A survey of models and algorithms for winter road maintenance. Part IV: Vehicle routing and fleet sizing for plowing and snow disposal

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Abstract

This is the last part of a four-part survey of optimization models and solution algorithms for winter road maintenance planning. The two first parts of the survey address system design problems for winter road maintenance. The third part concentrates mainly on vehicle routing problems for spreading operations. The aim of this paper is to provide a comprehensive survey of optimization models and solution methodologies for the routing of vehicles for plowing and snow disposal operations. We also review models for the fleet sizing and fleet replacement problems. © 2005 Elsevier Ltd. All rights reserved.

Keywords: Winter road maintenance; Snow removal; Snow disposal; Snow hauling; Arc routing; Operations research

0. Introduction

This is the last part of a four-part survey of optimization models and solution algorithms for winter road maintenance problems. The aim of this paper is to provide a comprehensive survey of optimization models and solution methodologies for the routing, the sizing, and the replacement of vehicle fleets for plowing and snow disposal operations. The third part of the survey [1] primarily addresses vehicle routing
and depot location problems for spreading chemicals and abrasives. The two first parts of the survey [2,3] address system design models for winter road maintenance.

Winter road maintenance planning involves a variety of decision-making problems relating to the routing of vehicles for spreading chemicals and abrasives, for plowing roadways and sidewalks, for loading snow into trucks, and for transporting snow to disposal sites. These problems are very difficult and site specific because of the diversity of operating conditions influencing the conduct of winter road maintenance operations and the wide variety of operational constraints. Spreading and plowing operations are usually performed on a regular basis in almost all rural and urban regions with frozen precipitation or significant snowfall. However, in urban areas with large snowfalls and prolonged subfreezing temperatures, not all the snow that is plowed to the roadside can remain there. The most common solution is to load snow into trucks for transport to disposal sites. Conversely, in rural regions, snow is often simply pushed to the sides of roadways without being removed and hauled. These operations consume over $2 billion in direct costs each year in the United States [4]. Indirect costs associated with corrosion and environmental impacts add at least $5 billion [4].

The paper is organized as follows. Section 1 describes the operations of plowing and snow disposal, and the vehicle routing, fleet sizing, and fleet replacement problems related to those operations. Models for the routing of vehicles for plowing and snow disposal operations are described in Sections 2 and 3, respectively. Models that address the sizing and replacement of vehicle fleets are reviewed in Section 4. Conclusions and future research paths in winter road maintenance planning are presented in Section 5.

1. Operations context and decision problems

This section contains a brief description of plowing, snow loading, and hauling operations and a discussion of associated problems of vehicle routing and fleet sizing and replacement. More detailed information on the state of the practice in managing winter road maintenance operations, including plowing and snow disposal operations, is presented in the synthesis report by Kuemmel [5]. For further details on winter road maintenance technologies, strategies, and tactics, see Minsk [6] and Blackburn et al. [7]. Decisions relating to the routing of vehicles for winter road maintenance usually belong to the operational planning level or real-time control, while decisions concerning the sizing and replacement of fleets for winter road maintenance vehicles pertain to the strategic or tactical planning levels.

1.1. Plowing and snow disposal operations

Snow falling on a paved surface may be removed by chemical, thermal, or mechanical techniques. Chemical methods include the external application of a freezing-point depressant and incorporation of the freezing-point depressant within the surface itself. Thermal methods involve applying heat to the surface from either above or below to remove snow and ice or to prevent its formation. Mechanical removal is the physical process of attempting to pick up the snow from the road, shearing it from the road if necessary, and casting it to a storage area off the road. Plowing and snow disposal operations fall into this category.

Plowing operations can be used alone or in conjunction with anti-icing, deicing, or abrasives spreading operations. Plowing operations alone are suitable for use during and/or after frozen precipitation has occurred at very low pavement temperatures (lower than about 12 °F), when it is too cold for chemicals
to work effectively, and on low-volume and unpaved roads. Anti-icing is the practice of attempting to prevent the formation of bonded snow and ice to a pavement surface by timely applications of a chemical freezing-point depressant. Chemical applications can be coordinated with timely plowing of snow and ice during anti-icing operations to produce the highest level of service during and after the precipitation. Deicing is necessary when ice or compacted ice is strongly bonded to the pavement and the bond has to be destroyed in order to remove the frozen layer. The practice of plowing operations in conjunction with deicing primarily results in controlling the depth of loose snow and ice on the roadway. Plowing operations together with abrasives spreading is a combination of winter road maintenance operations in which sand or other abrasives (or a mixture of abrasives and a chemical) are applied to the plowed or scraped roadway surface that may have a layer of compacted snow or ice already bonded to the pavement surface. This combination of operations is used to provide increased friction for vehicular traffic. However, abrasives are not chemicals and do not support the fundamental objective of either anti-icing or deicing operations. Plowing plus spreading with abrasives can be used in most snow and ice situations, particularly in very low pavement temperature situations where chemical spreading operations are not likely to be effective and on roads having a low traffic volume. This combination is also a viable option for unpaved roads if there is no, or very little, chemical in the mixture.

Plowing is most commonly accomplished by displacement plows mounted on the front, side, or beneath their truck carriers, or by rotary plows which pull the snow into a rotating element and cast it to the side. Blade plows are also used for scraping and cutting compacted snow and ice in the attempt to remove them. Frequently, the same equipment is used to remove both snow and ice. However, because of the high-strength adhesive bonds which may form between ice or compacted snow and pavement, specialized equipment is frequently required. Plowing operations are limited to one lane at a time. This contrasts with materials spreading operations where materials are spread onto the road through a spinner which can be adjusted so that more than one lane of a road segment can be treated in a single pass.

