Exchange Rates and Inflation Rates: Exploring Nonlinear Relationships[1]

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Abstract: This paper investigates the Purchasing Power Parity Theory (PPP) in the context of possible nonlinear relationships between prices and exchange rates of three key currencies. The main contribution of this paper is testing for nonlinearities and nonlinear relationships in a framework of information arrival. Three issues motivated the paper. First, research interest in exchange rate fluctuations, PPP, and exchange rate pass through. Second, market volatilities have triggered curiosity in nonlineraities in various economic and financial time series and nonlinear relationships and chaos in financial markets. For instance, the study of chaotic behavior may shed some light on the nature of latent nonlinearities. Third, developments in the econometrics of nonlinearity in the last three decades offer researchers new tools for detecting relationships that are inherently nonlinear and may not be conducive to various methodologies that are seeking to impose linear modeling on nonlinear relationships.

Our findings show strong evidence that the exchange series exhibit nonlinear dependencies that may be inconsistent with chaotic structure. We identify GARCH(1,1) process as the model that best explains the nonlinearities in the monthly exchange rates and inflation rates. Therefore, we propose and estimate bivariate GARCH(1,1) models of the variances to ascertain the flow of information between exchange rates and prices. Bivariate GARCH models show that, the volatility spillover and information arrival between exchange rates and price levels occur simultaneously. Thus, we find support for the PPP theory and exchange rate pass through in the economies under consideration. We conclude that two theories, i.e., the exchange rate pass through and PPP may simultaneously hold the key to exchange rate analysis.

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1. Introduction

A theory of exchange rate determination in the long-run, is the purchasing power parity theory (PPP). The basic idea behind the PPP is the law of one price (LOP). It holds that prices of tradable goods would equalize in all markets once they are converted to the same currency. If not, there are opportunities for arbitrage, which lead to adjustments in exchange rates until the law of one price is restored. Thus, in the most basic terms, the PPP links the exchange rates of currencies and the price of tradable goods in terms of various currencies. It also suggests an equilibrium value for currencies, which is the market exchange rate. The PPP has played a significant role in understanding exchange rate fluctuations. For instance, it has been employed to predict the equilibrium bilateral exchange rates between currencies based on the trends of consumer price indices or other measures of the general price levels.

The limitations of the LOP in stimulating effective arbitrage in international markets even for homogenous goods, may be one of the reasons why tests of PPP based on price indices produce disappointing results regarding the validity of the PPP. For instance, Huizinga (1987) and Meese and Rogoff (1988) fail to reject the hypothesis that exchange rates follow a random walk process. Frankel (1990) argues that if exchange rate deviations from PPP dampen slowly, it will be difficult to statistically or otherwise observe the convergence to rates justified by the PPP. Using a data set spanning 1869-1984 for dollar/pound exchange rates, he was able to reject the random walk hypothesis for this exchange rate series. In the 1990s, more studies (see Diebold et. al (1991) and Glen (1992)) and using data sets spanning longer time periods reject the random walk hypothesis and find support for mean reversion in exchange rates. However, these studies suffer from mixing the pre 1973 fixed exchange rates with floating exchange rates of the post 1973 era.

A survey of the literature on the law of one price (LOP) implies that the magnitude of discrepancies in prices are large, the speed of adjustment slow, and arbitrage processes that could potentially restore the exchange rate adjustment are imperfect. For instance, Isard(1976), Richardson (1978), Giovannini (1988), among others have shown that the deviations of prices from the LOP are large, persistent and correlated with exchange rate movements. Engel and Rogers (1995) and Engel (1993) show that price volatility within the same country and across borders are not consistent with the LOP. As we mentioned above, transportation costs, tariffs and non-tariff barriers, and marketing considerations surrounding pricing, contribute to factors that impede efficient arbitrage process and limit the effect of the process on equating prices in international markets.

Despite the theoretical soundness of the law of one price and the PPP, empirical findings have lent mixed support for the PPP. The empirical evidence may be divided into the studies of the 1990s and the decade of 2000. Several studies in 1990s support PPP at the level of heavily traded commodities such as gold and crude oil, among others. (See Froot and Rogoff (1996), Hakkio (1992), Frankel and Rose (1995), Mac Donald and Taylor (1992)).

Other studies have attempted to test the random walk hypothesis versus mean reversion based on the PPP using cross sectional data. For instance, Wei and Parsley (1995) test annual data for fourteen OECD countries for tradable goods. Consistent with previous studies they find that the exchange rate deviations from the PPP on average take around 4.5 years to dampen toward equilibrium rates set by the PPP. Thus, the long term and cross sectional data have reached a consensus by showing that the deviations of exchange rates from PPP tend to dampen out slowly and mean reversion is achieved, albeit at a slow rate.

