

# Linear and Nonlinear Supersymmetries

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**Abstract.** In this talk we use nonlinear realizations to study the spontaneous partial breaking of rigid and local supersymmetry.

## 1 Introduction

The winter of 1996 was a hard one for physics, bringing the untimely deaths of Professors Dmitrij Vasilievich Volkov and Victor Isaacovitch Ogievetsky. At this symposium it seems appropriate to celebrate the memories of both men, whose scientific achievements were so closely aligned, and whose inspiring presence is already acutely missed by their many friends and colleagues across the world.

In this talk we will discuss a subject close to their hearts: supersymmetry and its nonlinear realizations. In particular, we will consider the partial breaking of extended supersymmetry. For simplicity, we will restrict our attention to the case  $N = 2 \rightarrow N = 1$ , but many of our results can be readily extended to the case of higher supersymmetries, spontaneously broken to  $N = 1$ . It is a fitting memorial to see many of the ideas pioneered by Professors Volkov and Ogievetsky come into play.

The partial breaking of supersymmetry is of crucial importance to understanding the relation of theory to experiment. As theorists, we know in our bones that there is an ultimate theory, perhaps M theory, that exists at high energies. However, this theory is far removed from the physical world. To connect the two, we must integrate out the degrees of freedom associated with the high energies and construct a nonrenormalizable, effective field theory. This effective field theory should contain only those degrees of freedom that are relevant for physics in the world today.

Indeed, it is the point of view that underlies the effective field theory approach to pion dynamics. Below the scale of chiral symmetry breaking, we know that the interactions of pions and hadrons are governed by an effective field theory in which the unbroken isospin symmetry is realized linearly, but the spontaneously broken chiral symmetry is realized nonlinearly. The nonlinear symmetry is all that remains of the chiral symmetry below the scale where it is broken.

For the case at hand, we wish to construct a Lagrangian with two supersymmetries. The first supersymmetry, that of  $N = 1$ , is realized linearly, so it can be represented in terms of superfields. The second supersymmetry,  $N = 2$ , is realized nonlinearly on the superfields. In this way we can construct an effective

field theory of partial supersymmetry breaking. This theory is valid up to the scale where the second supersymmetry is spontaneously broken.

At first glance, it might seem impossible to partially break  $N = 2$  to  $N = 1$ . The argument runs as follows. Start with the  $N = 2$  supersymmetry algebra,

$$\begin{aligned} \{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} &= 2\sigma_{\alpha\dot{\alpha}}^m P_m \\ \{S_\alpha, \bar{S}_{\dot{\alpha}}\} &= 2\sigma_{\alpha\dot{\alpha}}^m P_m, \end{aligned} \quad (1)$$

where  $Q_\alpha$  and its conjugate  $\bar{Q}_{\dot{\alpha}}$  denote the first, unbroken supersymmetry, and  $S_\alpha, \bar{S}_{\dot{\alpha}}$  the second. Suppose that one supersymmetry is not broken, so

$$Q|0\rangle = \bar{Q}|0\rangle = 0. \quad (2)$$

Because of the supersymmetry algebra, this implies that the Hamiltonian also annihilates the vacuum,

$$H|0\rangle = 0. \quad (3)$$

Then, according to the supersymmetry algebra,

$$(\bar{S}S + S\bar{S})|0\rangle = 0. \quad (4)$$

The final step is to peel apart this relation and conclude that

$$S|0\rangle = \bar{S}|0\rangle = 0. \quad (5)$$

From this line of reasoning, one might think that partial breaking is impossible.

Fortunately, this argument has two significant loopholes. The first is that, technically-speaking, spontaneously-broken charges do not exist. Indeed, in a spontaneously broken theory, one only has the right to consider the algebra of the *currents*. For the case at hand, the current algebra can be modified as follows,

$$\begin{aligned} \{\bar{Q}_{\dot{\alpha}}, J_{\alpha m}^1\} &= 2\sigma_{\alpha\dot{\alpha}}^n T_{mn} \\ \{\bar{S}_{\dot{\alpha}}, J_{\alpha m}^2\} &= 2\sigma_{\alpha\dot{\alpha}}^n (v^4 \eta_{mn} + T_{mn}), \end{aligned} \quad (6)$$

where the  $J_{\alpha m}^i$  ( $i = 1, 2$ ) are the supercurrents and  $T_{mn}$  is the stress-energy tensor. Note that Lorentz invariance does not force the right-hand sides of the commutators to be the same. If there were no first supersymmetry, the  $v^4$  term in the second commutator could be absorbed in  $T_{mn}$ ; it would represent the scale of the supersymmetry breaking. Now, however, the first supersymmetry can be said to *define* the stress-energy tensor, in which case there is an extra term in the second commutator. This discrepancy prevents the current algebra from being integrated into a charge algebra, and the no-go theorem is avoided.

The second loophole involves the last step of the theorem. Even if the supercharges were to exist, it is only possible to extract (5) from (4) if the Hilbert space is positive definite. In covariantly-quantized supergravity theories, this is not the case: the gravitino  $\psi_{m\alpha}$  is a gauge field with negative-norm components.

There are, by now, many examples of partial supersymmetry breaking which exploit the first loophole. The first was given by Hughes, Liu and Polchinski

(1986) who showed that supersymmetry is partially broken on the world volume of an  $N = 1$  supersymmetric three-brane traveling in six-dimensional superspace. Since then there has been an explosion of interest in membranes, so the number of examples has grown substantially. [For another type of example, see Antoniadis, Partouche and Taylor (1996).]

The membrane approach leaves many open questions. For example, we would like to know all possible field-theoretic realizations of partial supersymmetry breaking, even those that do not originate with branes. We would also like to know whether the  $N = 2$  supersymmetry gives rise to any restrictions on matter couplings in the low-energy effective theory.

Finally, we would like to understand how partial breaking works in the presence of gravity. Gravity couples to the true stress-energy tensor, so it distinguishes between the right-hand sides of the commutators (6). Some early work on this question was done by Cecotti, Girardello and Porrati (1986) and by Zinov'ev (1987). These groups considered nonminimal cases and found that their gravitational couplings utilize the second loophole. One would like to reconcile their results with those above.

## 2 Coset Construction

In this talk we will take a bottom-up approach to the subject of partial supersymmetry breaking. We will use nonlinear realizations to describe the effective  $N = 1$  theory which holds below the scale of the second supersymmetry breaking. We will use the formalism of Coleman, Wess and Zumino (1969), as extended by Volkov (1973), to construct theories where the  $N = 1$  supersymmetry is manifest, and the second supersymmetry is nonlinearly realized.

The approach of Coleman, Wess, Zumino and Volkov is based on a coset decomposition of a symmetry group,  $G$ . We start with a group,  $G$ , of internal and spacetime symmetries, and partition the generators of  $G$  into three classes:

- $\Gamma_A$ , the generators of unbroken spacetime translations;
- $\Gamma_a$ , the generators of spontaneously broken internal and spacetime symmetries; and
- $\Gamma_i$ , the generators of unbroken spacetime rotations and unbroken internal symmetries.

The generators  $\Gamma_i$  close into the stability group,  $H$ .

