

Nonsingular Black Hole Evaporation and “Stable” Remnants

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We examine the evaporation of two-dimensional black holes, the classical space-times of which are extended geometries, like for example the two-dimensional section of the extremal Reissner–Nordstrom black hole. We find that the evaporation in two particular models proceeds to a stable end-point. This should represent the generic behavior of a certain class of two-dimensional dilaton-gravity models. There are two distinct regimes depending on whether the back-reaction is weak or strong in a certain sense. When the back-reaction is weak, evaporation proceeds via an adiabatic evolution, whereas for strong back-reaction, the decay proceeds in a somewhat surprising manner. Although information loss is inevitable in these models at the semi-classical level, it is rather benign, in that the information is stored in another asymptotic region.

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1. Introduction

One of the authors (M.O'L.) and T.Banks [1] had previously proposed that for a large class of modified scalar-gravity theories in which the classical geometries are all nonsingular, with causal structure identical to that of Reissner–Nordstrom, the Hawking evaporation to a final zero temperature remnant–like object could be studied without singularity, as opposed to the original CGHS (Callan-Giddings-Harvey-Strominger) [2] models in which a now well known singularity was found.

Here we report on calculations that support this picture. When in–falling matter perturbs one of these extremal solutions, two apparent horizons form. As the evaporation takes place, these apparent horizons approach each other. We find two distinct regimes, depending on whether the back–reaction is weak or strong in a certain sense. With weak back–reaction, an adiabatic approximation gives a correct description, and things settle back down to a stable remnant, with the apparent horizons meeting only after an *infinite* proper time. In the strong back–reaction regime, the apparent horizons meet after a *finite* proper time, and only after meeting do things settle back down to the extremal solution. Black holes in these models therefore evaporate in a completely nonsingular fashion, realizing the original objectives of CGHS. Information loss occurs at the semi–classical level, but only in a rather benign way.

In section 2 we introduce the models of interest. We discuss in some detail the behavior near the double horizon of the extremal static semi–classical space–time in section 3. In section 4 we describe the adiabatic approximation [3] for the nonsingular models. Section 5 contains the results of our numerical analysis and section 6 is devoted to our conclusions and a discussion of their implications.

2. The Models

Consider a Lagrangian taken from the general class of two–dimensional renormalizable generally covariant field theories [4],

$$\mathcal{L}_{cl} = \sqrt{-g}(D(\phi)R + G(\phi)(\nabla\phi)^2 + H(\phi)) . \quad (2.1)$$

We require that the potentials behave asymptotically like those of linear dilaton gravity [2],

$$D(\phi) \rightarrow \frac{G(\phi)}{4} \rightarrow \frac{H(\phi)}{4} \rightarrow e^{-2\phi} , \quad (2.2)$$

as $\phi \rightarrow -\infty$.

The renormalization group equations are hyperbolic on the two–dimensional target space of this model, and thus given a set of initial data one can consistently renormalize the model [5]. In the following, without loss of generality, we will restrict attention to the class of models satisfying $G(\phi) = -2D'(\phi)$. Other models may be obtained by a field redefinition of ϕ . Performing a Brans–Dicke transformation on the metric $\hat{g} = e^{-2\phi}g$ this Lagrangian may be rewritten in the simple form

$$\hat{\mathcal{L}}_{cl} = \sqrt{-\hat{g}}(D(\phi)\hat{R} + W(\phi)) \quad (2.3)$$

where we have defined $W(\phi) = e^{2\phi}H(\phi)$. This form of the Lagrangian is convenient for finding the classical solutions as described in [1], in which reference the extended space–time geometries are discussed.

All solutions are causally related to the two–dimensional $r - t$ section of the four–dimensional Reissner–Nordstrom (RN) black hole (see fig. 1 for the extremal geometry), but as noted above, the geometries in these generalized models may be nonsingular. The nonsingularity is achieved by requiring that as $\phi \rightarrow \infty$, $D \sim e^{n\phi}$ and $W \sim e^{m\phi}$, such that

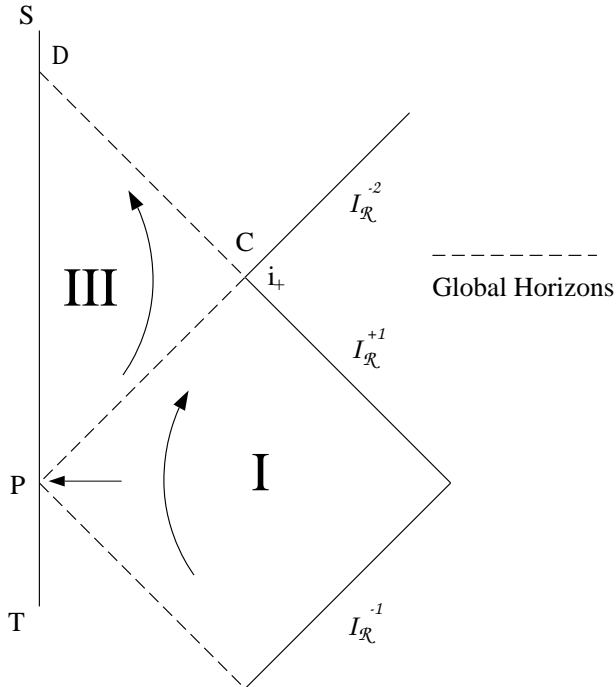


Fig. 1: Causal structure of the extremal zero temperature space–time geometries. The arrows show the flow of time. CD is the Cauchy horizon for regions I and III. The global horizons are at fixed r and the geometry near $r = r_h$ is the same in both the DW and RN cases (in particular the distance to P in the direction of the arrow is infinite). The line ST is singular and at finite spacelike distance for RN, nonsingular at infinite spacelike distance for DW.

