REVERSIBLE NON-EXPANSIVE SYMMETRIC CONVOLUTION FOR M-CHANNEL LIFTING BASED LINEAR-PHASE FILTER BANKS

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ABSTRACT
This paper presents an effective signal boundary solution in lossy-to-lossless image coding which is the unification of lossy and lossless image coding. Although M-channel filter banks (FBs) for lossy image coding have several effective signal boundary solutions, M-channel lifting based FBs (L-FBs) for lossless image coding do not have such an effective signal boundary solutions due to rounding error in each lifting step. This paper proposes reversible non-expansive symmetric convolution for M-channel lifting based linear-phase FBs (L-LPFBs) to apply lossy-to-lossless image coding. Our proposal is validated by comparing with the periodic extension in lossy-to-lossless image coding.

Index Terms—M-channel lifting based linear-phase filter bank (L-LPFB), lossy-to-lossless image coding, reversible non-expansive symmetric convolution

1. INTRODUCTION
Filter bank (FB) [1] technologies are frequently found in the field of signal processing applications. Especially, M-channel linear-phase FBs (L-PFBs) [2], include lapped transforms (LTs) [3], are well-known as one of the most useful transforms for image compression (coding). It is a reason that L-PFBs can adopt the symmetric extension [4] to overcome the signal boundary distortion problem. In this approach, the input signals are extended symmetrically in order to maintain the continuity at the signal boundaries. The output signals are reconstructed without the signal boundary distortion by using symmetry even if the extended signals are not transmitted. Since the symmetric extension improves the performance of image coding applications, most works in the field of FB systems are focused on L-PFBs.

Meanwhile, lifting based LPFBs (L-LPFBs) composed of lifting structures [5] and rounding operations are required for lossy-to-lossless image coding, which is the unification of lossy and lossless image coding, such as JPEG 2000. However, L-LPFBs (M > 2) generate the signal boundary distortion because the symmetric extension cannot be applied directly to L-LPFBs due to rounding error. Because of that, the periodic extension is often used for lossy-to-lossless image coding even if the FB without rounding operations has symmetry [6].

In this paper, the signal boundary problem in lossy-to-lossless image coding is solved by focusing on symmetry in each building block of a particular class of M-channel L-LPFBs again. The proposed reversible symmetric extension can obtain similar smoothness to the symmetric extension at the signal boundaries even if rounding operations are used. Moreover, the computational cost is less due to use of non-expansive convolution [7], i.e., the number of input signals does not increase even temporarily. Our proposal, which this paper calls the reversible non-expansive symmetric convolution, is validated by comparing with the periodic extension in lossy-to-lossless image coding.

Notations: I, J and \( \{ \cdot \}^T \) are an identity matrix, a reversal identity matrix and transpose of a matrix, respectively.

2. REVIEW
2.1. Linear-Phase Filter Banks (L-PFBs)
The polyphase representation of a typical structure of a FB is shown in Fig. 1. Using the lattice structure, the type-II analysis polyphase matrix \( \Phi(z) \) in an \( M \times MK \) LPFB can be presented as [8]

\[
\Phi(z) = \Phi_0 G_1(z) \cdots G_{K-2}(z) G_{K-1}(z)
\]

where

\[
\Phi_0 = \Phi_0 W, \quad G_k(z) = \Lambda(z) \Xi_k = \Lambda(z) W \Phi_k W,
\]

\[
\Phi_k = \begin{bmatrix} U_k & 0 \\ 0 & V_k \end{bmatrix}, \quad W = \frac{1}{\sqrt{2}} \begin{bmatrix} I & J \\ J & -I \end{bmatrix}, \quad \Lambda(z) = \begin{bmatrix} I & 0 \\ 0 & z^{-1}I \end{bmatrix}
\]

and \( U_k \) and \( V_k \) are \( M/2 \times M/2 \) arbitrary nonsingular matrices when \( M \) is even, respectively. (1) is shown in Fig. 2. Also, \( \Phi_k \) is usually replaced by \( I \) when \( k \geq 1 \). If \( \Phi(z) \) is invertible, the synthesis polyphase matrix \( \Phi(z) \) can be chosen as the inverse of \( \Phi(z) \), i.e., perfect reconstruction (PR) \( \Phi(z) = \Phi^T(z^{-1}) \) is achieved.