Given  $G$  and  $H$ , we define the coset  $G/H$  in terms of an equivalence relation on the elements  $\Omega \in G$ ,  $\Omega \sim \Omega h$ , with  $h \in H$ . The coset can be thought of as a section of a fiber bundle with total space,  $G$ , and fiber,  $H$ .

This equivalence relation suggests that we parametrize the coset as follows,

$$\Omega = \exp iX^A \Gamma_A \exp i\xi^a(X) \Gamma_a . \quad (7)$$

Physically, the  $X^A$  play the role of generalized spacetime coordinates, while the  $\xi^a(X)$  are generalized Goldstone fields, defined on the generalized coordinates

and valued in the set of broken generators  $\Gamma_a$ . There is one generalized coordinate for every unbroken spacetime translation, and one generalized Goldstone field for every spontaneously broken generator.

We define the action of the group  $G$  on the coset  $G/H$  by left multiplication,  $\Omega \rightarrow g\Omega = \Omega' h$ , with  $g \in G$ . In this expression,

$$\Omega' = \exp iX'^A \Gamma_A \exp i\xi'^a(X') \Gamma_a \quad (8)$$

and  $h = \exp i\alpha^i(g, X, \xi) \Gamma_i$ . The group multiplication induces nonlinear transformations on the coordinates  $X^A$  and the Goldstone fields  $\xi^a$ :

$$X^A \rightarrow X'^A, \quad \xi^a(X) \rightarrow \xi'^a(X'). \quad (9)$$

These transformations realize the full symmetry group,  $G$ . Note that the field  $\xi^a$  transforms by a shift under the transformation generated by  $\Gamma_a$ . This confirms that  $\xi^a$  is indeed the Goldstone field corresponding to the broken generator  $\Gamma_a$ .

An arbitrary  $G$  transformation induces a compensating  $H$  transformation which is required to restore the section. This transformation can be used to lift any representation,  $R$ , of  $H$ , to a nonlinear realization of the full group,  $G$ , as follows,

$$\chi(X) \rightarrow \chi'(X') = D(h)\chi(X). \quad (10)$$

Here  $D(h) = \exp(i\alpha^i T_i)$ , where  $\alpha^i$  was defined below (8), and the  $T_i$  are generators of  $H$  in the representation  $R$ .

To proceed further, it is helpful to have a vielbein, connection and covariant derivative, built from the Goldstone fields in the following way. One first computes the Maurer-Cartan form,  $\Omega^{-1}d\Omega$ , where  $d$  is the exterior derivative. One then expands  $\Omega^{-1}d\Omega$  in terms of the Lie algebra of  $G$ ,

$$\Omega^{-1}d\Omega = i(\omega^A \Gamma_A + \omega^a \Gamma_a + \omega^i \Gamma_i), \quad (11)$$

where  $\omega^A, \omega^a$  and  $\omega^i$  are one-forms on the manifold parametrized by the coordinates  $X^A$ .

The Maurer-Cartan form transforms as follows under a rigid  $G$  transformation,

$$\Omega^{-1}d\Omega \rightarrow h(\Omega^{-1}d\Omega)h^{-1} - dh h^{-1}. \quad (12)$$

From this we see that the fields  $\omega^A$  and  $\omega^a$  transform covariantly under  $G$ , while  $\omega^i$  transforms by a shift. These transformations help us identify

$$\omega^A = dX^M E_M^A \quad (13)$$

as the covariant vielbein,

$$\omega^a = dX^M E_M^A \mathcal{D}_A \xi^a \quad (14)$$

as the covariant derivative of the Goldstone field  $\xi^a$ , and

$$\omega^i = dX^M \omega_M^i \quad (15)$$

as the connection associated with the stability group,  $H$ . With these building blocks, it is easy to construct theories invariant under the full group  $G$ .

The coset construction is very general and very powerful. For the case of internal symmetries, it allows one to prove that any  $H$ -invariant action can be lifted to be  $G$ -invariant with the help of the Goldstone bosons. For  $N = 1$  supersymmetry, it can be used to show that any Lorentz-invariant action can be made supersymmetric with the help of the Goldstone fermion.

### 3 Nonlinear Supersymmetry

In this section we will show that any  $N = 1$  supersymmetric theory can be made  $N = 2$  supersymmetric with the help of an  $N = 1$  Goldstone superfield. We will find that the Goldstone superfield can contain either an  $N = 1$  chiral or vector multiplet (Bagger and Galperin, 1994, 1997a). [The case where the Goldstone superfield is an  $N = 1$  tensor multiplet can be obtained from the chiral case by a superspace duality transformation (Bagger and Galperin, 1997b).]

It is important to emphasize that the coset construction – while very useful and very general – does not tell us anything about the underlying theory in which both supersymmetries are linearly realized. Indeed, such a theory might not even exist. Therefore we shall resolutely insist that we are working in the context of an effective field theory, and leave to others the task of finding the more fundamental theory above the supersymmetry-breaking scale.

In what follows we shall first take a minimal approach, and choose the group  $G$  to be the supergroup whose algebra is (1). We will take the subgroup  $H$  to be the supergroup generated by  $P_a$ ,  $Q_\alpha$  and  $\bar{Q}_{\dot{\alpha}}$ . We parametrize the coset element  $\Omega$  as follows,

$$\begin{aligned} \Omega = & \exp i(x^a P_a + \theta^\alpha Q_\alpha + \bar{\theta}_{\dot{\alpha}} \bar{Q}^{\dot{\alpha}}) \\ & \times \exp i(\Psi^\alpha S_\alpha + \bar{\Psi}_{\dot{\alpha}} \bar{S}^{\dot{\alpha}}) . \end{aligned} \quad (16)$$

Here  $x$ ,  $\theta$  and  $\bar{\theta}$  are the coordinates of  $N = 1$  superspace, while  $\Psi^\alpha$  and its conjugate  $\bar{\Psi}_{\dot{\alpha}}$  are Goldstone  $N = 1$  superfields of (geometrical) dimension  $-1/2$ . These spinor superfields contain far too many component fields, so we need to find a set of consistent, covariant constraints to reduce the number of fields.

The correct constraints are most easily expressed in term of the  $N = 2$  covariant derivatives of the Goldstone superfield. The covariant derivatives can be found following the techniques of the previous section; they can be explicitly written as follows,

$$\begin{aligned} \mathcal{D}_\alpha &= D_\alpha - i(D_\alpha \Psi \sigma^a \bar{\Psi} + D_\alpha \bar{\Psi} \bar{\sigma}^a \Psi) \omega_a^{-1m} \partial_m \\ \bar{\mathcal{D}}_{\dot{\alpha}} &= \bar{D}_{\dot{\alpha}} - i(\bar{D}_{\dot{\alpha}} \Psi \sigma^a \bar{\Psi} + \bar{D}_{\dot{\alpha}} \bar{\Psi} \bar{\sigma}^a \Psi) \omega_a^{-1m} \partial_m \\ \mathcal{D}_a &= \omega_a^{-1m} \partial_m , \end{aligned} \quad (17)$$

where  $\omega_m^a \equiv \delta_m^a + i(\partial_m \Psi \sigma^a \bar{\Psi} + \partial_m \bar{\Psi} \bar{\sigma}^a \Psi)$  and  $D_\alpha$ ,  $\bar{D}_{\dot{\alpha}}$  are ordinary flat  $N = 1$  superspace spinor derivatives. The covariant derivatives obey the following

