ABSTRACT

We propose a receiver diversity based code-timing estimator for DS-CDMA systems. The systems are assumed to work in a flat fading and near-far environment, where an arbitrary antenna array is used at the receiver of the system to achieve the spatial diversity. The algorithm is derived by modeling the known training sequence as the desired signal and all other signals including the multiuser interfering signals and the additive noise as unknown colored Gaussian noise so that the knowledge of the number of active users is not required. We show that by utilizing the information collected via multiple antenna sensors, the length of the training sequences can be greatly reduced. Moreover, the algorithm does not require the search over a parameter space and the code-timing is obtained by rooting a second-order polynomial, which is computationally very efficient. Simulation results show that the algorithm is quite robust against the near-far problem and requires a much shorter training sequence than the existing estimators.

1. INTRODUCTION

The ability to achieve code synchronization in a near-far DS-CDMA (direct-sequence code division multiple access) environment has been determined to limit the capacity of communication systems [1]. As a result, code-timing estimation has received much attention in recent years. Several estimators have been proposed in the literature [2]. However, all of these estimators are developed based on a single antenna sensor. The problem of estimating the code-timing of a desired user for a receiver diversity DS-CDMA system that uses multiple antenna sensors at the receiver has not been well studied. In [3], we have proposed a code-timing estimation algorithm for receiver diversity DS-CDMA systems. The algorithm was developed by using the information collected by an arbitrary antenna array consisting of multiple multiantenna sensors and is referred to as MASE (Multiple Antenna Sensors Based Estimator).

The MASE algorithm is designed for the system where the number of antenna sensors is relatively small. Typically, a few sensors are used for a system with 20 to 30 active users. The performance of MASE cannot be significantly improved by further increasing the number of antenna sensors when, for fair comparisons with a single antenna based methods, we assume that the noise variance is proportional to the number of antenna sensors used in the receiver. In this paper, we propose another code-timing estimator when a relatively large number of antenna sensors is used in the receiver. This new algorithm takes further advantage of the spatial diversity and is referred to as REDIVE (Receiver Diversity Based Estimator). Both MASE and REDIVE are asymptotic maximum likelihood estimators and are derived by modeling the known training sequence as the desired signal and all other signals as unknown colored Gaussian noise. Also, both algorithms do not require the search over the parameter space. The code-timing estimates are obtained by rooting a second-order polynomial. However, the specifics of the data model used in REDIVE are different from those in MASE. Simulation results show that the amount of computations required by REDIVE is about the same as that required by MASE. The length of training sequences required by REDIVE is significantly shorter than that required by MASE, but at a cost of more antenna sensors.

2. PROBLEM FORMULATION

Consider an asynchronous BPSK (binary phase shift keying) DS-CDMA system. The \( k \)th user transmits a signal of the form

\[
\tilde{x}_k(t) = \sqrt{2P_t} s_k(t) \cos(\omega_c t + \tilde{\theta}_k),
\]

where \( P_t \) is the user’s transmitted power, \( \omega_c \) is the carrier frequency, \( \tilde{\theta}_k \) is a random carrier phase uniformly distributed between 0 and \( 2\pi \), and \( s_k(t) = \sum_{m=0}^{M-1} d_k(m) \tilde{c}_k(t - mT_s) \) with \( M \) being the number of the data bits considered, \( T_s \) denoting the data bit duration, \( d_k(m) \in \{-1, +1\} \) denoting the value of the \( m \)th data bit, and \( \tilde{c}_k(t) = \sum_{n=0}^{N-1} c_k(n) \Pi_{T_s}(t - nT_c) \) being the spreading waveform in which \( c_k(n) \in \{-1, +1\} \), \( N = T_k/T_c \), and \( \Pi_{T_s}(t) \) denoting a unit rectangular pulse over the chip period \( T_c \). We use an arbitrary antenna array consisting of \( L \) antenna sensors at the receiver of the system. We consider the case of flat fading, where for each user, the time-delay differences due to multipath are negligible. For this case, we model the signal received by the \( l \)th sensor as