Agencies employ different plow patterns for two-lane roads and multi-lane highways. For example, some agencies extend plows over the centerline of two-lane roads on the first pass or on the second pass, while other agencies plow the centerline on all passes or as often as possible without conflicting with traffic. Also, many agencies have developed tandem plow patterns in echelon formations for multi-lane highways given that plowing operations can treat only one lane at a time.

The large volumes of snow plowed from roads and walkways may exceed the available space along roadways and walkways for snow storage, and therefore require disposal by some means. Loading snow into trucks using snowblowers, rotary plows, or other types of snow loaders for transport to disposal sites is the most common solution. The trucks may be adjacent to, or in some cases following, the vehicle loading it, though adjacent trucks will further restrict traffic during the operation [6]. Loading and hauling of snow are generally post-storm operations, although they may be required during a snowfall to remove snow from areas, such as alleys or narrow channelled sections, with insufficient space for snow storage. These operations are usually performed in urban areas with heavy snowfalls and prolonged subfreezing temperatures. However, many metropolitan areas may undertake snow disposal following infrequent but very large storms. Parking regulations are generally put into effect during snow disposal to facilitate loading snow into trucks for hauling to disposal sites. Examples are regulations that prohibit street parking at all times on designated snow routes, allow alternate side street parking, prescribe alternate times for parking, and ban overnight parking.

Disposal sites are the destinations for snow hauling trucks originating in each snow removal sector, and must be visited many times during the snow disposal operations. There are several different types of
disposal sites that may be considered, including surface sites, quarry sites, sewer chutes, snow melting machines and water sites. Associated with every disposal site are a fixed location cost, an operating cost, and an annual capacity due to the limited space available to store snow. Each disposal site may also have an hourly capacity for unloading trucks depending on the configuration of the disposal site, and the available equipment and manpower. Surface sites typically require large plots of open land and may have very large capacities. They also may have other uses when snow is not present. Melters, in contrast, can be small mobile machines, but are typically quite expensive. Disposal in rivers or lakes represent the most economical disposal method, although disposal sites that allow melted snow to be processed in waste water treatment facilities provide environmental benefits.

In order to minimize the completion time for winter road maintenance, snowblowers (or other types of snow loaders) generally operate in a continuous process of loading trucks. In practice, there may be several empty trucks moving slowly in a queue alongside each snowblower to ensure that the snowblowers are never idle. As soon as a truck is filled with snow, it departs for the assigned disposal site while another truck takes its place to begin being filled. The truck that departed for the disposal site will travel to the disposal site, dumps its load of snow, possibly after waiting in line, and then return to the end of the queue alongside the assigned snowblower. This closed cyclic continuous system is illustrated in Fig. 1. There may be more than one such a cyclic closed system if a sector contains more than one snowblower or if a sector can be assigned to multiple disposal sites.

1.2. Vehicle routing problems for plowing and snow disposal

Plowing, snow loading, and hauling operations involve a number of vehicle routing problems where streets or roads have to be traversed by plows, snowblowers, and trucks. The plow routing and snowblower routing problems consist of determining a set of routes, each performed by a vehicle that starts and ends at its own depot, such that all road segments are serviced, all the operational constraints are satisfied, and the global cost is minimized. Another type of routing problem for snow disposal, called the truck routing problem, calls for the determination of a set of itineraries for the trucks filled with snow that travel from the assigned snowblower to disposal sites and back to the snowblower. This is a difficult shortest path problem, because of the movement of the snowblower while a truck travels to and from the disposal site. Truck routing also involves political and equity issues due to the disruptive nature of large numbers of trucks converging on disposal sites.
This section describes the typical characteristics of vehicle routing problems related to plowing and snow disposal operations by considering their main components (transportation network, road segments, sectors, disposal sites and vehicle depots, vehicles, and crews), the different operational constraints that can be imposed on the configuration of the routes, and the possible objectives to be achieved in the optimization process. These characteristics are summarized in Table 1. Models and algorithms proposed for the solution of vehicle routing problems related to plowing and snow disposal operations are reviewed in Sections 2 and 3, respectively. Vehicle routing problems related to spreading operations are described in the third part of the survey [1].

Vehicle routing problems related to winter road maintenance can be defined on directed, undirected, or mixed graphs depending on the topology of the transportation network and on the operating policies involved. As a rule, one-way streets are represented by arcs and two-lane, two-way streets (one lane each way) are represented by edges. If the two sides of the street can be serviced at the same time, as is often the case in spreading operations, the mixed graph can be the appropriate representation. Conversely, if the two sides of the street must be serviced separately, as is the case in plowing and snow disposal operations, arcs may have to be duplicated and edges are replaced by two arcs of opposite direction. The resulting graph is then directed. The transportation network can be urban or rural, depending on whether it is required to service all road segments or only a subset of road segments, respectively. The road segments that require to be serviced are called required road segments. Rural problems are often simpler due to the sparser road networks and the service requirement, in many cases, to remove snow only from the roadways and leave it beside the road to accumulate over the winter without hauling the snow to disposal sites. The transportation network is usually associated with a maximum time for completing spreading operations based on political and economic considerations. Since agencies have finite resources that generally do not allow the highest level of service on all roads, they must then prioritize their response efforts. The most common criterion for prioritizing response efforts is traffic volume. Typically, the roads of a network are partitioned into classes based on traffic volume which induce a service hierarchy, namely all roads carrying the heaviest traffic are given the highest level of service in order to provide safe roads for the greatest number of motorists, followed by medium-volume roads, and so on. Associated with each class of roads may be a maximum time for service completion.