$n \geq m - 2$. The multiple horizon structure is obtained by requiring that W have a zero. Henceforth this class of models will be referred to as DW models.

Just as in four–dimensions one believes that if charge cannot be radiated away then the zero temperature extremal RN space–time with $M^2 = Q^2$, where M is the mass and Q is the charge, will be an end–point for Hawking evaporation, it was conjectured that the zero temperature extremal limit of the DW models would also be a natural end–point for Hawking evaporation.

To compute the back–reaction we look at the trace anomaly produced by the quantum fluctuations of some conformally coupled (hence massless) matter fields. This is just the quantity calculated by Polyakov. We follow CGHS and study the back–reaction corrected

equations in the semi-classical limit, obtained by adding to the equations the terms corresponding to the trace anomaly of a large number N of the matter fields f_i and then taking N to infinity with $N\hbar$ fixed. Explicitly, the semi-classical action is

$$S = \int d^2x \mathcal{L}_{cl} - 2\kappa\partial_+\rho\partial_-\rho + \sum_{i=1}^N \partial_+f_i\partial_-f_i, \quad (2.4)$$

in conformal gauge ($g_{\pm\pm} = 0$, $g_{\pm\mp} = -\frac{1}{2}e^{2\rho}$, $x^\pm = x^0 \pm x^1$). Here we have defined $\kappa = N\hbar/12$.

By our choice of potentials, together with the condition $D' + 2\kappa < 0$ for all ϕ , we have avoided the singular kinetic term in field space which has been shown to be responsible for the singularity of the CGHS model.

Let us note in passing that the general solution to the CGHS equations that is static, with mass above that of the vacuum, actually has a weak coupling singularity on the outer horizon [6,7] (the quantum kink). This singular behavior arises through the interaction between the assumed dilaton-gravity asymptotics of the potentials (2.2), and the large N corrections. That is, this singularity is a weak coupling singularity that arises due to the assumed linear dilaton asymptotics of the Lagrangian, and so also will be present in our models.

Note that we may alternatively require the potentials to behave asymptotically as the spherically symmetric reduction of the four-dimensional Einstein equations,

$$D(\phi) \rightarrow \frac{G(\phi)}{2} \rightarrow e^{-2\phi}, \quad \text{and} \quad H(\phi) \rightarrow 2. \quad (2.5)$$

Here the two-dimensional metric is related to the four-dimensional metric via

$${}^{(4)}ds^2 = ds^2 + e^{-2\phi}d\Omega^2. \quad (2.6)$$

An example from this class of models, corresponding to the $r - t$ section of the four-dimensional RN black hole will also be considered in the following, with the same one-loop

back-reaction term included as above. This model has recently been considered in [3,8]. Unlike the DW models described above, this model is singular at the classical level and has a large N singularity at $e^{-2\phi} = \kappa/2$.

3. Static Solutions of the Generalized Models

The static solutions to the quantum equations of the DW models fall into three categories, those that qualitatively have the same structure as the quantum kink of early studies of the CGHS Lagrangian, an extremal state that approaches the classical extremal state (as $\kappa \rightarrow 0$), but with a singularity in the second derivative of the curvature on the horizon, and negative mass solutions that have the same structure as the classical negative mass solutions, in particular they are nonsingular. In the following we will refer to the extremal state as the vacuum of the two-dimensional theory, and measure mass relative to this vacuum state.

Thus at this level we have found that quantum corrections have made the classical phase space more singular. However, the vacuum state is nonsingular and given that the positive mass solutions will radiate, it is plausible that none of the positive mass singularities will appear in the dynamical collapse and evaporation.

In the next section we will introduce the adiabatic approximation used by Strominger and Trivedi to study the formally similar problem for a dimensional reduction of Reissner-Nordstrom geometry. The adiabatic approximation can be most easily investigated in light-cone gauge, so we now record the equations in that gauge.

We have the line element,

$$ds^2 = -h dv^2 + 2dr dv . \tag{3.1}$$

The scalar curvature is given by $R = -\partial_r^2 h$. To find the needed parts of the stress tensor,

we integrated the Bianchi identities $\nabla^\alpha T_{\alpha\beta} = 0$, using the equation for the trace anomaly [9],

$$T_\alpha^{\alpha m} = 2T_{rv}^m + hT_{rr}^m = \frac{\kappa}{4}R. \quad (3.2)$$

The components of interest to us are the trace, and $l^\alpha l^\beta T_{\alpha\beta}$, where $l = (h/2, 1)$.