\[
\tilde{y}_l(t) = \sum_{k=1}^{K} \tilde{u}_{l,k} \tilde{x}_k(t - \tau_k) + \tilde{n}_l(t), \quad l = 1, 2, \ldots, L,
\]

where \( K \) is the number of users, \( \tilde{u}_{l,k} \) is the fading coefficient, \( \tau_k \) is the propagation delay, and \( \tilde{n}_l(t) \) denotes the channel noise. Given sufficient physical separation among the constituent antennas, the fading coefficients \( \{ \tilde{u}_{l,k} \} \) can be modeled as mutually independent random variables [4].
We assume that \(\{\tilde{a}_{k,l}\}\) are also independent of the random carrier phase \(\{\tilde{\theta}_k\}\) and the channel noise \(\{n_i(t)\}\). In addition, we assume that the transmitter and receiver have aligned their clocks to roughly within a bit interval. This could be done, for example, on a side "signalling channel", where a call is initially set up. Hence, we consider only the relative propagation delay, that is, \(\tau_k \in [0, T_c]\).

Assume that the receiver front-end consists of an IQ-mixer followed by an integrate-and-dump filter with integration time \(T_c\). The equivalent received complex sequence of the \(l\)th sensor, \(y_l(i)\), is described as

\[
y_l(i) = \sum_{k=1}^{K} a_{l,k} \sqrt{T_c} e^{j\theta_k} \frac{1}{T_c} \int_{(i-1)T_c}^{iT_c} \tilde{\delta}_k(t-\tau_k) dt + n_l(i),
\]

where \(\theta_k = \tilde{\theta}_k - \omega \tau_k, n_l(i)\) denotes the noise term assumed to be zero-mean complex white Gaussian with variance \(\sigma_n^2\), and \(a_{l,k}\) is the fading coefficient assumed to be zero-mean complex Gaussian with variance \(\sigma_a^2\).

Let \(\tau_k = \tilde{\tau}_k + \tilde{\delta}_k\), where \(\tilde{\delta}_k \in [0, T_c]\). The integration in the right-hand side of (3) is then given by

\[
r_k(i) \triangleq \frac{1}{T_c} \int_{(i-1)T_c}^{iT_c} \tilde{\delta}_k(t-\tau_k) dt = \left(1 - \frac{\tilde{\delta}_k}{T_c}\right) c_k(i - m_1 - N - p_k - 1) + \left(\frac{\tilde{\delta}_k}{T_c}\right) c_k(i - m_2 - N - p_k - 2) - d_k(m_2),
\]

where \(m_1\) and \(m_2\) are integers such that \(0 \leq i - m_1 - N - p_k - 1 \leq N - 1\) and \(0 \leq i - m_2 - N - p_k - 2 \leq N - 1\). Let

\[
c_k = [c_k(N - 1), c_k(N - 2), \ldots, c_k(0)]^T,
\]

and

\[
r_k(m) = [r_k(mN + N) \ldots r_k(mN + 1)]^T
\]

where \((\cdot)^T\) denotes the transpose. We then have

\[
r_k(m) = [a_1(\tau_k), a_2(\tau_k)] z_k(m) \triangleq A(\tau_k) \tilde{z}_k(m),
\]

where

\[
z_k(m) \triangleq \begin{bmatrix} z_k^{(1)}(m) \\ z_k^{(2)}(m) \end{bmatrix}^T \triangleq \begin{bmatrix} \delta_k + \frac{\delta_k}{T_c} J_{++}^{[p_k]} \\ d_k(m_2) - d_k(m-1) \end{bmatrix}^T \triangleq \begin{bmatrix} 1 - \frac{\tilde{\delta}_k}{T_c} J_{+-}^{[p_k]} \\ \frac{\tilde{\delta}_k}{T_c} J_{-\pm}^{[p_k]} \end{bmatrix} \end{bmatrix} c_k,
\]

and

\[
a_1(\tau_k) = \left[\begin{array}{c} \left(1 - \frac{\tilde{\delta}_k}{T_c}\right) J_{++}^{[p_k]} + \frac{\tilde{\delta}_k}{T_c} J_{-\pm}^{[p_k]} \end{array}\right] c_k,
\]