Most policies define level of service for classes of highway based on their priority. Level of service policies tend to be results-oriented (e.g., bare pavement), resource-oriented (e.g., 24-h equipment coverage), or a combination of both. Models dealing with level of service policies are presented in the first part of the survey [2]. Associated with each road segment is a cost, which generally represents its length, and three traversal times, which are possibly dependent on the vehicle type: the time required to service the road segment, the time of deadheading the road segment if it has not yet been serviced, and the time of deadheading the road segment if it has already been serviced. Deadheading occurs when a vehicle must traverse a road segment without servicing it. In general, the longest operation consists of removing snow on a road segment, followed by deadheading an unserviced road segment, followed by deadheading a serviced road segment. With each road segment are generally associated parking regulations to facilitate loading snow into trucks for hauling to disposal sites. These restrictions are often limited to critical road segments that carry large traffic volume. Also associated with each road segment is a time interval, called service time window, during which the road segment can be plowed, which is possibly dependent on the parking restrictions or on the hierarchy of the network, and a service frequency (e.g., the road segment should be covered at least once every 2 h). Service time windows can also be associated with classes of roads or routes. Finally, since plowing operations are limited to one lane at a time,
Table 1
Characteristics of vehicle routing problems for plowing and snow disposal

<table>
<thead>
<tr>
<th>Components</th>
<th>Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transportation network</td>
<td>Undirected, directed or mixed</td>
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<tr>
<td></td>
<td>Urban or rural</td>
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<td>Required road segments</td>
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<td></td>
<td>Service hierarchy</td>
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<td></td>
<td>Maximum time for service completion</td>
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<tr>
<td>Road segments</td>
<td>Resource-oriented or results-oriented level of service policies</td>
</tr>
<tr>
<td></td>
<td>Length</td>
</tr>
<tr>
<td></td>
<td>Service and deadhead traversal times</td>
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<tr>
<td></td>
<td>Parking restrictions</td>
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<tr>
<td></td>
<td>Service time windows</td>
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<tr>
<td></td>
<td>Service frequencies</td>
</tr>
<tr>
<td></td>
<td>Service in tandem for multi-lane roads or separate passes</td>
</tr>
<tr>
<td>Sectors</td>
<td>Sector design</td>
</tr>
<tr>
<td></td>
<td>Number of sectors</td>
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<tr>
<td></td>
<td>Compactness or shape</td>
</tr>
<tr>
<td></td>
<td>Centrally located sites and depots relative to sectors</td>
</tr>
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<td></td>
<td>Balance in sector size or workload</td>
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<tr>
<td></td>
<td>Contiguity</td>
</tr>
<tr>
<td></td>
<td>Basic units</td>
</tr>
<tr>
<td>Snow disposal sites and vehicle depots</td>
<td>Disposal site and vehicle depot locations</td>
</tr>
<tr>
<td></td>
<td>Number of disposal sites and vehicle depots</td>
</tr>
<tr>
<td></td>
<td>Variable costs of disposal sites and vehicle depots</td>
</tr>
<tr>
<td></td>
<td>Fixed costs of disposal sites and vehicle depots</td>
</tr>
<tr>
<td>Plows, snowblowers and trucks</td>
<td>Home depot</td>
</tr>
<tr>
<td></td>
<td>Truck capacity</td>
</tr>
<tr>
<td></td>
<td>Vehicle type and road segment dependencies</td>
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<tr>
<td></td>
<td>One or multiple routes per vehicle</td>
</tr>
<tr>
<td></td>
<td>Variable costs of plows, snowblowers, and trucks</td>
</tr>
<tr>
<td></td>
<td>Fixed costs of plows, snowblowers, and trucks</td>
</tr>
<tr>
<td>Crews</td>
<td>Number of crews per sector</td>
</tr>
<tr>
<td></td>
<td>Maximum duration of driving and working periods</td>
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<tr>
<td></td>
<td>Number and duration of breaks</td>
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<tr>
<td></td>
<td>Overtime</td>
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<tr>
<td></td>
<td>Variable crew costs</td>
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<tr>
<td>Routes</td>
<td>Start and end locations of routes</td>
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<tr>
<td></td>
<td>Start times of routes</td>
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<td></td>
<td>Load balancing</td>
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<td></td>
<td>Class continuity</td>
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<td></td>
<td>Class upgrading</td>
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<td></td>
<td>Both-sides service</td>
</tr>
</tbody>
</table>
Table 1 (continued)

<table>
<thead>
<tr>
<th>Components</th>
<th>Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sector boundaries</td>
<td>One or multiple vehicles per route</td>
</tr>
<tr>
<td></td>
<td>Turn restrictions</td>
</tr>
<tr>
<td></td>
<td>Block design or length design</td>
</tr>
<tr>
<td>Objectives</td>
<td>Minimize deadheading</td>
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<tr>
<td></td>
<td>Minimize service completion time</td>
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<td></td>
<td>Minimize turn penalties</td>
</tr>
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<td></td>
<td>Minimize operational constraint violations</td>
</tr>
</tbody>
</table>

multi-lane road segments necessitate either multiple separate passes or tandem plow patterns in echelon formations.

Because of the difficulty and impracticability of organizing winter road maintenance operations in a wide transportation network, these operations are generally carried out concurrently by separate crews and equipment in many small subnetworks, called sectors. See Perrier et al. [2,3] for details on the design of sectors for plowing and snow disposal.

The routes performed for plowing and snow disposal operations start and end at one or more vehicle depots, located at the vertices of the graph. With every vehicle depot is associated a given number of vehicles of each type. In large cities, there are usually several snow disposal sites, possibly of different types, including surface sites, unused quarries, sewer chutes (openings into the storm sewer system), and water sites. Costs associated with disposal sites and vehicle depots include variable costs of operating disposal sites and vehicle depots, and fixed costs of acquiring disposal sites and vehicle depots.

