On the performance of Smith's rule in single-machine scheduling with nonlinear cost

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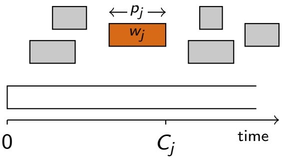
Generalized min-sum scheduling

Given: jobs $j = 1, \ldots, n$ with

- weight $w_j > 0$
- processing time $p_j > 0$



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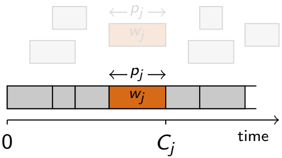

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Task: compute sequence with minimum cost $\sum_j w_j f(C_j)$

- C_j completion time of job j
- non-decreasing, non-negative cost function f

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- priorities and fairness

  $\rightsquigarrow$ L_k-norms/monomials compromise on worst and average case

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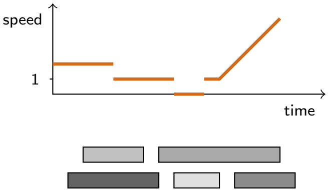

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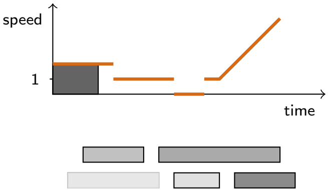

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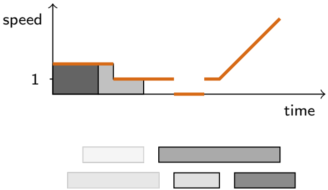

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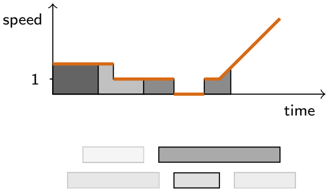

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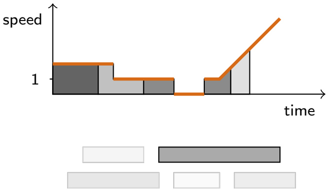

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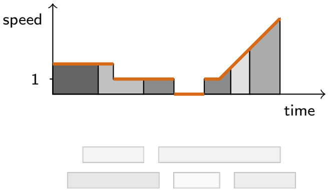

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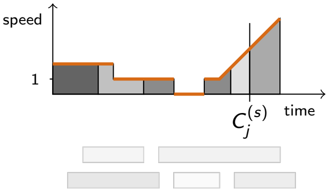

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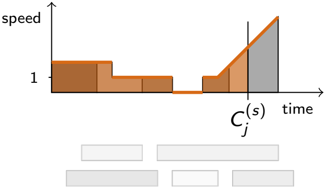

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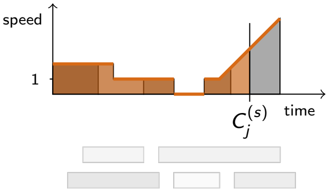

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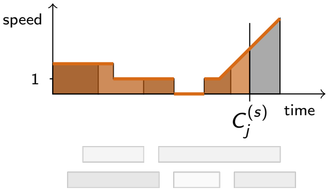

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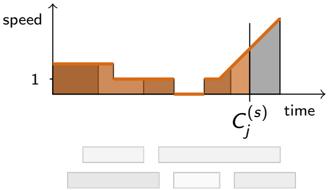

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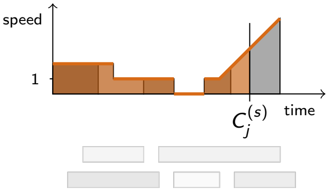

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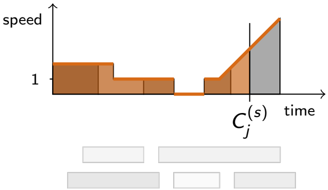

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Our main focus: convex / concave cost functions

Outline

1. Analysis of Smith's rule for convex (and concave) cost

2. Exact algorithms for monomials

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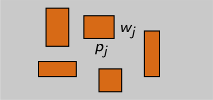

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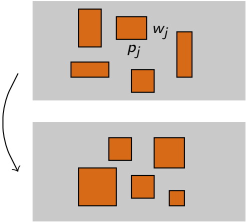

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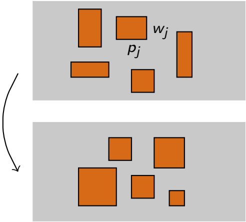