and

\[
a_2(\tau_k) = \left[\begin{array}{c} \left(1 - \frac{\tilde{\delta}_k}{T_c}\right) J_{++}^{[p_k]} - \frac{\tilde{\delta}_k}{T_c} J_{-\pm}^{[p_k]} \end{array}\right] c_k,
\]

with

\[
J_{ss}^{[p]} = \begin{bmatrix} 0 & 1_{p}^T \\ I_{p} & 0 \end{bmatrix}, \quad s = \pm 1,
\]

in which \(I_p\) denotes the \(p \times p\) identity matrix. Let

\[
y_l(m) = [y_l(mN + N) \ldots y_l(mN + 1)]^T
\]

and

\[
n_l(m) = [n_l(mN + N) \ldots n_l(mN + 1)]^T.
\]

Without loss of generality, assuming that the first user is the desired user, we can rewrite (3) as

\[
y_l(m) = a_{l,1} \sqrt{T_c} e^{j\theta_l} r_l(m) + e_l(m),
\]

where

\[
e_l(m) = \sum_{k=2}^{K} a_{l,k} \sqrt{T_c} e^{j\theta_k} r_k(m) + n_l(m)
\]

denotes the sum of the MAI and the additive noise. Let

\[
\beta_l = a_{l,1} \sqrt{T_c} e^{j\theta_l}.
\]

We then have

\[
y_l(m) = \beta_l A(\tau_l) \tilde{z}_l(m) + e_l(m), \quad l = 1, 2, \ldots, L.
\]

The problem of interest herein is to estimate \(\tau_l\) from \(\{\{y_l(m)\}_{m=0}^{M-1}\}_{l=1}^{L}\) assuming that \(\{c_i(n)\}_{n=0}^{N-1}\) and \(\{d_l(m)\}_{m=0}^{N-1}\) are known. Since the integer \(p_1\) has only \(N\) possible values \(\{0, 1, \ldots, N - 1\}\), which can be obtained by trying these values one by one, the problem becomes to estimate \(p_1 = \tilde{p}_1 / T_c\) with \(p_1\) being given.

3. THE REDIVE ALGORITHM

To utilize the receiver diversity, we arrange the output samples as follows:

\[
Y = \begin{bmatrix} y_1^T(1) & y_1^T(2) & \cdots & y_1^T(M) \\ \vdots & \vdots & \ddots & \vdots \\ y_L^T(1) & y_L^T(2) & \cdots & y_L^T(M) \end{bmatrix}, \quad L \times (M N).
\]

Let

\[
Z_l = \begin{bmatrix} z_l(0) & z_l(1) & \cdots & z_l(M - 1) \end{bmatrix}, \quad 2 \times M,
\]

and \(E\) be defined similarly to \(Y\). We have

\[
Y = \beta \text{vec} E^T \{A(\tau_l) Z_l\} + E,
\]

where \(\beta = [\beta_1, \beta_2, \ldots, \beta_L]^T\) and \(\text{vec}(X) = [x_1^T, x_2^T, \ldots, x_N^T]_{N \times N}^T\) with \(\{x_n\}_{n=0}^{N-1}\) being the columns of matrix \(X\). Let \(u = [1 - \mu, \mu]^T\). Then (20) can be written as

\[
Y = \beta u^T X + E,
\]

where

\[
X = \begin{bmatrix} J_{++}^{[p_1]} c_1 & J_{+\pm}^{[p_1]} c_1 \\ J_{--}^{[p_1]} c_1 & J_{-\pm}^{[p_1]} c_1 \end{bmatrix} (Z_1 \oplus I_N).
\]

Note that for a given \(p_1\), \(X\) is completely known. Let \(y_i, x_i,\) and \(e_i\) denote the \(i\)th columns of \(Y, X,\) and \(E\), respectively. Then

\[
y_i = (\beta u^T) x_i + e_i, \quad i = 1, 2, \ldots, (N M),
\]

where \(\{x_n\}_{n=0}^{N M}\) are known for a given \(p_1\). Due to the central limit theorem, we assume that \(e_i\) is independent of the desired signal and is a circularly symmetric complex Gaussian random vector with zero-mean and arbitrary covariance matrix \(Q\), that satisfies

\[
E \{e_i e_j^H\} = Q, \quad \forall i, j.
\]
where the unknown covariance matrix \( Q_s \) models both thermal noise and all other interference signals including MAI. It follows that the log-likelihood function is proportional to:

\[
C = - \ln |Q_s| - \text{tr} \left\{ Q_s^{-1} \frac{1}{NM} \sum_{i=1}^{NM} [y_i - Cx_i] [y_i - Cx_i]^H \right\},
\]