Plowing and snow disposal operations are performed using a fleet of plows, snowblowers, and trucks whose size and composition can be fixed or can be defined according to the level of service policies, the configuration of the streets and sidewalks, land use (e.g., residential or commercial) and density of development, and times for service completion for each class. A vehicle may end service at a depot other than its home depot. The capacity of the truck is expressed as the maximum volume of snow the truck can load for hauling to disposal sites. The subset of road segments of the transportation network which can be traversed by the vehicle is dependent on road segment widths and on the vehicle type. Large road segments may require large vehicles. Narrow road segments may require small or medium-sized vehicles. In some applications, each vehicle can cover multiple routes in the considered time period. Finally, associated with each vehicle type is a fixed leasing or acquisition cost and a variable cost that is proportional to the distance traveled. The variable cost component encompasses the costs of fuel and maintenance.

Crews of personnel assigned to sectors must satisfy several constraints from union contracts and agency or company regulations. Examples are working periods during the day, maximum duration of working and driving periods, number and duration of breaks during service, and overtime. A separate crew of personnel is usually assigned to each sector. Costs associated with crews depend on the pay structure (e.g., regular or premium time, single or dual working periods).

The routes must satisfy several operational constraints imposed by the level of service policies and the characteristics of the transportation network, road segments, sectors, vehicles, and crews. The routes can
start and end at one or more depot locations and each route can end service at a depot other than the original starting depot. In plowing operations, the route start times are dependent on the accumulation of snow and ice on the roadway surface. For example, in Montreal, snow plowing operations begin as soon as the accumulation of snow and ice reaches two centimeters and a half [8]. In Japan, snow plowing begins when accumulation reaches 5 cm [9]. To balance the workload across routes, they are often approximately the same length or duration. This helps ensure that plowing and snow disposal operations will be completed in a timely fashion. Since most arterial roads have multiple lanes that require separate passes, the total workload is usually measured in lane-kilometers. Class continuity requires that each route services road segments with the same priority classification. Thus, if a lower-class road is included in a route servicing higher-class roads, its service level may be upgraded. In plowing and snow disposal operations, it is often desirable that both sides of a two-lane, two-way road (one lane each way) be serviced by the same vehicle in a single route. The configuration of routes may also need to conform to existing sector boundaries. Routes crossing these boundaries must be avoided from an administrative standpoint. In some applications, each route can be operated by multiple vehicles. For example, if tandem service is required, the vehicles need to be assigned to a single route and operate in parallel. In practice, some operational constraints may often be treated as soft constraints or as terms in an objective function rather than hard constraints.

In urban areas, the impact of undesirable turns, such as U-turns and turns across traffic lanes, is generally greater in routing snow plows and snowblowers as compared to spreading operations. Since most plows are designed always to cast the snow to the right-hand side of the roadways, a left turn or a street crossing at an intersection results in a trail of snow in the middle of the intersection. Thus, the general guideline for constructing routes for snow plowing is that each plow should remain on the right-hand side of a roadway using a block pattern by accomplishing a series of right turns to avoid compromising safety. Conversely, for loading snow into trucks, each snowblower should cast the snow to a truck alongside or following directly behind using a length pattern where servicing one street at a time is preferred to frequent right turns. To deal with these situations, a penalty can be assigned to each type of turn (e.g., left, right, U-turn, and go straight).

Finally, several objectives can be considered for the routing of vehicles for plowing and snow disposal operations. Typical objectives are minimization of the distance covered by deadheading trips (or on the deadhead travel time); minimization of the time for service completion; minimization of the penalties associated with turns; minimization of the terms penalizing the violation of some operational constraints; or any weighted combination of these objectives.

1.3. Fleet sizing and replacement problems

Section 4 of this survey discusses models for fleet sizing and fleet replacement in the context of winter road maintenance. The most common types of mobile winter road maintenance equipment used on a routine basis, in approximate order of their total numbers, are trucks, plows, material spreaders, wheel loaders, motor graders, snowblowers or rotary plows, sweepers, and melters. Several factors determine the type, size, and design of appropriate equipment, including the frequency and severity of frozen precipitation, the nature and range of tasks, the environment, the level of service required, the type of road surface, and the extent of roads to be maintained, as well as geographic factors. In general, because winter road maintenance is seldom a constant, year-round, everyday activity, the vehicles and equipment used for winter road maintenance operations are typically powered equipment or vehicles primarily designed
for other activities such as concrete, asphalt, drainage, curb-cut, or traffic sign engineering. There are some conflicting objectives to consider in determining the sizes and replacement schedules of fleets for winter road maintenance vehicles. The important tradeoff in fleet sizing is that larger fleets are desired to more quickly clear the roadways, but larger fleets require greater expenditures. In fleet replacement, one usually wants to balance maintenance costs for keeping old vehicles and costs for acquiring new vehicles.

2. Vehicle routing models for plowing

Vehicle routing problems related to plowing operations are generally formulated as arc routing problems. An integrated overview of the most relevant operations research literature on arc routing was presented by Eiselt et al. [10,11]. A more extensive review of the literature in arc routing with special emphasis on applications was proposed by Assad and Golden [12]. More recently, a book on the subject was edited by Dror [13].

Several heuristics procedures have been proposed for the routing of vehicles for plowing operations. These can be broadly classified into three categories: constructive methods, composite methods, and adaptation of metaheuristics. These three classes of methods are covered in Sections 2.3–2.5, respectively. The characteristics of the contributions are then summarized in Table 2 at the end of the section. Some contributions emphasize both plowing and spreading operations, while others cover snow disposal in addition to plowing and spreading. Before, a brief review of simulation methods and rule-based decision support systems developed to help planners in making vehicle routing decisions for winter road maintenance operations is presented in Sections 2.1 and 2.2, respectively.

