LETTER

A New Sliding Surface Design Method of Linear Systems with Mismatched Uncertainties

Seung Ho JANG[†a)], ***Student Member and*** **Sang Woo KIM**[††b)], ***Member***

SUMMARY Sliding mode control (SMC) is known to be robust with respect to matched uncertainties. However, it does not guarantee stability of systems with mismatched uncertainties. In this paper, we propose a new method to design a sliding surface for linear systems with mismatched uncertainties. The proposed sliding surface provides a new stability criterion of the reduced-order system origin with respect to mismatched uncertainties. A numerical example is given to illustrate the effectiveness of the proposed method.

key words: *linear systems, sliding mode control, mismatched uncertainties*

1. Introduction

The interest of SMC for linear systems has been widely presented in the literature of robust control. Indeed, a salient feature is that the system behavior on sliding surface is unaffected by uncertainties satisfying the matching condition [1]. However, there are many systems affected by uncertainties which do not satisfy the matching condition. To solve this problem, various methods for systems with mismatched uncertainties have been recently proposed by Kwan [3], Shyu et al. [4], Zinober et al. [9] and Swaroop et al. [10]. Kwan used an adaptive technique to design SMC to alleviate mismatched uncertainties. To apply this method, matching conditions are required for the reduced-order system in the sliding mode. Shyu presented a method which guarantees asymptotic stability of the reduced-order system with mismatched uncertainties in sliding mode without Kwan's assumption. Zinober and Swaroop proposed SMC based on the backstepping design which was used to relax the matching conditions. However, since the integrator backstepping technique suffers from the problem of "explosion of terms.," an additional procedure is needed to solve this problem [10]. Shyu's stability criterion is simple but has redundancy since mismatched uncertainties are considered to be integrated.

In this paper, we divide mismatched uncertainties for the reduced-order system into a matched part and a mismatched part to reduce redundancy. Then, the sliding surface with a robust component, which is added to alleviate effect of matched part of the reduced system, is proposed.

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[†]The author is with the Electrical and Computer Engineering Division, POSTETH, Pohang, 790-784, Republic of Korea.
[††]The author is with the Faculty of Electrical and Computer Engineering Division, POSTECH, Pohang, 790-784, Republic of Korea.
a) E-mail: shjang@postech.ac.kr
b) E-mail: swkim@postech.ac.kr

This paper is organized as follows: Section 2 briefly introduces the uncertain systems considered in this paper and describes the proposed sliding surface. Section 3 presents variable structure control law to guarantee a reaching condition. In Section 4, an example is given to illustrate design procedure as well as its effectiveness. Finally, Section 5 serves as a conclusion.

2. Design Sliding Surface

Consider the following linear uncertain system:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ B_2\end{bmatrix}\{(I+\Delta B_2)u + \Delta f_m\} + \begin{bmatrix}\Delta A_{11} & \Delta A_{12}\\ 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}, \tag{1}$$

where $x_1 \in R^{n-m}$ and $x_2 \in R^m$ are the state vectors, $u \in R^m$ is control input and $\Delta f_m \in R^m$ represent the lumped matching uncertainties. $\Delta(\cdot)$ denotes uncertainties in $(\cdot)$. A_{11}, A_{12}, A_{21}, A_{22} and B_2 are known constant matrices with appropriate dimensions. Furthermore, we assume that the followings are valid.

Assumption 1: Δf_m, ΔB_2, ΔA_{11} and ΔA_{12} are bounded in Euclidean norm by known functions as, $\forall(x,t) \in R^n \times R^+$, $\|\Delta f_m\| \le \rho_m(x)$, $\|\Delta B_2\| \le 1-\epsilon_b$, $\|\begin{bmatrix}\Delta A_{12} & \Delta A_{12}\end{bmatrix}\| \le \eta$ and $\|\Delta A_{12}\| < \|A_{12}\|$ where ϵ_b and η are positive constants.

Assumption 2: The pair (A, B)

$$A = \begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix},\ B = \begin{bmatrix}0\\ B_2\end{bmatrix}, \tag{2}$$

is controllable, and the matrix B_2 has full rank.