Minimizing (25) with respect to \( Q_s \) yields the ML estimate \( \hat{Q}_s \) of \( Q_s \):

\[
\hat{Q}_s = \frac{1}{NM} \sum_{i=1}^{NM} [y_i - Cx_i] [y_i - Cx_i]^H.
\]  

Inserting \( \hat{Q}_s \) into (25), we note that the estimate \( \hat{C} \) of \( C \) is determined by minimizing the following cost function

\[
C_1 = \left\| \frac{1}{NM} \sum_{i=1}^{NM} [y_i - Cx_i] [y_i - Cx_i]^H \right\|^2.
\]  

Let

\[
\hat{R}_{xx} = \frac{1}{NM} \sum_{i=1}^{NM} x_i x_i^H,
\]

\[
\hat{R}_{xy} = \frac{1}{NM} \sum_{i=1}^{NM} x_i y_i^H,
\]

\[
\hat{R}_{yy} = \frac{1}{NM} \sum_{i=1}^{NM} y_i y_i^H.
\]

The matrix in the right-hand side of (27) can be described as

\[
F \triangleq \frac{1}{NM} \sum_{i=1}^{NM} [y_i - Cx_i] [y_i - Cx_i]^H
\]

\[
= \left[ C - \hat{R}_{xx} \hat{R}_{xx}^{-1} \right] \hat{R}_{xx} \left[ C - \hat{R}_{xx} \hat{R}_{xx}^{-1} \right]^H
\]

\[
+ \hat{R}_{yy} - \hat{R}_{xx} \hat{R}_{xx}^{-1} \hat{R}_{xy}.
\]

Hence, the unstructured ML estimate of \( C \), which does not use the structure of \( C \), is given by

\[
\hat{C} = \hat{R}_{xx} \hat{R}_{xx}^{-1}.
\]

Using the \( \hat{C} \) in (33), we obtain the unstructured estimate \( \hat{Q}_s \) of \( Q_s \):

\[
\hat{Q}_s = \hat{R}_{yy} - \hat{R}_{xx} \hat{R}_{xx}^{-1} \hat{R}_{xy}.
\]

Consider now the structure of \( C \). The \( C_1 \) in (27) can be rewritten as

\[
C_1 = \| \left( C - \hat{C} \right) \hat{R}_{xx} (C - \hat{C})^H + \hat{Q}_s \|
\]

\[
= \| \hat{Q}_s \| \left[ 1 + \hat{R}_{xx}^{-1} (C - \hat{C}) (C - \hat{C})^H \right].
\]

Minimizing \( C_1 \) in (35) with respect to the unknown parameters in \( C \) requires a multidimensional search over the parameter space, which is computationally prohibitive. Hence, instead of determining the exact ML estimates of the unknown parameters, we determine the large sample approximate ML estimates as follows. By neglecting the second- and higher-order terms in the Taylor expansion of the gradient of \( \ln(C_1) \) with respect to the unknowns, we can show that minimizing \( \ln(C_1) \) is asymptotically (for large \( NM \)) equivalent to minimizing \([5]\)

\[
C_2 = \text{tr} \left[ \hat{R}_{xx} (\beta \hat{C} - \hat{C})^H \hat{Q}_s^{-1} (\beta \hat{C} - \hat{C}) \right].
\]  

Note that the number of known samples here is \( NM \), which is easily much larger than \( L \) for a moderate \( M \) when \( L \) is about the same as \( N \).