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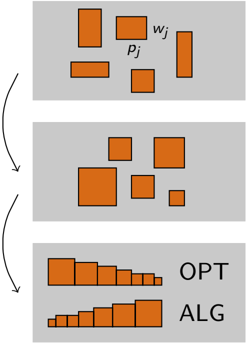

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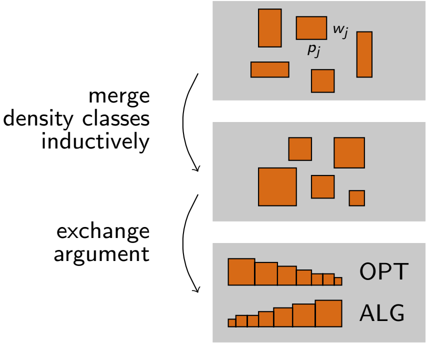

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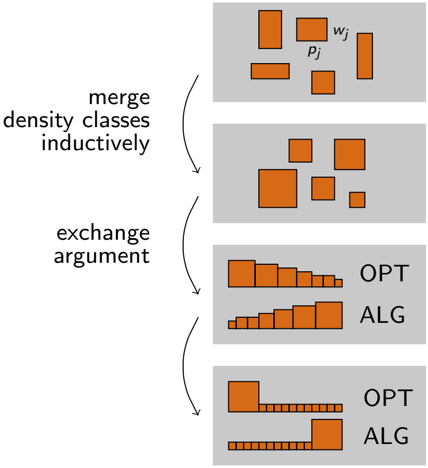

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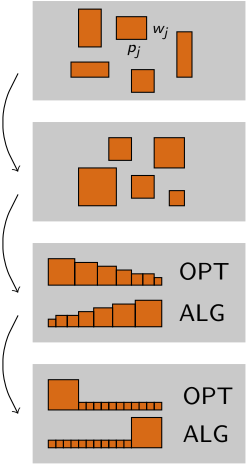

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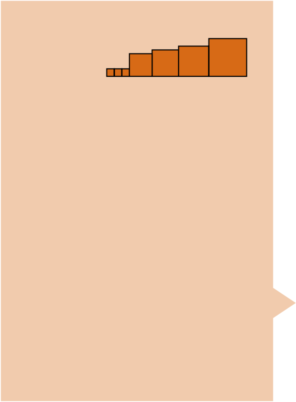

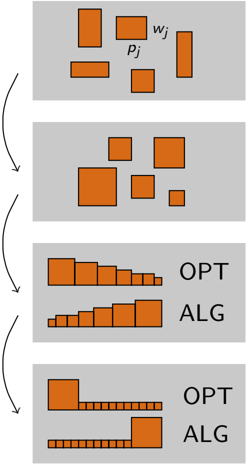

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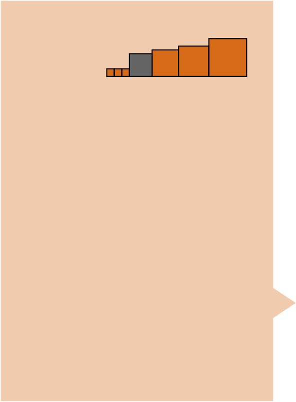

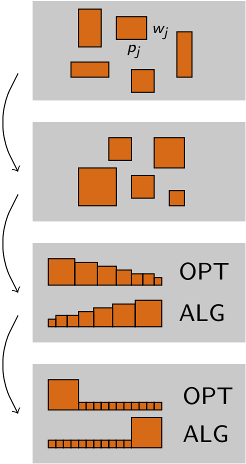

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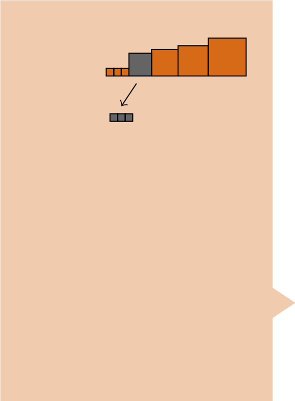