More recent studies in the last decade have applied cointegration tests to test for PPP. For instance, Hong and Phillips (2005) apply a modified cointegration test to examine the linear purchasing power parity (PPP) specification using US, Japan, and Canada monthly data after Bretton Woods. While commonly used ADF and PP cointegration tests give mixed results on the presence of cointegration, the modified test rejects the null of cointegration. Similarly, Bahmani-Oskooee and

Goswami (2005) employ monthly data from eight developing Asian countries over a thirty-one-year period to test for the purchasing power parity (PPP) in the black market . They show that while the model variables are cointegrated in a Johansen-Juselius framework, the domestic price and the foreign price are not weakly exogenous in many countries, and because of that a direct test provides the rejection of the PPP hypothesis.

Koekijk,Tims and van Dijk (2004) analyze purchasing power parity (PPP) for the euro zone. They test the PPP hypothesis for a panel of real exchange rates within the euro area over the period 1973-2003. Their empirical findings lend support for the PPP hypothesis for the full panel of real exchange rates. They conclude that the process of economic integration in Europe has accelerated convergence toward PPP within the euro area.

Baum, Caglayan, and Barkoulas (1998) model the dynamics of adjustment to long-run purchasing power parity (PPP) over the post-Bretton Woods period in a nonlinear framework. They estimate exponential smooth transition autoregressive (ESTAR) models of deviations from PPP using both CPI- and WPI-based measures for a broad set of U.S. trading partners. Their findings show evidence of a mean-reversion which support the PPP.

The survey of literature indicates that the empirical evidence on the PPP theory is far from conclusive. This paper continues our research on the behavior of bilateral daily exchange rates. The premise of our paper is that the previous research might have overlooked the possibilities of nonlinearities in the exchange rate and inflation rate series. We investigate nonlinearities and chaotic patterns in three major bilateral exchange rates versus the dollar and inflation rates in the countries under study. While we are basing our paper on the PPP theory, we examine a less restrictive relationship between inflation rates and exchange rates than the one implied by the PPP. The main contribution of this paper is testing for nonlinearities and nonlinear relationships in the framework of information arrival and exchange.

Our paper is motivated by the following issues. First, economists have long been interested in exchange rate fluctuations, PPP, and exchange rate pass through. However, as we have shown, the empirical evidence on PPP is mixed and further investigation is necessary given the importance of the topic.

Second, the volatility in financial and currency markets has generated interest in investigating various forms of nonlinearities and chaotic behavior in the exchange markets. For instance, the study of the chaotic behavior may shed some light on the performance of technical analysis in financial markets. Technical analysis has been used in forecasting other financial time series and may be successful in forecasting short-term fluctuations in the dollar if the series is nonlinear and/or chaotic (see for example, Blume, Easley, and O'Hara (1994), Bohan (1981), Brock, Lakonishok, and LeBaron (1992), Brush (1986), Clyde and Osler (1997), LeBaron (1991), Pruitt and White (1988, 1989), Taylor (1994), among others).

Third, developments in the econometrics of nonlinearity in the last three decades offer researchers new tools for detecting relationships that are inherently nonlinear and may not be conducive to various methodologies that are seeking to impose linear modeling on nonlinear relationships. The recent studies using cointegration tests would fall in this category.

Typical linear models assume that time series under study are linearly related to underlying shocks that form the series. However, if there are nonlinearities, the time series and past shocks are related through a nonlinear relationship. In these cases the time series may be nonlinear in mean or variance or both. Time series that are nonlinear in mean allow for nonzero higher moments. Those with nonlinearities in variance, under certain conditions, posses higher order moments with nonzero values. Various ARCH and GARCH models may be capable of explaining these nonlinearities.

While there have been advances in modeling deterministic nonlinear systems, their application in economics and finance has been limited for several reasons. First, unlike natural sciences, economic theory does not provide specific nonlinear functional forms in modeling the time series behavior. Second, controlled experiments are almost impossible in economics, preventing economists from deriving the parameters of deterministic non-linear systems underlying relationships among economic variables. Despite the above limitations, testing for nonlinearities and chaotic structures has made inroads in financial and economic research.

Drawing on existing tests of nonlinearities and chaos, we first investigate the existence of chaotic behavior as the source of nonlinearities in the monthly bilateral exchange rates of the Swiss franc, the British pound, and the Japanese yen against the US dollar and the price levels in each country. To accomplish this task, we estimate AR(1) and GARCH(1,1) models for each series. The filtered series, i.e., the model residuals are tested for chaos to see if there are any lingering nonlinearities originating from chaotic behavior in the series. If so, one would conclude that methods of investigation that are inherently seeking to establish liner relationship between price levels and exchange rates would fail to ferret out the underlying nonlinear relationships. These methods would include estimating correlation coefficient, linear regressions, and cointegration tests. If on the other hand, chaos is not the source of nonlinearities, then models that properly capture the underlying nonlinearities may be better-suited to explain the relationship between the variables.

