Annual Distribution Budget in the Beverage Industry: a case study
(Authors’ names blinded for peer review)

Nowadays Unicer can improve its tactical distribution planning decisions and study several alternative scenarios for its supply strategies and network configuration thanks to an operations research driven process. In this paper we present the decision support system responsible for this new methodology in the major Portuguese beverage company. At the core of this system there is a mathematical programming-based heuristic that has decision variables related to transportation and inventory management problems. The company runs a set of production and distribution platforms with different characteristics to fulfill customers demand. The main challenge of this work was to render a tactical distribution plan, also known in the company by annual distribution budget, as realistic as possible without jeopardizing the nature of the strategic/tactical tool. The company presents a very complex tactical distribution planning due to the increasing variety of stock-keeping-units and to the need of a very flexible distribution network to satisfy customers, who demand a very fragmented product basket. One of the main causes of this complexity is the existence of uncommon flows of finished products from the distribution centers to the production platforms. These movements yield an intricate supply chain that needs to be properly handled.

The quality of the solutions provided and the implementation of a user friendly interface and very readable and editable inputs/outputs for the decision support system gave the necessary motivation for its wide use by the company’s practitioners. The corollary of the utilization of this tool translates on a potential cost reduction of about 2M euro per year, on the quality of information made available to decision makers and on their engagement in looking to operations from a different perspective having a more operations research reasoning.

Key words: Tactical Distribution Planning; Mixed-Integer Programming; Decision Support System; Beverage Industry; Supply Chain Management

Introduction
Beverage and food industries have a huge impact on the European Union (EU) economy. In fact, EU is the largest producer of food and beverages, ranking first in terms of sales and exports (in value). In Portugal, the scenario is not different and the food industry generates a market value of over 10.6 billion of euros, which is approximately 7.3 percent of the total gross domestic product (Instituto Nacional de Estatística 2011). The beverage industry faces ever increasing competition and companies that want to strive have to excel
in terms of price, quality and customer service. To that end, distribution efficiency and efficacy has become a major point. Indeed, many authors claim that in order to build up a sustainable competitive advantage there is just about supply chain management to achieve it (e.g. Fearne and Hughes 1999).

Generally, beverage companies supply a variety of products that range from wine to beer or water. In fact, these companies pertain to the more general fast moving consumer goods industry and are affected by the same issues. Hence, the beverage market is becoming more and more demanding. This fact translates into an increasing variety of stock-keeping-units as well as on the need for a very flexible distribution network to fulfill customers’ demand, which relates to a very fragmented product basket. The combination of a large products portfolio, complex distribution networks and demanding customers give rise to very intricate supply chains that need to be properly handled. The problems that these companies face can be found on different levels. On a strategic level it can be important to know where to locate production or distribution platforms; on a tactical level, a company may be faced upon the decision of choosing which logistics providers to select and with which kind of contract, or in which days the clients should be visited; on a more operational level, there exists the daily problem of the design as well as the consolidation of routes to serve customers previously assigned to that day based on their demand orders.

This paper describes the work done closely with Unicer, the major Portuguese beverage company part of the Carlsberg group, with revenue of around 500 million euros. Unicer’s operations include the production, commercialization and distribution of beer, plain and sparkling water, soft drinks and wine. The project aim was to design, develop and implement an operations research (OR)-based approach to support managers to take their tactical decisions concerning distribution planning. However, the operational complexity is not completely put aside. Unicer has used and still runs the tool that emerged from this project not only to perform its annual budgeting of the distribution operation at a tactical level, but also to model and test alternative strategies for the supply chain. The operational level is then managed with the help of software packages specialized in vehicle routing and load planning.

Next we detail the problem addressed, the solution approach developed and discuss the impact of our work in the company. We also discuss important aspects of applying OR to practical problems. We finish by withdrawing some concluding remarks and future project developments.
The Challenge

Unicer holds some of the Portuguese most popular brands of beer, soft drinks, plain and sparkling water. Today the company sells more than 380 SKUs to over 19,000 different clients across the globe.

Tactical distribution planning is a vital step at Unicer’s planning tasks as transport costs correspond to a significant share of the total product cost. This process is under the responsibility of the logistics department director, the main stakeholder of the plans. The logistics director reports the achieved results directly to the company’s board, namely the Chief Operations Office - COO. Tactical distribution planning is important at two phases of the company’s planning process.

The first concerns the creation of the annual distribution budget (DB). DB is part of the company’s annual budgeting process. The budgeting is a vital tool to align company’s goals and translate the strategy defined into the next 12 months. This process starts in mid-September and lasts until late October. The first main task is the creation of an annual sales budget (SB). SB is responsibility of the sales department and defines a monthly sales forecast for each product for the following year. With this input the production planning department works on the annual production budget (PB) which defines the total production quantities of each product at each production platform for the entire planning horizon of the 12 months. The results of these two steps define the input required for the DB.

The second important phase of application of the tactical distribution planning involves the validation of strategy changes in the supply chain configuration, product portfolio, clients supply mode and negotiations with the outsourced companies responsible for the transports. Whenever these situations happen, the company studies their impact in terms of future distribution costs. The process is similar to the creation of the DB, however the planning horizon usually increases and data is often more aggregated.

The challenge present in this project is therefore twofold. First, we aim to create a tactical distribution plan for the next 12 months, the so-called DB. The plan details the flow of the finished products among the different locations of the supply chain, by knowing the production plan for a set of production facilities and the customers demand for the next year. Simultaneously, it defines the supply chain configuration by deciding which platforms are operating and their respective activity level. Second, we intent to build an approach capable of modeling various scenarios for the supply chain network to provide a flexible
tool. To better understand the problem at hand, in the following subsections we describe
the main entities and movements in the supply chain (Platforms, Clients and Transports)
associating the expected outputs in each area both at tactical and strategic level.

Platforms
The company has nine production platforms spread across the country and specialized in
producing different types of product families. The production facilities dedicated to beer
and soft drinks are strategically located close to the geographical center of demand, while,
mineral and sparkling water plants are restricted to be placed near a water source often
distant from the final consumer.

Distribution platforms are used to storage purposes, consolidate shipments and perform
picking operations. The company holds two major distribution platforms, one is located
near the Oporto region and the other in the region of Lisbon, the two main consumption
areas in Portugal. Other distribution platforms are available across the country, but are
much smaller.

Additionally, some of the production platforms also have available areas to store products
and supply clients’ orders. Therefore, they act as both production and distribution sites.
Such feature is uncommon and introduces an additional level of complexity to the supply
chain management.

Platforms have limited capacity for storage, pallet movement, picking operations and
loading shipping containers. Storage capacity is detailed into three different types of pallet
storage: (1) drive-in, (2) racks and (3) floor stacking. In drive-in and rack pallet storage,
the amount of slots available for pallets is strictly defined. Whereas, for the floor stacking
capacity one has to take into account the number of stacking levels that a given product
pallet allows for. The available capacities are determined by the activity level selected.
Figure 2 depicts possible cost curves for operations at different activity levels. Further-
more, some platforms may only operate during some months remaining idle for the rest
of the year. This corresponds to the filled dot depicted in every plot of Figure 2 and the
cost corresponds to the fixed cost without activity. Figures 2a and 2b may represent an
outsourced platform. In the first case, the same unitary cost is paid for any quantity moved
/ stored, whereas, in the second case, there are contracted levels of activity for which fixed
and different unitary costs have to be paid. The cost structure depicted in Figure 2c is
more likely to occur for platforms managed by Unicer.
Figure 1  Geographic location of Unicer’s main production and distribution platforms. The darker the area in the map the higher the population density of the region. In addition there are other smaller platforms not shown in this figure.