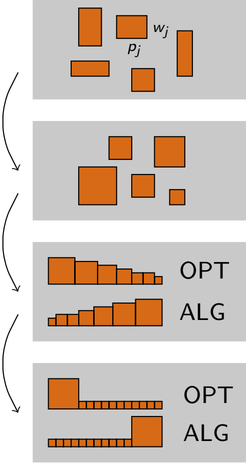

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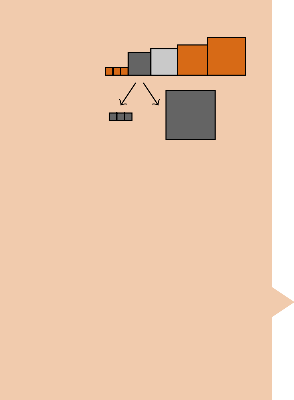

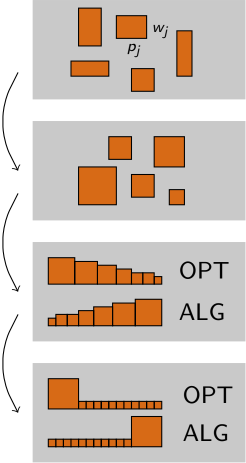

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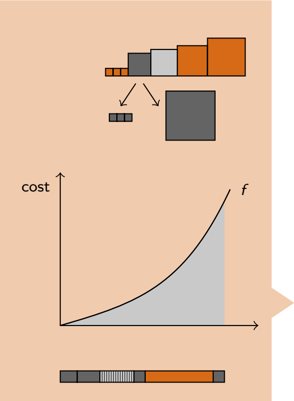

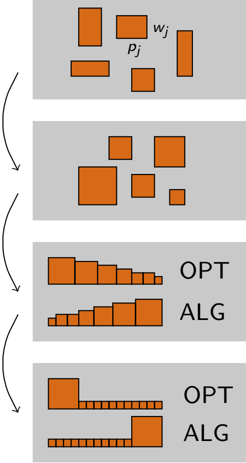

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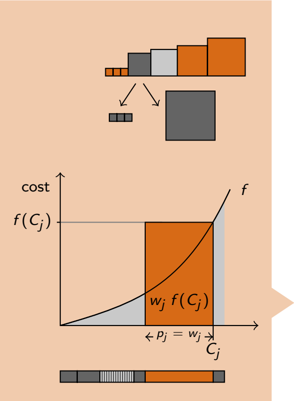

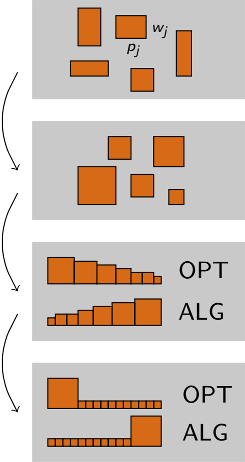

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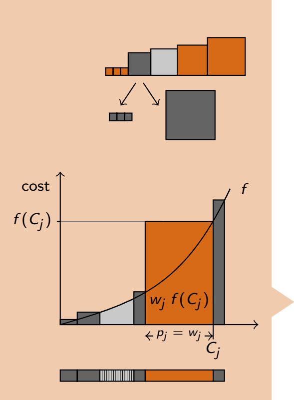

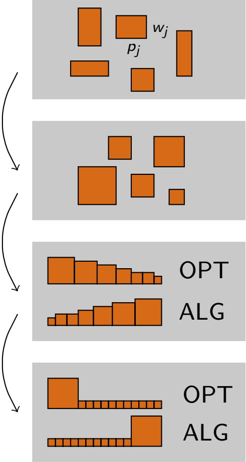

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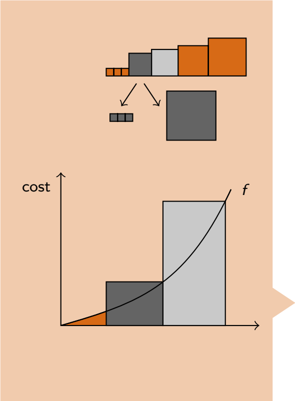