Our findings show strong evidence that the three bilateral exchange rate series exhibit nonlinear dependencies. However, we also find evidence that the series behavior may be inconsistent with chaotic structure. We identify a GARCH(1,1) process as the model that best explains the nonlinearities in the monthly exchange rates and inflation rates. Therefore, we propose a bivariate GARCH(1,1) model for exchange and inflation rates to ascertain the flow of information between exchange rates and prices. We show that the information flow between exchange rates and prices occurs simultaneously through volatility spill-over in both directions. This result confirms that the information transmission between prices and exchange rates occur simultaneously giving support for both the PPP and exchange rate pass through theory. Our findings are particularly compelling because they confirm the power of commonly regarded nonlinear models in explaining the behavior of the exchange rates.

This paper is organized as follows. First we discuss the methodology of the paper. Then we proceed to explain the sources of data and present the summary statistics in Section III. Section IV offers our main empirical findings. A brief summary and conclusion is presented in Section V.

2. Methodology and tests for chaos

This paper starts out by analyzing exchange rates and inflation rates for stationarity, non-linearities, and ARCH effects based on commonly know statistics. As mentioned above, we are particularly concerned with observing and detecting the sources of nonlinearities in each time series process. To rule in or out the existence of chaotic behavior, we apply the Brock, Dechert, and Scheinkman (1987) test (BDS) and Correlation Dimension tests of chaos to each series. We find that while nonlinearities are present, these nonlinearities are not consistent with chaotic patterns. We propose and estimate autoregressive models for the exchange rates and inflation rates, along with bivariate GARCH(1,1) models of variances for the three exchange rate and inflation rate series and show evidence that volatility spillovers occur across prices and exchange rates.

The common tests of chaos are discussed in Adrangi et al. (2001a), Adrangi et al. (2001b), and Adrangi et al. (2004). We present them briefly in this paper to inform the readers. There are two tests that we employ here: (i) the Correlation Dimension of Grassberger and Procaccia (1983) and Takens (1984), (ii) the BDS statistic of Brock, Dechert, and Scheinkman (1987).

2.1 Correlation dimensions

Consider the stationary time series x_t, t =1...T. One imbeds x_t in an m-dimensional space by forming M-histories starting at each date t: $x_t^2 = \{x_t, x_{t+1}\}$,.., $x_t^M = \{x_t, x_{t+1}, {}_{t+2},......,x_{t+M-1}\}$. One employs the stack of these scalars to carry out the analysis. If the true system is n-dimensional, provided M ≥ 2n+1, the M-histories can help recreate the dynamics of the underlying system, if they exist (Takens (1984)). For a given embedding dimension M and a distance ε, the correlation integral is given by

$$C^M(\varepsilon) = \lim_{T\to\infty} \{\text{the number of (i,j) for which } \left\|x_i^M - x_j^M\right\| \le \varepsilon\}/T^2 \quad (1)$$

where $\|\cdot\|$ is the distance induced by the norm. For small values of ε, one has $C^M(\varepsilon)$~ε^D where D is the dimension of the system (see Grassberger and Procaccia (1983)). The Correlation Dimension in embedding dimension M is given by

$$D^M = \lim_{\varepsilon\to 0}\lim_{T\to 0}\{\ln C^M(\varepsilon)/\ln\varepsilon\} \quad (2)$$

and the Correlation Dimension is itself given by

$$D = \lim_{M\to 0} \ln D^M \quad (3)$$

We estimate the statistic

$$SC^M = \frac{\{\ \ln C^M(\varepsilon_i) - \ln C^M(\varepsilon_{i-1})\}}{\{\ \ln(\varepsilon_i) - \ln(\varepsilon_{i-1})\}} \quad (4)$$

for various levels of M (e.g., Brock and Sayers (1988)). The SC^M statistic is a local estimate of the slope of the C^M versus e function. Following Frank and Stengos (1989), we take the average of the three highest values of SC^M for each embedding dimension.

2.2 BDS statistics

Brock, Dechert and Scheinkman (1987) employ the correlation integral to obtain a statistical test that has been shown to have strong power in detecting various types of nonlinearity as well as deterministic chaos. BDS show that if x_t is (i.i.d) with a nondegenerate distribution,

$$C^M(\varepsilon) \to C^1(\varepsilon)^M \quad ,as \quad T \to infinity \quad (5)$$

for fixed M and ε. Employing this property, BDS show that the statistic

$$W^M(\varepsilon) = \sqrt{T}\{[C^M(\varepsilon) - C^1(\varepsilon)^M]/\sigma^M(\varepsilon)\} \quad (6)$$

where σ^M, the standard deviation of [·], has a limiting standard normal distribution under the null hypothesis of IID. W^M terms the BDS statistic. Nonlinearity will be established if W^M is significant for a stationary series void of linear dependence. The absence of chaos will be suggested if it is demonstrated that the nonlinear structure arises from a known non-deterministic system.

