on the determination of the energization sequence of the motor, its effect on electromagnetic torque production and the utilization of the torque production mechanism for the classification of the brushless dc motor.

## **2. Determination of Energization Sequence for Brushless DC Motors**

Three-phase brushless dc motors are operated by energizing two of its three phase windings at a time. However, for continuous operation of the motor in a particular direction of rotation, the pair of windings to be energized is dependent on the rotor position. The dependence of phase winding energization on rotor position lies in the fact that the rotor magnet of the motor induces voltages in the phase windings during rotation, and efficient motor operation is accomplished when the energized windings are experiencing their steady or non-varying back emf. Hence, knowledge of the back emf of each phase winding as a function of rotor position is necessary in the determination of the phase winding energization sequence.

The determination of phase winding back emf as a function of rotor position for a three-phase brushless dc motor was obtained by operating the brushless dc machine in generator mode. In this test, a brush dc motor was used as a prime mover to drive the three-phase, two-pole brushless dc machine at constant speed in an anti-clockwise direction. The apparatus used is shown in Fig. 1. The rotating flux of the two-pole brushless dc machine rotor induces voltages in each phase winding. For a three wire star connected brushless dc machine, the star point is not

accessible and the three resistors labeled  $R$ , in star connection were used to obtain machine phase voltages from line to the star point  $n$  formed by the three resistors.

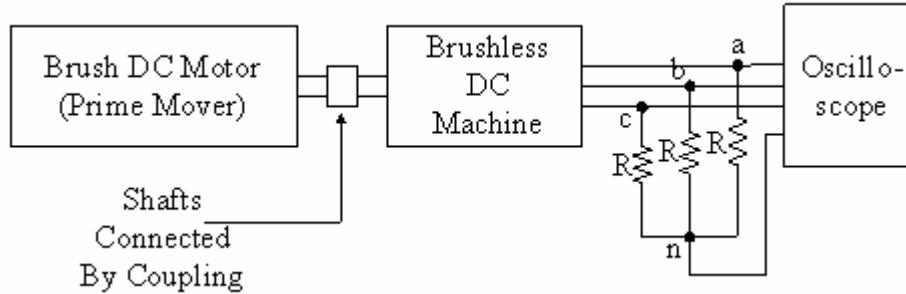


Fig. 1 Brushless DC Machine Operated as a Generator

The resulting generated phase voltages as functions of rotor position  $\theta$ , relative to the stator for the two-pole, three-phase brushless dc machine are shown in Fig. 2(a). These generated phase voltages are trapezoidal in nature, having flat tops of 120 electrical degrees and positive and negative slopes each of 60 electrical degrees. Their magnitudes for a particular brushless dc machine are dependent on the speed of rotation of the machine.

The three generated phase voltages  $e_{an}$ ,  $e_{bn}$  and  $e_{cn}$  are displaced 120 electrical degrees from each other as shown in Fig. 2(a), and their variations are dependent on rotor position, since,

$$e = \frac{d\lambda}{dt} = \frac{d\theta}{dt} \cdot \frac{d\lambda}{d\theta} = \omega \frac{d\lambda}{d\theta} \quad (1)$$

where,  $e$  is the generated voltage,  $\lambda$  is the flux linkage,  $\theta$  is the rotor position and  $\omega$  is the angular velocity of the rotor. From Eq. (1), the generated voltage waveform is a function of rotor position, thereby providing an indication of the rotor position at any time. The waveforms of Fig. 2 reveal that for a two-pole machine, one electrical cycle of generated waveform was completed in one mechanical revolution of the rotor. However, in the case of a four-pole rotor, there would be two electrical cycles of generated voltage waveform for one mechanical revolution of the rotor.

Fig. 2(a) reveals that two phase voltages are of constant value for 60 electrical degrees and for a star connected stator as shown in Fig. 3, line voltage waveforms can be drawn from two phase voltages. These line voltage generated waveforms  $e_{bc}$ ,  $e_{ca}$  and  $e_{ab}$  are shown in Fig. 2(b). Since two phase windings of a star connected brushless dc motor are experiencing a constant

generated line voltage for 60 electrical degrees, then efficient operation of the motor is obtained when the two energized windings are experiencing their constant back emf.