2.2. Lifting Structure
The lifting structure [5], also known as the ladder structure, is a special type of lattice structure. It is implemented by cascading elementary matrices - identity matrices with a single nonzero off-diagonal element.

Fig. 1. M-channel FB (\( \downarrow \), \( \uparrow \) and \( z^{-1} \) means downsampling, upsampling and delay operation, respectively).
2.3. Symmetric Extension

In lossy compression without rounding operations, the symmetric extension for LPFB in (1) can be implemented by using symmetry in each building block. Fig. 4 shows the analysis part of the symmetric extension in case of $K = M$. Let $l$-th $(l \geq 1)$ $M \times 1$ input signals for $\Xi_k$ be $X_{k,l} = [s_{k,l}, t_{k,l}]^T$ where $s_{k,l}$ and $t_{k,l}$ are $M/2 \times 1$ vectors. If their reflected signals are $JX_{k,l}$, symmetry is achieved as

$$\Xi_k X_{k,l} = \Xi_k [s_{k,l}] \triangleq Y_{k,l}$$

$$\Xi_k JX_{k,l} = \Xi_k [Jt_{k,l}] = JY_{k,l}$$

where

$$Y_{k,l} = \frac{1}{2} \left[ U_k (s_{k,l} + Jt_{k,l}) + JV_k (s_{k,l} - t_{k,l}) \right].$$

Since $s_{k,0} = Jt_{k,0}$ when the process is stepping over just signal boundaries, i.e., $l = 0$, the input and output signals are expressed as

$$X_{k,0} = [Jt_{k,0}, t_{k,0}] = JX_{k,0} \text{ and } Y_{k,0} = \left[ U_k Jt_{k,0} \right] = JY_{k,0},$$

respectively. In the synthesis part, perfect reconstruction is achieved by using such symmetry without receiving redundant signals. As a matter of course, the symmetric extension is also easily applied to the opposite signal boundaries. Note that if the output signals of the input signals $X_{k,l}$ are $Y_{k,l}$ when lifting structures and rounding operations are used in $\Xi_k$, the output signals of the reflected input signals $JX_{k,l}$ are NOT $JY_{k,l}$ due to rounding error, i.e., symmetry is not achieved. Hence, the symmetric extension cannot be directly applied to L-LPFBs.

3. L-LPFBS AND REVERSIBLE NON-EXPANSIVE SYMMETRIC CONVOLUTION

3.1. L-LPFBs

This paper presents L-LPFBs based on [8] and [9]. First, a building block $G_k(z)$ can be represented by

$$G_k(z) = \Lambda(z) \Xi_k \triangleq \Lambda(z)W_L \Phi_k W_R,$$

where

$$W_L = W \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 \\ 0 & -\frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} I & 0 \\ J & I \end{bmatrix} \begin{bmatrix} I & \frac{1}{2}J \\ 0 & I \end{bmatrix}$$

and

$$W_R = \begin{bmatrix} \sqrt{2}I & 0 \\ 0 & \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} I & 0 \\ J & I \end{bmatrix} \begin{bmatrix} I & \frac{1}{2}J \\ 0 & I \end{bmatrix}.$$
Fig. 5. Building blocks of $M \times KM$ L-LPFB: (top) $\Xi_k$, (bottom) $J\hat{\Xi}_kJ$ for boundary area.

Fig. 6. Analysis part of the reversible symmetric extension by $M \times 4M$ L-LPFB (dashed line means signal boundary).

Fig. 7. Analysis part of the reversible non-expansive symmetric convolution by $M \times 4M$ L-LPFB (dashed line means signal boundary).

and $\hat{V}_k$ mean matrices factorized $U_k$ and $V_k$ into single-row elementary reversible matrices (SERMs) by [9] with the fewest rounding operations. Also, $W$ in the last block $E_0$ is factorized into lifting structures by lifting factorization of Givens rotation matrix as

$$W = \begin{bmatrix} I & (1 - \sqrt{2}) J \\ 0 & I \end{bmatrix} \begin{bmatrix} I & 0 \\ 1 / \sqrt{2} J & I \end{bmatrix} \begin{bmatrix} I & (1 - \sqrt{2}) J \\ 0 & I \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix}. $$

3.2. Reversible Symmetric Extension

The loss of symmetry due to rounding error can be solved by a simple matrix manipulation. Fig. 6 shows a realization of symmetry with rounding operations.