3. Data and summary statistics

We utilize the monthly bilateral dollar exchange rate series from January, 1974 through May, 2009, thereby covering the time period when the value of the dollar has been determined in a free float foreign exchange market system. Although we would prefer high frequency data, we leave that for a future work because daily inflation rates are not normally considered in many contexts. Percentage changes in exchange rates and price levels are obtained by taking the ratio of log of the exchange rates as in $R_t = (\ln(P_t/P_{t-1}))\cdot 100$, where P_t represents the monthly values at month t.

Table 1 presents the diagnostics for the R_t series. The returns series are found to be stationary employing the Augmented Dickey Fuller (ADF) statistics. There are linear and nonlinear dependencies as indicated by the Q and Q^2 statistics, and Autoregressive Conditional Heteroskedasticity (ARCH) effects are suggested by the ARCH(6) chi-square statistic. Table 1 summarizes our findings as follows: (i) there are clear indications that nonlinear dynamics are generating the monthly exchange values, (ii) these nonlinearities may be explained by ARCH effects, and (iii) whether these dynamics are chaotic in origin is the question that we turn to next.

Table 1. Diagnostics

Table 1 presents the percentage return diagnostics for three exchange rates (monthly data). Returns are given by $R_t = \ln(P_t/P_{t-1}) \cdot 100$, where P_t represents closing exchange rate on day t. ADF represents the Augmented Dickey Fuller tests (Dickey and Fuller (1981)) for unit root. The Q(12) and $Q^2(12)$ statistics represent the Ljung-Box (Q) statistics for autocorrelation of the R_t and R_t^2 series respectively. The ARCH(6) statistic is the Engle (1982) test for ARCH (of order 6) and is χ^2 distributed with 6 degrees of freedom. [a] and [b] represent significance levels of .01 and .05, respectively .

Panel A: Exchange Rates, Monthly

	Yen	S Franc	Pound
Interval: January 1974-May 2009			
N = 424			
Mean	0.108	0.042	-0.110
SD	1.277	1.097	1.217
ADF	-4.80[a]	-4.61[a]	-4.11[a]
Q(12)	186.42[a]	4.96[a]	60.671[a]
$Q^2(12)$	27.51[a]	13.36	48.56[a]
ARCH (6)	12.99[a]	12.24[a]	18.34[a]

Panel B: Inflation Rates, Monthly

	Japan	Switzerland	UK
Interval: January 1974-April 2009			
N = 423			
Mean	0.028	0.041	0.183
SD	0.230	0.164	0.445
ADF	-4.46[a]	-3.89 [a]	-4.03[a]
Q(12)	382.43[a]	205.59[a]	69.40[a]
$Q^2(12)$	42.42[a]	30.42[a]	3.72 [a]
ARCH (6)	26.15[a]	61.22[a]	82.44[a]

To capture the linear structure, we first estimate an autoregressive model of exchange rates and inflation rates as follows:

$$ER_t = \sum_{i=1}^{p} \pi_i ER_{t-i} + \varepsilon_t \qquad (7)$$

and

$$INF_t = \sum_{i=1}^{p} \gamma_i INF_{t-i} + \xi_t$$

where, ER represents percentage changes in the exchange rates, and INF measures monthly inflation rates. The lag length for each series is selected based on the Akaike (1974) criterion. The residual term (ε_t) represents the index movements that are purged of linear relationships and seasonal influences.

The analysis of the summary statistics indicates that the exchange rates of the pound and yen show a higher degree of volatility as evidenced by the standard deviation of the monthly exchange rate variations of the percentage returns. As for inflation rates, the inflation rate in Britain indicates a higher price variation than the other economies in the sample. Monthly exchange rate variations and inflation rate series are stationary, but there are linear and nonlinear dependencies in the series shown by the Q and Q^2 statistics. The Lagrange multiplier tests show that the nonlinearities in both the exchange rate and inflation rate series may be due to the ARCH effects.

Now, we turn our attention to testing for the source of these nonlinearities. The correlation dimension and BDS statistics are employed to see if the nonlinearities are consistent with chaos.

4. Empirical findings

4.1 Correlation dimension estimates

Table 2 reports the Correlation Dimension (SC^M) estimates for various exchange rate and price series alongside that for the Logistic series that we developed.

Table 2. Correlation Dimension Estimates, Exchange Rates and Inflation Rates

This table reports SC^M statistics for the Logistic series (w=3.750, n=2000), monthly percentage changes in exchange rates and inflation rates over four embedding dimensions: 5, 10, 15, 20. AR(1) represents autoregressive order one residuals.