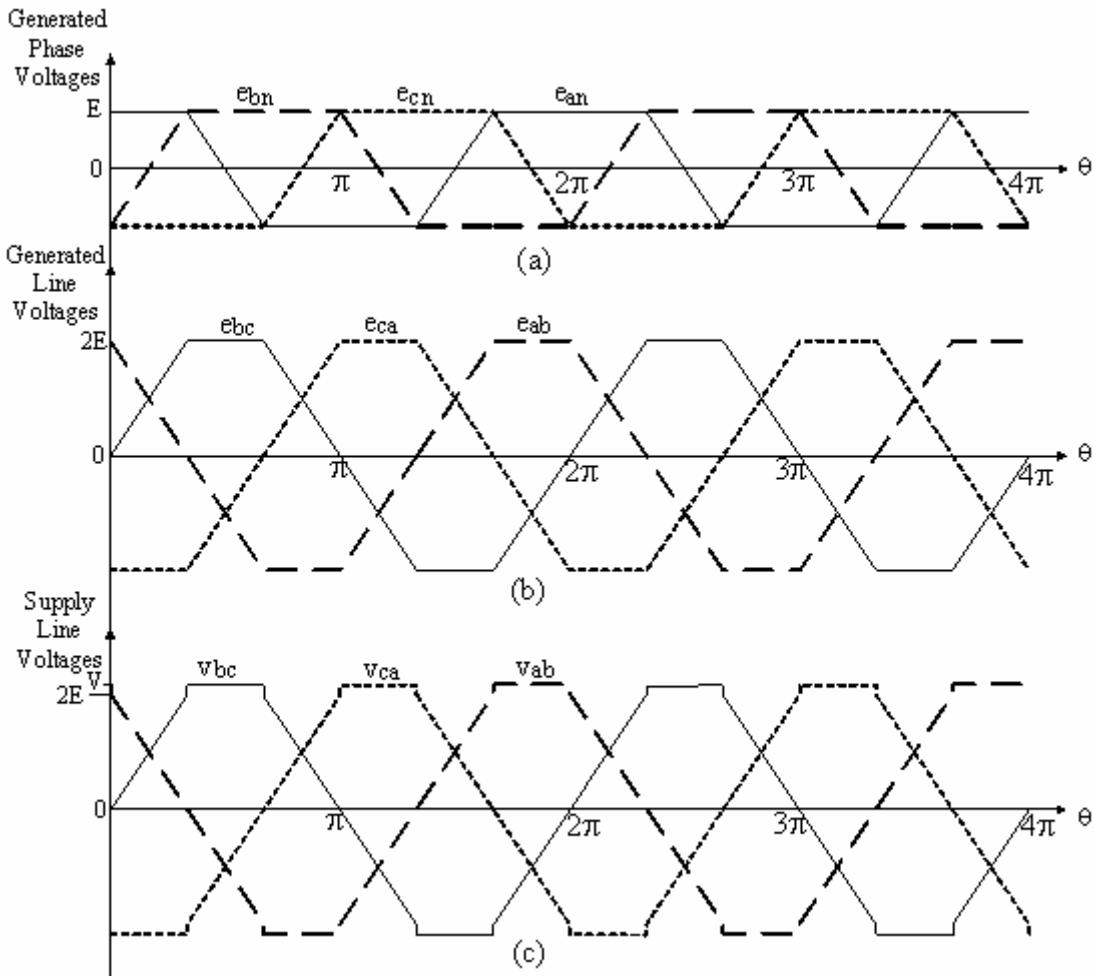


Fig. 2 Brushless DC Motor Voltages (a) Generated Phase Voltages  $e_{an}$ ,  $e_{bn}$  and  $e_{cn}$  (b) Generated Line Voltages  $e_{bc}$ ,  $e_{ca}$  and  $e_{ab}$  (c) Supply Line Voltages  $V_{bc}$ ,  $V_{ca}$  and  $V_{ab}$

Hence, the generated line voltage waveforms shown in Fig. 2(b), which are functions of rotor position  $\theta$ , are used to determine the sequence of energization of the motor windings for a particular direction of rotation. Therefore, for anti-clockwise operation of the brushless dc motor, using Fig. 2(b), and starting with rotor position at  $\theta = 0^\circ$ , the winding pairs should be energized in the sequence *ac*, *bc*, *ba*, *ca*, *cb*, *ab* and *ac* again, with each winding pair being energized for 60 electrical degrees [8-9]. It must be noted that for clockwise operation of the brushless dc motor, the sequence of energization of the winding pairs must be reversed and would take the form *ab*, *cb*, *ca*, *ba*, *bc* and *ac*.