Figure 2  Examples of possible cost functions for platform activities. (a) initial fixed cost and linear variable costs; (b) piecewise fix cost activity costs; (c) initial fix cost and variable piecewise activity costs.

Both the definition of the operating platforms and the adjustment of platforms capacity are particularly important in the beverage industry due to the high seasonality of sales. The sales profile of these products presents peaks of demand at Easter, Christmas, and especially summer. On the other hand, production capacity remains almost constant
throughout the year. This fact forces the industry to work on a make-to-stock basis as the capacity in the peak of sales is insufficient to match the demand, thus stressing the supply chain. Traditionally, during the low season (December to March) the company makes use of the idle platforms to store the seasonal stock, and adjusts the activity level of the remaining platforms to increase their operational capacity during summer.

Concerning the platforms, the tactical distribution plans should define for each platform at each month:

- If it should be operating or not;
- If operating, what is the optimal activity level;
- The utilization of each type of storage;
- The total number of pallets handled;
- The total amount of picking operations;
- The total number of containers shipped;
- The activity costs. Depending on the cost function associated with the platform it may be proportional to the number of products stored, pallets handled, picking operations and containers shipped.

On a simulation perspective it is important that the approach can model new scenarios for the opening and closing of production and distribution platforms.

Customers

Customers can be categorized into four sales groups: “Capilar”, Retailers, Strategic and Exports. This distinction is important as the different customer groups present distinct relationships in the supply chain.

Customers belonging to the “Capilar” group are located in the regions of Oporto and Lisbon and the company supplies them with a door-to-door delivery system. They range from small to large restaurants, coffees shops, bakeries, bars and related establishments which serve food and beverages. Their orders are rather small, generally less than a pallet, but of a very diverse product basket requiring a complex picking operation.

Retailers are companies with special commercial contracts with Unicer that perform their own door-to-door delivery, especially in the regions outside Oporto and Lisbon. Orders of these clients are restricted in size to 33 pallets or 25.5 tons, which ensures the full use of a large truck. Furthermore, picking operations are not allowed to this type of customers.
Strategic clients consist of modern retail chains, wholesalers and chains of restaurants, hotels and other businesses dedicated to commercialize food and beverage. All have several stores spread all over the country. This sales group is the most heterogeneous, thus there is not a typical order, both in terms of quantities and product mix. Nevertheless, these clients are particularly important as stockouts in their stores have a huge impact in the brands visibility and recognition.

Finally, Exports clients are located outside Portugal. This segment represents over 40% of the total sales volumes with Spain and Angola as the main destinations, although Unicer sells its products to over 40 different countries. Most of the orders of these customers are large amounts of a single or two products and are shipped in containers.

In relation to the customers, tactical plans define for each one:

- The total quantity of each product sent from each platform;
- The total supply costs.

From a simulation point of view, it is interesting to study the effect of adding a new set of customers to the network or to assess the impact of changing the order policy of a given group of customers.

**Transports**

Production platforms, distribution platforms and customers form a three echelon distribution network. The first echelon is composed by the production sites and the second one by distribution platforms. Customers in the downstream echelon can be supplied by both upstream echelons. Figure 3 sketches an example to better understand the dynamics of Unicer’s supply chain. This example considers two production platforms (\(PP_1\) and \(PP_2\)), two distribution platforms (\(PD_1\) and \(PD_2\)) and three customers (\(C_1\), \(C_2\) and \(C_3\)). In this representation of the supply chain nodes are locations and arcs represent the flow of finished products. Only the arcs used to supply client \(C_2\) are shown. We distinguish between two type of flows: direct supply transportation movements which aim to supply client orders (depicted as solid arcs) and transportation movements intended to reallocate the stock among the facilities (depicted as dashed arcs and called inverse movements hereafter).

Usually, inverse movements are not considered in distribution planning as often supply chains are acyclic networks, where production platforms can only send its products either to a distribution platform or directly to a client and distribution platforms only deliver to
clients. The situation present in this case study is far more complex as the finished products can flow among production platforms and distribution platforms, and distribution platforms can also send products back to production platforms. These inverse movements aim to deliver clients orders more efficiently. Before introducing the different supply strategies to the different customer groups, we first introduce some more general aspects of their supply.

Apart from “Capilar” customers which are supplying using the company’s fleet, Unicer subcontracts the services of trucking companies or third party logistics providers (3PLs) to deliver its products. These companies use trucks able to carry up to 33 pallets or a maximum weight of about 25.5 tons. Hereafter the term truck will be used to refer to these large vehicles.

We use the term full pallet to refer a pallet loaded with a single product, in opposition to the term picking that is used to mention units (boxes) of products or pallets with several products. A picking operation (only possible at distribution platforms) takes full pallets which originally came from the production lines and coverts them into separate product units or rearranges them to form pallets with several products (mix pallets).

The supply of “Capilar” clients starts by sending full pallets from production platforms to distribution platforms. At the distribution platforms the small sized heterogeneous orders are picked and loaded into the company’s vehicles to which are assigned routes visiting several customers.

Retailers receive their large orders in trucks completely loaded with full pallets directly from a production platform. All the products in the order are commonly produced in just
one production center from which the shipping is made. If so no further transport movements occur. Exceptionally, these orders can also include very small amounts of products produced in other platforms or have some picking units. In case this happens the following cases may occur. If some products are not produced at the platform, full pallets are sent from a distribution platform or from the production platform responsible for their production. In the case of the presence of picking in the order, these operations are performed at a distribution platform which sends the units of picking back to the production platform to deliver the order.

Due to the orders diversity coming from Strategic clients, their supply triggers the most complex movements in the supply chain. This is explained by different inventory management strategies adopted by these clients, which can be classified into: centralized and decentralized. Clients with a centralized strategy have their own central distribution platforms. Stores send their requirements to these depots which are responsible for sending orders to Unicer and for receiving the products to later send to the stores. This creates large orders which may be adequate to be supplied directly from production platforms, like in Retailers. However, if the product mix is too wide and no particular production platform produces the majority of the products or the amount of picking operations required is high, the orders are served from a distribution platform. In decentralized strategies the stores send orders directly to Unicer that is responsible for shipping products directly to them, resulting in much smaller orders. Therefore, these orders must be consolidated at a distribution platform to achieve an efficient use of the truck capacity, which afterwards performs a route over several stores.

Shipments to Exports customers can travel by land or by sea, but in both cases they are performed using containers. These deliveries are made from the production platforms and are mostly composed of full pallets. Exceptionally in the case of orders from these clients, the production platform can also perform limited picking operations.

Table 1 summarizes the different delivery modes for the different customer groups.

