Linear Precoding with Low Complexity MMSE Turbo-Equalization and Application to the Wireless LAN System

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Abstract—An improved turbo-equalization process is proposed for linear precoding with iterative decoding. Using recent results in turbo-equalization, we derive the equalization coefficients for an optimized precoding decoder thanks to the SINR criterion. The proposed method keeps a low complexity even with large size precoders and high order modulations. Theoretical and simulation results show that full diversity is achieved after three or four iterations over a flat Rayleigh fading channel. An application of this transmission scheme is presented in the context of the OFDM wireless LAN 802.11a. We observe a gain of several decibels compared to the conventional unprecoded transmission scheme.

I. INTRODUCTION

Diversity is an efficient solution to exploit fading in wireless mobile communications. Space diversity is commonly utilized thanks to space-time block coding (STBC) for multi-antenna systems [1], [2]. In reception, full diversity is obtained with a maximal ratio combining (MRC) receiver for an arbitrary number of receive antennas [3]. On the other hand, linear precoding (LP) can be used to exploit time or frequency diversity over a fading channel [4]. Hadamard codes are well known in the context of code division multiple access (CDMA), but they have also been applied as linear precoding in multicarrier systems [5]. This application, known as OFDM-CDM (OFDM code division multiplexing), has been generalized to complex matrices under the name of LP-OFDM (linearly precoded OFDM) [6]. In addition, iterative decoding schemes have been developed for joint channel coding and linear precoding [5], [7] and [8]. Particularly in [5], Kaiser proposed a sub-optimal low complexity turbo-equalizer. In [7], Qu et al. also investigated a turbo-equalization structure for LP-OFDM. This structure, based on the single carrier turbo-equalization of [9], used two switching sets of equalization coefficients. This proposed scheme looked very promising as good performance is achieved with quite low complexity. However, we observed that the equalization step has a critical impact on the performance. Consequently, in this paper we propose a new derivation of the equalization coefficients from the maximum signal to interference plus noise ratio (SINR) after the precoding decoder. These developments are derived from the recent results in single carrier turbo-equalization presented in [10]. We will show that this leads to optimal equalization coefficients along the iterations, unlike [5] and [7], and without complexity increase. In addition, we derive a lower bound of our system that emphasizes an analogy between an MRC receiver for multiple antennas and our single antenna transmission scheme.

The paper is organized as follows. Section II describes the proposed transmission scheme, where the bit interleaved coded modulation (BICM) and the linear precoding are concatenated to an OFDM modulation. In section III, the proposed linear precoding decoder is detailed, including the derivation of the equalization coefficients, the soft bit mapping/demapping and a brief discussion about complexity. The system performance over a flat fading Rayleigh channel are simulated and presented in section IV. In addition, an application to the wireless local area network (WLAN) 802.11a system [11] exploiting time and frequency diversity is presented.

II. LINEAR PRECODING WITH ITERATIVE DECODING

A. System Model

Data information is encoded using a bit interleaved coded modulation (BICM) scheme with a convolutional encoder. Coded bits are mapped onto complex symbols \(s_k\) chosen in the constellation \(\chi\) (BPSK, QPSK, 2\(^q\)-QAM...) using the Gray mapping. A time or frequency diversity is obtained.
thanks to linear precoding. Linear precoding involves spreading each information symbol over several time intervals or subcarriers. Each vector of symbols \( s = (s_0, \ldots, s_{M-1})^T \) of size \( M \) is multiplied by an \( M \times M \) precoding matrix \( D_M = (d_{k,l}) \). This operation can be viewed as a block code of size \( M \) with a coding rate of 1.