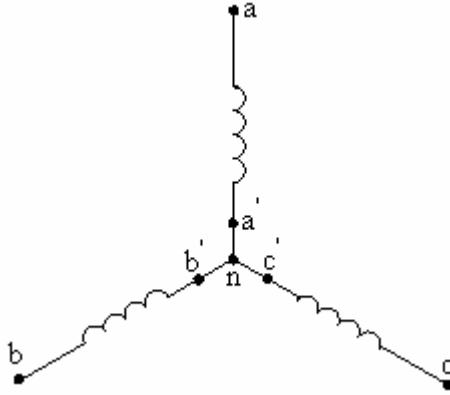


Fig. 3 Star Connected Brushless DC Motor

Fig. 2(c) shows the line voltages for efficient motor operation, placing the supply voltages in phase with the generated or back emf line values. The line supply voltages are greater than the line back emfs to ensure that electromagnetic torque is developed by the machine.

The development of the energization sequence for a three-phase brushless dc motor, as a function of rotor position and hence back emf, for rotation in a particular direction has been lacking in the literature. The material presented above can be used in the absence of the manufacturer's data to determine the energization sequence of a brushless dc motor.

### 3. Torque Production and Operation of BLDCM Using Vector Analysis

The theory of vector analysis of a three-phase stator, justifying the existence and location of vector currents and voltages and the equality of scalar and vector current magnitudes was presented in [10].

A cross sectional view of a two-pole, three-phase brushless dc motor is shown in Fig. 4. The rotor magnet is shown with a reduced diameter and hence an enlarged air-gap for illustration purposes. The two-pole rotor is assumed to be rotating at a constant angular velocity  $\omega$  rad/sec in an anticlockwise direction. At the instant of observation in Fig. 4, it's d-axis which is defined as the centre of the south pole is at the position  $\theta = 0^\circ$ , which corresponds to the  $\theta = 0^\circ$  point on the horizontal axes of the waveforms in Fig. 2. At this rotor position  $\theta = 0^\circ$ , winding pair  $ac$  would begin to experience their constant back emf  $E_{ac}$  due to the effect of the rotor magnet on the stator windings as shown in Fig. 2(b). Hence, at this rotor position  $\theta = 0^\circ$ , winding pair  $ac$  must be energized with a supply voltage of  $V_{ac}$  volts to oppose the constant back emf  $E_{ac}$  presently

experienced by the windings. The magnitude of the supply voltage must be greater than the constant back emf developed by the windings as shown in Fig. 2(c) and sufficient to develop electromagnetic torque to sustain the rotor speed.