In resume, to fulfill customers’ orders a transportation movement has to initiate at a production platform afterwards many options are available. We can distinguish between three different general paths to fulfill a customer order: (1) these movements serve a given customer directly from the production platform; (2) these movements start by sending full pallets products to a distribution platform that are then partially picked and sent back to
Table 1  Main features of the different supply strategies of the customer groups.

<table>
<thead>
<tr>
<th>Customer Group</th>
<th>“Capilar”</th>
<th>Retailers</th>
<th>Strategic</th>
<th>Exports</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supply mode</td>
<td>door-to-door from distribution platform</td>
<td>directly from production platforms</td>
<td>Both from production and distribution platforms</td>
<td>directly from production platforms</td>
</tr>
<tr>
<td>Truck Utilization</td>
<td>LTL(^1)</td>
<td>FTL(^1)</td>
<td>FTL(^1)/LTL(^2)</td>
<td>Containers</td>
</tr>
<tr>
<td>Full Pallets</td>
<td>No</td>
<td>Yes</td>
<td>Yes/No</td>
<td>Yes</td>
</tr>
<tr>
<td>Picking</td>
<td>Product units</td>
<td>Rarely</td>
<td>Product units and mix pallets</td>
<td>At the production platform</td>
</tr>
<tr>
<td>Product Mix</td>
<td>Complex</td>
<td>Simple</td>
<td>Complex</td>
<td>Simple</td>
</tr>
</tbody>
</table>

\(^1\)FTL - Full Truck Load, \(^2\)LTL - Less than full Truck Load.

a production platform so they are, finally, shipped to the customer; (3) in these movements a distribution platform receives products from different production platforms, consolidates the orders and send them to the final customers.

With respect to transports we are particularly interested in capturing the flow among the different platforms, including the inverse movements. The tactical distribution plans created should define:

- The quantities to be sent through the different platforms (full pallets and picking);
- The total inter-platform costs.

Transportation management also rises important simulation questions, such as what happens in case of a more flexible distribution network achieved by adding more transportation lines linking the existing points (i.e. production platforms, distribution platforms and clients) as by definition of the problem not all possible transportation lines are used by the company. It is also important to understand the net effect of negotiating some present transportation tariffs.

**Initial Situation**

Up to the beginning of this project, the main purpose of the DB was to estimate the cost that the logistic department would incur in the following year. It was a common practice at the department to further divide the DB into the platforms budget and transports budget. Platforms budget estimated the expenditures related to the operation of the platforms, namely its activity costs, and the transportation budget projected the total costs in terms of inter-platform movements and client supply. Both these processes were performed manually with the help of spreadsheets and were strongly dependent on the managers’ experience. These estimates were based on past data, since no plan for the distribution was created.
Managers looked at total volumes per month as a detailed sales budget per client or client category was not available, and tried to detect discrepancies in comparison to the previous year. If such differences were found or the managers acknowledged some significant change in past assumptions, for example new important clients or products, or changes in the delivery mode for a set of customers, they would adapt last year’s budget to reflect the new reality.

To validate the different strategical choices, the procedure embedded the same idea of looking to the past and compared it to the new scenario estimating the future costs.

The company was aware that its approach had some drawbacks:

1. It was heavily dependent on the experience of the managers assigned to the tasks.
2. A significant deal of manual effort was required to conduct the spreadsheet-based planning, leading to a hard time-consuming task. This limited the number of scenario analysis performed to evaluate the performance of different solutions or to assess the sensitivity of the plans to the input data.
3. The current plans did not ensure the capacity constraints identified at the platform level. Furthermore, as the plans did not detail the flows of finished products it was hard to identify and evaluate the bottleneck activities.
4. Finally, no optimization of costs used to take place, both when choosing the platforms activity level and when selecting the supply mode of clients. These decisions enclose trade-offs which were not being explicitly considered, such as increasing the level of activity of one platform compared to opening for a short period a platform previously inactive or defining the platform to supply a given client.

This project seeks to develop a new process to breach the deficiencies detected at the current methodology. Furthermore, this new process aims at increasing the detail of the decisions and the quality of information made available to decision makers to support better managerial decisions.

**Distribution Planning in the Fast Moving Consumer Goods Industry**

In the fast moving consumer goods industry the transportation process is usually managed by third party logistics providers (3PL). It is widely acknowledged that by outsourcing the transportation services, an increase in the truck utilization is achieved since the 3PL can consolidate several shipments from different clients. Moreover, the efficient utilization of the truck capacities results in a means of reducing freight costs (Stank and Goldsby 2000).
Our case study is not an exception and the company has contracts with several 3PLs. Nevertheless, the transportation planning is completely under their control. Crainic and Laporte (1997) distinguish between three different levels of transportation planning:

- **Strategical transportation planning** that encompasses a long planning horizon and is responsible for defining the distribution network structure and for defining the customer service levels.

- **Tactical transportation planning** that still uses aggregate information to define the best affectation of resources. Hence, having the distribution network fixed, this level aims at introducing activities on the fixed facilities.

- **Operational transportation planning** that deals with the detailed planning of vehicle loads, routing and platform management. This planning level has to be very agile in adapting to a very dynamic setting.

The decision support system of this case study helps in making decisions both at the strategic and tactical levels. However, its underlying mathematical model has a clear tactical scope. Many of the mathematical models available in the literature for tactical transportation planning in a multi-echelon network also include production decisions as shown in the review by Mula et al. (2010). Although capturing more upstream decisions, they have a lower level of detail in the downstream echelons when compared to formulation presented in this paper. For example, an application in the chemical process industry is presented by Timpe and Kallrath (2000). Distribution and marketing decisions are planned simultaneously with batch and campaign production in a multi-plant production system. In their case study, the network is composed of 4 plants and 4 sales points which is significantly smaller than ours. We can find a distribution network similar to the one in this paper in the work of Bassett and Gardner (2010). In their study authors formulated two mathematical models, the first for a three echelon distribution network and the second adding an extra production echelon to form a four echelon network. Even though the structure of the network in this problem resembles ours, both the decision level of the model (strategical vs. tactical) and distance in the location of the facilities (long vs. short) are clearly different in comparison to the present case study.

To the best of our knowledge, the work by Kreipl and Pinedo (2004) is the one sharing more common features with ours. It also covers a tactical problem faced by a beverage company, namely Carlsberg A/S beer brewer in Denmark, and the model considers a three
echelon distribution network in which customers can be supplied from both upstream echelons. However, contrarily to our work, it defines production decisions and no inverse movements can occur. Furthermore, a single distribution platform composes the second echelon and inventory can not be kept at production platforms. Finally, the activity level of the platforms remains constant throughout the planning horizon.

For the aforementioned reasons, besides its relevance in the context of the case study, we consider that the mathematical formulation developed in this project is an important contribution. The main innovations rely on the consideration of the inverse movements in the network, the several activity levels of the platforms and the introduction of operational insight at a tactical level.

The Solution Approach
Our solution strategy to the tactical distribution planning problem relies on a heuristic solution based on the mathematical formulation of the problem. This is due to the large scale of the instances resulting from the case study which prohibited the use of a commercial solver on the complete mathematical formulation to achieve an optimal or quasi-optimal solution. In our first experiments we had problems loading the complete model into the solver for memory reasons or the solver took a prohibitively large amount of computational time to provide the first feasible solution. Below we start by describing in general lines the mixed integer programming (MIP) model for the tactical distribution planning problem, given in the Appendix, and later we specify how we solve it heuristically to find good solutions to the problem.