2.1. Simulation methods

Most early systems for the routing of vehicles for winter road maintenance operations relied in large part on simulation techniques either as a tool to assist planners in constructing feasible routing plans or as a tool to evaluate the quality of the configuration of a given set of routes and to help guide manual modifications.

Brown [14] proposed an interactive computer-simulation tool, called AID, to model vehicle movements and interactions for plowing and snow disposal operations in small urban areas. Inputs of the system are road segment characteristics (length, width, snow depth, etc.), vehicle characteristics (weight, hourly cost, speed, etc.), vehicle routes, and climatic and weather conditions. A vehicle can be assigned to more than one route. The system, which can handle multiple winter seasons, allows the user to specify maintenance actions (plow snow or do nothing) and starting times for plowing, select vehicle types, and assign vehicles to routes. The system can also run without user interaction by using decision rules resident in the simulation model. Routes can be added or modified during the simulation run. The output of the model is the total cost defined by accident costs, delay costs, vehicle operating costs, lost wages, and productivity costs. The model may also be used for defining the level of service, partitioning the road network into priority classes and sizing and replacing vehicle fleets by evaluating the consequences of modifying road segment and vehicle characteristics. The accuracy of the model was validated using data from Hanover, New Hampshire. The simulation model was also useful in analyzing a variety of scenarios related to the modification of road segment and vehicle parameters such as snow depth, snow density, operator wage rate, and vehicle hourly cost.
<table>
<thead>
<tr>
<th>Authors</th>
<th>Problem type</th>
<th>Problem characteristics</th>
<th>Objective function</th>
<th>Model structure</th>
<th>Solution method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marks and Stricker [33]</td>
<td>Plow routing</td>
<td>Two or four passes per road and service hierarchy</td>
<td>Min deadheading</td>
<td>$m$-vehicle undirected Chinese postman problem</td>
<td>Cluster first, route second</td>
</tr>
<tr>
<td>Moss [35]</td>
<td>Plow and spreader routing</td>
<td>Service hierarchy, class continuity, and maximum route durations</td>
<td>Min deadheading</td>
<td>Directed Chinese postman problem</td>
<td>Cluster first, route second</td>
</tr>
<tr>
<td>Transport Canada [36]</td>
<td>Plow routing</td>
<td>Multiple passes per road and service hierarchy</td>
<td>Min deadheading</td>
<td>$m$-vehicle undirected Chinese postman problem</td>
<td>Cluster first, route second</td>
</tr>
<tr>
<td>Lemieux and Campagna [38]</td>
<td>Plow routing</td>
<td>Service hierarchy and turn restrictions</td>
<td>Min number of U-turns</td>
<td>Directed hierarchical postman problem</td>
<td>Constructive method</td>
</tr>
<tr>
<td>Atkins et al. [39]</td>
<td>Plow routing</td>
<td>Two passes per road and balance in route length</td>
<td>Min plowing completion time</td>
<td>2-vehicle directed Chinese postman problem</td>
<td>Cluster first, route second</td>
</tr>
<tr>
<td>Chernak et al. [40]</td>
<td>Plow routing</td>
<td>Two or four passes per road, balance in route length and service hierarchy</td>
<td>Min plowing completion time and deadheading</td>
<td>2-vehicle directed Chinese postman problem</td>
<td>Constructive method</td>
</tr>
<tr>
<td>Robinson et al. [41]</td>
<td>Plow routing</td>
<td>Two passes per road and balance in route length</td>
<td>Min plowing completion time and number of U-turns</td>
<td>2-vehicle directed Chinese postman problem</td>
<td>Cluster first, route second, cluster second</td>
</tr>
<tr>
<td>Hartman et al. [42]</td>
<td>Plow routing</td>
<td>Two passes per road and balance in route length</td>
<td>Min plowing completion time, number of U-turns</td>
<td>2-vehicle directed Chinese postman problem</td>
<td>Constructive method</td>
</tr>
<tr>
<td>Authors</td>
<td>Problem Description</td>
<td>Objectives</td>
<td>Algorithm</td>
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<tr>
<td>Salim et al. [44,45]</td>
<td>Plow and spreader routing, Service hierarchy, maximum route service times, and one or multiple vehicles (routes) per route (vehicle)</td>
<td>Min deadheading</td>
<td>Arc routing problem</td>
<td>Constructive method</td>
<td></td>
</tr>
<tr>
<td>Haslam and Wright [46]</td>
<td>Plow routing, Multiple vehicles, class continuity and maximum route length</td>
<td>Min deadheading</td>
<td>Directed capacitated arc routing problem</td>
<td>Composite method</td>
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<td>Campbell and Langevin [48]</td>
<td>Plow routing, spreader routing, and snowblower routing, Time windows, service frequency, vehicle capacities, spreading rates, turn restrictions, street segment dependencies, and both-sides service</td>
<td>Min multicriteria additive function</td>
<td>Arc routing problem</td>
<td>Composite method</td>
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<td>Gendreau et al. [53]</td>
<td>Plow routing, Turn restrictions</td>
<td>Min deadheading and turn penalties</td>
<td>Mixed rural postman problem with turn penalties</td>
<td>GeoRoute</td>
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<td>Kandula and Wright [56]</td>
<td>Plow and spreader routing, Class continuity, maximum route length, and both-sides service</td>
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<td>Capacitated Chinese postman problem</td>
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<td>Wang and Wright [58]</td>
<td>Plow and spreader routing, Time windows, class continuity and class upgrading</td>
<td>Min deadheading, time windows violations and class continuity violations</td>
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Tucker and Clohan [15] developed an interactive program to assist planners in constructing feasible routing plans for plowing operations. The number of routes to build must correspond to the number of vehicles available for plowing. Also, each route must start and end at the same location and routes must have nearly equal service traversal times with minimum route overlap. Analytical formulas are derived to estimate the service traversal time per route when plows have the same or different blade widths. In addition, multiple passes may be required to clear a road segment, depending on the segment width and the effective width of the plow blade. Finally, penalties are used to restrict the total number of U-turns and left turns. Computer graphics are used to display the road network, and feasible routes are constructed one at a time on the screen using a sequential insertion heuristic [16]. The interactive program is embedded into a simulation program to model plow movements and interactions. The simulation program incorporates meteorological conditions (storm length, rate of snowfall, snow density, storm starting time), plow characteristics (fleet size, plow weight, plow width, plowing speed) and route configuration characteristics (snow accumulation before starting plowing, route starting time). The simulation model was validated using data from the town of Newington, Connecticut. The model can be used to assess the relative efficiency of various routing plans by comparing route duration, number of U-turns and left turns, as well as distance covered by deadheading trips. The authors also demonstrated how their simulation model may be used to analyze the sensitivity of plowing time to storm length, accumulation rate and snow accumulation before starting plowing.