Let

\[
\hat{u} = \hat{R}_{xx} \hat{u}, \quad \hat{v} = [\hat{\mu}_1, \hat{\mu}_2]^T,
\]

and

\[
\hat{C} = \hat{C} \hat{R}_{xx} = [\hat{c}_1, \hat{c}_2].
\]

Then (36) can be written as

\[
C_2 = \left( \beta - \hat{C} \hat{u}/\|\hat{u}\|^2 \right)^H \|\hat{u}\|^2 \hat{Q}_s^{-1} \left( \beta - \hat{C} \hat{u}/\|\hat{u}\|^2 \right)
\]

\[
- \left( \hat{C} \hat{u} \right)^H \hat{Q}_s^{-1} \left( \hat{C} \hat{u} \right)/\|\hat{u}\|^2 + \text{constant},
\]

where \( \cdot \) denotes the complex conjugate. The minimization of \( C_2 \) is achieved when

\[
\hat{\beta} = \frac{\hat{C} \hat{u}}{\|\hat{u}\|^2} = \frac{\hat{C} \hat{R}_{xx} \hat{u}}{u^H R_{xx} u}
\]

and

\[
\hat{\mu} = \arg\max_{\mu} \left\{ u^H \hat{R}_{xx} \hat{C} \hat{R}_{xx} \hat{u}/u^H R_{xx} u \right\},
\]

which is obtained by rooting a second order polynomial.

4. CRB of the Parameter Estimates

Let

\[
\eta = [\mu \quad \text{Re}(\beta) \quad \text{Im}(\beta)]^T
\]

where \( \text{Re}(X) \) and \( \text{Im}(X) \), respectively, denote the real and imaginary part of \( X \). The CRBs for the parameter estimates of \( \eta \) can be written as

\[
\text{CRB}(\eta) = 2 \text{Re} \left\{ \left[ F_{\mu\beta} \quad F_{\beta\beta} \right] \right\}^{-1}
\]

where

\[
F_{\mu\beta} = \sum_{i=1}^{NM} x_i^H \left[ \begin{array}{cc} 1 & -1 \\ 1 & 1 \end{array} \right] x_i \beta^H Q_s^{-1} \beta,
\]

\[
F_{\beta\beta} = \sum_{i=1}^{NM} x_i^H \left[ \begin{array}{c} -\beta^T \\ \mu^T \end{array} \right] x_i \left[ \begin{array}{cc} \beta^H Q_s^{-1} -1_i & \cdots & \beta^H Q_s^{-1} -1_2 \end{array} \right],
\]

and

\[
F_{\beta\beta} = \sum_{i=1}^{NM} x_i^H \left[ \begin{array}{c} \mu^T \end{array} \right] Q_s^{-1}.
\]
which is computationally very efficient. Simulation results

and REDIVE with different $L$ along with the MMSE estimator. It is seen that MMSE performs poorly. It is also seen that REDIVE is significantly better than MASE. Yet the average numbers of MATLAB flops required by REDIVE with $L = 20$ and 30, respectively, are about 0.8 and 1.5 times as much as those required by MASE.

In Figure 2, the performance of the REDIVE algorithm is compared with the MASE algorithm and the CRB as a function of $M$. It is seen that as $M$ increases, the performance of the REDIVE algorithm approaches the CRB. The RMSE (root mean-squared error) of REDIVE is much smaller than that of MASE.

6. CONCLUSIONS

We have proposed a code-timing estimator for receiver diversity DS-CDMA systems. The systems are considered working in a flat fading and near-far environment. The algorithm has been derived by modeling the known training sequence as the desired signal and all other signals including the multiuser interfering signals and the additive noise as unknown colored Gaussian noise so that the knowledge of the number of active users is not required. We have shown that by utilizing the information collected via multiple antenna sensors, the length of training sequences can be greatly reduced. The algorithm is an asymptotic maximum likelihood estimator. As a result, the mean-squared error of the code-timing estimates obtained by the algorithm approaches the Cramer-Rao lower bound as the length of the training sequence increases. Moreover, the algorithm does not require the search over a parameter space and the code-timing is obtained by rooting a second-order polynomial, which is computationally very efficient. Simulation results

have shown that the algorithm is quite robust against the near-far problem and requires a much shorter training sequence than the existing estimators.

REFERENCES