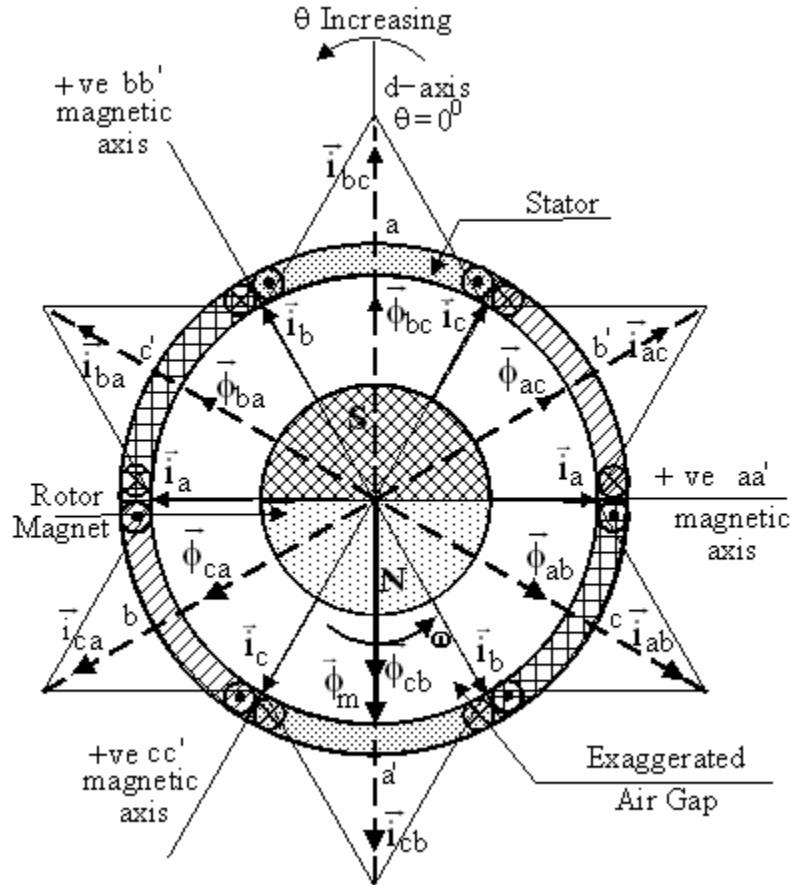


Fig. 4 Stator And Rotor Vectors For Two-Pole, Brushless DC Motor

The energization of stator winding pair  $ac$  with supply voltage  $V_{ac}$  and the resulting phase currents are shown in Fig. 5(a). The energization of winding pair  $ac$  results in the currents  $i_a$  through winding  $aa'$  and  $i_c$  through winding  $cc'$  respectively, where,  $i_a = i_c$ . These currents establish stationary current vectors  $\vec{i}_a$  and  $\vec{i}_c$  along the positive magnetic axis of winding  $aa'$  and negative magnetic axis of winding  $cc'$  respectively [10]. The vector addition of these two stationary current vectors  $\vec{i}_a$  and  $\vec{i}_c$ , results in the resultant stationary current vector  $\vec{i}_{ac}$  as shown in Fig. 4.

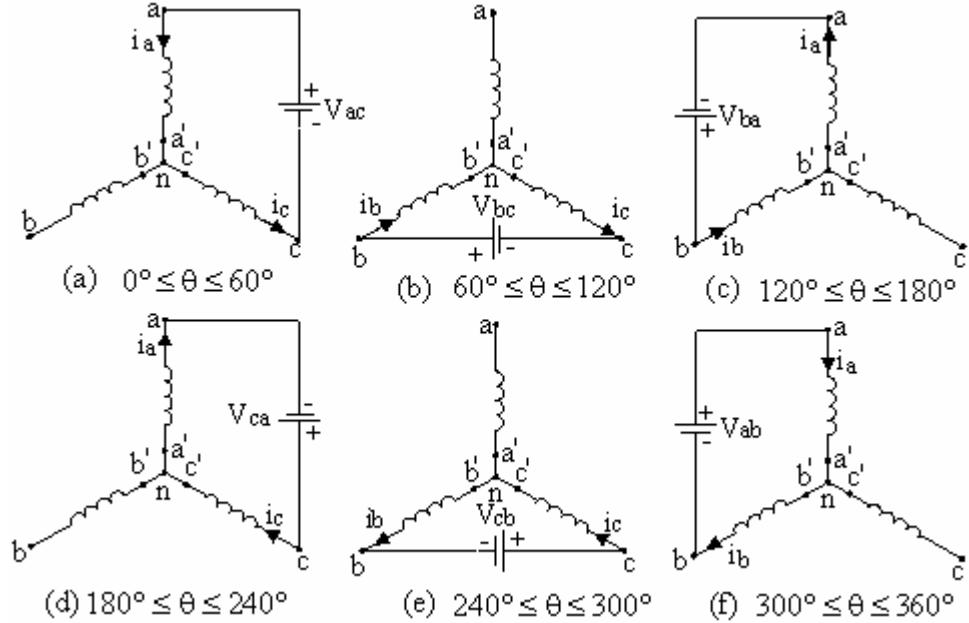


Fig. 5 Energization Sequence of Two-Pole, Three-Phase Stator

At this rotor position  $\theta = 0^\circ$ , the resultant stationary current vector  $\vec{i}_{ac}$ , is displaced from the rotor flux vector  $\vec{\phi}_m$  by an angle of 120 electrical degrees. The resultant stationary stator flux vector  $\vec{\phi}_{ac}$ , produced by current vector  $\vec{i}_{ac}$ , establishes a magnetic south pole at the arrow head of the  $\vec{\phi}_{ac}$  vector, while a north magnetic pole exist at the arrow head of the rotor flux vector  $\vec{\phi}_m$ . The interaction of these flux vectors  $\vec{\phi}_{ac}$  and  $\vec{\phi}_m$  develops electromagnetic torque, resulting in the rotor and its flux vector being pulled towards the resultant stationary stator flux vector  $\vec{\phi}_{ac}$ , causing rotation of the motor in an anti-clockwise direction. The electromagnetic torque  $\vec{T}_e$  developed by the machine is given by the cross product of peak flux linkage vector  $\vec{\lambda}_m$  (where,  $\vec{\lambda}_m = N \vec{\phi}_m$ ) and current vector  $\vec{i}_{ac}$  [11], hence,