Case 1 ($l \neq 0$):

If $l$-th input signals are $X_{k,l}$ and rounding operations are considered, it is expressed as

$$\hat{E}_k X_{k,l} \triangleq Y_{k,l} \neq Y_{k,l}. $$

According to (2) and (3), $\hat{E}_k$ can be represented by

$$\hat{E}_k = J\hat{E}_kJ.$$  (4)

This means that each building block $\hat{E}_k$ for extended signals can be replaced by $J\hat{E}_kJ$. This relationship is preserved even if $\hat{E}_k$ is used in place of $E_k$, i.e.,

$$\hat{E}_k = J\hat{E}_kJ.$$  (5)

By the replacement of building blocks on boundary area and (4), the implementation in case of the reflected input signals $JX_{k,l}$ is similarly expressed as follows:

$$J\hat{E}_kJ \cdot JX_{k,l} = J\hat{E}_k X_{k,l} = JY_{k,l}.$$  (6)

where $J \cdot J = I$. Fig. 5 shows symmetry in building blocks $\hat{E}_k$ and $J\hat{E}_kJ$ of $M \times KM$ L-LPFB. As a result, it is clear that symmetry can be satisfied by a simple matrix manipulation even if the implementation has rounding operations in case of $l \neq 0$.

Case 2 ($l = 0$):

This case structurally achieves symmetry. Let the input signals $X_{k,0}$ be $[(Jt_{k,0})^T, t_{k,0}^T]^T$ for simplicity. (4) is rewritten as

$$\hat{E}_k X_{k,0} = \begin{bmatrix} \hat{U}_k Jt_{k,0} \\ J\hat{U}_k Jt_{k,0} \end{bmatrix} = Y_{k,0} = JY_{k,0}.$$  (7)

It is clear that symmetry is also satisfied even if the implementation has rounding operations in case of $l = 0$.

Consequently, the symmetric extension is applicable to L-LPFBs in (1) by replacing $\hat{E}_k$ at boundary area to $J\hat{E}_kJ$.

3.3. Reversible Non-Expansive Symmetric Convolution

It is important that the input and output signals for $\hat{E}_k$ always have symmetry as already indicated in Sec. 3.2. Additionally, note that only half of the output signals processed by $\hat{E}_k$ is used in case of $l = 0$, i.e., the signal boundaries. Consequently, the reversible symmetric extension in the above subsection can be replaced by the non-expansive convolution [7] as shown in Fig. 7. This structure, which this paper calls reversible non-expansive symmetric convolution, has less computational cost because it does NOT need extension of the input signals at the signal boundaries temporarily. Also, since $U_k$ ($k \neq 0$) usually adopts $I$ as discussed in Sec. 2.1, $JU_k$ and $U_kJ$ are replaced by simple $J$.

4. RESULTS

$8 \times 16$ paraunitary LPFB (PULPFB) and $8 \times 24$ PULPFB, which have $U_k = I (k \neq 0), U_0^{-1} = U_0^T$, and $V_k^{-1} = V_k^T$, were designed based on Sec. 3.1 for simplicity. We optimized design parameters by
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Fig. 8. Comparison of a particular area of the image Barbara reconstructed by 8 × 24 PULPFBs (bit rate: 0.25 [bpp]): (left) the periodic extension, (right) the proposed method.

5. CONCLUSION

This paper presented the reversible non-expansive symmetric convolution for M-channel lifting based linear-phase filter banks (L-LPFBs) to apply lossy-to-lossless image coding. Since the proposed method has similar smoothness to the symmetric extension at the signal boundaries, it does not generate the signal boundary distortion even if rounding operations are used. Moreover, the computational cost is less due to non-expansive convolution. As result, it achieved better coding performance than the periodic extension.

6. REFERENCES