A Model for the Tactical Distribution Planning Problem
The aforementioned three echelon distribution network can be described as a graph $G = (V,A)$. The vertex set is composed by the reunion of the set $P$ of available production platforms (echelon 1), the set $D$ of distribution platforms (echelon 2) and the set $C$ of customers (echelon 3). The arc set explicitly defines the possible paths among vertices corresponding in practice to the transportation lines used by the company. These lines can be split into connections between platforms ($I$) and the paths linking platforms to customers ($A$).

The problem is to define the flow of finished products $K$ from the production platforms to the customers over the planning horizon $T$, in order to satisfy the customers demand
at minimum cost. It can be understood as the integration of two subproblems: a transportation and an inventory management subproblem. The first subproblem resembles a multi-commodity, multi-echelon, multi-period transportation problem. In this scope, the model has to decide about the quantity shipped in each period from each platform to another and from each platform to the set of final customers. The second subproblem handles all the activities within a platform subject to capacity constraints. Hence, the model decides about handled pallets, units of picking and shipping containers, as well as it controls the inventory and allocation of products to different storage types. Of course, these subproblems are deeply intertwined because the transportation quantities decided will have a direct impact on the amount of products handled/stored.

Next we present the main decisions taken at each entity of the supply chain.

Platforms
Platforms can work on different activity levels \( \mathcal{N} \) that allow for different capacity restrictions. The definition of these activity levels is crucial to attain a realistic representation of the functioning of a platform. The different activity levels have a set of related costs and capacities that incorporate directly the possibility of hiring additional employees to picking or container loading operations, extra forklifts to increase pallet movement or even the creation of new working shifts. In the original case study formulation, we made further distinctions on the abilities of each platform, such as the ability of loading or not maritime containers. Nevertheless, for the sake of clarity we will not present these details.

Regarding platforms the main decision involves setting the platform activity level in each period. To do so we use a binary variable associated to each activity level, \( a_i^n \) which takes the value 1, if the (production or distribution) platform \( i \) is at activity level \( n \) in period \( t \). We model the inactivity of a platform using an artificial activity level 0. At this level of activity a fixed cost can be incurred but other costs and capacities are set to zero.

To capture the stock level at the end of each period, we define \( s_{i,kt}^e \) as the number of pallets of product \( k \) stored in platform \( i \) at storage of type \( e \) in period \( t \).

Customers
In our mathematical formulation we grouped customers belonging to the set \( \mathcal{C} \) according to transportation type into maritime \( (\mathcal{M}) \) or terrestrial \( (\mathcal{R}) \) to capture the requirements in terms of containers. Customers are divided by a second criteria into national \( (\mathcal{F}) \) or international \( (\mathcal{E}) \). We assume to know \( D_{j,kt}^e \) representing the demand of customer \( j \) for product
k in the palletization type b in period t. Palletization types correspond to the previously defined full pallets (b = 1) and picking (b = 2). This immediately suggests the following decision variables \( x_{ijkbt} \) defined as the number of pallets of product k with palletization type b transported from platform i to customer j in period t to catch the supply decisions. However, these variables are insufficient to translate the reality into the model as they do not capture the real operational move in the tactical model, especially the inverse moves.

To overcome this, both the parameter \( D_{jkbt} \) and the decision variable \( x_{ijkbt} \) were refined. For this purpose we rely on the historical data for customer’s demand orders. The demand of customer j for each product k in a given period t is split into types of orders where it is inserted. The orders are classified into types according to: order size (total order weight measured in tons), production platform producing the majority of products that appear in this order and the magnitude of this majority (a percentage of the total order weight). We define a finite set \( q \in Q \) to classify orders according to their size and called it tonnages. Similarly, the magnitude of the majority of products belonging to a single production platform is also classified by intervals \( p \in G \) denoted as percentages. In Figure 4 an example of the demand conversion for a customer j for 10,000 pallets of Product k in March is given.

Hence, we also need to detail \( x \) and introduce a new decision variable \( f \):
$x_{ijkbwt}$ number of pallets of product $k$ with palletization $b$ transported from platform $i$ to customer $j$ in period $t$ to supply an order with the majority of products from production platform $w$

$f_{jit}$ binary variable which takes the value 1, if demand orders of customer $j$ in period $t$ with a majority of products from production platform $i$ having a percentage $p$ and a tonnage $q$ are satisfied directly from $i$, or 0 if these orders are satisfied through a distribution platform from the set $D$

Ensuring the link between $x$ and $f$ allows us to capture the operational behavior of deliveries in our tactical model.

**Transports**
The last decision variable details the inter-platform movements. $z_{iukbd}$ represents the number of pallets of product $k$ with palletization $b$ transported from platform $i$ to platform $w$ in period $t$.

Having presented the main entities of our model, we can now describe the objective function and the main constraints.

**Objective function**
The objective function minimizes the total distribution costs over the whole planning horizon. These total costs correspond to: platform fixed activity costs, platform storage costs, pallet moving costs, picking moving costs, shipping container loading costs, transportation costs between platforms and transportation costs to deliver orders to customers.

Notice that all the transportation costs take into account the possibility of dealing with returnable products in routes that may be subject to such accounting. Furthermore, for FTL cost calculation one has to take into consideration the transportation mode used in a given transportation line from platform $i$ to client $j$, i.e. either trucks or containers.

**Constraints**

**Demand fulfillment constraints:** All orders from customers should be delivered without any delays, i.e. in the same period as they have occurred. In other words, backlogging is not allowed.

**Demand supply strategy constraints:** For each client the model assigns demand of orders types $(i,q,p)$ either to the corresponding production platform $i$ or to the set of
distribution platforms $D$. To give coherence to the plans and comply with the planners reasoning we also enforce that as soon as a given order type $(i, q, p)$ is fulfilled through the corresponding production platform $i$, then all demand orders having either a heavier tonnage $q$ or a higher percentage $p$ to be also fulfilled from $i$. Thus, this corresponds to the selection of a cut-off point both in terms of tonnage and percentage above which orders are consider properly to be supplied from a production platform.

As an example, Table 2 presents the impact on the distribution paths for a given customer in a given time period after fixing the supply strategy of order type $(i, 10 – 15t, 80 – 60\%)$ to be fulfilled directly from the production platform.

**Inventory balance constraints (production platforms):** The inventory balance constraints related to the production platforms characterize the movements that are allowed on these platforms. We distinguish between the inventory balance constraints for products produced in the respective production platform or not. We also have into consideration that, due to custom duties, at the production platforms picking can only be done to satisfy international customers. Finally, every unit of picking entering the platform is forced to leave the platform in the same period in order to satisfy national customers demand.

**Inventory balance constraints (distribution platforms):** The inventory balance constraints at distribution platforms show their flexibility. In fact, contrarily to the production platforms they can process any entering pallet into picking and dispatch in all palletization forms to the connected customers.