$$\vec{T}_e = \vec{\lambda}_m \times \vec{i}_{ac} = N(\vec{\phi}_m \times \vec{i}_{ac}) \quad (2)$$

$$\vec{T}_e = N |\vec{\phi}_m| |\vec{i}_{ac}| \sin\alpha \vec{k} \quad (3)$$

where,  $\vec{\phi}_m$  is the rotor flux vector,  $\alpha$  is the angle between vectors  $\vec{\phi}_m$  and  $\vec{i}_{ac}$  in Fig. 4, and  $\vec{k}$  is the unit vector whose direction is perpendicular to the plane in which  $\vec{\phi}_m$  and  $\vec{i}_{ac}$  exists. Eq. (3)

indicates that the developed electromagnetic torque varies with the magnitude of the resultant stationary current vector  $|\vec{i}_{ac}|$  and  $\sin\alpha$ , since,  $N$  and  $\vec{\phi}_m$  are constants.

The utilization of the developed energization sequence for the brushless dc motor, for rotor positions in the range  $0^\circ \leq \theta \leq 60^\circ$  resulted in the electromagnetic torque equation presented in Eq. (3). The electromagnetic torque developed by the motor produced anticlockwise rotation, resulting in an increase in the angle  $\theta$  from its  $0^\circ$  position. This increase in the angle  $\theta$  in an anticlockwise direction, results in a decrease of angle  $\alpha$  from its initial  $120^\circ$ , thereby increasing the electromagnetic torque developed by the motor, provided there is no decrease in the magnitude of the resultant stationary current vector  $|\vec{i}_{ac}|$ . When  $\alpha = 90^\circ$ , the developed torque is maximum, but as  $\alpha$  decreases and reaches  $60^\circ$ , the developed electromagnetic torque decreases to the value when  $\theta$  was  $0^\circ$ . When  $\theta > 60^\circ$  in Fig. 2(b), the back emf in winding pair  $ac$  is no longer at its constant value for this speed of operation, and the electromagnetic torque developed for  $\alpha < 60^\circ$ , would be less than the values obtained for  $0^\circ \leq \theta \leq 60^\circ$  and  $60^\circ \leq \alpha \leq 120^\circ$ . If winding pair  $ac$  remains energized up to the point where  $\theta = 120^\circ$ , the angle between the vectors  $\vec{\phi}_m$  and  $\vec{i}_{ac}$  would be  $\alpha = 0^\circ$ , and the electromagnetic torque developed using Eq. (3) would be zero. In addition to zero torque being developed at  $\theta = 120^\circ$ , the rotor would be locked in this zero torque position, since the north pole of the rotor magnet would be aligned with the south pole produced by resultant stationary flux vector  $\vec{\phi}_{ac}$ . Hence, for continuous torque production and rotation of the motor and efficient energy conversion from electrical to mechanical, winding pair  $ac$  must not remain energized for  $\theta > 60^\circ$ . Examination of Fig. 2(b), reveals that at  $\theta = 60^\circ$ , winding pair  $bc$  has just begun to experience its constant back emf  $E_{bc}$ , hence, winding  $aa'$  must be commutated and winding  $bb'$  brought into conduction with winding  $cc'$ . That is, winding pair  $bc$  must be energized with  $V_{bc}$  at  $\theta = 60^\circ$  as shown in Figs. 2(c) and 5(b).

The energization of winding pair  $bc$  with supply voltage  $V_{bc}$  results in the current  $i_b$  through winding  $bb'$  and  $i_c$  through winding  $cc'$ . These currents establish stationary current vectors  $\vec{i}_b$  and  $\vec{i}_c$ , along the positive magnetic axis of winding  $bb'$  and negative magnetic axis of winding  $cc'$  respectively. The vector addition of these two current vectors  $\vec{i}_b$  and  $\vec{i}_c$ , results in the resultant stationary current vector  $\vec{i}_{bc}$  as shown in Fig. 4. At this rotor position  $\theta = 60^\circ$ , the

resultant stationary current vector  $\vec{i}_{bc}$ , is displaced from the rotor flux vector  $\vec{\phi}_m$  by an angle of 120 electrical degrees. The interaction of these vectors  $\vec{i}_{bc}$  and  $\vec{\phi}_m$  develops electromagnetic torque, resulting in the rotor and its flux vector being pulled towards the resultant stationary stator flux vector  $\vec{\phi}_{bc}$ , causing rotation to continue in an anti-clockwise direction. The process of torque production continues until  $\theta = 120^\circ$  and a new winding pair  $ba$  is brought into conduction as shown in Fig. 2(c).