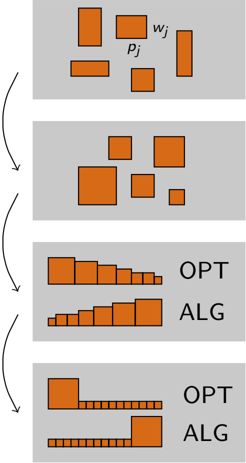

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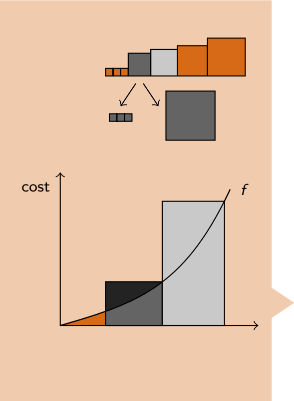

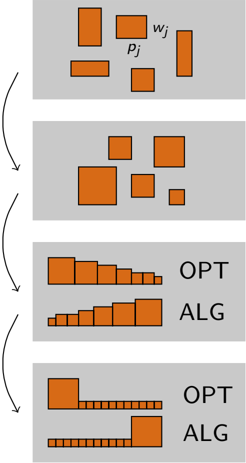

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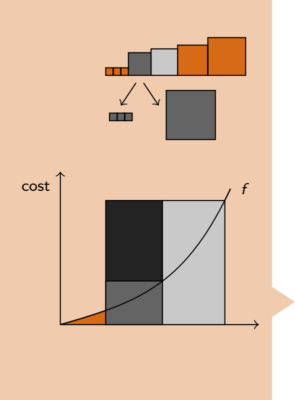
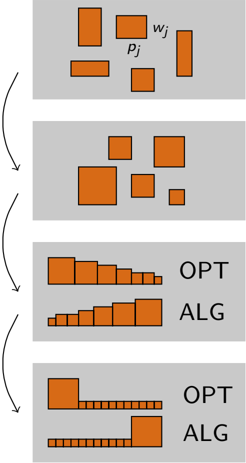
3. one big job & several very small jobs

Analysis of Smith's rule

Narrow space of worst-case instances for convex cost:





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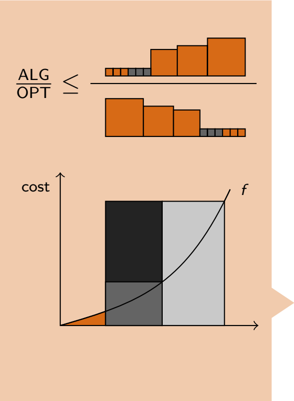

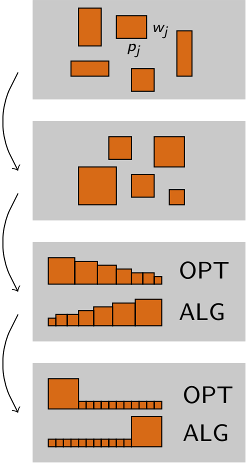

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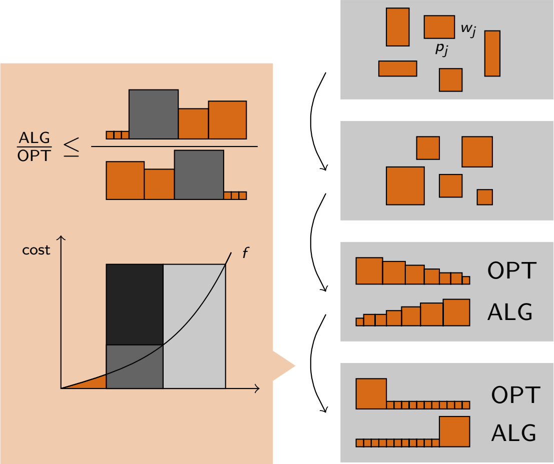

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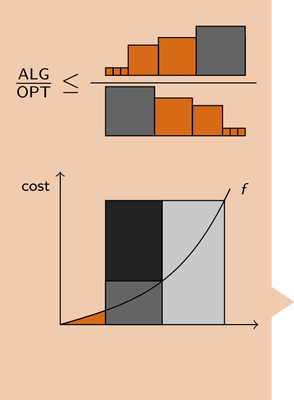

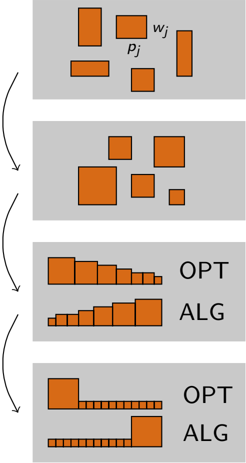

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Theorem

The tight approximation ratio of Smith's rule for fixed convex f is

$$\sup_{0<q,p} \frac{\int_0^q f(t)dt + p \cdot f(q+p)}{p \cdot f(p) + \int_p^{p+q} f(t)dt} \ .$$