The ϕ and trace equations may be written as,

$$\nabla^2 \phi = \frac{(e^{-2\phi}(D'W + \kappa(W - \frac{1}{2}W')) - D''(D' + \kappa)(\nabla\phi)^2)}{D'(D' + 2\kappa)}, \quad (3.3)$$

$$R = -\partial_r^2 h = \frac{(-e^{-2\phi}(2W + W') + 2D''(\nabla\phi)^2)}{(D' + 2\kappa)}, \quad (3.4)$$

where $\nabla^2 \phi = \partial_r(2\partial_v \phi + h\partial_r \phi)$, and $(\nabla\phi)^2 = 2\partial_r \phi \partial_v \phi + h(\partial_r \phi)^2$. The remaining local linear combination of the constraint equations is (conveniently the linear combination needed for matching across the shock wave),

$$\begin{aligned} l^\alpha l^\beta T_{\alpha\beta} &= 2D'(\partial_v \phi + \frac{1}{2}h\partial_r \phi)^2 + (\frac{1}{2}h\partial_r + \partial_v - \frac{1}{2}(\partial_r h))(\frac{1}{2}\partial_r + \partial_v)D \\ &+ \frac{\kappa}{4}(\frac{h}{2}\partial_r^2 h + \partial_v \partial_r h - \frac{1}{4}(\partial_r h)^2) + t_r(r) \end{aligned} \quad (3.5)$$

where t_r is determined by the initial state of the quantum vacuum, and for our collapse, and for static solutions with no net flux at infinity, we set t_r to zero in coordinates that are asymptotic to the linear dilaton vacuum.

For static solutions we put $h = h(r)$, $\phi = \phi(r)$. With linear dilaton asymptotics at $\phi \rightarrow -\infty$, $W \rightarrow 4$, $D \rightarrow e^{-2\phi}$, we find to lowest order³,

$$\begin{aligned} h(r) &= 1 - 2Me^{-2r} \\ \phi(r) &= -r. \end{aligned} \quad (3.6)$$

One can numerically integrate the static equations using the above as boundary conditions at $r \rightarrow \infty$.

³ Certain choices of the potentials D and W will give an additional term in the asymptotics of h proportional to $r \exp(-2r)$.

Now let us consider the behavior of fields near the horizon, $r = r_h$, of the extremal solution. We know from the classical analysis in [1], that the classical extremal solution has to lowest order $\phi = \phi_0$ and $h = h_0 x^2$ near the horizon, where $x = r - r_h$. Let us look at the quantum corrections to these formulae,

$$\begin{aligned}\phi &= \phi_0 + \beta x^{1+\delta} \\ h &= \alpha_1 x^2 + \alpha_2 x^{3+\eta}.\end{aligned}\tag{3.7}$$

Plugging this into the static equations we find (evaluating all functions at $\phi = \phi_0$),

$$\begin{aligned}D'W &= \kappa\left(\frac{W'}{2} - W\right) \\ \alpha_1 &= \frac{e^{-2\phi_0}(W + \frac{1}{2}W')}{D' + 2\kappa} \\ &= \frac{e^{-2\phi_0}W}{\kappa} \\ \eta &= \delta \\ (\delta + 1)(\delta + 2) &= \frac{D''W + D'W - \kappa(\frac{1}{2}W'' - W')}{D'(W + \frac{1}{2}W')}.\end{aligned}\tag{3.8}$$

β and α_2 are related by

$$\beta = \frac{-\kappa\alpha_2(\delta + 2)}{2\alpha_1\delta D'}.\tag{3.9}$$

Notice that the solutions have two obvious extensions through $\phi = \phi_0$ (see [3] for a similar discussion). To study perturbations by shock waves of matter we will restrict to the odd extension which is the smoother of the two continuations and is the one that is a deformation of the classical geometry,

$$\begin{aligned}\phi &= \phi_0 + \beta x|x|^\delta \\ h &= \alpha_1 x^2 + \alpha_2 x|x|^{2+\delta}.\end{aligned}\tag{3.10}$$

This also means that immediately above the shock wave we will find two apparent horizons at $\phi_+ < \phi_0$ and at $\phi_- > \phi_0$, with a geometry qualitatively the same as the positive mass classical solutions.

We can evaluate the parameters in the expansion near the horizon of the extremal solution and observe that the quantum vacuum near the horizon is indeed a small κ deformation of the classical vacuum. Let us fix

$$D = e^{-2\phi} - \gamma^2 e^{2\phi}, \quad \text{and} \quad W = 4 - \mu^2 e^{4\phi}. \quad (3.11)$$

For small κ and large μ , we find,

$$e^{-2\phi_0} = \frac{\mu}{2} + \frac{\kappa}{2} \\ \alpha_1 = 4, \quad \delta = \frac{2\kappa}{\mu}, \quad \frac{\beta}{\alpha_2} = \frac{1}{8}. \quad (3.12)$$

For definiteness we put $\beta = -1/\mu$ in the following.

To check that the quantum behavior near $\phi = \phi_0$ that we have displayed is consistent with the linear dilaton at infinity we numerically integrated out from $\phi = \phi_0$ to $\phi = \pm\infty$, and indeed have observed the linear dilaton vacuum for $\phi \rightarrow -\infty$ and the large ϕ classical behavior in the other asymptotic regime.