Similarly, the energization of the other phase windings shown in Figs. 5(c) to (f), results in the production of resultant stationary current vectors  $\vec{i}_{ba}, \vec{i}_{ca}, \vec{i}_{cb}$  and  $\vec{i}_{ab}$  respectively as shown in Fig. 4. These resultant stationary current vectors occupy a fixed position in the stator. They are displaced from each other by an angle of 60 electrical degrees and their magnitudes are dependent on the current flowing in the phase windings.

The electromagnetic torque developed by the machine is not constant throughout each  $60^\circ$  movement of the rotor and is given by

$$\vec{T}_e = \lambda_m |\vec{i}_{xy}| \sin \alpha \vec{k} \quad (4)$$

where, x is the phase winding terminal connected to the positive end of the supply voltage, y is the other phase winding terminal connected to the negative end of the supply voltage and  $\alpha$  is in the range  $60^\circ \leq \alpha \leq 120^\circ$ . The electromagnetic torque developed by the motor for a fixed stator winding current  $I$  for one revolution of the rotor and ignoring the electromagnetic torque developed during commutation is shown in Fig. 6 [11].

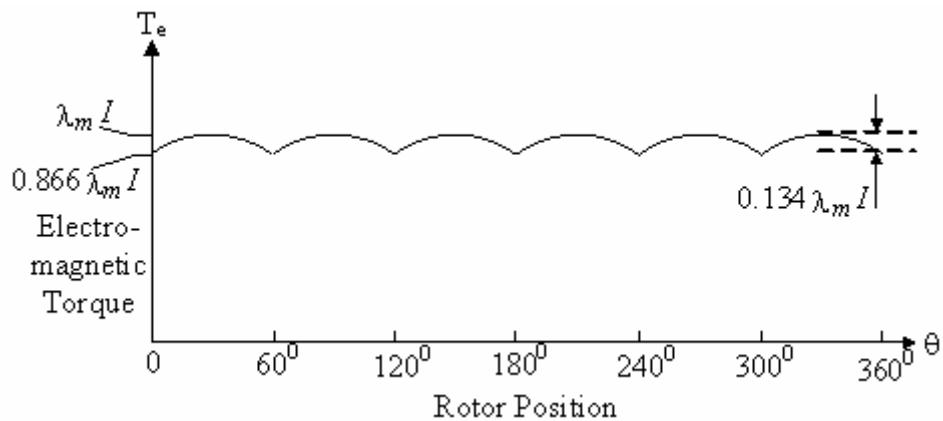


Fig. 6 Electromagnetic Torque Developed in One Revolution

The events described above for efficient operation of the two-pole, three-phase brushless dc motor, showing the range of rotor positions for a pair of windings to remain energized, the corresponding back emf of the energized windings and the corresponding electromagnetic torque developed are summarized in Table 1 below.

Table 1 Rotor Position, Constant Back EMF, Windings Energized and Torque Developed

Rotor Position Range	Constant Back EMF	Windings Energized	Electromagnetic Torque Developed $60 \leq \alpha \leq 120$
$0 \leq \theta \leq 60^\circ$	$E_{ac}$	ac	$\lambda_m  \vec{i}_{ac}  \sin \alpha$
$60 \leq \theta \leq 120$	$E_{bc}$	bc	$\lambda_m  \vec{i}_{bc}  \sin \alpha$
$120 \leq \theta \leq 180$	$E_{ba}$	ba	$\lambda_m  \vec{i}_{ba}  \sin \alpha$
$180 \leq \theta \leq 240$	$E_{ca}$	ca	$\lambda_m  \vec{i}_{ca}  \sin \alpha$
$240 \leq \theta \leq 300$	$E_{cb}$	cb	$\lambda_m  \vec{i}_{cb}  \sin \alpha$
$300 \leq \theta \leq 360$	$E_{ab}$	ab	$\lambda_m  \vec{i}_{ab}  \sin \alpha$