M =	5	10	15	20
Logistic	1.02	1.00	1.03	1.06
Logistic AR	0.96	1.06	1.09	1.07
UK				
Pound AR(1)	3.26	6.07	9.75	13.73
Infl. AR(1)	3.78	6.85	9.53	11.79
Switzerland				
Sfranc AR(1)	3.84	7.09	10.02	12.07
Infl. AR(1)	3.77	6.82	9.24	10.93
Japan				
Yen AR(1)	4.12	4.06	2.55	4.75
Infl. AR(1)	2.45	4.37	5.72	7.08

The values of the correlation dimension for chaotic series and its filtered version shown in the first two rows of the Table do not show an explosive trend. For instance, SC^M estimates for the logistic map stay around one as the embedding dimension rises. Furthermore, the estimates for the logistic series are not sensitive to the AR transformation, consistent with chaotic behavior.

For the exchange rate and inflation rate series, on the other hand, SC^M estimates show inconsistent behavior with chaotic structures. For instance, the SC^M does not settle. The estimates for the AR transformation do not change results much, but are mostly larger and do not settle with increasing of the embedding dimension. These initial indicators suggest that the series under consideration are not showing signs of chaos.

4.2 BDS Test results

Tables 3 and 4 report the BDS statistics (Brock, Dechert and Scheinkman (1987)) for [AR(p)] series, and standardized residuals ($\varepsilon/\sqrt{h}$) from the GARCH(1,1) models, respectively. The BDS statistics are evaluated against critical values obtained by bootstrapping the null distribution for each of the GARCH models. The critical values for the BDS statistics are reported in Adrangi et al. (2001a), Adrangi et al. (2001b), and Adrangi et al. (2004).

The BDS statistics strongly reject the null of no nonlinearity in the [AR(1)] errors for all of the return series. However, BDS statistics for the standardized residuals from the GARCH-type models are mostly insignificant at the .01 and .05 levels. On the whole, the BDS test results provide compelling evidence that the nonlinear dependencies in the exchange rate and inflation rate series arise from GARCH-type effects, rather than from a complex, chaotic structure.

Table 3. BDS statistics for AR(1) residuals

The figures are BDS statistics for the AR(p). Here, the upper marks [a], [b], and [c] represent the significance levels of .01, .05, and .10, respectively.

Panel A: Exchange Rates

	M			
ε/σ	2	3	4	5
BP				
0.50	1.757	2.529 [b]	3.826 [a]	4.985 [a]
1.00	2.585 [b]	3.729 [a]	4.978 [a]	5.567 [a]
1.50	2.640 [b]	3.451 [a]	4.454 [a]	4.848 [a]
2.00	2.501 [b]	3.305 [a]	4.289 [a]	4.715 [a]
SF				
0.50	0.245	1.968	2.615 [b]	3.487 [a]
1.00	0.401	1.353	1.873 [c]	2.186 [c]
1.50	-0.293	0.520	0.984	1.012
2.00	-0.564	0.109	0.508	0.554
JY				
0.50	1.423	1.682	1.498	1.057
1.00	1.710	1.468	1.030	1.250
1.50	1.813 [c]	1.628	1.421	1.614
2.00	1.850 [c]	1.360	1.026	1.348

Panel B: Inflation Rates

	M			
ε/σ	2	3	4	5
UK				
0.50	0.147	0.174	0.202	0.242
1.00	0.133	0.153	0.170	0.187
1.50	0.123	0.142	0.154	0.162
2.00	0.108	0.102	9.265 [a]	8.492 [a]
SW				
0.50	7.416[a]	0.380	0.763	-0.433
1.00	-1.184	2.610[b]	7.170[a]	-6.004 [a]
1.50	0.081	1.111	4.493[a]	0.130
2.00	2.948[b]	0.942	1.251	1.350
JP				
0.50	4.213[a]	6.265[a]	8.133[a]	9.309[a]
1.00	4.989[a]	5.945[a]	6.978[a]	7.415[a]
1.50	6.490[a]	7.099[a]	7.646[a]	7.894[a]
2.00	7.177[a]	7.606[a]	8.191[a]	8.297[a]

Notes: UK, SW, and JP represent the United Kingdom, Switzerland, and Japan. BP, SF, and JY stand for the British Pound, Japan yen and Swiss franc.

From the BDS statistics presented in Table 4 on the next page, it is apparent that the variations of the GARCH model may explain the nonlinearities in the dollar series.

Table 4. BDS Statistics for GARCH(1,1) standardized residuals

The figures are BDS statistics for the standardized residuals from GARCH(1,1) models. The BDS statistics are evaluated against critical values obtained from Monte Carlo simulations (available from authors). The upper marks [a], [b], and [c] represent the significance levels of .01, .05, and .10, respectively.