The linear sliding surface is usually defined as

$$\sigma(x) = Cx,$$

or without loss of generality

$$\sigma(x) = Kx_1 + x_2, \tag{3}$$

where $C \in R^{m\times n}$ and $K \in R^{m\times n-m}$. In sliding mode, $\sigma(x) = 0$, the reduced-order system is governed by the following equation [1]

$$\dot{x}_1 = (A_{11} + \Delta A_{11})x_1 - (A_{12} + \Delta A_{12})Kx_1. \tag{4}$$

If mismatched uncertainties do not exist, K can be selected so that $A_{11} - A_{12}K$ is asymptotically stable. However, since ΔA_{11} and ΔA_{12} are mismatched uncertainties, stability of reduced-order system (4) is affected. Shyu's stability criterion of the reduced-order system with mismatched uncertainties is presented by Lemma 2.1 [4].

Lemma 2.1: Consider the reduced-order system with uncertainty described by

$$\dot{x}_1 = (A_{11} - A_{12}K)x_1 + \bar{f}(x_1), \tag{5}$$

where $\bar{f}(x_1) = (\Delta A_{11} - \Delta A_{12}K)x_1$. Then, if $\bar{f}(x_1)$ satisfies the uniform Lipschitz condition $\|\bar{f}(x_1^1) - \bar{f}(x_1^2)\| \le b\|x_1^1 - x_1^2\|$ where $0 \le b < 0.5\lambda_{min}(\bar{Q})/\|\bar{P}\|$ with $\bar{P}, \bar{Q} \in R^{(n-m)\times(n-m)}$ which are symmetric, positive definite matrices satisfying the Lypunov equation $(A_{11} - A_{12}K)^T\bar{P} + \bar{P}(A_{11} - A_{12}K) = -\bar{Q}$, then the reduced-order system (5) is asymptotically stable.

The scenario of our proposed scheme is that the mismatched uncertainties can be divided into the matched part which is in the range space of A_{12} and the rest mismatched part for the reduced-order system (4). Then, after we include an additional robust component for the matched part of the reduced-order system, we apply Shyu's method to the rest mismatched part. The mismatched uncertainties ΔA_{11} and ΔA_{12} in reduced-system can be divided such as

$$\Delta A_{11} = A_{12}\bar{\Delta}A_{11} + \Delta A_{ns1}, \tag{6}$$
$$\Delta A_{12} = A_{12}\bar{\Delta}A_{12} + \Delta A_{ns2}. \tag{7}$$

Consequently, the reduced-order system (4) has the following form :

$$\begin{aligned}\dot{x}_1 &= A_{11}x_1 + A_{12}\left\{-(I + \bar{\Delta}A_{12})Kx_1 + \Delta f_{rm}\right\} \\ &\quad +\Delta f_{ru},\end{aligned} \tag{8}$$

where $\Delta f_{rm} = \bar{\Delta}A_{11}x_1$ and $\Delta f_{ru} = (\Delta A_{ns1} - \Delta A_{ns2}K)x_1$. Furthermore, we make the following assumption.

Assumption 3: There exists $\epsilon > 0$ satisfying the following condition $\|\bar{\Delta}A_{12}\| \le 1-\epsilon$. And, Δf_{rm} and Δf_{ru} are bounded in Euclidean norm as $\|\Delta f_{rm}\| \le \rho_{rm}\|x_1\|$ and $\|\Delta f_{ru}\| \le \rho_{ru}\|x_1\|$, respectively.

This paper proposes the following sliding surface with additional component $\bar{K}(x_1)$:

$$\sigma(x) = Kx_1 + \bar{K}(x_1) + x_2, \tag{9}$$

where the additional component provides robustness against the matched part of the reduced-order system which is in the range space of A_{12}. Hence the reduced-order system is rewritten as

$$\begin{aligned}\dot{x}_1 &= (A_{11} - A_{12}K)x_1 + A_{12}\left\{-(I + \bar{\Delta}A_{12})\right. \\ &\quad \left.\bar{K}(x_1) - \bar{\Delta}A_{12}Kx_1 + \Delta f_{rm}\right\} + \Delta f_{ru}.\end{aligned} \tag{10}$$

To show stability of system (10), we define a candidate of Lypunov function $V(x_1)$ mapping from R^{n-m} to R such as

$$V(x_1) = x_1{}^T P x_1, \tag{11}$$

where $P \in R^{(n-m)\times(n-m)}$ is a positive definite matrix. And, for simplicity of notation, we define a new function as follows

$$\phi(x_1) = A_{12}^T P x_1. \tag{12}$$

In the following theorem, based on the new sliding surface (9), a new stability criterion for the reduced-order system (10) is presented.