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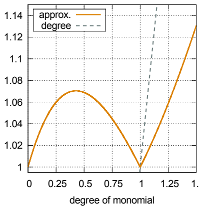
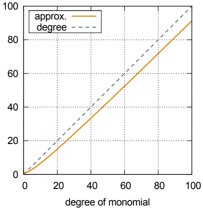
Corollary

If f is a polynomial of degree k with non-negative coefficients then the tight approximation ratio is

$$\alpha_k \ := \max_{0.5\le p<1} \frac{(1-p)^{k+1} \ + \ (k+1)p}{kp^{k+1} \ + \ 1} \ .$$

Tight approximation ratios for polynomials





cost function	ratio
square root	1.07
degree 2 polynomials	1.31
degree 3 polynomials	1.76
degree 4 polynomials	2.31
degree 5 polynomials	2.93
degree 6 polynomials	3.60
degree 10 polynomials	6.58
degree 20 polynomials	15.04
exponential	∞

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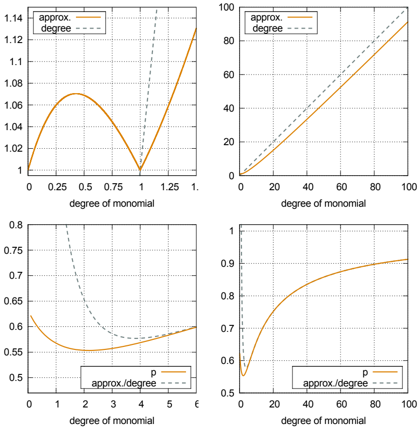

Observation: $\frac{\alpha_k}{k} \approx p_k \quad \left(\substack{\text{length of big job} \\ \text{corresponding to } \alpha_k}\right)$

Bounding the approximation ratio

Theorem

For cost function $f(t) = t^k$, the tight approximation factor α_k of Smith's ruler observes the following for $k \geq 4$:

- $\lim\limits_{k \to \infty} \left(p_k - \sqrt[k+1]{\frac{1}{k^2}} \right) = 0$,

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- $k - \alpha_k \;\geq\; \ln k - \frac{1}{2k}$.

Related computational results

Approximation ratio in experiments:

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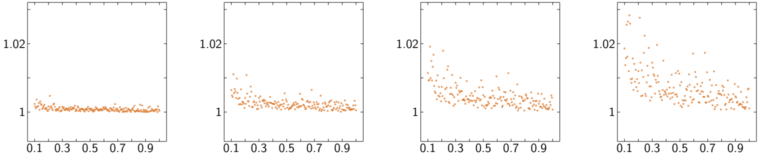

t^2 tight ratio 1.31 | t^3 tight ratio 1.76 | t^4 tight ratio 2.31 | t^5 tight ratio 2.93

x-value $\rightsquigarrow$ correlation of w_j and p_j

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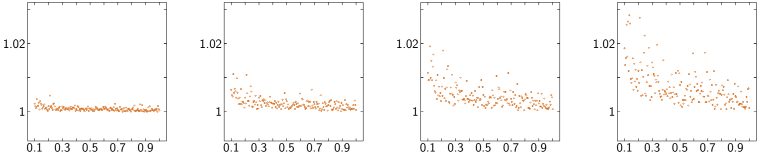

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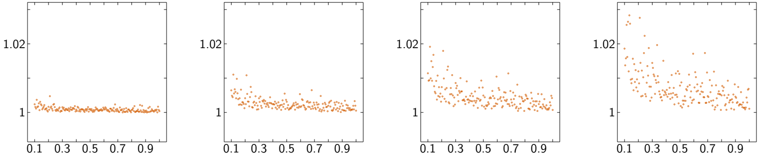

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x-value $\rightsquigarrow$ correlation of w_j and p_j

- $\rightsquigarrow$ experimental performance much better than worst-case
- $\rightsquigarrow$ more realistic analysis for processing times $1, 2, \ldots, p_{\max}$ and given $\sum p_j$