Panel A: Exchange Rates

	M			
ε/σ	2	3	4	5
BP_gar11				
0.50	-0.275	-0.372	-0.107	-0.132
1.00	-0.676	-0.370	-0.414	0.798
1.50	-0.467	-0.383	0.107	0.163
2.00	-0.290	-0.429	-0.124	-0.116
SF_gar11				
0.50	1.439	3.016[a]	3.509[a]	4.645[a]
1.00	1.349	2.315	2.795[b]	3.127[a]
1.50	0.683	1.515	1.912[c]	1.912[c]
2.00	0.123	0.937	1.314	1.336
JY_gar11				
0.50	1.028	0.167	0.169	-0.122
1.00	0.736	0.030	-0.536	-0.542
1.50	0.696	0.139	-0.229	-0.252
2.00	1.000	0.174	-0.356	-0.257

Panel B: Inflation Rates

	M			
ε/σ	2	3	4	5
UK_gar11				
0.50	7.381[a]	9.695[a]	11.453[a]	12.873[a]
1.00	7.346[a]	8.943[a]	10.225[a]	11.138[a]
1.50	7.591[a]	8.640[a]	9.398[a]	9.879[a]
2.00	7.516[a]	8.326[a]	8.907[a]	8.854[a]
SW_gar11				
0.50	-0.130	-0.511	-0.507	-0.208
1.00	-0.129	-1.649	-0.780	0.041
1.50	-1.204	-1.318	-0.439	0.122
2.00	-0.720	-0.590	0.172	0.653
JP_gar11				
0.50	0.766	1.713	2.281[b]	2.403[b]
1.00	0.104	0.605	1.621	1.749
1.50	-0.580	-0.491	-0.438	-0.706
2.00	-1.018	-1.137	-1.335	0.213

4.3 Bi-variate GARCH models

To model the relationship between the exchange rate and inflation rate while simultaneously accounting for the nonlinearities stemming from GARCH effects, we estimate three VAR models in a bivariate GARCH context. Zellner and Palm (1974) and Zellner (1979) show that a VAR represents a flexible approximation to any wide range of simultaneous structural models and may be viewed as Taylor series approximation for nonlinear models as well. Thus, we propose the following VAR model for the remainder of our empirical investigation.

$$ER_t = \sum_{i=1}^{p} \pi_{t-i}\, ER_{t-i} + \sum_{i=1}^{p} \lambda_i INf_{t-i} + \varepsilon_t$$
$$INF_t = \sum_{i=1}^{p} \gamma_i\, INF_{t-i} + \sum_{i=1}^{p} \eta_i\, ER_{t-i} + \xi_t \qquad (8)$$

As indicated above, the variance of the error terms in the above VAR equations are time -varying.

Theory suggests that informed trading in financial assets, including currencies, will induce persisting changes in the volatility of those series (Kyle (1985)), and there is a great deal of evidence that many financial price series exhibit time varying volatility. Specific to debt securities, several researchers have argued that interest rate risk premia are time variant (for instance, Shiller (1979) and Singleton (1980)). Weiss (1984), Engle, Ng, and Rothschild (1990), and Engle, Lilien, and Robins (1987) find significant ARCH effects or serial correlation in variances in short term rates over several decades. In the present study, variance persistence or clustering may arise from market features unique to exchange rate markets.

There is also reason to suspect that these variance effects are correlated across the two variables. For example, Engle, Ng, and Rothschild (1990) indicate that the underlying forces behind volatility for shorter end of term structure are common across different rates - indicative of co-persistence of variance. Such co-persistence will have important implications for an empirical analysis of variance behavior.

While currencies and other asset prices may exhibit high variance persistence in their univariate representations, this persistence may be common across different and related series, so that linear combinations of the variables show lesser volatility persistence. Ross (1989) argues that volatility may be regarded as a measure of information flow. Thus, if information arrives first in one market, one should see a volatility spillover from that market to others. Therefore, to study the exchange rate and inflation rate dynamics, an appropriate extension to the above VAR model will be employed to simultaneously allow for time varying volatility and volatility spillovers between exchange rates and the general price levels.

The statistics in Table 1 justify some of the above suspicions in the relationship between exchange rates and inflation rates. The Ljung-Box Q(12) and Q ʔ(12) statistics indicate significant levels of serial correlation in the returns and the square of the returns. These statistics indicate linear and nonlinear dependencies in monthly inflation levels and exchange rates. Test statistics for ARCH errors (Engle (1982)) further suggest serial correlation in the errors. On the other hand, there is less evidence of serial dependencies in the standardized residuals from fitting the returns to a GARCH(1,1) model.[2] The Q(12) statistics are substantially smaller and the Q ʔ(12) statistics are smaller or insignificant. Such evidence indicates that the most basic GARCH model effectively captures the nonlinearities in the data. Moreover, the standardized residuals exhibit relatively smaller kurtosis, further evidence of the GARCH model providing a superior fit to the data (Hsieh (1989)).

[2] We choose Bollerslev's (1986) GARCH (1,1) model over higher order ARCH or GARCH models due to the strong support found for this model in recent work. Moreover, the GARCH (1,1) model, with its fewer parameters, is more viable in a multivariate setting (Baillie and Bollerslev (1990)).