commutation relations,

$$\begin{aligned}
\{\mathcal{D}_\alpha, \mathcal{D}_\beta\} &= -2i(\mathcal{D}_\alpha \Psi^\gamma \mathcal{D}_\beta \bar{\Psi}^{\dot{\gamma}} + (\alpha \leftrightarrow \beta)) \mathcal{D}_{\gamma\dot{\gamma}} \\
[\mathcal{D}_\alpha, \mathcal{D}_a] &= -2i(\mathcal{D}_\alpha \Psi^\gamma \mathcal{D}_a \bar{\Psi}^{\dot{\gamma}} + (\alpha \leftrightarrow a)) \mathcal{D}_{\gamma\dot{\gamma}} \\
\{\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot{\beta}}\} &= 2i\sigma_{\alpha\dot{\beta}}^a \mathcal{D}_a - 2i(\mathcal{D}_\alpha \Psi^\gamma \bar{\mathcal{D}}_{\dot{\beta}} \bar{\Psi}^{\dot{\gamma}} \\
&\quad + (\alpha \leftrightarrow \dot{\beta})) \mathcal{D}_{\gamma\dot{\gamma}} ,
\end{aligned} \tag{18}$$

where  $\mathcal{D}_{\alpha\dot{\alpha}} \equiv \sigma_{\alpha\dot{\alpha}}^a \mathcal{D}_a$ .

One set of constraints is simply (Bagger and Galperin, 1994)

$$\begin{aligned}
\bar{\mathcal{D}}\bar{\mathcal{D}}\Psi_\alpha &= \mathcal{O}(\Psi^3) \\
\mathcal{D}_\alpha \Psi_\beta + \mathcal{D}_\beta \Psi_\alpha &= \mathcal{O}(\Psi^3) .
\end{aligned} \tag{19}$$

The right-hand side of this equation must be adjusted for consistency with (18). Remarkably, this can be done using the dimensionless invariants  $\bar{\mathcal{D}}_{\dot{\alpha}}\Psi_\alpha$  and  $\mathcal{D}_\alpha\Psi_\beta$  (together with their complex conjugates). It turns out that there is a unique, consistent solution order-by-order in powers of the Goldstone field.

The solution to the constraints (19) is easy to find in perturbation theory. To lowest order, it is just the chiral multiplet  $\Phi$ ,

$$\begin{aligned}
\Psi_\alpha &= D_\alpha \Phi + \mathcal{O}(\Psi^3) \\
\bar{\mathcal{D}}_{\dot{\alpha}}\Phi &= \mathcal{O}(\Psi^3) .
\end{aligned} \tag{20}$$

In this expression,  $D_\alpha$  is the ordinary  $N = 1$  superspace spinor derivative.

A second set of constraints is (Bagger and Galperin, 1997a)

$$\begin{aligned}
\bar{\mathcal{D}}_{\dot{\alpha}}\Psi_\alpha &= \mathcal{O}(\Psi^3) \\
\mathcal{D}^\alpha \Psi_\alpha + \bar{\mathcal{D}}_{\dot{\beta}} \bar{\Psi}^{\dot{\beta}} &= \mathcal{O}(\Psi^3) .
\end{aligned} \tag{21}$$

As above, the right-hand side must be adjusted for consistency with the algebra of covariant derivatives. Again, there is a unique, consistent solution. To lowest order in perturbation theory, it is

$$\begin{aligned}
\Psi_\alpha &= W_\alpha + \mathcal{O}(\Psi^3) \\
W_\alpha &= -\frac{1}{4} \bar{\mathcal{D}}\bar{\mathcal{D}} D_\alpha V + \mathcal{O}(\Psi^3) ,
\end{aligned} \tag{22}$$

where  $V$  is a real  $N = 1$  vector superfield. We see that the chiral and vector Goldstone multiplet can each be obtained to lowest order in perturbation theory. In fact, the consistency of the multiplets survives to all orders in perturbation theory.

The Goldstone action can be constructed order-by-order in the Goldstone fields. For the chiral case, it is simply (Bagger and Galperin, 1994)

$$S = v^4 \int d^4x d^2\theta d^2\bar{\theta} E [\Phi^+ \Phi + \mathcal{O}(\Phi^4)] . \tag{23}$$

In this expression,  $E = \text{Ber}(E_M^A)$  is the superdeterminant of the vielbein, and  $v$  is the constant of dimension one which corresponds to the scale of the supersymmetry breaking. The action (23) is invariant under the full  $N = 2$  supersymmetry.

For the vector multiplet, the Goldstone action is just (Bagger and Galperin, 1997a)

$$S = \frac{v^4}{4} \int d^4x d^2\theta \mathcal{E} W^2 + \text{h.c.} \\ + \int d^4x d^4\theta E \mathcal{O}(W^4) . \quad (24)$$

This action is invariant under  $N = 2$  supersymmetry. It is also gauge-invariant. Curiously enough, the gauge field contribution to the Goldstone action coincides with the expansion of the Born-Infeld action.

Having constructed the  $N = 2$  Goldstone action, we are now ready to add  $N = 2$  covariant matter. The basic ingredients are  $N = 2$  nonlinear generalizations of  $N = 1$  chiral and vector superfields. The generalized chiral superfields are defined by the constraint  $\bar{D}_{\dot{\alpha}}\chi = 0$ . This constraint is consistent for either type of Goldstone multiplet.

The matter action is easy to write down for either Goldstone multiplet. The kinetic term is

$$S = \int d^4x d^4\theta E K(\chi^+, \chi) \quad (25)$$

while the superpotential term is

$$S = \int d^4x d^2\theta \mathcal{E} P(\chi) . \quad (26)$$

As before,  $E$  and  $\mathcal{E}$  are superdeterminants of the supervielbein  $E_M^A$ . They can be adjusted to preserve the condition

$$\int d^4x d^4\theta E F(\chi) = 0 . \quad (27)$$

This allows the matter action to be Kähler invariant, so the matter couplings are described in terms of Kähler manifolds, just as for  $N = 1$ .

It is not hard to generalize these results to include vector superfields. The general conclusion is that any  $N = 1$  invariant theory can be lifted to be  $N = 2$  supersymmetric with the help of a Goldstone superfield. Furthermore, the Goldstone superfield can be either an  $N = 1$  chiral or vector multiplet.

Now that we have two explicit realizations of partial supersymmetry breaking, we can ask how they avoid the no-go argument discussed above. In each case, the nonlinear theory exploits the loophole of Hughes, Liu and Polchinski (1986). For example, in the vector case the second supercurrent goes like  $J_{\alpha}^m \sim v^4 \sigma_{\alpha\dot{\alpha}}^m \bar{\lambda}^{\dot{\alpha}}$ , so its commutator with the second supercharge reproduces the algebra (6).

## 4 Geometry

The fact that the constraints need to be adjusted order-by-order in  $\Psi_\alpha$  hints that a deeper structure underlies partial supersymmetry breaking. The  $N = 2$  supersymmetry does not provide enough symmetry to uniquely fix the covariant derivatives and the associated constraints. This intuition is borne out for the case of the chiral multiplet, where a much deeper set of symmetries acts on the Goldstone multiplet (Bagger and Galperin, 1994).