Theorem 2.1: Based on Assumption 3, consider the following additional surface component

$$\bar{K}(x_1) = \frac{\bar{\rho}_{rm}^2}{2\epsilon\xi}\phi(x_1), \tag{13}$$

where ξ is a positive scalar and $\bar{\rho}_{rm} = (1-\epsilon)\|K\| + \rho_{rm}$. Then, the reduced-order system (10) is globally asymptotically stable, if $\rho_{ru} < 0.5(\lambda_{min}(Q) - \xi)/\|P\|$ with $P, Q \in R^{(n-m)\times(n-m)}$ are positive definite matrices satisfying the Lypunov equation

$$(A_{11} - A_{12}K)^T P + P(A_{11} - A_{12}K) = -Q. \tag{14}$$

Proof: Consider the following time derivative of the Lypunov function (11) associated with the reduced-order system (10).

$$\begin{aligned}\dot{V}(x_1) &= x_1^T(A_{11} - A_{12}K)^T P x_1 + x_1^T P(A_{11} - A_{12} \\ &\quad K)x_1 + 2x_1^T P A_{12}\left\{-(I + \bar{\Delta}A_{12})\bar{K}(x_1)\right. \\ &\quad \left.-\bar{\Delta}A_{12}Kx_1 + \Delta f_{rm}\right\} + 2x_1^T P \Delta f_{ru}.\end{aligned} \tag{15}$$

Using Assumption 3 and (13),

$$\begin{aligned}&2x_1^T P A_{12}\left\{-(I + \bar{\Delta}A_{12})\bar{K}(x_1)\right. \\ &\quad \left.-\bar{\Delta}A_{12}Kx_1 + \Delta f_{rm}\right\} \\ &\le -2\phi^T(I + \bar{\Delta}A_{12})\bar{K}(x_1) + 2\|\bar{\Delta}A_{12}K\| \\ &\quad \|\phi\|\|x_1\| + 2\|\phi\|\|\Delta f_{rm}\| \\ &\le -2\phi^T(I + \bar{\Delta}A_{12})\bar{K}(x_1) + 2(1-\epsilon)\|K\| \\ &\quad \|\phi\|\|x_1\| + 2\rho_{rm}\|\phi\|\|x_1\| \\ &\le -2\frac{\epsilon\bar{\rho}_{rm}^2}{2\epsilon\xi}\phi^T\phi + 2\left\{(1-\epsilon)\|K\| + \rho_{rm}\right\} \\ &\quad \|\phi\|\|x_1\| \\ &= -\frac{\bar{\rho}_{rm}^2}{\xi}\|\phi\|^2 + 2\bar{\rho}_{rm}\|\phi\|\|x_1\| \\ &\le \xi\|x_1\|^2 = \xi x_1^T x_1.\end{aligned} \tag{16}$$

Therefore, we obtain the following condition from (15) and (16)

$$\begin{aligned}\dot{V}(x_1) &\le x_1^T(A_{11} - A_{12}K)^T P x_1 + x_1^T P(A_{11} \\ &\quad - A_{12}K)x_1 + \xi x_1^T x_1 + 2x^T P \Delta f_{ru} \\ &= -x_1^T Q x_1 + \xi x_1^T x_1 + 2x^T P \Delta f_{ru}.\end{aligned} \tag{17}$$

By the fact that $x_1^T Q x_1 \ge \lambda_{min}(Q)\|x_1\|^2$ and $\|x_1^T P \Delta f_{ru}\| \le \rho_{ru}\|P\|\|x_1\|^2$, we have $\dot{V} \le -\lambda_{min}(Q)\|x_1\|^2 + \xi\|x_1\|^2 + 2\rho_{ru}\|P\|\|x_1\|^2$. Hence, $\dot{V}$ is negative if it is satisfied $\rho_{ru} < 0.5(\lambda_{min}(Q) - \xi)/\|P\|$. The proof is complete. □

Remark 1: It is clear that if ξ of (13) approaches the zero, upper bounds of b and ρ_{ru} are same as $0.5\lambda_{min}(Q)/\|P\|$ to confirm asymptotic stability. And, there exists ρ_{ru} which is less than or equal to b since $\|\Delta f_{ru}\|$ is less than or equal to $\|\bar{f}(x_1)\|$ and $\bar{f}(x_1)$ satisfies the uniform Lipschitz condition. Consequently, the proposed sliding surface (9) improves robustness with respect to mismatched uncertainties in the uncertain system (1).