The OFDM channel depicted in Fig. 1 includes a symbol interleaving \( \Pi_2 \), the OFDM modulation/demodulation and the multipath frequency-selective channel. OFDM operations such as the cyclic prefix insertion prevent intersymbol interference (ISI) and intercarrier interference (ICI). As a consequence, the relationship between the transmit symbols \( s \) and the output \( r \) of the OFDM channel can be written as

\[
\mathbf{r} = \mathbf{H}\mathbf{D}_M s + \mathbf{n}
\]

where \( \mathbf{H} = \text{diag}(h_0, \ldots, h_{M-1}) \) is a fading coefficient matrix and \( \mathbf{n} = (n_0, \ldots, n_{M-1})^T \) an additive white Gaussian noise with variance \( \sigma_n^2 \). The interleaving \( \Pi_2 \) maps precoded symbols onto different subcarriers and OFDM symbols. Thus, our linear precoding scheme can exploit both time and frequency diversity.

At the receiver, the precoding decoder module performs the symbol estimation \( \hat{s} \) from the received data \( r \) and the \( a \ priori \) information \( \hat{s} \). In addition, this module requires knowledge of the channel matrix \( \mathbf{H} \). This matrix is considered to be perfectly known only at the receiver side.

The module called soft demapper computes the log likelihood ratio (LLR) required by the channel decoder from the estimated symbols. The LLRs at the output of the decoder are used by the soft mapper to produce the estimated symbols for the next iteration.

Before decoding, deinterleaving \( \Pi_1^{-1} \) permutes the LLRs to decorrelate them. Similarly, the interleaving \( \Pi_1 \) after decoding leads to the independence of modulated bits.

### B. Precoding Matrix

Joint coding-precoding schemes have been proposed in [5] and [8]. In [5] the system called OFDM-CDM is based on an iterative MMSE equalizer with interference canceler and uses a Hadamard matrix. The Hadamard matrices are recursively defined for each \( M \) power of two by

\[
\mathbf{D}_M = \frac{1}{\sqrt{2}} \begin{pmatrix} \mathbf{D}_{M/2} & \mathbf{D}_{M/2} \\ \mathbf{D}_{M/2} & -\mathbf{D}_{M/2} \end{pmatrix} \quad \text{with} \quad \mathbf{D}_1 = 1. \quad (2)
\]

The use of a fast Hadamard transform (FHT) leads to a very simple implementation of the multiplication by \( \mathbf{D}_M \) and its inverse.

In [8], the LP-OFDM system is based on a maximum \( a \ posteriori \) (MAP) decoding and uses the complex matrices of size \( 2 \times 2 \) or \( 4 \times 4 \) defined by

\[
\mathbf{D}_2 = \begin{bmatrix} 1 & e^{j \pi/4} \\ 1 & -e^{j \pi/4} \end{bmatrix},
\]

\[
\mathbf{D}_4 = \begin{bmatrix} 1 & e^{j \pi/4} & e^{j \pi/2} & e^{j 3\pi/4} \\ 1 & e^{j \pi/4} & e^{j \pi/2} & e^{j 3\pi/4} \\ 1 & e^{j 2\pi/4} & e^{j 2\pi/2} & e^{j 2\pi 3/4} \\ 1 & e^{j 2\pi/4} & e^{j 2\pi/2} & e^{j 2\pi 3/4} \end{bmatrix}. \quad (3)
\]

The choice of (3) responds to the criterion of maximum diversity gain with a maximum likelihood (ML) or MAP decoding. Both types of matrix are tested in this paper.

### III. LINEAR PRECODING DECODER

#### A. Improved Equalization

In this section, we adapt the results of [10] to optimize the equalization step inside the precoding decoder. Fig. 2 shows the precoding decoder with \( a \ priori \) information adapted from [5] and [10]. OFDM with a cyclic prefix prevents received samples from ISI and ICI. As a consequence, the channel matrix \( \mathbf{H} \) is diagonal as the equalization matrix \( \mathbf{P} = \text{diag}(p_0, \ldots, p_{M-1}) \). Its computation is then much easier than in classical turbo-equalization.