4. The Adiabatic Approximation

In order to study the semi-classical stability of the static extremal solutions of the DW models, consider sending in a matter shock wave $l^\mu l^\nu T_{\mu\nu}^f = 2M\delta(v)$. We define

$$\Sigma = 2\partial_v \phi + h\partial_r \phi, \quad (4.1)$$

so that the future apparent horizon is the locus of $\Sigma = 0$. The discontinuity in Σ across the shock is

$$\delta\Sigma = \frac{4M}{\sqrt{D'(D' + 2\kappa)}}. \quad (4.2)$$

To obtain explicit expressions we use the potentials of equation (3.11) and make a large μ , small κ , and small M expansion. Specifically, we have $M/\mu \ll \kappa/\mu \ll 1$. We may

calculate the positions of the apparent horizons which turn out to be

$$r_{\pm} = r_h \pm \sqrt{\mu} \left(\frac{M}{\mu} \right)^{\frac{1}{2+\delta}} \quad (4.3)$$

which is obtained by setting $\Sigma + \delta\Sigma = 0$. This implies that

$$\partial_r \Sigma(r_{\pm}) = \mp \frac{8}{\sqrt{\mu}} \left(\frac{M}{\mu} \right)^{\frac{1+\delta}{2+\delta}}. \quad (4.4)$$

The discontinuity in h across the shock is given by

$$\delta h = \frac{8M}{\kappa} \int dr \left(-1 - \frac{D'}{\sqrt{D'(D' + 2\kappa)}} \right). \quad (4.5)$$

By inserting this expression into the constraint equation we find $\partial_v \Sigma$ at the apparent horizons,

$$\partial_v \Sigma = -\frac{8M\kappa}{\mu^2}. \quad (4.6)$$

One may then try to make an adiabatic approximation to compute the relative positions of the horizons as a function of v , by assuming the equations (4.4) and (4.6) continue to hold for all v if things are changing slowly enough:

$$\partial_v \hat{r}^{\pm} = -\frac{\partial_v \Sigma}{\partial_r \Sigma} = \mp \frac{\kappa}{\mu} (\hat{r}^{\pm} - r_h) \quad (4.7)$$

which gives

$$\hat{r}^{\pm}(v) = r_h + r_0^{\pm} e^{-\frac{\kappa}{\mu}(v-v_0)} \quad (4.8)$$

or expressing things in terms of an effective mass, which measures the difference between the actual mass and the extremal mass

$$M(v) = M_0 e^{-\frac{2\kappa}{\mu}(v-v_0)}. \quad (4.9)$$

The adiabatic approximation will break down when the energy flux due to Hawking radiation is comparable to $M(v)$, so will be valid as long as $\kappa/\mu \ll 1$.

These expressions are almost identical to ones obtained in [3] in the case of the spherically symmetric reduction of the Reissner–Nordstrom black hole, apart from slight changes in numerical coefficients. The RN case is obtained by picking different potentials D and W . In this case, the adiabatic approximation is valid as long as $\kappa/Q^2 \ll 1$, where Q is the charge of the extremal black hole.

5. The Numerical Collapse

When an extremal solution is perturbed by an incoming flux of matter the picture one expects is the following: above the shock wave two apparent horizons form on either side of the line ϕ_0 , and as the black hole evaporates these horizons approach each other, with the solution settling back down to the extremal solution at \mathcal{I}_R^+ . In this section we describe numerical solutions for a shock wave of in-falling matter impinging upon the extremal solution of the DW model described in section 3. We also consider the analogous calculation for the RN type model. Our aim is to test whether the extremal black hole is stable, and in what manner the evaporation proceeds.

Another approach would be to do a linear stability analysis of the problem, where one might hope to be able to understand things analytically. In fact, it seems this is not so for the back-reaction corrected equations. A numerical analysis of this problem is tractable, but is of comparable difficulty to numerically solving the full nonlinear equations, so we choose to do the latter.

In the context of the CGHS model, numerical results have previously been obtained in [10,11,12]. Here we are solving equations of precisely the same form, but with somewhat different potentials. See [11] for a description of the numerical algorithm used in this paper, and also appendices A and B, where the gauge and coordinate choices, and equations of