**Activity levels constraints:** Each platform at each period can only operate at one activity level.

**Platform activity cost constraints:** Platforms costs depend on the activity level selected, thus we have to link them.
**Platform capacity constraints:** Similar to the platform costs, the amount of activity performed in each platform depends on the decided activity level. Hence, we have to impose the corresponding limits to the number of shipping containers loaded, pallets stored, number of pallets moved and amount of picking performed in the platforms.

In this section (together with the Appendix) the general mathematical formulation for our problem was introduced. It is important to clarify that we presented here the simplified version of the model on top of which the heuristic embedded in our optimization tool is built and, therefore, only focuses on the key modeling characteristics that can be replied in similar situations.

**The Heuristics**
The MIP model is not solvable for the large size instances of the case study. It suffers from its computational intractability especially because of the large number of demand and flow variables. Therefore, solving this problem requires the use of efficient solution approaches. Mathematical programming-based heuristics, also known as matheuristics (James and Almada-Lobo (2011), Ball (2011), Maniezzo et al. (2010)), are algorithms which seek the best trade-off between the effectiveness of exact approaches and the efficacy of meta-heuristics. We based our solution strategy in MIP-based heuristics which are a class of matheuristics relying on the heuristic solution of the mathematical formulation.

The MIP-based heuristic designed has two phases: construction and improvement. Each phase uses a decomposition of the original mathematical formulation by time period. At each iteration of the construction phase we solve a single-period version of our MIP model. We start by solving the subproblem corresponding to the first time period, then we fix the solution of this period and set the final stock decisions as an input to the following subproblem, in this case the second period. We repeat this process and progressively move towards the end of the planning horizon. Once the solution of the last period is finished a feasible solution to the problem has been achieved.

The improvement phase of the heuristic seeks to increase the solution quality of the feasible solution at hand. Solving a single-period version of our model turns the final inventory decisions at each platform myopic since no further information on the demand is considered at the model. To overcome this and other potential limitations of the single-period version model, at each iteration of the improvement phase a two-period model is
solved. These two time periods must be adjacent on time. Once again we start at the beginning of the planning horizon and re-optimize the solution corresponding to the first two time periods obtained in the construction phase. In next iteration, we fix the solution for the first time period and re-optimize time periods two and three together, and so forth until the final period in reached again. By keeping some overlap among successive iterations we guarantee a less myopic heuristic and potentially reduce the solution cost. Figure 5 depicts a visual interpretation of the heuristics.

The use of MIP-base heuristics for a practical case study offers several advantages:

- With some expertise and compared to the traditional heuristic approaches, these heuristics are more easily implemented (i.e. require less parameters, less effort in tuning parameters and validating solutions). Moreover, they are rather problem-independent.
- They take advantage of the computational efficiency of modern commercial solvers.
• Despite being based on the model, they can cope with models extensions such as new constraints or even new decisions variables with limited or none changes in the heuristic.

• It has been proved in the literature that their performance often achieves quasi-optimal solution for a variety of different problems, which for the majority of the companies is more than enough.

However, it should be noticed that these heuristics rely on a decomposition of a larger problem, expecting that the resulting subproblems are easier to solve than the main one. When this is not the case, these heuristics can either lose their efficiency or fail to deliver a feasible solution to the problem. This is an important risk to manage.

Summing its pro and cons we believe that the use of MIP-based heuristics pays-off its use by appearing as a more flexible approach to cope with future changes of the problem.

Decision Support System

This section describes the decision support system that wraps around the optimization tool using the solution strategy of the core problem described in the previous section. In Figure 6 the relation between the building blocks of this decision support system (detailed in the next sections) and other software owned by the company are presented.

This decision support system works through an on-line platform that can be accessed by any computer connected to the Internet. To develop this tool several programming
languages were used. The browser interface is coded in JavaScript and the communication with the dedicated server is established through C#. The core optimization tool uses C++ to read the data, execute the solution strategy and output the solution. The mathematical models are solved with the help of a commercial mathematical programming solver. Finally, an add-in coded in Visual Basic for Applications (VBA) uses the raw output data to build user-friendly reports and extract information from the output solution.

**Master Data**

The Master Data input corresponds to a spreadsheet which organizes most of the parameters of the model. Hence, the user feeds a list of products with all the required characteristics, a list of clients and their information, a list of platforms and their abilities, the allowed activity levels for each platform and the possible transportation arcs with the respective costs. As shown in Figure 6 a major part of the information is gathered from Unicer’s ERP system SAP R/3 and loaded into a spreadsheet for further validation and modification. SAP APO system can also be useful for loading the current configuration of the supply chain, namely the production and distribution platforms and the used transportation lines.

This considerable amount of data is permanently stored at the decision support system’s server and it can be changed on an incremental fashion. Due to the strong interaction between these different data fields, this input is very prone to yield consistency errors as reported by other authors such as Farasyn et al. (2008). To circumvent this fact, we have implemented a VBA add-in that identifies all potential errors and missing data. Moreover, it points towards intelligent suggestions to rectify the incoherent and missing values. This add-in may for example indicate that a client has no transportation lines from any of the platforms and suggest some possible corrections. This type of information revealed itself to be crucial for the good usage of the tool and it saved considerable time to the analyst setting up the decision support system.

Finally, the use of spreadsheets to input the data required constitutes an easy and inexpensive way for defining new planning scenarios, one of the goals of the project. Inserting new products is a straightforward operation. Adding new customers or platforms, or even changing their location can be more time demanding due to the number of transportation lines affected, but the process is as simple as adding or editing lines in the master data template.
Demand forecast by order type
This block is responsible for creating the demand parameter. Two distinct inputs are needed to calculate such parameter. The demand forecast per client and per time period of the planning horizon and the deliveries history of a similar horizon in the past which is available from SAP R/3. For the DB demand forecast corresponds to monthly estimates for the next year and the delivery history of the current year. With the demand forecast we obtain total demand per client, product and period. The deliveries history allows us, with the necessary preprocessing calculations, to achieve the disaggregation detail of demand by order types based on past customer demand orders.

Production budget
The production plan that feeds the distribution planning defines products’ production quantities at each production platform in each time period. The core problem consists of assigning and scheduling production lots in a multi-plant environment, where each plant has a set of filling lines that bottle and pack drinks. The work of Guimarães et al. (2012) proposes a method to create these plans. The output of this method or other solution approaches enter directly as parameter in the distribution planning.

Optimization tool
The optimization tool is responsible for feeding the information to our solution strategy, applying the heuristics and feeding forward the output to be decoded by the VBA add-in. Meanwhile, during the execution, it also sends feedback to the user on the current solution status. The optimization tool is triggered by the user through the on-line interface after setting up the input. Afterwards the heuristic starts with its linkage to the mixed-integer programming.

KPIs and Reports
After processing the raw output by the developed add-in, the decision makers have available graphical Key Performance Indicators (KPI) and extensive reports to perform their analysis and make informed decisions. We have implemented 7 different KPIs that cover the main areas of influence of the decision support system as follows:

- **KPI 1: Aggregated Costs: Platforms vs. Transportation** - This KPI reports the main partition of total costs between platform and transportation costs.
• **KPI 2: Platform Costs by Process** - This KPI disaggregates the overall platform costs by activity cost, storage costs, pallet moving cost, picking moving cost and shipping container loading cost.