Let \( \mathbf{G} = (g_{k,l}) \) be defined as

\[
\mathbf{G} = \mathbf{D}_M^T \mathbf{P} \mathbf{H} \mathbf{D}_M. \quad (4)
\]

The diagonal coefficients of \( \mathbf{G} \) are all equal and we note

\[
g_0 = \frac{1}{M} \sum_{k=0}^{M-1} p_k h_k = g_{m,m}, \quad m = 0 \ldots M - 1. \quad (5)
\]

The upper branch in Fig. 2 represents the evaluation of the interference term from the \( a \ priori \) information \( \hat{s} \). The lower branch is the equalization step followed by the inverse precoding. Thus, the output of the precoding decoder can be written as

\[
\hat{s}_l = g_0 s_l + \sum_{k=0}^{M-1} d_l(k)p_k n_k + \sum_{m=0}^{M-1} g_{m,l} (s_m - \hat{s}_m) \ . \quad (6)
\]

The matrix \( \mathbf{P} \) is derived according to the maximisation of the signal to noise plus interference ratio SINR after the precoding decoder. Under the assumption of a Gaussian distribution of the interference, the SINR is given from (6) by

\[
\text{SINR} = \frac{\sigma_0^2 |g_0|^2}{\sigma_{IN}^2} \quad (7)
\]

where \( \sigma_{IN}^2 \) is the variance of the interference plus noise denoted by

\[
\sigma_{IN}^2 = \mathbf{E}\{\hat{s}_l - g_0 s_l|^2\}. \quad (8)
\]

Following the demonstration of [10], the maximum of the SINR is obtained for the following value of the equalization coefficients

\[
p_k = \frac{\lambda}{(\sigma_s^2 - \sigma_{IN}^2)} h_k^* \quad (9)
\]
where $\lambda$ is a positive real number.

Minimization of the mean-square error criterion is used to fix an optimal value for $\lambda$. The minimum of the mean-square error (MSE)

$$\text{MSE} = \mathbb{E}\{|\hat{s}_i - s_i|^2\}$$  \hspace{1cm}(10)

is obtained for

$$\lambda = \frac{\sigma_s^2}{1 + \beta\sigma_s^2}$$  \hspace{1cm}(11)

with

$$\beta = \frac{1}{M} \sum_{k=0}^{M-1} \frac{|h_k|^2}{(\sigma_s^2 - \sigma_s^2)|h_k|^2 + \sigma_s^2}. \hspace{1cm}(12)$$

Equation (9) provides an easy way to evaluate the equalization coefficients, but it requires the knowledge of $\sigma_s^2$. The variance $\sigma_s^2$ is estimated over a decoded block after the soft mapper for each iteration using the approximation [10]

$$\sigma_s^2 = \mathbb{E}\{|\hat{s}_i|^2\} \approx \frac{1}{N} \sum_{i=0}^{N-1} |\hat{s}_i|^2 \hspace{1cm}(13)$$

where $N$ is the block size.

Thus, for each decoding iteration, $\sigma_s^2$ is evaluated and the equalization matrix $P$ is updated.

At the first iteration, we have $\sigma_s^2 = 0$ and $\lambda = \sigma_s^2$ which results in classical MMSE equalization without interference cancellation. Now, if we consider the case of perfect channel decoding (error-free feedback), we obtain $\sigma_s^2 = \sigma_s^2$. The equalization coefficients become

$$p_k = \frac{\lambda}{\sigma_s^2} h_k^* \hspace{1cm}(14)$$

and the residual interference is equal to zero. The new expression for the estimated symbol is then

$$\hat{s}_i = s_i + \sum_{k=0}^{M-1} p_k h_k + \sum_{k=0}^{M-1} d_{k,l} p_k n_k. \hspace{1cm}(15)$$

In the case of a flat-fading Rayleigh channel, a useful comparison between the proposed system and transmission with spatial diversity can be made. Indeed, the analytical expression of (15) with equalization coefficients given by (14) is identical to the output of the MRC receiver described in [3] with spatial diversity $d=M$ ($M$ receive antennas). This shows that the performance bound of our system is equal to a transmission scheme with the same channel coding and $M$ receive antennas exploiting spatial diversity.

### B. Soft Bit Mapping/Demapping

The precoding decoder deals with complex symbols and the input/output of the channel decoder are LLRs. Thus, a soft demapper converts the estimated symbols $\hat{s}_k$ into LLR on encoded bits $b^*_k$.