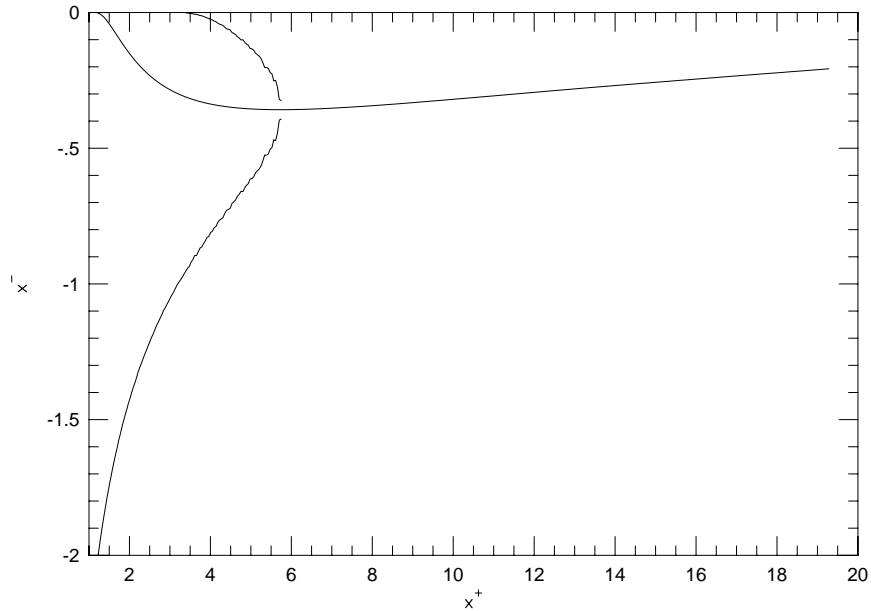


Fig. 2: Plot of motion of apparent horizons for DW model, in the strong back-reaction regime. The lower line is the outer apparent horizon, the middle line is $\phi = \phi_0$, and the upper line is the inner apparent horizon.

motion are stated. For numerical purposes it is convenient to work on a grid of null lines, so conformal gauge is appropriate. This should be borne in mind when making quantitative comparisons with the results of the previous section, where it was necessary to use light-cone gauge to get analytical results.

For the DW model of equation (3.11), with $\kappa = 10$, $\mu = 15$, $\gamma = 8$, and shock mass $M_s = 1.5$, the results are plotted in fig. 2. Note that here $\kappa/\mu > 1$ so one is in the strong back-reaction regime and the adiabatic approximation of the previous section is not expected to hold. The integration is stopped near $x^- = 0$ where the line $\phi = \phi_0$ starts out. This means these calculations will hold for either of the extensions described in the previous section. The horizons meet at finite proper time, and the line ϕ_0 goes from being spacelike to timelike at this point. Note the equations of motion imply that $\partial_+ \partial_- \phi = 0$ when an apparent horizon (where $\partial_+ \phi = 0$) intersects $\phi = \phi_0$. The position of the

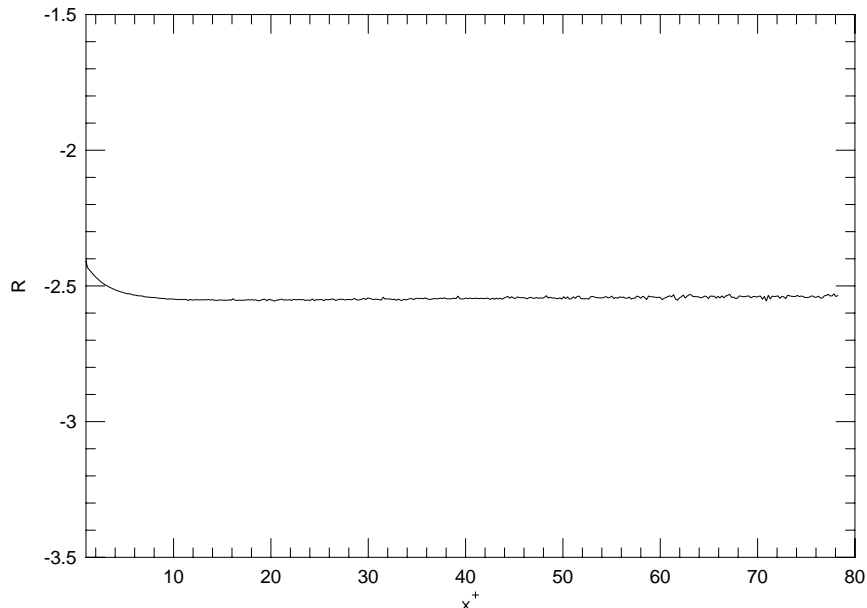


Fig. 3: Plot of the curvature along a line of constant $\phi = -1.2$, for the DW model, in the strong back-reaction regime. Here the range of x^+ extends much further out. The curvature approaches a constant value which is the same as the initial curvature of the extremal solution.

apparent horizons $\hat{x}^-(x^+)$ may be found by solving

$$\frac{\partial \hat{x}^-}{\partial x^+} = -\frac{\partial_+^2 \phi}{\partial_- \partial_+ \phi} \quad (5.1)$$

which indicates that $\partial \hat{x}^- / \partial x^+$ blows up as $\partial_+ \partial_- \phi \rightarrow 0$, as long as $\partial_+^2 \phi$ remains finite. This is precisely what is happening at the meeting point of the apparent horizons, consistent with the numerical results. Also note that the wiggles in the path of the outer horizon are due to $\partial_- \partial_+ \phi$ becoming very small in this region. This causes the error in the path of the line $\partial_+ \phi = 0$ to be much larger than, say, the error in the path of a line of constant ϕ . The conclusive indication that the horizons meet comes from the observation that the line $\phi = \phi_0$ becomes timelike. Numerical convergence has been checked by varying the stepsize, and varying the position of the initial surface.