Using the result of Theorem 2.1, the proposed sliding surface (9) can be rewritten as

$$\begin{aligned}\sigma(x) &= Kx_1 + \frac{\bar{\rho}_{rm}^2}{2\epsilon\xi}\phi(x_1) + x_2 \\ &= \left(K + \frac{\bar{\rho}_{rm}^2}{2\epsilon\xi}A_{12}^T P\right)x_1 + x_2 \\ &= K_{net}x_1 + x_2, \end{aligned} \tag{18}$$

where $K_{net} \triangleq K + \frac{\bar{\rho}_{rm}^2}{2\epsilon\xi}A_{12}^T P$.

3. Variable Structure Control

In Section 2, when the uncertain system (1) is in the sliding mode, we proved the asymptotic stability of the reduced-order system. Next, we need to find control law u to drive state trajectories of the system onto the sliding surface (18). This means that the control law is designed to satisfy reaching condition.

Theorem 3.1: For the system (1) satisfying assumptions 1 and 2, the following control law is considered:

$$u = \begin{cases} u_{eq} & \text{if } \sigma = 0 \\ u_{eq} - \rho(x)\frac{B_2\sigma}{\|B_2\sigma\|} & \text{if } \sigma \neq 0 \end{cases} \tag{19}$$

where

$$u_{eq} = -B_2^{-1}\begin{bmatrix} K_{net} \\ I \end{bmatrix}^T \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} x$$

$$\rho(x) > \frac{(1-\epsilon_b)\|u_{eq}\| + \eta\|B_2^{-1}K_{net}\|\|x\| + \rho_m(x)}{\epsilon_b}.$$

Then, the reaching condition of the sliding surface (18) is satisfied.

Proof: Consider the following time derivative of the sliding surface (18).

$$\begin{aligned}\dot{\sigma}(x) &= K_{net}\dot{x}_1 + \dot{x}_2 \\ &= \begin{bmatrix} K_{net} \\ I \end{bmatrix}^T \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} x + K_{net}\begin{bmatrix} \Delta A_{11} \\ \Delta A_{12} \end{bmatrix}^T x \\ &\quad + B_2\left\{(I + \Delta B_2)u + \Delta f_m\right\}. \end{aligned} \tag{20}$$

If $\sigma(x) \neq 0$, choose a Lypunov function candidate as $V_\sigma(t) = \frac{1}{2}\sigma^T\sigma$. From (19) and (20), we have

$$\begin{aligned}\dot{V}_\sigma &= \sigma^T\dot{\sigma} \\ &= \sigma^T B_2\left[B_2^{-1}K_{net}\begin{bmatrix} \Delta A_{11} \\ \Delta A_{12} \end{bmatrix}^T x + \Delta B_2 u_{eq}\right. \\ &\quad \left. - (I + \Delta B_2)\rho(x)\frac{B_2^T\sigma}{\|B_2^T\sigma\|} + \Delta f_m\right]. \end{aligned} \tag{21}$$

Using Assumption 1, we can get the following condition

$$\begin{aligned}\dot{V}_\sigma &\leq \|B_2^T\sigma\|\left[\eta\|B_2^{-1}K_{net}\|\|x\| + (1-\epsilon_b)\|u_{eq}\|\right. \\ &\quad \left. + \rho_m(x)\right] - \rho(x)(I + \Delta B_2)\frac{\|B_2\sigma\|^2}{\|B_2\sigma\|}. \end{aligned} \tag{22}$$

Since $\rho(x) > \frac{(1-\epsilon_b)\|u_{eq}\| + \eta\|B_2^{-1}K_{net}\|\|x\| + \rho_m(x)}{\epsilon_b}$, we have $\dot{V}_\sigma < 0$. The proof is complete. □

4. Numerical Example

Consider the following uncertain system:

$$\begin{aligned}\dot{x} &= \begin{bmatrix} a_{11} + \Delta a_{11} & a_{12} + \Delta a_{12} & a_{13} + \Delta a_{13} \\ a_{21} + \Delta a_{21} & a_{22} + \Delta a_{22} & a_{23} + \Delta a_{23} \\ a_{31} + \Delta a_{31} & a_{32} + \Delta a_{32} & a_{33} + \Delta a_{33} \end{bmatrix} x \\ &\quad + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\{(1 + \Delta B)u + f\} \end{aligned} \tag{23}$$