To see this, let us first extend the  $N = 2$  algebra by a complex central charge,  $Z$ :

$$\begin{aligned} \{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} &= 2\sigma_{\alpha\dot{\alpha}}^a P_a & \{S_\alpha, \bar{S}_{\dot{\alpha}}\} &= 2\sigma_{\alpha\dot{\alpha}}^a P_a \\ \{Q_\alpha, S_\beta\} &= 2\epsilon_{\alpha\beta} Z & \{\bar{Q}_{\dot{\alpha}}, \bar{S}_{\dot{\beta}}\} &= 2\epsilon_{\dot{\alpha}\dot{\beta}} \bar{Z}. \end{aligned} \quad (28)$$

We then consider a coset where the group  $G$  contains not only  $N = 2$  supersymmetry, but also its maximal automorphism group,  $SO(5, 1) \times SU(2)$ , where the  $SU(2)$  acts on the two supersymmetry generators, and  $SO(5, 1)$  is the  $D = 6$  Lorentz group. (Under  $SO(5, 1)$ , the generators  $P_a$  and  $Z$  form a  $D = 6$  vector, while the two supercharges form a single  $D = 6$  Majorana-Weyl spinor). Let us take  $H$  to be  $SO(3, 1) \times SO(2) \times U(1)$ , where  $SO(3, 1) \times SO(2) \subset SO(5, 1)$ ,  $U(1) \subset SU(2)$ , and  $SO(3, 1)$  is the  $D = 4$  Lorentz group.

Our parametrization of the coset  $G/H$  involves the  $N = 1$  superspace coordinates, as well as different Goldstone superfields for each of the broken symmetries,

$$\begin{aligned} \Omega &= \exp i(x^a P_a + \theta^\alpha Q_\alpha + \bar{\theta}_{\dot{\alpha}} \bar{Q}^{\dot{\alpha}}) \\ &\quad \times \exp i(\Phi Z + \bar{\Phi} \bar{Z} + \Psi^\alpha S_\alpha + \bar{\Psi}_{\dot{\alpha}} \bar{S}^{\dot{\alpha}}) \\ &\quad \times \exp i(\Lambda^a K_a + \bar{\Lambda}^{\dot{a}} \bar{K}_{\dot{a}} + \Xi T + \bar{\Xi} \bar{T}). \end{aligned} \quad (29)$$

Here  $\Lambda^a, \bar{\Lambda}^{\dot{a}}$  are the Goldstone superfields associated with the generators  $K_a, \bar{K}_{\dot{a}}$  of  $SO(5, 1)/SO(3, 1) \times SO(2)$ . Similarly,  $\Xi, \bar{\Xi}$  are the Goldstone superfields for the broken generators  $T, \bar{T}$  of  $SU(2)/U(1)$ .

As before, the  $N = 1$  Goldstone superfields contain far more components than the minimal Goldstone multiplet. This motivates us to impose the following consistent set of constraints:

$$\begin{aligned} \bar{\mathcal{D}}_{\dot{\alpha}} \Phi &= 0, & \mathcal{D}_\alpha \Phi &= 0, & \mathcal{D}_a \Phi &= 0 \\ \mathcal{D}_\alpha \Psi^\beta &= 0, & \bar{\mathcal{D}}_{\dot{\alpha}} \Psi^{\dot{\beta}} &= 0. \end{aligned} \quad (30)$$

These constraints allow us to express the Goldstone superfields  $\Psi^\alpha, \Lambda^a$  and  $\bar{\Xi}$  in terms of a single superfield  $\Phi$ . [This way of eliminating Goldstones was called the ‘‘inverse Higgs effect’’ by Ivanov and Ogievetsky (1975).] To lowest order, we find  $\Psi^\alpha = -\frac{1}{2} D^\alpha \Phi$ ,  $\Lambda_a = -\partial_a \Phi$ , and  $\bar{\Xi} = \frac{1}{4} D^2 \Phi$ . The constraint  $\bar{\mathcal{D}}_{\dot{\alpha}} \Phi = 0$  reduces  $\Phi$  to an  $N = 1$  chiral superfield.

The remarkable fact about this construction is that it reveals a geometrical role for each component of the chiral Goldstone multiplet. The scalar field,  $A$ , is



the complex Goldstone boson associated with the spontaneously broken central charge symmetry. Its derivative,  $\partial_m A$ , is the Goldstone boson associated with  $SO(5, 1)/SO(3, 1) \times SO(2)$ . The  $F$ -component of  $\Phi$  is the complex Goldstone boson associated with the  $SU(2)/U(1)$ . Finally, the spinor is the Goldstone fermion that arises from the partially broken supersymmetry.

The action (23) turns out to be invariant under  $SO(5, 1)$ , but it explicitly breaks  $SU(2)$  down to  $U(1)$ . Furthermore, any  $R$ -invariant  $N = 1$  matter action can be lifted to be  $SO(5, 1)$  invariant. These facts hint that the Goldstone action might be related to the six-dimensional membrane of Hughes, Liu and Polchinski (1986). Indeed, it is not hard to show that the chiral Goldstone action is precisely the gauge-fixed membrane action.

The geometry that underlies the vector case is presently under study. The Born-Infeld form of the gauge action suggests that it might be related to some sort of D-brane. The fact that there are no “transverse” scalars hints that the action might be that of a space-filling D3-brane. In any case, one would like to find the Goldstone-type symmetries associated with the gauge field strength and the auxiliary field of the Goldstone multiplet.

In fact, the  $D$ -component of the Goldstone multiplet can be interpreted as the Goldstone boson associated with the following  $U(1)$  subgroup of the  $SU(2)$  automorphism symmetry:  $\delta\theta^\alpha = i\eta\Psi^\alpha$ ,  $\delta\Psi^\alpha = i\eta\theta^\alpha$ . Under such a transformation, the  $D$ -component is shifted by the constant parameter  $\eta$ .

If we were to extend  $G$  in  $G/H$  by this  $U(1)$ , we would eliminate the dimensionless invariant  $\mathcal{D}^\alpha\Psi_\alpha$  in favor of the corresponding Goldstone superfield. Even then, there would still be a dimensionless invariant associated with the gauge field strength,  $\mathcal{D}_{(\alpha}\Psi_{\beta)}$ . This suggests that there is an extension of  $N = 2$  supersymmetry which associates a Goldstone-like symmetry with this field strength.

Moreover, gauge fields themselves can be interpreted as Goldstone fields associated with infinite-dimensional symmetry groups (Ivanov and Ogievetsky, 1976). This leads us to wonder whether the full symmetry of the new multiplet is some infinite-dimensional extension of  $N = 2$  supersymmetry.

## 5 Supergravity

We have just seen that there are two independent Goldstone realizations of partial supersymmetry breaking in four dimensions. (A third is related by duality.) Both give rise to the current algebra (6). Because the spontaneous breaking relies on the curious shift in the “second” stress-energy tensor, one would like to see what happens when the Goldstone multiplets are coupled to supergravity.