Following a line of constant ϕ as x^+ becomes large, one finds the curvature approaches

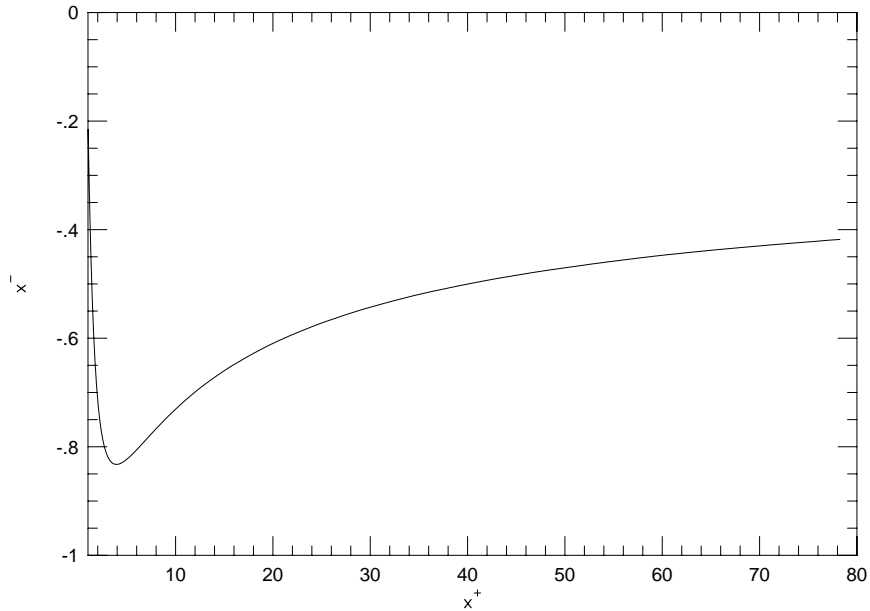


Fig. 4: Plot of the path of a line of constant $\phi = -1.2$, for the DW model, in the strong back-reaction regime. As x^+ becomes large the line appears to asymptotically approach a null line.

a constant which equals the initial curvature of the extremal solution, as shown in fig. 3 . This indicates that despite the fact that the apparent horizons have met, the solution still settles back down to the zero temperature extremal state.

Lines of constant ϕ appear to approach a null line $x^- = x_0^-$, which becomes a global horizon, as shown in fig. 4 . It is difficult to tell from the numerics whether $x_0^- = 0$, or whether it is shifted out to more negative x^- by the in-falling matter. More sophisticated numerical calculations are needed to answer this question definitively.

Simulations were also performed for the weak back-reaction regime where $\kappa/\mu \ll 1$, when the adiabatic approximation is expected to hold. Here potential numerical errors were somewhat larger, but the results were found to be consistent with the adiabatic approximation. The apparent horizons persisted for very large x^+ , approaching the critical line $\phi = \phi_0$.

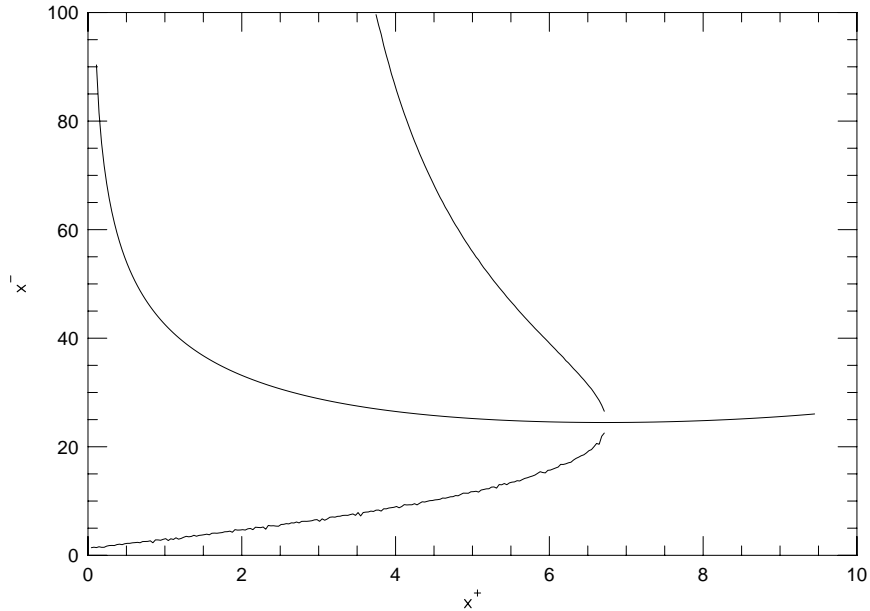


Fig. 5: Plot of motion of apparent horizons for RN model, in the strong back-reaction regime. The lower line is the outer apparent horizon, the middle line is $\phi = \phi_0$, and the upper line is the inner apparent horizon.

Qualitatively similar results are obtained for the case of the RN model. In the strong back-reaction regime where $\kappa/Q^2 > 1$, with $\kappa = 200$, $Q^2 = 60$, and shock mass $M_s = .08$, results are shown in fig. 5 . One difference here is that coordinates are chosen so the line $\phi = \phi_0$ starts out at $x^- = +\infty$. Again the solution appears to settle back down to the extremal solution as x^+ becomes large, after the apparent horizons have met. In the weak back-reaction regime ($\kappa/Q^2 \ll 1$), results consistent with the adiabatic approximation were found, with the apparent horizons approaching the the critical line out to large values of x^+ .