• **KPI 3: Transportation Costs by Process** - This KPI decomposes the overall transportation costs in its main components: transportation costs to serve customers and between platforms.

• **KPI 4: Platform Utilization** - Based on the amount of pallets distributed by each platform, this KPI shows their relative importance to satisfy customers demand among all platforms and allows to identify the major bottlenecks in the distribution network.

• **KPI 5: Platform Costs by Platform** - This KPI separates the overall platform costs by platform. This allows the decision maker to have a quick view over the platforms yielding higher costs.

• **KPI 6: Transportation Costs by Type of Client** - Unlike KPI 3, the disaggregation of the transportation costs in this case is done by type of client.

• **KPI 7: Expeditions per Platform** - The last KPI has a more operational character. For each platform we assess the amount of pallets sent, which are split by those shipped to clients and to other platforms.

Beyond the general information that the KPIs display, the decision maker has the possibility of digging further into the results through the seven implemented reports. The following description gives a hint on the type of information made available within our decision support system.

• **Report 1: Total Costs** - This report gives the same information of KPI 1, 2 and 3 but, moreover, it is possible to see the period (monthly) evolution of these costs.

• **Report 2: Platform Costs by Process** - This report breaks down KPI 2 and, therefore, the process costs are split by platform and by month.

• **Report 3: Transportation Costs by Type of Client** - Similarly, in this report, KPI 6 is broken down per month.

• **Report 4: Transportation Costs by Client** - This report goes further in detail than Report 3 and details costs for each client independently and, moreover, the costs are also split by product transported.

• **Report 5: Movements Report** - This is the most important report that summarizes the activity of all platforms. For each platform we have the monthly evolution of the
activity levels, stock, entries and deliveries. Moreover, utilization rates are also available for each resource. Figure 7 shows the details of this report for one platform.

- **Report 6: Activity Levels** - Activity levels for all platforms throughout the months are given in this report.

- **Report 7: Stock Report** - This report shows the amount of stock at each platform and in each type of storage for all months.

**Interface**

The final block of our decision support system is the interface. The on-line interface has three main areas: (1) data files upload, (2) tool execution and (3) history of solutions. Figures 8 and 9 show the graphical interface and the expected interactions with it, respectively. The left column in the graphical interface is responsible to manage the data files of the run, in the central column the user can launch new runs of the tool and has access to the log of the incumbent and previous runs. Finally, in the last column the solution files are available to be downloaded.
Validation

Together with Unicer’s planning team we validated our approach in October 2012 during the creation of the DB. This was also a phase of intensive training of the decision support system future users. We helped them in defining the master data spreadsheet and ensuring a proper set up to the production plans and demand forecast required. In this master data the original 380 SKUs sold by the company were clustered into around 120 product clusters by merging products with similar physical and demand properties. Similarly, the customers
were also clustered into client clusters by using the aforementioned customers categories and the district in which they are located. This reduces the original 19,000 clients to about 200 client clusters. Clustering guarantees the tractability of the MIP models in our solution strategy.

In the 2012 DB, a total of twenty one platforms were to be planned: nine production platforms, two major distribution platforms and the remaining locations are auxiliary distribution platforms. Over 200 transportation lines were available among platforms and more than 1300 lines connecting platforms and client clusters were defined as supply alternatives. Concerning activity levels, larger platforms could operate in three to four levels, while the auxiliary platforms usually presented only two possible levels of activity (active or inactive).

Our decision support system converted the SB per client into a detailed forecast by order typology considering a total of 180 possible types defined by the nine product platforms, five tonnages and four percentages. This process took 300 seconds since over 30,000 demand orders have to be analyzed in every month. At this point the main goal was to evaluate the plans created by the tool from the business perspective. This was done in several meetings and the costs analyzed had a similar order of magnitude to the estimates and business sense of the managers. Moreover, a deeper analysis of the plans established full confidence in the tool as these suggestions embedded important insights on how to operate the distribution process.

We have repeated this process at the beginning of 2013 to compare the yearly plan defined by Unicer to the year (without any use of the new tool) against the potential solution provided by the decision support system from a cost-efficiency point of view. The real plan defined by the operations over the year was evaluated according to the costs defined in the master data file and set the base total cost of the operation. We compared this plan with the ones obtained by:

- Solving the complete mathematical model formulation present in Appendix;
- The construction heuristic;
- The construction heuristic followed by the improvement heuristic.

We used an Intel i7-3630QM processor with 16,0 GB of RAM in our tests. All the heuristic approaches had their running times limited to one hour. Solving the complete model revealed to be inadequate as the solution provided was of very poor quality. The
Table 3 Comparison of the MIP models.

<table>
<thead>
<tr>
<th>Model</th>
<th>Variables (Binary)</th>
<th>Constraints</th>
<th>Non-Zeros</th>
<th>Avg. Solution Time (secs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Complete Model</td>
<td>830,000 (30,000)</td>
<td>335,000</td>
<td>3,900,000</td>
<td>28800(^1)</td>
</tr>
<tr>
<td>Single Period Model</td>
<td>66,000 (2,400)</td>
<td>26,000</td>
<td>300,000</td>
<td>30</td>
</tr>
<tr>
<td>Two Period Model</td>
<td>135,000 (4,900)</td>
<td>51,000</td>
<td>595,000</td>
<td>115</td>
</tr>
</tbody>
</table>

\(^1\) maximum time limit

Figure 10 Comparison of the two heuristic plans and the original company’s plan in the different cost categories.

Both the storage capacity violation and the movement capacity violation are depicted considering a secondary axis to highlight the differences between plans.

Figure 11 Year-to-date total cost and total cost in each month of the different plans.

solver stopped its search at a very early stage, only 55 nodes of the search tree were explored when the maximum running time (8 hours) was achieved and the solution had no value in practice. Table 3 presents the average model size and solution time for the different
models. The single period model corresponds to the model used in the construction phase and the two period model to the one used in the improvement phase.

Both versions of our heuristic procedures finished long before the time limit and delivered better solutions than the one by the company. The plan obtained by the constructive heuristic was obtained in 354s and reduced the cost of the company’s plan by 5.25%. Applying the improving heuristic on top of the initial solution further reduced the total cost up to 6.8% below the original company’s plan. These improvements in the total cost correspond to reductions of approximately 1.7M and 2.2M euros, respectively. Figures 10 and 11 depict in more detail the differences among the alternative plans. Both heuristic procedures trade-off the costs categories involved differently from the company’s perspective: they increase the transportation costs among platforms and the storage costs in order to induce a significant reduction in the customers’ supply cost (see Figure 10) which is responsible for the largest cost share. Essentially, the plans suggested indicate that repositioning the stock along the supply chain can lead to important savings when supplying the clients’ demand. We stress the fact that the total stock on the supply chain is the same across the plans, the differences in terms of storage costs are due to the increased use of more expensive facilities. Moreover, Figure 11 shows that the cost difference between plans was mostly explained by the behavior in the peak months of the Summer, when the capacity of the supply chain is more taken and, therefore, decisions have a higher impact which persists until the end of the year. Another important aspect to emphasize is that both plans obtained by the heuristics resulted in less transgression of both the storage and the movement capacities (see Figure 10). We had to allow these violations in the model (penalized in the objective function), otherwise it would be infeasible due to the low capacities defined for these resources in the master data file. These low capacities are explained by the fact that managers often underestimate them to ensure feasibility when performing the operational planning of the distribution as shown by their level of over utilization in the actual plan. Figure 12 highlights the benefits of the heuristics over the planning horizon considering only tangible costs. The apparent bad performance of the plans suggested by the heuristics in the month of April in Figure 12 is indeed the result of an effort in lowering the storage and movement capacity violations, which are substantially reduced when compared to the actual company plan.
Conclusions and Future Work