Using (6) and the “max-log” approximation $\ln(e^x + e^y) \approx \max(x, y)$, we deduce the value of the LLR $L(b^*_k)$. If $\chi_0$ is the subset of $\chi$ for which $b^*_k = b$ and according to the Gaussian assumption at the equalizer output, we have

$$L(b^*_k) = \frac{1}{\sigma_{IN}} \ln \left( \min_{s \in \chi_0} \left( |\hat{s}_k - g_0 s|^2 \right) - \min_{s \in \chi_0} \left( |\hat{s}_k - g_0 s|^2 \right) \right), \hspace{1cm}(16)$$

with

$$\sigma_{IN}^2 = g_0 (1 - g_0) \sigma_s^2 \quad \text{and} \quad g_0 = \lambda \beta. \hspace{1cm}(17)$$

The soft mapper provides the inverse transformation at the output of the channel decoder. If we denote $(b^0, \ldots, b^{N-1})$ the bits that constitute the $2^N$-ary symbol $s \in \chi$, an estimation of the complex symbols is given by

$$\hat{s}_k = \sum_{s \in \chi} s \prod_{i=0}^{n-1} \Pr\{b^i_k = b^i\} \hspace{1cm}(18)$$

where the probability $\Pr\{b^i_k = b^i\}$ is deduced from the LLR $L_{dec}(b^*_k)$ calculated by the channel decoder

$$\Pr\{b^i_k = 1\} = \frac{\exp(L_{dec}(b^*_k))}{1 + \exp(L_{dec}(b^*_k))}, \hspace{1cm}(19)$$

$$\Pr\{b^i_k = 0\} = 1 - \Pr\{b^i_k = 1\}. \hspace{1cm}(20)$$

### C. Complexity

The overall complexity of the decoding scheme is essentially dominated by the channel decoding and remains low even with large size precoders.

Indeed, the treatment of a precoded vector involves only a few matrix multiplications. Moreover, the multiplications by $P$ and $H$ require only $O(M)$ operations as they are diagonals. Using the FHT, the multiplications by $D_M$ and $D_M^H$ have a complexity of $O(M \log_2(M))$ additions when a Hadamard matrix is used and $O(M^2)$ multiplications with a complex matrix. This does not depend on the constellation size, as the precoding decoder operates only on complex symbols. The modulation order influences the soft bit mapping/demapping, but with a low impact on the overall complexity. In addition, $P$ must be updated for each decoding iteration, which requires $O(M)$ operations per precoded vector.

This can be compared to an ML based scheme as proposed in [8]. The complexity of an exhaustive search is $O((2^N)^M)$. In this case, the parameters of the system are limited to low order modulations and small size precoders, typically a BPSK with $M=4$. The iterative process in both cases implies an increase in decoding complexity relative to the system without precoding.

### IV. Simulation Results

This section presents the performance of our system for different coding rates ($1/2$, $2/3$ and $3/4$) and modulations (BPSK, QPSK, 16-QAM). The parameters of the system have been chosen to be close to the 802.11a wireless LAN standard [11], except for the channel encoder which involves a circular recursive systematic convolutional (CRSC) code with constraint length $K=7$. The code is punctured for the rates $2/3$ and $3/4$ and performs close to the convolutional code used in 802.11a. The channel decoder uses the soft-input soft-output max-log-MAP algorithm.
A. Rayleigh Fading Channel

Fig. 3-5 present the performance results over a flat fading Rayleigh channel which represents an OFDM transmission with perfect time/frequency interleaving.

The bit error rate (BER) is presented as a function of signal to noise ratio Eb/N0.

As a reference, we simulate the BICM scheme over the Rayleigh (no precoding curve) and the Gaussian (AWGN curve) channels. The performance of the coded MRC receiver for eight antennas is also represented (MRC curve, diversity gain d=8) on Fig. 3. The genie-aided curve corresponds to the performance of our system with error-free feedback and the equalization coefficients given in (14). Finally, the label iter 1 to 5 indicates the performance of our receiver with improved equalization for one to five decoding iterations and a precoder size M=8.