6. Discussion and Conclusions

The picture of the end-point of the evaporation of these two-dimensional extremal black holes which is consistent with the numerical calculations is shown in fig. 6 . In

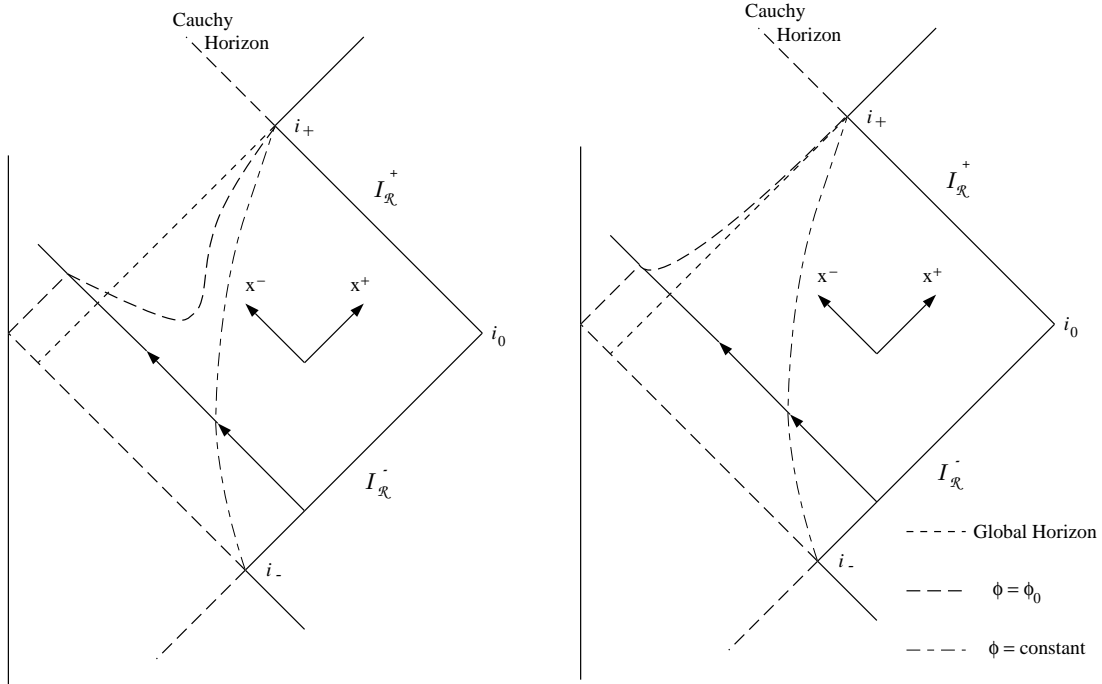


Fig. 6: Penrose diagrams showing the end-points of the black hole evaporation. The case of strong back-reaction is shown on the left, the case of weak back-reaction is shown on the right.

the strong back-reaction regime, lines of constant ϕ approach a global horizon, and the solution settles back down to the extremal one. After the apparent horizons meet the line $\phi = \phi_0$ turns timelike and becomes asymptotically null as it approaches the global horizon. In the weak back-reaction regime the apparent horizons meet at \mathcal{I}_R^+ , and the critical line $\phi = \phi_0$ remains spacelike.

Our results indicate the presence of two qualitatively different regimes separated by some critical value of κ/μ in the DW case, and κ/Q^2 in the RN case. This should not be too surprising, since a similar phenomena occurs in the case of the damped harmonic oscillator, where there is a critical value of the damping separating two regimes of behavior. Although we have studied two particular models, we believe that this behavior will be generic to the class of models in which $W(\phi)$ has a simple zero, which includes the nonsingular models discussed in section 2.

Because for large κ/μ the apparent horizons collide, locally, the end-point of the evaporation of these extremal solutions looks like an $M^2 < Q^2$ static solution in the case of the RN model. One may wonder if some two-dimensional quantum positive mass theorem prevents such an unexpected occurrence. However, positive mass theorems only give us information about the asymptotic structure of the space-time and certainly do not preclude the type of relaxation that we observe here. Classically the DW and RN models obey positive mass theorems [13] with the mass defined on space-like surfaces that become asymptotically null along \mathcal{I}_R^+ and have their left boundary at the meeting of the two global horizons of the extremal solution below the shock wave. Unfortunately the quantum positive mass theorems [14] do not appear to give any useful information in their present formulation due to the quantum corrections to the vacuum. Further, quantum positive mass theorems require a nonsingular spacelike surface that is asymptotically linear dilaton vacuum at both ends, see [14,15,16]. In our situation this of course can never be the case, since the left boundary of our spacelike hypersurfaces meets the horizon of the extremal solution which looks distinctly not like the LDV.

These results appear to confirm the conjecture that zero temperature extremal (in the sense of RN) two-dimensional black holes are semi-classically stable. In the case of the DW models the evaporation of nonextremal black holes proceeds in a completely nonsingular way, realizing the original objectives of CGHS. Information loss is inevitable in these models at the semi-classical level⁴, since an in-falling flux of matter will always produce correlations with the region behind the global horizon, and hence inaccessible to an observer who finds themselves at i_+ after an infinite proper time. The information loss is of a somewhat benign type though. For observers outside the global horizon the

⁴ See [1,3,17] for related discussions.

quantum mechanics that they participate in is unitary, indeed the spacetime before the Cauchy horizon can (by definition) be foliated by space-like Cauchy hypersurfaces. The information that is no longer accessible to them is stored in a stable remnant, one of an infinitely degenerate set of possible final states. This infinity of states corresponds to all the possible field configurations on a space-like slice through region III (fig. 1). These remnants avoid the problems of overproduction in external fields and divergences in virtual loops by nature of their large internal geometry [17] .