#### 4. Classification of Brushless DC Motor

Electric motors are classified into two main categories, namely brush dc and ac brushless motors as shown in Fig. 7 and presented in [1]. Brush dc motors are made up of stators consisting of poles produced by permanent magnets or dc excited magnets, which give rise to static magnetic fields across the rotor. The rotor of these brush dc motors consists of windings connected to mechanical commutators to facilitate the application of a dc power source. Current flow through these rotor windings takes place through carbon brushes which make contact with the commutators, thereby producing a magnetic field and a current vector which remains in a relatively fixed position relative to the stator. The relatively stationary current vector of the rotor interacts with the stationary magnetic field of the stator, developing electromagnetic torque given by the cross product of these two vectors. Fig. 8(a) shows the brush dc machine stationary flux

linkage vector  $N\vec{\phi}_s$  and the relatively stationary rotor current vector  $\vec{i}_r$ , separated by an angle  $\alpha$ .

The electromagnetic torque developed by the machine is given by:

$$\vec{T}_e = N(\vec{i}_r \times \vec{\phi}_s) . \quad (5)$$

Three-phase ac machines are divided into two categories, synchronous and asynchronous. The stators of synchronous and asynchronous ac machines are supplied with three-phase ac voltages and the resulting three-phase ac currents produce a rotating current vector and magnetic field, both of which are of constant magnitude and rotate at the angular velocity of the supply voltage.

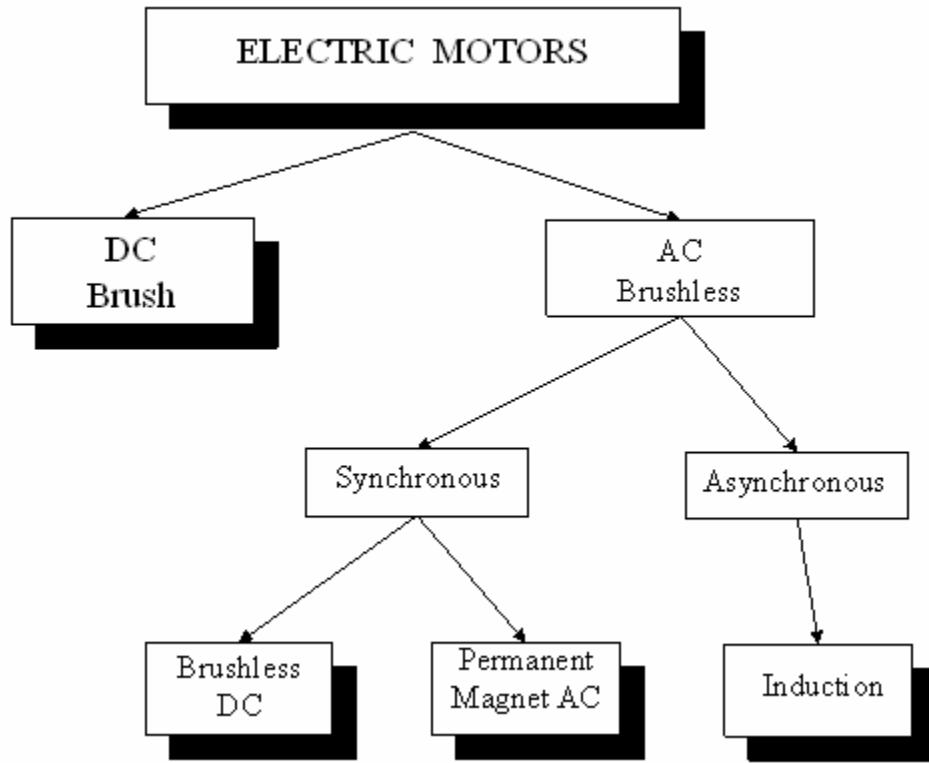


Fig. 7 Classification of Electrical Motors

The difference between synchronous and asynchronous machines lies in the fact that the rotor of an asynchronous machine derives its electrical energy from the stator by the process of induction to produce the rotor's current vector and magnetic field. This process results in the speed of the rotor being lower than that of the stator's rotating magnetic field. However, the rotors of synchronous machines possess their own magnetic field in the form of fixed magnets or dc excited magnets and do not depend on induced currents from the stator's magnetic field. This results in the rotor of synchronous machines having the same speed as the stator rotating current

vector and magnetic field. Although the rotor speed of asynchronous machines are lower than the speed of the stator rotating flux, the speed of the rotor flux is the same as that of the stator flux. The stator current vectors, rotor flux vectors and their speeds, together with the rotor speed of synchronous and asynchronous machines are represented in Figs. 8(b) and 8(c).