Simulation models have also been developed to help planners for the strategic planning of winter road maintenance operations. For example, Pruett and Kuong Lau [17] proposed a computer-aided simulation model to assist planners at the Louisiana Department of Transportation and Development in making decisions about manpower, equipment, and materials utilization for the strategic planning of many types of highway maintenance activities including snow removal. Also, Wells [18] proposed a discrete event simulation approach to aid planners in metropolitan and larger urban areas for the strategic planning of various maintenance activities including salt and sand spreading, snow plowing, and snow disposal. The main goal of her approach is to help in evaluating the consequences of variations in the level of service, availability of resources, budget, and weather conditions. The approach is also proposed as a way to control maintenance activities by setting standards for crew performance. Inputs of the system are data regarding resource levels and costs, workload standards, performance characteristics, desired level of service, and weather conditions. Outputs of the system include activity delays caused by adverse weather conditions, resource shortages, times dedicated to each activity, expenditures on manpower, equipment and materials, as well as frequency of activity, total working hours and total workload for each activity. The accuracy of the simulation model was validated using historic data from a Midwestern city in the US. The system also permitted useful scenarios analysis related to outputs versus resources expended.

2.2. Rule-based decision support systems

More recently, several rule-based decision support systems were proposed to assist planners in making vehicle routing decisions for plowing and snow disposal. For example, Fukuchi et al. [19] described a rule-based decision support system for planning of snow removal operations in Japan. The system provides instructions regarding operation details, vehicle types, fleet size, and personnel to be put on alert up to 24 h in advance of need by following the rules of a knowledge database based on input weather forecasts. The knowledge database includes data on mobilization conditions of vehicles distinguished by type, snow removal capabilities, operation patterns for different weather conditions, standard operation hours,
mobilization priority, conditions for altering operations for situations requiring temporary allocation of
a large number of vehicles, and conditions for changes in commuting time zones. The system may also
be used at the real-time level to provide new instructions directly to vehicle operators by employing
satellite circuits in response to sudden changes in weather and road surface conditions. Also, Kanemura
[20] described a decision support system that has been implemented in the city of Sapporo, Japan. The
system, which relies on the city’s meteorological observation facilities, collects, analyzes, and provides
information on snowfall conditions, meteorological conditions, short-range forecasts, and status of snow
removal operations (e.g. being conducted or completed) in various parts of the city. These data are used
to estimate the starting and completion time for road maintenance as well as to determine if a road
surface improvement service should be performed, if personnel and equipment should be kept waiting
for mobilization on the same day, and if the current road maintenance service should stop or continue.

Some agencies have developed rule-based decision support systems that emphasize both plowing and
spreading operations. Grenney and Marshall [21] described a prototype rule-based decision support sys-
tem developed to help planners in the western US decide when to dispatch crews and equipment for
plowing and sand spreading operations. The Minnesota Department of Transportation developed a winter
road maintenance management system to help in planning, organizing, directing and controlling win-
ter road maintenance operations by determining resource requirements, allocating resources, scheduling
crews, and evaluating performance [22]. The system is based on the application of management principles
dictated by experience to winter road maintenance operations. Inputs of the system are employee hourly
and overtime rates, equipment and materials unit costs and inventories, as well as service levels, road
segment characteristics, and routes for spreading and plowing. For each sector, for each road class and for
each route, outputs of the system are quantities of sand and salt to spread, application rates and material
inventory balances, as well as labor, materials and equipment costs. Alfeler et al. [23] described a pro-
totype decision support system developed by the Minnesota Department of Transportation for defining,
collecting, analyzing, and applying performance and evaluation data to manage maintenance activities
such as sanding and plowing, as well as to help efficiently allocate labor, equipment, and material re-
sources. The categories of maintenance activities were initially developed by using internal department
knowledge of customer needs and were verified through direct customer research. The system incorpo-
rates analytical models that calculate the value added to customers in terms of road-user costs (i.e. travel
time and accident costs) to help operations managers respond to customer needs.