where

$$\begin{aligned} &a_{11} = 0.1, \quad a_{12} = 0.8, \quad a_{13} = 0.2 \\ &a_{21} = 1.2, \quad a_{22} = 2.6, \quad a_{23} = 1.0 \\ &a_{31} = -0.4, \quad a_{32} = 1.7, \quad a_{33} = 0.6 \\ &\Delta a_{11} = 0.05 + 0.02\sin(\pi t) + 0.12\sin(10\pi t) \\ &\Delta a_{12} = -0.1 + 0.01\cos(\pi t) \\ &\Delta a_{13} = 0.006 + 0.014\sin(2\pi t) \\ &\Delta a_{21} = 0.25 + 0.10\sin(\pi t) \\ &\Delta a_{22} = -0.5 + 0.05\cos(\pi t) + 0.2\cos(5\pi t) \\ &\Delta a_{23} = 0.03 + 0.07\sin(2\pi t) \\ &\Delta a_{31} = 0.5\sin(\pi t), \quad \Delta a_{32} = 0, \quad \Delta a_{33} = 0 \\ &\Delta B = 0.1\sin(\pi t), \quad f = 3\sin(\pi t). \end{aligned}$$

In the sliding mode, the reduced-order system with mismatched uncertainties of (23) can be described as

$$\begin{aligned}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} &= \begin{bmatrix} 0.1 & 0.8 \\ 1.2 & 2.6 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \Delta a_{11} & \Delta a_{12} \\ \Delta a_{21} & \Delta a_{22} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \\ &\quad + \begin{bmatrix} 0.2 \\ 1.0 \end{bmatrix}x_3 + \begin{bmatrix} \Delta a_{13} \\ \Delta a_{23} \end{bmatrix}x_3. \end{aligned} \tag{24}$$

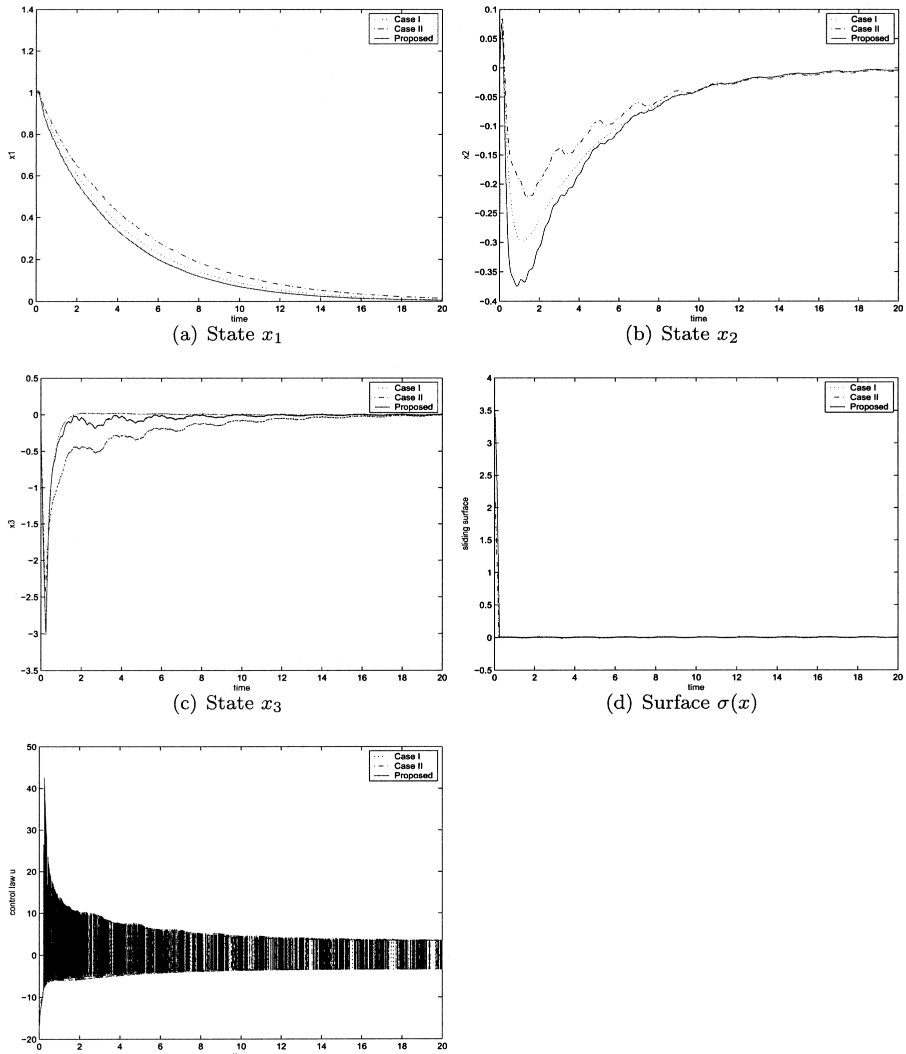
If we design the sliding surface $\sigma(x) = 2.33x_1 + 5.43x_2 + x_3$, mismatched uncertainties satisfy the following condition

$$\begin{aligned} &\left\|\left(\begin{bmatrix} \Delta a_{11} & \Delta a_{12} \\ \Delta a_{21} & \Delta a_{22} \end{bmatrix} - \begin{bmatrix} \Delta a_{13} \\ \Delta a_{23} \end{bmatrix}\begin{bmatrix} 2.33 & 5.43 \end{bmatrix}\right)\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right\| \\ &\leq 1.2090\left\|\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right\|. \end{aligned}$$