To be able to investigate the volatility spillovers and information arrival in the context of our paper, we propose the VAR model of equation (8) while simultaneously controlling for the likely variance and covariance persistence via the bivariate GARCH model (similar models have been employed by Hamao, Masulis and Ng (1990), Chan, Chan and Karolyi (1991), and Chatrath and Song (1998), among others). The following three equations achieve this goal.

$$\sigma_{ex,t} = \alpha_0 + \alpha_1 \sigma_{ex,t-1} + \alpha_2 \varepsilon^2_{ex,t-1} + \alpha_3 \varepsilon^2_{p,t-1} \quad (9)$$

$$\sigma_{p,t} = \beta_0 + \beta_1 \sigma_{p,t-1} + \beta_2 \varepsilon^2_{p,t-1} + \beta_3 \varepsilon^2_{ex,t-1} \quad (10)$$

$$\sigma_{\exp,t} = \alpha_0 \beta_0 + \alpha_1 \beta_1 \sigma_{\exp,t-1} + \alpha_2 \beta_2 \varepsilon_{ex,t-1} \varepsilon_{p,t-1} \quad (11)$$

Assuming that the vector of error terms is t-distributed with zero expected values and time-varying covariance matrix, i.e.:

$$\begin{pmatrix} \hat{\varepsilon}_{ex,t} \\ \hat{\varepsilon}_{p,t} \end{pmatrix} | \Omega_{t-1} \_ Studentt(\begin{pmatrix} 0 \\ 0 \end{pmatrix} \begin{pmatrix} \sigma_{ex,t} & \sigma_{\exp,t} \\ \sigma_{\exp,t} & \sigma_{p,t} \end{pmatrix}, \Theta)$$

where $\sigma_{ex,t}$ and $\sigma_{p,t}$ are the variance functions of $\varepsilon_{ex,t}$ and $\varepsilon_{p,t}$, respectively, conditional on information set (Θ) available up to time t-1; $\sigma_{\exp,t}$ represents the conditional covariance given by an autoregressive linear function of the cross product in the past squared errors; u_{it} are the randomly distributed regression errors; Ω is the inverse of the degrees of freedom in the Student t distribution, and the conditional correlation,

$$\rho_{\exp,t} = \sigma_{\exp,t} (\sigma_{ex,t} \sigma_{p,t})^{-\frac{1}{2}}$$

is allowed to vary over time.

The parameters α_2 and $ß_2$ in (9) and (10) are the measures of volatility persistence in the exchange rates, and exchange rates respectively, with a large value indicating that the conditional variance remains elevated for extended periods of time following return shocks. The parameters $\alpha_3$and $ß_3$ are intended to capture the volatility spillovers between markets. For instance, α_3=0 and $ß_3$>0 would be consistent with the hypothesis that the volatility spills over from the exchange rates to the price levels, and not *vice versa*. In this example, the bivariate GARCH model results would be interpreted as evidence that supports the exchange rate pass through.

Unlike cointegration vectors or VAR models, bivaraite GARCH models are well positioned to filter nonlinearities and possibly shed light on the information flow between sectors of the economy or series of interest. For instance, in this study we are interested in spillover of volatility between exchange rates and domestic inflation rates. The question is directly related to the exchange rate pass-through theory.

In the following discussion we offer the estimation results of the bivaraite GARCH(1,1) models of equation (9-11). The focus of this segment of the empirical results is the volatility spillovers between the two series. Thus, we do not present the results of the estimation of the VAR systems.

The direction and intensity of the volatility spillovers are analyzed by examining the size and the significance of the cross equation squared lagged residuals. The coefficients of interest in these results are α_3 and β_3.

Table 5 presents the estimation results of three bivariate GARCH models for the three currencies and price levels. Bivariate GARCH(1,1) models appear to capture volatility in each series quite well. Most model coefficients are statistically significant at commonly expected levels of significance. The coefficients of the lagged squared residuals of the inflation equations (α_3) are all positive and statistically significant. The implication is that volatility spillovers from inflation to exchange rates are beyond being random and do occur consistently and significantly. The spillover of volatility in this manner suggests that price movements inform currency markets and currency movements, supporting the purchasing power parity theory. It should be noted that the change in volatility in exchange rates due to shocks to price levels, is dampened to some degree as shown by the less than one coefficient. The magnitudes of these coefficients may be consistent with some of the research findings that restoration of exchange rates to those suggested by the PPP theory may be slow.

Table 5. Bivariate GARCH Model results

Estimation results of equations (9) and (10)

Here, the upper marks [a], [b], and [c] represent the significance levels of .01, .05, and .10, respectively.