The Swedish Road and Transport Research Institute (VTI) developed a rule-based expert system to
assist planners in making real-time winter maintenance operation and scheduling decisions [24]. The
system focuses on choice of operation (plowing, sanding, or salting), starting time, and material type
(dry, prewetted or brine) and quantity for different road classes. Besides weather conditions and road
weather forecasts, data for the proposed system consist of information about the network, including
available vehicles and equipment, and a knowledge database of maintenance practices gathered from
literature studies and interviews with experts. For further details on the VTI system, see the review by
Perchanok et al. [25]. Recently, Mahoney and Myers [26] described a project initiated by the FHWA to
develop a prototype winter Maintenance Decision Support System (MDSS) to provide winter maintenance
decision makers with real-time road maintenance guidelines (e.g., chemical use, plowing, timing of
operation, and location) regarding vehicle routes. The system merges weather forecasting with road
condition information, chemical concentration algorithms, and anti-icing and deicing rules of practice.
The system allows users to select treatment constraints (chemical used, route times, application rates) for
each route. In addition, users can tailor the rules of practice for each route.
Computer-assisted route design systems have also been developed to assist planners in constructing feasible plow and/or spreader routes. For example, Lappalainen [27] described a computerized route design system to minimize the number of plows and the distance covered by deadheading trips in Finland. Predieri [28] described a computerized road map used for routing vehicles for various municipal services, including plowing, salt spreading, and anti-icing in the city of Bologna. The system allows the user to construct routes by selecting road segments on the screen. A similar computer-aided system for the design of snow plow routes in Finland was described by Korhonen et al. [29]. The roads of the Finnish network are partitioned into several classes based on traffic volume, each with different service levels and time limits for service completion. The system is based on the use of a digital map that allows the user to construct routes by pointing and clicking with a mouse on roads of the map while providing information such as route length, route duration, distance covered by deadheading trips, and service-level data. Martikainen and Keranen [30] described a project supported by the Minnesota Department of Transportation to develop an automated route planning system for summer and winter maintenance operations. Farkas and Corbley [31] mentioned the use of a GIS-based snow tracking system for plowing operations in the city of Newark, New Jersey. Besides interactive on-screen route design, the system permits display of road segment conditions (e.g., plowed to pavement, in progress, plowed but snow-packed, or unplowed) at any given time on the screen, thereby ensuring real-time control. Finally, Cortina and Low [32] described an interactive route design system, called Snowman, to build spreader and plow routes in the town of Brighton, New York. Each route must start and end at the same depot and each road segment must be covered by the required number of passes. The system, which relies in large part on the individual expertise of the planner, helps the user construct one route at a time interactively by pointing and clicking with a mouse on road segments of the schematic transportation network, thereby adding each road segment to the current route. As a route is created, on-screen statistical information regarding distance traveled, elapsed time, and materials utilization can be viewed. Once all road segments are covered by the appropriate number of passes, the planner can attempt to interactively improve any route. Several criteria are used to evaluate the quality of the routes generated by the planner: the minimization of total deadhead travel time, balance in route times, the number of passes for each road segment, and violations of acceptable times for service completion for each class.

2.3. Constructive methods

One of the first contributions dealing with the plow routing problem within the context of arc routing is due to Marks and Stricker [33]. Given a fleet of $m$ homogeneous plows, the problem considered is to design a set of $m$ plow routes such that each road segment is cleared within either two or four passes, depending on its width, while minimizing the distance covered by deadheading trips. Multiple pass requirements are taken into account by duplicating each road segment as many times as the required number of passes on the road segment. The problem is modeled as a $m$-vehicle undirected Chinese postman problem. Two approaches are presented for solving the problem. In the first approach, the transportation network is partitioned into $m$ subnetworks by solving a districting problem, and a Chinese postman problem is solved for each of them using a decomposition heuristic. In the second approach, a unicursal graph is first derived from the original network, and arbitrarily partitioned into $m$ mutually exclusive, collectively exhaustive subgraphs of approximately the same size so that an Eulerian cycle can be defined for each of them without additional duplication of edges. For details, see Stricker [34]. The first approach was tested on real data from the city of Cambridge, Massachusetts, involving one vehicle for urban waste
collection but not for snow plowing. The authors also suggested three strategies to handle the hierarchy of the network when class connectivity is satisfied. The first strategy tries to allow the highest level of equipment usage on road segments of highest priority by multiplying the length of each road segment by its priority (with 1 being the highest priority) and solving a Chinese postman problem using these weighted lengths so as to favour the duplication of edges associated with road segments of highest priority. The second strategy solves a Chinese postman problem on each connected subgraph induced by the set of edges of a specific priority class and assigns exactly one vehicle to each postman tour. Finally, the last strategy generates several Eulerian cycles while disregarding road priorities, and chooses the cycle which best adheres to the hierarchy of the network.

Moss [35] proposed a cluster-first, route-second approach to solve the vehicle routing problem for plowing and spreading operations in Centre County, Pennsylvania. Road segments are first organized into balanced sectors, and a vehicle route is obtained for each of them by solving a directed Chinese postman problem. Each direction of a two-lane, two-way road (one lane each way) can be serviced by a different vehicle. The cluster phase tries to ensure that the graph generated by the edges of each sector is Eulerian to reduce deadheading in the routing phase. Service hierarchy, class continuity for each sector, and maximum route times are enforced.

The Bureau of Management Consulting, Transport Canada [36], modeled a plow routing problem, with a homogeneous fleet of plows and multiple pass requirements for large road segments, as a multi-vehicle undirected Chinese postman problem. Multiple pass requirements are taken into account by duplicating each road segment the required number of times. The problem is solved using a cluster first, route second heuristic, based on earlier work by Stricker [34]. The cluster phase breaks the original graph into small subgraphs according to several rules so as to enable routes with less deadheading. In particular, each subgraph should contain an even number of odd degree nodes forming as compact and centralized a location as possible. The route phase then solves an undirected Chinese postman problem in each subgraph and Fleury’s algorithm [37, p. 309] is used for determining an Eulerian cycle in the resulting Eulerian subgraph. The heuristic was tested on real data from a major Canadian city. The Bureau of Management Consulting also proposed to handle the hierarchy of the network and the direction of the traffic flow directly within Fleury’s algorithm by selecting, at each iteration, the next edge of highest priority whose removal does not disconnect the Eulerian subgraph, while trying to respect the direction of the traffic flow.