We see that the genie-aided system and the MRC receiver with spatial diversity have exactly the same performance, as predicted in III-A. With our system, we observe a convergence towards the genie-aided curve after three or four iterations when the signal to noise ratio is sufficiently high. This can be observed for different coding rates, modulation orders and precoding sizes. This allows us to conclude that, unlike the system of [5], the diversity gain d=M is achieved with our proposed system.

Fig. 5 shows BER performance for a precoding size M=4 in order to compare both types of matrices defined in (2) and (3). If we observe close performance for the coding rate 1/2, the convergence is faster with complex than with Hadamard matrices for the highest rate. However, a better convergence is observed with a Hadamard matrix of size M≥8.

B. Application to WLAN

Fig. 6 and Fig. 7 present the performance results with the OFDM modulation of the 802.11a standard. 48 data symbols and 4 pilots are transmitted in each OFDM symbol. The total duration of one OFDM symbol is 4 µs, including a cyclic prefix of 0.8 µs.

The multipath channel model is the channel BRAN B (τrms=100 ns) described in [12]. Each of the 16 taps suffers independent Rayleigh fading with a mean corresponding to an exponential decaying average power delay profile. In order to explore both frequency and time diversity, the Rayleigh fading follows the Jakes model for the speed of 3 m/s and 60 m/s and a carrier frequency of 5.2 GHz (Doppler frequency 52 Hz and 1040 Hz).

The channel state information is assumed to be perfectly known, and the results are given in the form of frame error rate (FER) with a frame size of 1152 data bytes.

For all the cases tested, the BICM scheme is compared with the system using a precoder based on a Hadamard matrix of size M=8. Interleaver Π1 and Π2 operates on the whole frame, which allows linear precoding to exploit both time
and frequency diversity. For the simulations, $\Pi_1$ and $\Pi_2$ are randomly chosen, without other optimizations.

FER performance was measured for at least 1000 channel realizations (one realization per frame) and 20 erroneous frames. Gains brought by the new precoding decoder compared to the system without precoding are given in Table I for a FER of $2 \times 10^{-2}$ on both simulated channels. We observe a gain comprised between 2.5 dB and over 4.3 dB for the modes tested (BPSK with 1/2 rate, QPSK with 2/3 rate and 16-QAM with 3/4 rate). The global performance is better for the fast varying channel (60 m/s), where there is more potential diversity.

### Table I

<table>
<thead>
<tr>
<th>Modulation, coding rate</th>
<th>Frame duration</th>
<th>Gain over channel B, 3 m/s</th>
<th>Gain over channel B, 60 m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>BPSK, 1/2</td>
<td>1536 $\mu$s</td>
<td>3.2 dB</td>
<td>2.5 dB</td>
</tr>
<tr>
<td>QPSK, 2/3</td>
<td>576 $\mu$s</td>
<td>4.3 dB</td>
<td>4.0 dB</td>
</tr>
<tr>
<td>16-QAM, 3/4</td>
<td>256 $\mu$s</td>
<td>4.2 dB</td>
<td>4.2 dB</td>
</tr>
</tbody>
</table>

### V. Conclusion

We have presented an improved equalization scheme for linear precoding with iterative decoding. The equalization coefficients are optimized to maximize the SINR after the precoding decoder.

We have confirmed the good performance achievement of turbo-equalization scheme with a BPSK modulation as mentioned in [7]. Particularly, we have observed a convergence toward the performance bound after three or four iterations. Furthermore, we have shown that similar performance is also achieved with a 16-QAM modulation and Hadamard matrices.

The comparison with a multi-antenna MRC receiver shows that the diversity offered by the system is fully exploited. Moreover, we have observed the ability to exploit time and frequency diversity in the context of the wireless LAN 802.11a with varying channels. Finally, these results show and confirm that the turbo-equalization scheme for OFDM-CDM and LP-OFDM is a very promising technique for wireless channels. Further work should include turbo-coding and a study about the influence of the channel estimation.

### References