In this paper we describe the real-world tactical distribution problem faced by Unicer, the major Portuguese beverage production company. The literature tackling tactical distribution problems with the features of this real-world application is sparse. However, we built on existing concepts from transportation and inventory a new mixed-integer programming model having as a key feature the insights of operational practice at a tactical level. The model is the basis of the solution strategy designed and implemented in a decision support system which is being used by the company. We scrutinized what is behind the main build-
ing blocks in order to increase the awareness of important factors which can give interested readers a basis to build something similar and shown its potential cost reduction impact.

Today Unicer uses OR in their tactical distribution decisions, which are now based on automated, detailed and accurate tactical distribution plans improving this planning step at the company. The Decision Support System is being used to evaluate different logistic scenarios and to help in preparing the annual budget. The budget for 2013 was already validated using this new tool. The attained benefits of using the Decision Support System are evident not only by its potential of cost reduction but also by the easiness of simulation of multiple logistic scenarios and by the time saved in preparing the annual budget. Today Unicer can analyze virtually all possible distribution scenarios. This is of great value to a company that needs to challenge its practices very frequently. Moreover, the new planning methodology makes the process more transparent and the lead time to deliver the plans has decreased enormously. Analysts recognized that the Decision Support System has an underlying optimization model that retrieves solutions that were hard to grasp with the previous empirical methods, dotting the decision maker with information and perspectives that he did not have prior to the project.

Since the decision support system is built on a modular basis with very tunable blocks, it has the potential to be rolled-out to other facing similar real-world problems. Of course, the most straightforward step would be to adapt this approach to other beverage companies having similar distribution problems. However, other fast moving consumer goods companies seem also a natural extension as they also handle a vast product portfolio, many clients and a dynamic distribution network.

Future work could be devoted to integrate distribution and production tactical planning as they are intrinsically correlated. It is also interesting to extend the decision support tool to accommodate customer service levels and give more empowerment to the decision maker about the sense of the solution. Letting, for example, the possibility to adding some ceilings on key costs, such as transportation costs between platforms.

Appendix. The MIP Formulation

The parameters related to the platforms needed to formulate the problem are as follows:
Costs

\[ F_i^n \] fixed activity cost in platform \( i \in \mathcal{P} \cup \mathcal{D} \) for activity level \( n \)
\[ uH_i^n \] unitary storage cost in platform \( i \in \mathcal{P} \cup \mathcal{D} \) for activity level \( n \)
\[ uM_i^n \] unitary pallet moving cost in platform \( i \in \mathcal{P} \cup \mathcal{D} \) for activity level \( n \)
\[ uP_i^n \] unitary picking moving cost in platform \( i \in \mathcal{P} \cup \mathcal{D} \) for activity level \( n \)
\[ uC_i^n \] unitary shipping container loading cost in platform \( i \in \mathcal{P} \cup \mathcal{D} \) for activity level \( n \)

Capacities

\[ rH_{ie}^n \] storage capacity in number of stack positions in platform \( i \in \mathcal{P} \cup \mathcal{D} \) for activity level \( n \) and storage type \( e \)
\[ rM_{ie}^n \] capacity for pallet movements in platform \( i \in \mathcal{P} \cup \mathcal{D} \) for activity level \( n \)
\[ rP_{ie}^n \] capacity for picking movements in platform \( i \in \mathcal{P} \cup \mathcal{D} \) for activity level \( n \)
\[ rC_{ie}^n \] capacity for loading shipping containers in platform \( i \in \mathcal{P} \cup \mathcal{D} \) for activity level \( n \)

To correctly assess the available storage capacity we further have to take into consideration the number of pallets that we can stack as this depends on the product and storage type. The parameter \( \nu_{ie}^k \) sets the maximum number of pallet stacking levels of product \( k \) at storage type \( e \). For production platforms the planned production quantities are also an input to the model. Let \( P_{jk} \) define the quantity of product \( k \) produced at platform \( j \in \mathcal{P} \) in period \( t \).

The demand parameter used in the mathematical model is as follows:

\[ D_{jkt}^{eb} \] demand of client \( j \) in period \( t \) for product \( k \) with palletization \( b \) within an order with the majority of products from \( i \in \mathcal{P} \) having a percentage \( p \) and a tonnage \( q \)

The following parameters are necessary to capture the transportation costs among the different locations of the supply chain:

\[ fT_{ij} \] full truck load (FTL) cost for traveling from \( i \) to \( j \)
\[ vT_{ij} \] less than full truck load (LTL) cost for traveling from \( i \) to \( j \)

To describe the MIP model formulated for the tactical distribution problems the following additional parameters are required:

\[ \alpha_k \] cost factor to account for the inverse logistics of product \( k \)
\[ \beta_{ij} \] cost factor to account for the inverse logistics of passing in arc \((i, j) \in \mathcal{A} \cup \mathcal{I})\)
\[ \gamma_i^R \] land container capacity if only product \( k \) is transported
\[ \gamma_i^M \] maritime container capacity if only product \( k \) is transported
\[ \zeta_k \] weight of each pallet of product \( k \)
\[ \delta_k \] number of product units in a pallet of product \( k \)
\[ \mu_k \] factor for converting pallets of product \( k \) into full pallets of product \( k \)

We also have to introduce auxiliary decision variables to linearize the piecewise cost functions at platforms which are defined as follows:

\[ cH_{it} \] storage cost in platform \( i \in \mathcal{P} \cup \mathcal{D} \) in period \( t \)
\[ cM_{it} \] pallet moving cost in platform \( i \in \mathcal{P} \cup \mathcal{D} \) in period \( t \)
\[ cP_{it} \] picking moving cost in platform \( i \in \mathcal{P} \cup \mathcal{D} \) in period \( t \)
\[ cC_{it} \] shipping container loading cost in platform \( i \in \mathcal{P} \cup \mathcal{D} \) in period \( t \)

The overall MIP model reads:

\[
\min \sum_{i \in \mathcal{P} \cup \mathcal{D}, n, t} F_i^n a_{it}^n + \sum_{i \in \mathcal{P} \cup \mathcal{D}, t} (cH_{it} + cM_{it} + cP_{it} + cC_{it})
\]
The following auxiliary constraints to quantify the different platform costs that depend on the platform activity level. Constraints (2) quantify the storage cost of each platform in each period. This cost is incurred for every full pallet stored and depends on the platform activity level. Note that \( M \) denotes a big number.