Parametrized analysis of Smith's rule

Theorem

The tight approximation ratio of Smith's rule for convex f and fixed parameters $p_{\max}$ and $\sum_j p_j$ is

$$\sup\left\{ \frac{\mathrm{INC}(p, p_{\max}, \sum_j p_j)}{\mathrm{DEC}(p, p_{\max}, \sum_j p_j)} \,\middle|\, p = 0, 1, 2, \ldots, \sum_j p_j \right\} .$$

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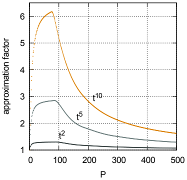
valuable lower bound for exact computations



$p_{\max} = 50$

Exact algorithms for quadratic cost

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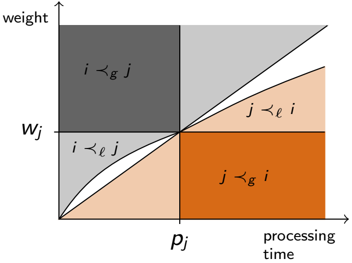

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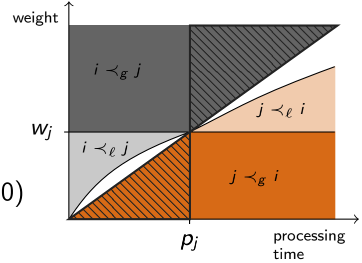

Conjecture Mondal, Sen (2000)

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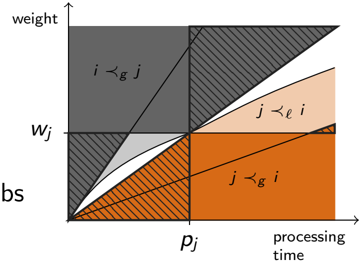
H., Jacobs



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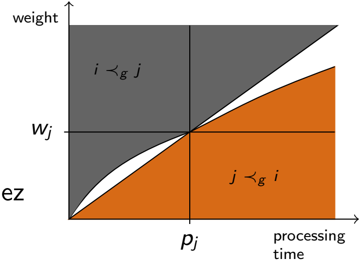

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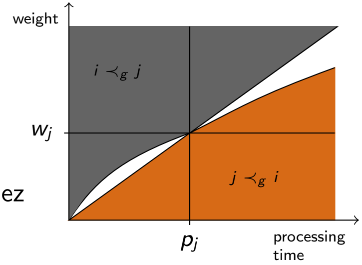

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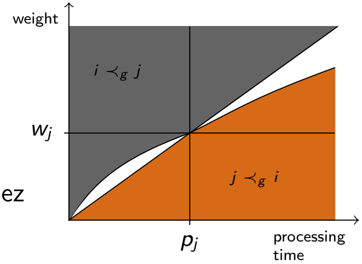

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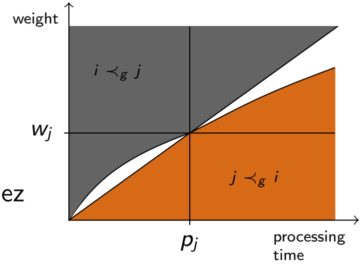

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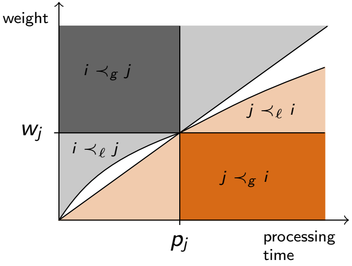

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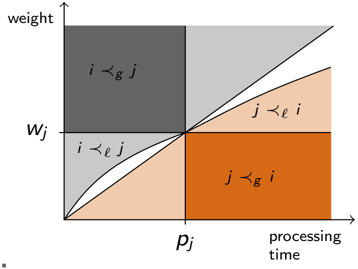

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(joint work with Jens Schulz & Daniela Luft)

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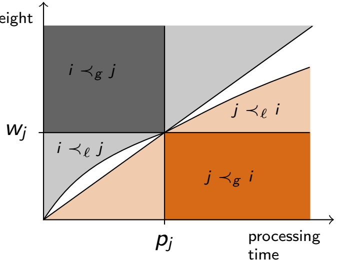

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