In Fig. 8(b), the rotor flux vector and stator current vector of the asynchronous machine are rotating at the angular velocity of the supply voltage  $\omega_s$ , while the rotor rotates at an angular velocity  $\omega_m$  which is lower than  $\omega_s$ . However, the rotor's flux vector and stator current vector and the rotor of a synchronous machine all rotate and the angular velocity of the supply voltage  $\omega_s$  as shown in Fig. 8(c).

The electromagnetic torque developed for synchronous and asynchronous machines in Fig. 8 is given by

$$\vec{T}_e = N(\vec{i}_s \times \vec{\phi}_r). \quad (6)$$

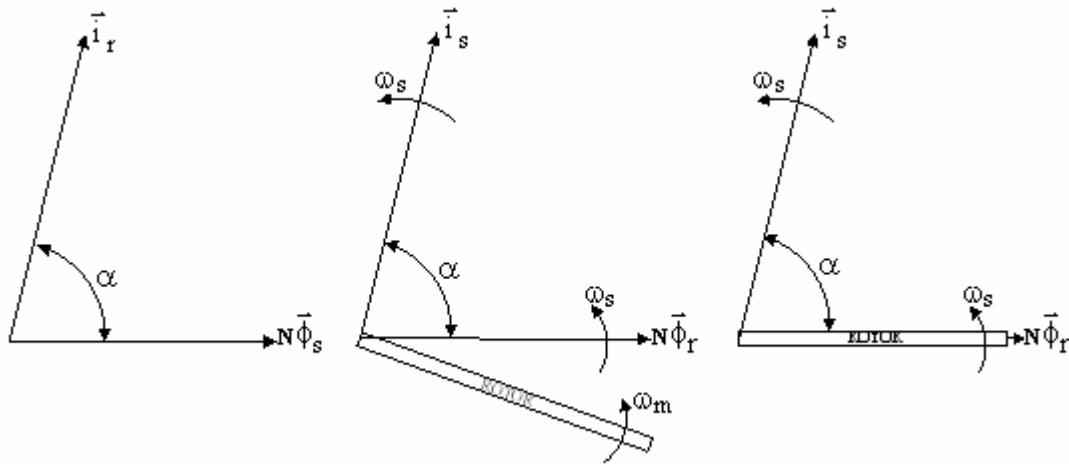


Fig. 8 Flux and Current Vectors of Electrical Machines (a) Brush DC Motor (b) Asynchronous Machine (c) Synchronous Machine

It is clear from Fig. 8, that dc machines are characterized by stator flux vector and rotor current vector both occupying a relatively fixed position in the machine space, while, ac machines are characterized by rotor flux vector and stator current vector both rotating at the angular velocity of the stator supply voltage within the machine space. The energization of a pair of stator phase windings of a brushless dc motor for 60 electrical degrees results in the production of a rotating

rotor flux vector and a stationary stator current vector. This clearly does not fit the classification of a dc machine, although the stator windings are energized with a dc supply during this interval. In addition, examination of the rotor flux vector and the stator current vector over a cycle as shown in Fig. 4, also reveal their non-stationary nature, with the stator current vector always leading the rotor flux vector and they complete an electrical cycle in the same time. Further to this, the line voltage waveforms of Fig. 2 are not dc, but alternating in nature and are similar to the trapezoidal back emf of the motor when operated as a generator. These properties clearly indicate that the brushless dc motor is an ac synchronous motor although the stator windings are energized by dc voltages and the torque-speed characteristics of the motor is similar to that of the brush dc motor [12].

## 5. Conclusion

Smooth and efficient operation of the brushless dc motor relies on the knowledge of the energization sequence of the windings. This sequence which is supplied by motor manufacturers, can be obtained by operating the motor in generator mode and employing the technique presented in this paper. The application of this energization sequence, which is a function of rotor position, produces resultant stationary current vectors, which interact with the rotor flux linkage vector to develop electromagnetic torque. Hence the vector method of analysis, when applied to brushless dc motors, displays the torque production mechanism in the motor and presents a powerful insight into the physical phenomena occurring in the motor. In addition, the cross product used in the computation of the developed electromagnetic torque of the motor, revealed the importance and necessity of phase winding commutation in order to sustain continuous motor operation. Finally, the characterization of the brushless dc motor as an ac or dc motor was addressed. The characteristics of the motor clearly indicate that it is an ac synchronous motor, although the stator windings are energized by dc voltages and the torque-speed characteristics of the motor is similar to that of the brush dc motor.

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