In this section, we will work backwards, and start by constructing two Lagrangians and two sets of supersymmetry transformations for the massive  $N = 1$  spin-3/2 multiplet. We will then “unHiggs” the theories by adding appropriate Goldstone fields and coupling gravity. In this way we will find the supergravities associated with each of the Goldstone multiplets. (The work in this section was done in collaboration with Richard Altendorfer and Samuel Osofsky.)

We will see that the second Lagrangian corresponds to an alternative representation for the  $N = 1$  massive spin-3/2 multiplet, one which was originally found by Ogievetsky and Sokatchev (1977). When coupled to gravity, this representation gives rise to a new  $N = 2$  supergravity with a modified  $N = 2$  supersymmetry algebra.

### 5.1 The Massive $N = 1$ Spin-3/2 Multiplet

The starting point for the supergravity coupling is the massive  $N = 1$  spin-3/2 multiplet. This multiplet contains six bosonic and six fermionic (on-shell) degrees of freedom, arranged in states of the following spins,

$$\begin{pmatrix} \frac{3}{2} \\ 1 \quad 1 \\ \frac{1}{2} \end{pmatrix}. \quad (31)$$

The traditional representation of this multiplet contains the following fields (Ferrara and van Nieuwenhuizen, 1983): one spin-3/2 fermion, one spin-1/2 fermion, and two spin-one vectors, each of mass  $m$ . The Ogievetsky-Sokatchev representation has the same fermions, but just one vector plus one antisymmetric tensor. As we shall see, each representation has a role to play in the theory of partial supersymmetry breaking.

The traditional representation is described by the following Lagrangian (Ferrara and van Nieuwenhuizen, 1983):

$$\begin{aligned} \mathcal{L} = & \epsilon^{mnpq} \bar{\psi}_m \bar{\sigma}_n \partial_p \psi_q - i \bar{\zeta} \bar{\sigma}^m \partial_m \zeta - \frac{1}{4} \mathcal{A}_{mn} \bar{\mathcal{A}}^{mn} \\ & - \frac{1}{2} m^2 \mathcal{A}_m \bar{\mathcal{A}}^m + \frac{1}{2} m \zeta \bar{\zeta} + \frac{1}{2} m \bar{\zeta} \zeta \\ & - m \psi_m \sigma^{mn} \psi_n - m \bar{\psi}_m \bar{\sigma}^{mn} \bar{\psi}_n. \end{aligned} \quad (32)$$

Here  $\psi_m$  is a spin-3/2 Rarita-Schwinger field,  $\zeta$  a spin-1/2 fermion, and  $\mathcal{A}_m = A_m + iB_m$  a complex spin-one vector. This Lagrangian is invariant under the following  $N = 1$  supersymmetry transformations,

$$\begin{aligned} \delta_\eta \mathcal{A}_m &= 2\psi_m \eta - i \frac{2}{\sqrt{3}} \bar{\zeta} \bar{\sigma}_m \eta - \frac{2}{\sqrt{3}m} \partial_m (\zeta \eta) \\ \delta_\eta \zeta &= \frac{1}{\sqrt{3}} \bar{\mathcal{A}}_{mn} \sigma^{mn} \eta - i \frac{m}{\sqrt{3}} \sigma^m \bar{\eta} \mathcal{A}_m \\ \delta_\eta \psi_m &= \frac{1}{3m} \partial_m (\bar{\mathcal{A}}_{rs} \sigma^{rs} \eta + 2im \sigma^n \bar{\eta} \mathcal{A}_n) - \frac{i}{2} (H_{+mn} \sigma^n + \frac{1}{3} H_{-mn} \sigma^n) \bar{\eta} \\ &\quad - \frac{2}{3} m (\sigma_m{}^n \bar{\mathcal{A}}_n \eta + \bar{\mathcal{A}}_m \eta), \end{aligned} \quad (33)$$

where  $H_{\pm mn} = \mathcal{A}_{mn} \pm \frac{i}{2} \epsilon_{mnr s} \mathcal{A}^{rs}$ .

The alternative Ogievetsky-Sokatchev representation has the following Lagrangian,

$$\begin{aligned}
 \mathcal{L} = & \epsilon^{pqrs} \bar{\psi}_p \bar{\sigma}_q \partial_r \psi_s - i \bar{\zeta} \bar{\sigma}^m \partial_m \zeta - \frac{1}{4} A_{mn} A^{mn} + \frac{1}{2} v^m v_m \\
 & - \frac{1}{2} m^2 A_m A^m - \frac{1}{4} m^2 B_{mn} B^{mn} + \frac{1}{2} m \zeta \zeta + \frac{1}{2} m \bar{\zeta} \bar{\zeta} \\
 & - m \psi_m \sigma^{mn} \psi_n - m \bar{\psi}_m \bar{\sigma}^{mn} \bar{\psi}_n , \tag{34}
 \end{aligned}$$

where  $A_{mn}$  is the field strength associated with the real vector field  $A_m$ , and  $v_m = \frac{1}{2} \epsilon_{mnr s} \partial^n B^{rs}$  is the field strength for the antisymmetric tensor  $B_{mn}$ . This Lagrangian is invariant under the following  $N = 1$  supersymmetry transformations,

$$\begin{aligned}
 \delta_\eta A_m &= (\psi_m \eta + \bar{\psi}_m \bar{\eta}) + \frac{i}{\sqrt{3}} (\bar{\eta} \bar{\sigma}_m \zeta - \bar{\zeta} \bar{\sigma}_m \eta) - \frac{1}{\sqrt{3} m} \partial_m (\zeta \eta + \bar{\zeta} \bar{\eta}) \\
 \delta_\eta B_{mn} &= \frac{2}{\sqrt{3}} \left( \eta \sigma_{mn} \zeta + \frac{i}{2m} \partial_{[m} \bar{\zeta} \bar{\sigma}_{n]} \eta \right) + i \eta \sigma_{[m} \bar{\psi}_{n]} + \frac{1}{m} \eta \psi_{mn} + \text{h.c.} \\
 \delta_\eta \zeta &= \frac{1}{\sqrt{3}} A_{mn} \sigma^{mn} \eta - \frac{i m}{\sqrt{3}} \sigma^m \bar{\eta} A_m - \frac{1}{\sqrt{3}} m \sigma_{mn} \eta B^{mn} - \frac{1}{\sqrt{3}} v_m \sigma^m \bar{\eta} \\
 \delta_\eta \psi_m &= \frac{1}{3m} \partial_m (A_{rs} \sigma^{rs} \eta + 2i m \sigma^n \bar{\eta} A_n) - \frac{i}{2} (H_{+mn}^A \sigma^n + \frac{1}{3} H_{-mn}^A \sigma^n) \bar{\eta} \\
 & - \frac{2}{3} m (\sigma_m{}^n A_n \eta + A_m \eta) + \frac{1}{3m} \partial_m (2v_n \sigma^n \bar{\eta} - m \sigma^{rs} \eta B_{rs}) \\
 & - \frac{2i}{3} (v_m + \sigma_{mn} v^n) \eta - \frac{i m}{3} (B_{mn} \sigma^n \bar{\eta} + i \epsilon_{mnr s} B^{nr} \sigma^s \bar{\eta}) , \tag{35}
 \end{aligned}$$

where the square brackets denote antisymmetrization, without a factor of  $1/2$ .