	Yen	Pound	Swiss Franc
α_0	0.084	0.467	0.148
	(11.760)[a]	(5.119)[a]	(1.012)
α_1	0.663	0.818	0.988
	(10.182)[a]	(12.966)[a]	(60.358)[a]
α_2	0.573	0.387	0.099
	(8.934)[a]	(6.979)[a]	(1.974)[b]
α_3	0.722	0.739	0.818
	(2.105) [a]	(1.866) [c]	(1.965) [b]
β_0	0.089	0.003	-0.003
	(2.158)[a]	(0.586)	(-0.294)
β_1	0.925	0.847	0.853
	(21.451)[a]	(0.586)	(33.840)[a]
β_2	0.235	0.445	0.406
	(5.023)[a]	(8.735)[a]	(17.211)[a]
β_3	-4.001	0.796	-0.496
	(3.215)[a]	(4.209)[a]	(2.561)[a]
Log likelihood	-485.175	-445.574	-529.180
Akaike info criterion	2.336	2.149	2.544
Schwarz criterion	2.423	2.235	2.631
Hannan-Quinn criterion	2.371	2.183	2.579

The spillover of volatility from exchange rates to price levels is measured by the magnitude and the statistical significance of the coefficient β_3. This coefficient is less than one in two out of three cases and negative in two out of three. The statistical significance of this coefficient indicates that the volatility spillovers show information transmission from exchange rates to price levels, demonstrating that there is a feedback. The negative signs may be interpreted as heightened volatility in the exchange rates leading to less volatility in the opposite direction in prices. This observation is noteworthy in the context of exchange rate pass through. For instance, significant volatility in the exchange rates, especially if it is due to currency appreciation, could be contributing to lower domestic prices, and lower inflation levels. Equally possible, markets maybe expecting central bank interventions if the large magnitude of the volatility stems from large currency depreciation. This is consistent with the theory of rational expectations in the case of most major currencies. Conversely, lower volatility levels in the exchange rates may be interpreted as stable exchange rates. Under certain conditions, this could lead to rising domestic prices. For instance, during periods of strong demand for imports in the world, import price hikes may not be the dampening the counter effect of a large appreciation in exchange rates. Overall, our bivariate GARCH model results could be interpreted as shocks to exchange rates having the opposite effect on price levels and inflation rates.

This interpretation would be consistent with the exchange rate pass through theory. For example, there has been a considerable debate on the causes of low pass-through from exchange rates to consumer prices. Devereux and Yetman (2002) develop a simple model of a small open economy in which exchange rate pass-through is determined by the frequency of price changes of importing firms. Their model implies that there should be a positive, but nonlinear, relationship between pass-through and mean inflation, and a positive relationship between pass through and exchange rate volatility. In a sample of 122 countries, this is strongly supported by the data. On the other hand, Herzberg, Kapetanios and Price (2003) find that the appreciation of sterling that began in 1996 appeared to pass through into import prices very slowly, an apparent example of incomplete exchange rate pass-through. Incomplete pass-through has typically been explained by a combination of sticky prices and pricing to market. They suggest that some of the factors supporting pricing to market may also introduce non-linear responses to exchange rate shocks but find no evidence of non-linearity.

5. Summary and conclusions

This paper investigates the nonlinearities in the behavior of the bilateral monthly exchange rates of three currencies against the dollar in relation to inflation rates of their domestic economies. We suggest that previous research might have overlooked the possibilities of nonlinearities in the exchange rate and inflation rate series. While we are basing our paper on the PPP theory, we examine a less restrictive relationship between inflation rates and exchange rates than the one implied by the PPP. The main contribution of this paper is testing for nonlinearities and nonlinear relationships in a framework of information arrival.

Our paper is motivated by three issues. First, economists have long been interested in exchange rate fluctuations, PPP, and exchange rate pass through. Second, the volatility in financial markets has generated interest in applying chaos theory to these markets including movements in currency exchange rates. The study of the chaotic behavior may shed some light on the performance of technical analysis in financial markets. Third, developments in the econometrics of nonlinearity in the last three decades offer researchers new tools for detecting relationships that are inherently nonlinear and may not be conducive to various methodologies that are seeking to impose linear modeling on nonlinear relationships.

Drawing on existing tests of nonlinearities and chaos, we first investigate the existence of chaotic behavior as the source of nonlinearities in the monthly bilateral exchange rates of the Swiss franc, the British pound, and the Japanese yen against the US dollar and the price levels in each country. To accomplish this task, we estimate AR (1) and GARCH(1,1) models for each series. The filtered series, i.e., the model residuals are tested for chaos to see if there are any lingering nonlinearities originating from chaotic behavior in the series.

Our findings show strong evidence that the exchange series exhibit nonlinear dependencies. However, we find evidence that the series behavior may be inconsistent with chaotic structure. We identify GARCH(1,1) process as the model that best explains the nonlinearities in the monthly exchange rates and inflation rates. Therefore, we propose and estimate bivariate GARCH(1,1) models of the variances to ascertain the flow of information between exchange rates and prices. Estimation results of the bivariate GARCH models offer evidence that the shock transmission between domestic prices and exchange rates occur in both directions. Thus, while there is some support for the PPP theory, there is evidence that exchange rate pass through also occurs in the economies under consideration. This finding indicates that the two theories, that is, the exchange rate pass through and PPP may simultaneously hold the key to exchange rate analysis.

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