Let us note that the above discussion is based on semi-classical reasoning. It is still possible that when space-time is properly second quantized the information loss problem will be cured. Highly nonlocal quantum gravity effects may wind up giving the remnants a very long, but finite lifetime. If this lifetime is of order the age of the universe, experimenters making measurements over shorter times will still see an effective loss of information, which they would attribute to the existence of “stable” remnants.

The conjectured extremal state possesses a Cauchy horizon as shown in fig. 1. We would like to comment on the sense in which this horizon is traversable⁵ and the possibility of information loss for observers who traverse it. We should first note that the Cauchy horizon in our model appears to be a double horizon, and as discussed in [8] has a softened divergence of the stress tensor as compared to the inner horizon of a nonextremal space-time. It is also possible that the disturbance produced by the shock wave will separate the local horizon and the global Cauchy horizon in a manner similar to the mass inflation models of Poisson and Israel [19]. In mass inflation it was shown that an infinite tidal force appears along the Cauchy horizon, but the singularity is weak [20] in the sense that an observer can traverse it without getting stretched infinitely.

⁵ See [18] and references therein for earlier discussions.

Let us assume then that a traversable Cauchy horizon is present. An observer who passes into the region above this Cauchy horizon faces a potential loss of unitarity associated with the lack of a global foliation of the space–time by space–like Cauchy hypersurfaces. The existence of a Cauchy horizon, its nature if it exists and the necessity of a unitarity restoring mechanism for those who cross it, are the subject of our ongoing investigations. The answers to these questions should shed light on an S–matrix description of quantum gravity in the presence of remnant–like objects.

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Appendix A. Numerical method for DW model

The numerics are performed in conformal gauge

$$g_{++} = g_{--} = 0, \quad g_{+-} = g_{-+} = -\frac{1}{2}e^{2\rho} . \quad (\text{A.1})$$

The linear dilaton vacuum in the coordinates we choose is

$$\phi = \rho = -\frac{1}{2} \log(-x^+ x^-) \quad (\text{A.2})$$

while a solution with mass M looks like

$$\phi = \rho = -\frac{1}{2} \log(M - x^+ x^-) \quad (\text{A.3})$$

in the limit $x^+ x^- \rightarrow -\infty$. The equations to be solved are

$$\begin{aligned} \partial_+ \partial_- \rho &= \frac{-D'' \partial_+ \phi \partial_- \phi - \frac{1}{4} e^{2\rho-2\phi} (W + \frac{W'}{2})}{D' + 2\kappa} \\ \partial_+ \partial_- \phi &= -\frac{D'' \partial_+ \phi \partial_- \phi + \frac{1}{4} e^{2\rho-2\phi} W + \kappa \partial_+ \partial_- \rho}{D'} . \end{aligned} \quad (\text{A.4})$$

Here $D = e^{-2\phi} - \gamma^2 e^{2\phi}$ and $W = 4 - \mu^2 e^{4\phi}$. The boundary conditions are that along $x^+ = 1$ the solution correspond to the extremal solution discussed in section 3. Above the shock wave along \mathcal{I}^- the solution should agree with the classical shock solution. This amounts to setting

$$\partial_+ \phi = \partial_+ \rho \sim \frac{x^- + M}{2(M - x^+ x^- - Mx^+)} \quad (\text{A.5})$$

as $x^- \rightarrow -\infty$. In practice a large negative initial value of x^- is chosen.

Appendix B. Numerical method for the RN model

Here the numerics are also performed in conformal gauge (A.1) . Asymptotically as $x^+ - x^- \rightarrow \infty$ the solutions approach the vacuum

$$\phi = -\log(\frac{1}{2}(x^+ - x^-)), \quad \rho = 0 . \quad (\text{B.1})$$

The equations to be solved are

$$\begin{aligned} \partial_+ \partial_- \rho &= \frac{\partial_+ \phi \partial_- \phi + \frac{1}{4} e^{2\rho} (e^{2\phi} - 2Q^2 e^{4\phi})}{1 - \frac{\kappa}{2} e^{2\phi}} \\ \partial_+ \partial_- \phi &= \partial_+ \partial_- \rho + \partial_+ \phi \partial_- \phi + \frac{Q^2}{4} e^{2\rho+4\phi} . \end{aligned} \quad (\text{B.2})$$

Here the boundary conditions are that along $x^+ = 0$ the solution match onto the quantum corrected extremal solution [8], while above the shock wave along \mathcal{I}^- the solution agree with a classical shock solution. This means

$$\partial_+ \phi \sim -\frac{1}{x^+ - x^-} + \frac{4M}{(x^+ - x^-)^2} \left(1 - \log\left(\frac{1}{2}(x^+ - x^-)\right)\right), \quad \partial_+ \rho \sim \frac{2M}{(x^+ - x^-)^2} \quad (\text{B.3})$$

as $x^- \rightarrow -\infty$.

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