These Lagrangians describe the free dynamics of massive spin-3/2 and 1/2 fermions, together with their supersymmetric partners, massive spin-one vector and tensor fields. They can be thought of as ‘‘unitary gauge’’ representations of theories with additional symmetries: a second supersymmetry for the massive spin-3/2 fermion, and additional gauge symmetries associated with the massive gauge fields.

## 5.2 The Supergravity Coupling

To study partial breaking, we need to ‘‘unHiggs’’ these Lagrangians by including appropriate gauge and Goldstone fields. In each case we need to add a Goldstone multiplet and gauge the full  $N = 2$  supersymmetry. The supersymmetric partners of the Goldstone fermion will turn out to be the Goldstone bosons that restore the gauge symmetries associated with the massive bosonic fields. At the end of the day, we will find two theories with  $N = 2$  supersymmetry nonlinearly realized, but  $N = 1$  represented linearly on the fields. The resulting effective field theories describe the physics of partial supersymmetry breaking, well below the scale where the second supersymmetry is broken.

The trick to this construction is to add the right fields. Because  $N = 1$  supersymmetry is not broken, the Goldstone fermion must belong to an  $N = 1$  supersymmetry multiplet. For the two cases of interest, we shall see that the Goldstone fermion must belong to the chiral or the vector multiplet, discussed above.

Let us first consider the chiral case. Under the first supersymmetry, a complex boson  $\phi$  transforms into a Weyl fermion  $\chi$ ,

$$\delta_{\eta^1}\phi = \sqrt{2}\eta^1\chi. \quad (36)$$

If  $\chi$  is the Goldstone fermion, it shifts under the second supersymmetry,

$$\delta_{\eta^2}\chi = \sqrt{2}v^2\eta^2 + \dots, \quad (37)$$

where  $v$  is the scale of the second supersymmetry breaking. Therefore the closure of the two supersymmetries on  $\phi$  gives

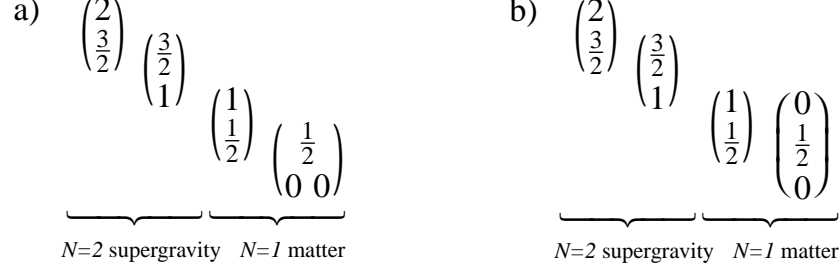
$$[\delta_{\eta^2}, \delta_{\eta^1}]\phi = 2v^2\eta^1\eta^2 + \dots \quad (38)$$

The complex scalar  $\phi$  undergoes a constant shift. This is in accord with our previous result: The field  $\phi$  is itself a Goldstone boson, corresponding to a complex central charge. It expects to be eaten by a complex vector field, which suggests that the chiral Goldstone multiplet should be associated with the traditional representation for the massive spin-3/2 multiplet.

As shown in Figure 1(a), the degree of freedom counting works out just right. We start with the  $N = 1$  chiral Goldstone multiplet and add an  $N = 1$  vector multiplet. We then add the gauge fields of  $N = 2$  supergravity. As we will see, the full set of fields can be used to construct a Lagrangian which is invariant under  $N = 2$  supersymmetry. The final results look complicated, but they are actually very simple: In unitary gauge, the two vectors eat the two scalars, while the Rarita-Schwinger field eats one linear combination of the spin-1/2 fermions. This leaves the massive  $N = 1$  multiplet coupled to  $N = 1$  supergravity.

With that said, we now present the Lagrangian (Altendorfer, Bagger, Osofsky, 1998):

$$\begin{aligned} e^{-1}\mathcal{L} = & -\frac{1}{2\kappa^2}\mathcal{R} + \epsilon^{mnr s}\bar{\psi}_{mi}\bar{\sigma}_n D_r\psi_s^i - i\bar{\chi}\bar{\sigma}^m D_m\chi - i\bar{\lambda}\bar{\sigma}^m D_m\lambda - \mathcal{D}^m\phi\overline{\mathcal{D}_m\phi} \\ & -\frac{1}{4}\mathcal{A}_{mn}\overline{\mathcal{A}^{mn}} - \left(\frac{1}{\sqrt{2}}m\psi_m^2\sigma^m\bar{\lambda} + im\psi_m^2\sigma^m\bar{\chi} + \sqrt{2}im\lambda\chi + \frac{1}{2}m\chi\chi\right. \\ & + m\psi_m^2\sigma^{mn}\psi_n^2 + \frac{\kappa}{4}\epsilon_{ij}\psi_m^i\psi_n^j\overline{H^+} + \frac{\kappa}{\sqrt{2}}\chi\sigma^m\bar{\sigma}^n\psi_m^1\overline{\mathcal{D}_n\phi} \\ & \left. + \frac{\kappa}{2\sqrt{2}}\bar{\lambda}\bar{\sigma}_m\psi_n^1\overline{H^-} + \frac{\kappa}{\sqrt{2}}\epsilon^{mnr s}\bar{\psi}_{m2}\bar{\sigma}_n\psi_r^1\overline{\mathcal{D}_s\phi} + \text{h.c.}\right), \quad (39) \end{aligned}$$



**Fig. 1.** The unHiggsed versions of the (a) traditional and (b) alternative representations of the  $N = 1$  massive spin-3/2 multiplet. The traditional representation contains the degrees of freedom associated with an  $N = 1$  chiral multiplet. The alternative representation exchanges the chiral multiplet for its dual, an  $N = 1$  tensor multiplet.

where  $\kappa$  denotes Newton's constant,  $m = \kappa v^2$ , and

$$\begin{aligned}
 \mathcal{A}_m &= A_m + iB_m \\
 \mathcal{A}_{mn} &= \partial_m \mathcal{A}_n - \partial_n \mathcal{A}_m \\
 H_{\pm mn} &= \mathcal{A}_{mn} \pm \frac{i}{2} \epsilon_{mnr s} \mathcal{A}^{rs} .
 \end{aligned} \tag{40}$$

The supercovariant derivatives are as follows,

$$\begin{aligned}
 \hat{\mathcal{D}}_m \phi &= \partial_m \phi - \frac{\kappa}{\sqrt{2}} \psi_m^1 \chi - \frac{1}{\sqrt{2}} \kappa v^2 \mathcal{A}_m \\
 \hat{\mathcal{A}}_{mn} &= \mathcal{A}_{mn} + \kappa \psi_{[m}^2 \psi_{n]}^1 - \frac{\kappa}{\sqrt{2}} \bar{\lambda} \bar{\sigma}_{[n} \psi_{m]}^1 .
 \end{aligned} \tag{41}$$