\[
cH_{it} \geq \sum_{k, n} uH^i_{tn} \frac{s_{ik}}{\mu_k} + M(a^n_{ti} - 1) \quad \forall i \in \mathcal{P} \cup \mathcal{D}, n \in \mathcal{N}, t \in \mathcal{T}
\]

Constraints (3) account for the full pallet moving costs. These costs have to consider all pallets handled either when receiving or sending products.

\[
cM_{it} \geq \sum_{(i,j) \in \mathcal{A}, k, b, w} uM^n_{it} x_{ijkbw} + \sum_{(i,j) \in \mathcal{I}, k} uM^n_{it} z_{ijk} + \sum_{(i,j) \in \mathcal{A}, k, t} uM^n_{it} \frac{z_{ijk}}{\mu_k} + M(a^n_{ti} - 1) \quad \forall i \in \mathcal{P} \cup \mathcal{D}, n \in \mathcal{N}, t \in \mathcal{T}
\]

On the other hand, to obtain the picking costs (constraints (4)) it is only valued the amount of units of picking exiting the platform.

\[
cP_{it} \geq \sum_{(i,j) \in \mathcal{A}, k, w} uP^n_{it} x_{ijkw} + \sum_{(i,j) \in \mathcal{I}, k} uP^n_{it} z_{ijk} + \sum_{(i,j) \in \mathcal{I}, k} uP^n_{it} \frac{z_{ijk}}{\mu_k} + M(a^n_{ti} - 1) \quad \forall i \in \mathcal{P} \cup \mathcal{D}, n \in \mathcal{N}, t \in \mathcal{T}
\]

The final cost constraints refers to the loading shipping containers cost that is obtained through constraints (5).

\[
cC_{it} \geq \sum_{(i,j) \in \mathcal{A}, k, b, w} uC^n_{it} x_{ijkbw} + M(a^n_{ti} - 1) \quad \forall i \in \mathcal{P} \cup \mathcal{D}, n \in \mathcal{N}, t \in \mathcal{T}
\]

Next we introduce demand fulfillment constraints. The first constraints of this group (6) state that the customer’s demand has to be completely satisfied.

\[
\sum_{(i,j) \in \mathcal{A}} x_{ijkbw} = \sum_{q, p} D^n_{jkbw} \quad \forall j \in \mathcal{C}, k \in \mathcal{K}, b \in \mathcal{B}, w \in \mathcal{P}, t \in \mathcal{T}
\]

Constraints (7) and (8) make use of decision variables \( f^n_{jxt} \) to assign demand order typologies to a certain distribution echelon (production or distribution platforms).

\[
x_{ijkbt} = \sum_{q, p} D^n_{jkbt} f^n_{jxt} \quad \forall (i, j) \in \mathcal{A}, i \in \mathcal{P}, j \in \mathcal{C}, k \in \mathcal{K}, b \in \mathcal{B}, t \in \mathcal{T}
\]

\[
\sum_{(i,j) \in \mathcal{A}, i \in \mathcal{D}} x_{ijkbt} = \sum_{q, p} D^n_{jkbt}(1 - f^n_{jxt}) \quad \forall j \in \mathcal{C}, k \in \mathcal{K}, b \in \mathcal{B}, w \in \mathcal{P}, t \in \mathcal{T}
\]
Constraints (9) and (10) define the cut-off point for supplying orders from the production platforms.

\[ f_{jt}^p - f_{jt}^{pp} \geq 0 \quad \forall j \in C, i \in P, q, q' \in Q : q' \geq q, p \in G, t \in T \]  
\[ f_{jt}^{pp} - f_{jt}^p \geq 0 \quad \forall j \in C, i \in P, q \in Q, p, p' \in G : p' \geq p, t \in T \]  

The inventory balance constraints related to the production platforms are expressed in (11)-(13). Constraints (11) and (12) distinguish between the inventory balance constraints for products produced in the respective production platform or not (making use of set \( K^j \) that stands for the set of products belonging to platform \( j \in P \)), respectively. These equations show that picking at the production platforms can only be done to satisfy international customers.

\[
\sum_{(j,i) \in A:i \in N,k,j} x_{jk1jt} + \sum_{(j,i) \in A:i \in E,k,h,j} x_{jk2jt} \quad \forall j \in P, k \in K, t \in T
\]  
\[
\sum_{(j,i) \in A:i \in N,k,j} z_{ijk1t} = \sum_{(j,i) \in A:i \in I} z_{ijkht} + \sum_{(j,i) \in A:i \in I} z_{jkhbt} + \sum_{(j,i) \in A:i \in I} z_{ijkbt} + \sum_{(j,i) \in A:i \in I} z_{jk1t} + \sum_{(j,i) \in A:i \in I} z_{jk2t} \quad \forall j \in P, k \in K \setminus K^j, t \in T
\]  

Constraints (13) force every unit of picking entering the platform to leave the platform in the same period in order to satisfy national customers demand.

\[
\sum_{(i,j) \in A:i \in D} z_{ijk2t} = \sum_{(i,j) \in A:i \in N,j} x_{jk2jt} \quad \forall j \in P, k \in K, t \in T
\]  

The inventory balance constraints at distribution platforms is given in (14).

\[
\sum_{(i,j) \in A:i \in N,k,j} z_{ijkht} + \sum_{(i,j) \in A:i \in I} z_{ijk1t} = \sum_{(i,j) \in A:i \in I} z_{jkhbt} + \sum_{(i,j) \in A:i \in I} z_{jktb} + \sum_{(i,j) \in A:i \in I} x_{jk2t} \quad \forall j \in D, k \in K, t \in T
\]  

The following capacity constraints limit the amount of activity performed in each platform depending on the decided activity level: shipping containers loaded (15), pallets stored (16), pallets moved (17), picking performed in production platforms (18) and picking performed in distribution platforms (19).

\[
\sum_{(j,i) \in A:i \in M \cup D,k,b,w} x_{jkhus} \leq \sum_{n} rC_{jw}a_{jt}^n \quad \forall j \in P \cup D, t \in T
\]  
\[
\sum_{k \in K} s_{jk1t}/\mu_k/\nu_{k}^{n} \leq \sum_{n} rH_{jw}a_{jt}^n \quad \forall j \in P \cup D, t \in T
\]  
\[
\sum_{(i,j) \in A:i \in K} x_{jk1w1i}/\mu_k + \sum_{(i,j) \in A:i \in K} z_{jk11i}/\mu_k \leq \sum_{n} rM_{jw}a_{jt}^n \quad \forall j \in P \cup D, t \in T
\]  
\[
\sum_{(i,j) \in A:i \in E,k,b,w} x_{jk2ws} \leq \sum_{n} rP_{jw}a_{jt}^n \quad \forall j \in P, t \in T
\]
\[
\sum_{(i,j) \in A:i \in E,k,b,w} z_{jk2iws} - \sum_{(i,j) \in A:i \in E,k,b,w} z_{ijk2iws} \leq \sum_{n} rP_{jw}a_{jt}^n \quad \forall j \in D, t \in T
\]  

Finally, equations (20) ensure that each platform only operates at a single activity level in each period.

\[
\sum_{n} a_{jt}^n = 1 \quad \forall j \in P \cup D, t \in T
\]
References