Choosing $Q = I$, we get

$$P = \begin{bmatrix} 2.1951 & -0.2685 \\ -0.2685 & 0.2038 \end{bmatrix}. \tag{25}$$

Since $0.5\lambda_{min}(Q)/\|P\| = 0.2242$, Shyu's stability criterion,



(a) State x_1 (b) State x_2

(c) State x_3 (d) Surface $\sigma(x)$

(e) Control Input

Fig. 1 Simulation results of case I, case II and our proposed method.

which is described by Lemma 2.1, cannot be applied. However, if the mismatched uncertainties are divided into the matched part which is in the range of $[0.2\ \ 1.0]^T$ and the rest mismatched part, the reduced-order system can be rewritten as

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0.1 & 0.8\\ 1.2 & 2.6\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0.2\\ 1.0\end{bmatrix}\{(1+\bar{\Delta}A_{12})x_3 + \Delta f_{rm}\} + \Delta f_{ru},$$

where

$$\bar{\Delta}A_{12} = 0.03 + 0.07\sin(\pi t)$$

$$\Delta f_{rm} = \begin{bmatrix}0.25 + 0.1\sin(\pi t)\\ -0.5 + 0.05\cos(\pi t)\end{bmatrix}^T\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

$$\Delta f_{ru} = \begin{bmatrix}0.12\sin(10\pi t) & 0\\ 0 & 0.2\cos(5\pi t)\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}.$$

Here, we can easily show that the Assumption 3 is valid. And, the values of ϵ, ρ_{rm} and ρ_{ru} is 0.9, 0.5535 and 0.2, respectively. Our sliding surface is defined as

$$\sigma(x) = 2.33x_1 + 5.43x_2 + \bar{K}(x_1, x_2) + x_3, \tag{26}$$

where $\bar{K}(x_1, x_2) = \frac{\bar{\rho}_{rm}^2}{2\epsilon\xi}\begin{bmatrix}0.2\\ 1.0\end{bmatrix}^T P\begin{bmatrix}x_1\\ x_2\end{bmatrix}$, $\bar{\rho}_{rm} = 1.1444$, $\xi = 0.09$ and P is defined by (25). Since $\rho_{ru} \leq 0.5(\lambda_{min}(Q) - \xi)/\|P\| = 0.2040$, the reduced-order system (24) with mismatched uncertainties is asymptotically stable by Theorem 2.1. Also, by Theorem 3.1 in Section 3, we can derive a sliding mode control law to drive $\sigma(x)$ to zero in finite time.

Simulation is performed for three cases: The one, referred to as case I, is that the conventional sliding mode control is applied for the uncertain system (23) without the mismatched uncertainties, the other, referred to as case II, is that the conventional sliding mode is applied for the uncertain system (23) with mismatched uncertainties, and the third is the proposed method. In the case I and the case II, the sliding surface is defined as

$$\sigma(x) = 2.33x_1 + 5.43x_2 + x_3. \tag{27}$$

The proposed sliding surface with additional robust component is defined as (26). And ϵ_b, η and ρ_m in the controller (19) are chosen to be 0.9, 0.6526 and $3+0.5\|x\|$, respectively. Simulation results are plotted in Fig. 1. It is shown that the system behavior of the case II is distorted by mismatched uncertainties even though it is still stable. However, it can be seen that our method has a quite close response as the case I with removal of mismatched uncertainties. It depicts that the robustness improvement over the conventional sliding mode control with mismatched uncertainties.

5. Conclusion

In this paper, we have proposed a new design method of the sliding surface for systems with mismatched uncertainties. Also, we derive a new stability criterion for the reduced-order system using on the proposed sliding surface. The proposed scheme enables us to handle mismatched uncertainties by means of offering robustness for the matched part which is in the range space of the input matrix of the reduced-order system. Effectiveness of the proposed method is demonstrated by a numerical example.

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