This Lagrangian is invariant under two independent abelian gauge symmetries, as well as the following supersymmetry transformations,

$$\begin{aligned}
 \delta e_m^a &= i\kappa(\eta^i \sigma^a \bar{\psi}_{mi} + \bar{\eta}_i \bar{\sigma}^a \psi_m^i) \\
 \delta \psi_m^i &= \frac{2}{\kappa} D_m \eta^i \\
 &\quad + \left( -\frac{i}{2} \hat{H}_{+mn} \sigma^n \bar{\eta}_1 + \sqrt{2} \overline{\hat{\mathcal{D}}_m \phi} \eta^1 - \kappa \psi_m^1 (\bar{\chi} \bar{\eta}_1) + i v^2 \sigma_m \bar{\eta}_2 \right) \delta_2^i \\
 \delta \mathcal{A}_m &= 2\epsilon_{ij} \psi_m^i \eta^j + \sqrt{2} \bar{\lambda} \bar{\sigma}_m \eta^1 \\
 \delta \lambda &= \frac{i}{\sqrt{2}} \bar{\mathcal{A}}_{mn} \sigma^{mn} \eta^1 - i\sqrt{2} v^2 \eta^2 \\
 \delta \chi &= i\sqrt{2} \sigma^m \hat{\mathcal{D}}_m \phi \bar{\eta}_1 + 2v^2 \eta^2 \\
 \delta \phi &= \sqrt{2} \chi \eta^1 ,
 \end{aligned} \tag{42}$$

for  $i = 1, 2$ . This result holds to leading order, that is, up to and including terms in the transformations that are linear in the fields. Note that this representation

is irreducible in the sense that there are no subsets of fields that transform only into themselves under the supersymmetry transformations. (Because of this, the multiplet structure outlined in Fig. 1 is slightly misleading.)

Let us now consider the case of the vector Goldstone multiplet. Under the first supersymmetry, the real vector  $B_m$  of a vector multiplet transforms into a Weyl fermion  $\lambda$ ,

$$\delta_{\eta^1} B_m = \sqrt{2}i (\lambda \sigma_m \bar{\eta}^1 - \eta^1 \sigma_m \bar{\lambda}) . \quad (43)$$

If  $\lambda$  is the Goldstone fermion, it shifts under the second supersymmetry. Therefore the closure of the two supersymmetries on  $B_m$  gives

$$[\delta_{\eta^2}, \delta_{\eta^1}] B_m = 2iv^2 (\eta^2 \sigma_m \bar{\eta}^1 - \eta^1 \sigma_m \bar{\eta}^2) + \dots \quad (44)$$

From this we see that the real vector  $B_m$  is a Goldstone boson. It expects to be eaten by an antisymmetric tensor field. This suggests that the vector Goldstone multiplet should be associated with the alternative representation for the massive spin-3/2 multiplet.

The degree of freedom counting is shown in Figure 1(b). As before, we include the  $N = 2$  supergravity multiplet. This time, however, the matter fields include the  $N = 1$  vector Goldstone multiplet, together with an  $N = 1$  tensor multiplet. In unitary gauge, one vector eats one scalar, while the antisymmetric tensor eats the other vector. [The massless antisymmetric tensor field contains one degree of freedom. It was introduced by Ogievetsky and Polubarinov (1966), who called it the “notoph,” or inverse photon.] These are the minimal set of fields that arise when coupling the Ogievetsky-Sokatchev spin-3/2 multiplet to  $N = 2$  supergravity.

The Lagrangian for this system can be worked out following the same procedure described above. One finds (Altendorfer, Bagger, Osofsky, 1998):

$$\begin{aligned} e^{-1} \mathcal{L} = & - \frac{1}{2\kappa^2} \mathcal{R} + \epsilon^{pqrs} \bar{\psi}_p \bar{\sigma}_q D_r \psi_s^i - i \bar{\chi} \bar{\sigma}^m D_m \chi - i \bar{\lambda} \bar{\sigma}^m D_m \lambda - \frac{1}{2} \mathcal{D}^m \phi \mathcal{D}_m \phi \\ & - \frac{1}{4} \mathcal{F}_{mn}^A \mathcal{F}^{Amn} - \frac{1}{4} \mathcal{F}_{mn}^B \mathcal{F}^{Bmn} + \frac{1}{2} v^m v_m - \left( \frac{1}{\sqrt{2}} m \psi_m^2 \sigma^m \bar{\lambda} + m i \psi_m^2 \sigma^m \bar{\chi} \right. \\ & + \sqrt{2} m i \lambda \chi + \frac{1}{2} m \chi \chi + m \psi_m^2 \sigma^{mn} \psi_n^2 + \frac{\kappa}{2\sqrt{2}} \epsilon_{ij} \psi_m^i \psi_n^j \mathcal{F}_-^{Amn} \\ & + \frac{\kappa}{2} \chi \sigma^m \bar{\sigma}^n \psi_m^1 \mathcal{D}_n \phi + \frac{\kappa}{2} \bar{\lambda} \bar{\sigma}_m \psi_n^1 \mathcal{F}_+^{Bmn} + \frac{\kappa}{2} \epsilon^{pqrs} \bar{\psi}_p^2 \bar{\sigma}_q \psi_r^1 \mathcal{D}_s \phi \\ & \left. - i \frac{\kappa}{2} \chi \sigma^m \bar{\sigma}^n \psi_m^1 v_n - i \frac{\kappa}{2} \epsilon^{pqrs} \bar{\psi}_p^2 \bar{\sigma}_q \psi_r^1 v_s + \text{h.c.} \right) \end{aligned} \quad (45)$$

where, as before,  $m = \kappa v^2$ , and

$$\begin{aligned} \mathcal{D}_m \phi &= \partial_m \phi - \frac{m}{\sqrt{2}} (A_m + B_m) \\ \mathcal{F}_{mn}^A &= \partial_{[m} A_{n]} + \frac{m}{\sqrt{2}} B_{mn} \\ \mathcal{F}_{mn}^B &= \partial_{[m} B_{n]} - \frac{m}{\sqrt{2}} B_{mn} . \end{aligned} \quad (46)$$

This Lagrangian is invariant under an ordinary abelian gauge symmetry, an anti-symmetric tensor gauge symmetry, as well as the following two supersymmetries,

$$\begin{aligned}
 \delta_\eta e_m^a &= i\kappa(\eta^i \sigma^a \bar{\psi}_{mi} + \bar{\eta}_i \bar{\sigma}^a \psi_m^i) \\
 \delta_\eta \psi_m^1 &= \frac{2}{\kappa} D_m \eta^1 \\
 \delta_\eta A_m &= \sqrt{2} \epsilon_{ij} (\psi_m^i \eta^j + \bar{\psi}_m^i \bar{\eta}^j) \\
 \delta_\eta B_m &= \bar{\eta}^1 \bar{\sigma}_m \lambda + \bar{\lambda} \bar{\sigma}_m \eta^1 \\
 \delta_\eta B_{mn} &= 2\eta^1 \sigma_{mn} \chi + i\eta^1 \sigma_{[m} \bar{\psi}_{n]}^2 + i\eta^2 \sigma_{[m} \bar{\psi}_{n]}^1 + \text{h.c.} \\
 \delta_\eta \lambda &= i\hat{\mathcal{F}}_{mn}^B \sigma^{mn} \eta^1 - i\sqrt{2} v^2 \eta^2 \\
 \delta_\eta \chi &= i\sigma^m \bar{\eta}^1 \hat{\mathcal{D}}_m \phi - \hat{v}_m \sigma^m \bar{\eta}^1 + 2v^2 \eta^2 \\
 \delta_\eta \psi_m^2 &= \frac{2}{\kappa} D_m \eta^2 + iv^2 \sigma_m \bar{\eta}^2 - \frac{i}{\sqrt{2}} \hat{\mathcal{F}}_{+mn}^A \sigma^n \bar{\eta}^1 \\
 &\quad + \hat{\mathcal{D}}_m \phi \eta^1 + \kappa ((\bar{\psi}_m^1 \bar{\chi}) \eta - (\bar{\chi} \bar{\eta}) \psi_m^1) - i\hat{v}_m \eta^1 \\
 \delta_\eta \phi &= \chi \eta^1 + \bar{\chi} \bar{\eta}^1
 \end{aligned} \tag{47}$$

up to linear order in the fields. The supercovariant derivatives are given by

$$\begin{aligned}
 \hat{\mathcal{D}}_m \phi &= \mathcal{D}_m \phi - \frac{\kappa}{2} (\psi_m^1 \chi + \bar{\psi}_m^1 \bar{\chi}) \\
 \hat{\mathcal{F}}_{mn}^A &= \mathcal{F}_{mn}^A + \frac{\kappa}{\sqrt{2}} (\psi_{[m}^2 \psi_{n]}^1 + \bar{\psi}_{[m}^2 \bar{\psi}_{n]}^1) \\
 \hat{\mathcal{F}}_{mn}^B &= \mathcal{F}_{mn}^B - \frac{\kappa}{2} (\bar{\lambda} \bar{\sigma}_{[n} \psi_{m]}^1 + \bar{\psi}_{[m}^1 \bar{\sigma}_{n]} \lambda) \\
 \hat{v}_m &= v_m + \left( i\kappa \psi_n^1 \sigma_m{}^n \chi - \frac{i\kappa}{2} \epsilon_m{}^{nrs} \psi_n^1 \sigma_r \bar{\psi}_s^2 + \text{h.c.} \right).
 \end{aligned} \tag{48}$$

These fields form an irreducible representation of the  $N = 2$  algebra.

### 5.3 The SuperHiggs Effect

Each of the two Lagrangians presented above has a full  $N = 2$  supersymmetry (up to the appropriate order). The first supersymmetry is realized linearly, so it is not broken. The second is realized nonlinearly, so it is spontaneously broken. In each case, the transformations imply that

$$\zeta = \frac{1}{\sqrt{3}} (\chi - i\sqrt{2}\lambda) \tag{49}$$

does not shift, while

$$\nu = \frac{1}{\sqrt{3}} (\sqrt{2}\chi + i\lambda) \tag{50}$$

does. Therefore  $\nu$  is the Goldstone fermion for  $N = 2$  supersymmetry, spontaneously broken to  $N = 1$ .

In the chiral case, we find

$$\begin{aligned} [\delta_{\eta_1}, \delta_{\eta_2}] \phi &= 2\sqrt{2}v^2 \eta_1 \eta_2 \\ [\delta_{\eta_1}, \delta_{\eta_2}] \mathcal{A}_m &= \frac{4}{\kappa} \partial_m \eta_1 \eta_2 . \end{aligned} \quad (51)$$

The complex scalar  $\phi$  is indeed the Goldstone boson for a gauged central charge. Moreover, in unitary gauge, where

$$\phi = \nu = 0 , \quad (52)$$

this Lagrangian reduces to the usual representation for a massive  $N = 1$  spin-3/2 multiplet.

In the vector case, we have

$$\begin{aligned} [\delta_{\eta^2}, \delta_{\eta^1}] A_m &= \frac{2\sqrt{2}}{\kappa} \partial_m (\eta^1 \eta^2 + \bar{\eta}^1 \bar{\eta}^2) - \sqrt{2} i v^2 (\eta^2 \sigma_m \bar{\eta}^1 - \eta^1 \sigma_m \bar{\eta}^2) \\ [\delta_{\eta^2}, \delta_{\eta^1}] B_m &= \sqrt{2} i v^2 (\eta^2 \sigma_m \bar{\eta}^1 - \eta^1 \sigma_m \bar{\eta}^2) \\ [\delta_{\eta^2}, \delta_{\eta^1}] B_{mn} &= \frac{2i}{\kappa} D_{[m} (\eta^2 \sigma_n] \bar{\eta}^1 - \eta^1 \sigma_n] \bar{\eta}^2) . \end{aligned} \quad (53)$$

We see that the real vector  $-(A_m - B_m)/\sqrt{2}$  is the Goldstone boson for a gauged *vectorial* central extension of the  $N = 2$  algebra. In addition, the real scalar  $\phi$  is the Goldstone boson associated with a single *real* gauged central charge. In unitary gauge, with

$$-\frac{1}{\sqrt{2}}(A_m - B_m) = \phi = \nu = 0 , \quad (54)$$

this Lagrangian reduces to the Ogievetsky-Sokatchev representation for the massive  $N = 1$  spin-3/2 multiplet.

Now that we have two explicit realizations of partial supersymmetry breaking, we can go back and see how they avoid the no-go argument presented in the introduction. We first compute the second supercurrent. In each case it turns out to be

$$J_{m\alpha}^2 = v^2 (\sqrt{6} i \sigma_{\alpha\dot{\alpha}m} \bar{\nu}^{\dot{\alpha}} + 4 \sigma_{\alpha\beta mn} \psi^{2n\beta}) \quad (55)$$

plus higher-order terms. Computing, we find

$$\begin{aligned} \{ \bar{Q}_{\dot{\alpha}}, J_{m\alpha}^1 \} &= 2 \sigma_{\alpha\dot{\alpha}}^n T_{mn} \\ \{ \bar{S}_{\dot{\alpha}}, J_{m\alpha}^2 \} &= 2 \sigma_{\alpha\dot{\alpha}}^n T_{mn} . \end{aligned} \quad (56)$$

In the presence of supergravity, there is no confusion about the stress-energy tensor. There is just one such tensor, and it shows up on the right-hand side of the current algebra.

For the case at hand, however,  $J_{\alpha m}^i$  and  $T_{mn}$  contain contributions from *all* of the fields, including the second gravitino. When covariantly-quantized, the



second gravitino gives rise to states of negative norm. Indeed, it is not hard to check that

$$(\bar{S}S + S\bar{S})|0\rangle = 0, \quad (57)$$

even though

$$S|0\rangle \neq 0 \quad \bar{S}|0\rangle \neq 0. \quad (58)$$

The supergravity couplings exploit the second loophole to the no-go theorem!

The Lagrangian in the chiral case is a truncation of the supergravity coupling found by Cecotti, Girardello and Porrati (1986) and by Zinov'ev (1987). Their results were based on *linear*  $N = 2$  supersymmetry; they involved  $N = 2$  vector- and hyper-multiplets. The Lagrangian for the vector case is new. It contains a new realization of  $N = 2$  supergravity. In each case, the couplings presented here are minimal and model-independent. They describe the superHiggs effect in the low-energy effective theories that arise from partial supersymmetry breaking.

Thus we have seen that there is no obstacle to partial supersymmetry breaking in the presence of gravity. Indeed, each of the two Goldstone multiplets give rise to its own massive spin-3/2 multiplet. Of course, the connection between these results and the theory of membranes and D-branes is an urgent and open question.

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