Lemieux and Campagna [38] studied the problem of determining a plow route that starts at a depot, traverses every road segment only once in both directions, and ends at the depot, while respecting precedence relation constraints and U-turn restrictions. The problem is modeled as a directed hierarchical postman problem. Let \( G = (V, A) \) be a directed network with counterpart arcs in opposite directions between intersection nodes. Since \( G \) is symmetric, an Eulerian circuit in \( G \) can be determined by means of Fleury’s algorithm [37, p. 309]. The authors proposed to handle precedence relation constraints directly within Fleury’s algorithm by selecting the next arc (whose removal does not disconnect graph \( G \)) so as to cover high-class road segments as quickly as possible. However, since there is no guarantee that there exists an Eulerian circuit in \( G \) that strictly satisfies precedence relation constraints without introducing any deadheading, these constraints are treated as soft constraints. Soft precedence relation constraints allow a low-class road segment to be plowed before a high-class road segment. Let \( G' = (V', A') \) be the subgraph induced by the subset of arcs not yet included in the circuit. Given a partial circuit that ends at arc \((u_i, u_j)\), the authors always choose the arc \((u_j, u_k)\) with highest priority so that \( G' = (V', A'\setminus((u_j, u_k))) \) is connected. However, if \( u_k = u_i \) (implying a U-turn), the two following rules are used for attempting to
restrict the number of U-turns:

**Rule 1:** Choose the arc \((v_j, v_l)\), \(v_l \neq v_i\), with the same priority class as that of arc \((v_j, v_k)\) so that \(G' = (V', A'\setminus((v_j, v_l)))\) is connected. If such an arc does not exist, choose the arc \((v_j, v_k)\).

**Rule 2:** Choose the arc \((v_j, v_l)\), \(v_l \neq v_i\), with highest priority so that \(G' = (V', A'\setminus((v_j, v_l)))\) is connected.

With the first rule, the heuristic tries to cover the arcs with the highest priorities as quickly as possible, even if it implies U-turns. In contrast, with the second rule, the heuristic can compromise the importance of observing priorities so as to avoid U-turns. The authors applied the heuristic to a very small hypothetical problem.

In a mathematical modeling competition in 1990, students from various colleges and universities in the US studied the problem of designing routes for two plows to clear the county roads in a district of Wicomico County, Maryland. The county roads are two-way with one lane in each direction and form a strongly connected, directed graph \(G\) with 139 vertices and 374 arcs. Each plow can service exactly one lane at a time and each route must start and end at the same location. The objective considered is to minimize the plowing completion time. Since \(G\) is unicursal, the plow routing problem considered consists of determining two Eulerian circuits of approximately the same length in \(G\) to minimize the plowing completion time.

A first solution method was that of Atkins et al. [39] who developed a traditional cluster-first, route-second heuristic. The graph \(G\) is first partitioned into two subgraphs of approximately the same size (in terms of total road lengths). This is done by iteratively choosing a two-way road and adding the two associated arcs of opposite direction to the subgraph that has the smaller current total length. An Eulerian circuit is then constructed for each subgraph by tracing a closed walk on a spanning tree of the associated undirected graph, into which the non-tree edges are inserted in a simple fashion.

Chernak et al. [40] studied a more realistic problem in which the service hierarchy of the transportation network is also considered, with an objective of minimizing the distance covered by deadheading trips, in addition to minimizing the plowing completion time. This problem is solved using a heuristic approach that constructs, for each plow, a primary route servicing roads of highest priority and a second route servicing the other roads. The heuristic was also used to evaluate the impacts of changing the width of the plow blade on service completion times.

Robinson et al. [41] suggested a cluster-first, route-second method and a route-first, cluster-second method for determining two Eulerian circuits of approximately the same length in \(G\). In the cluster-first, route-second method, roads are first organized into two balanced subgraphs by means of the procedure described by Atkins et al. [39], and an Eulerian circuit is constructed in each subgraph in a depth-first search fashion. In the route-first, cluster-second method, an Eulerian circuit \(C = (x, \ldots, y, \ldots, x)\) is first built on \(G\) in a depth-first search fashion and is then segmented into two infeasible routes \(R_1 = (x, \ldots, y)\) and \(R_2 = (y, \ldots, x)\), where \(x\) and \(y\) are the starting points of routes \(R_1\) and \(R_2\), respectively. Comparisons showed that the cluster-first, route-second method produced better routes than the route-first, cluster-second method, in terms of both the number of U-turns implied and the times for plowing completion.

Finally, Hartman et al. [42] extended the end-pairing algorithm [43] to simultaneously build two Eulerian circuits of approximately the same length in \(G\).

In a series of two papers, Salim et al. [44,45] proposed the Snow Removal Asset Management (SRAM) system to solve the vehicle routing problem for plowing and spreading operations in Black Hawk County, Iowa. The SRAM system can deal with service hierarchy and maximum route service times. Although
the system relies in large part on decision rules drawn from interviews with experts, it also uses a simple constructive method that builds feasible routes one at a time for each class of roads using the following greedy criterion. Given a partial route that ends at a road of a given priority class, choose the nearest road of the same class that fits within the maximum route service time limit. For each class of roads, the optimal assignment of vehicles to routes is then found by solving a transportation problem with supply nodes representing vehicles and demand nodes representing either plowing routes or spreading routes. The assignment variables correspond to the time spent by a vehicle on a route. A vehicle can be assigned to more than one route, and a route can have more than one vehicle assigned to it. The quantity of materials required for each spreading route is determined last. Related field testing showed that the system was useful in analyzing a variety of scenarios concerning the number of required road segments, vehicles and drivers and the status of the materials inventory, and reduced the total traversal time (service and deadheading) by 1.9–9.7% (depending on snowfall conditions) over the solution in use by the county.

2.4. Composite methods

Haslam and Wright [46] developed an interactive route generation procedure for the plow routing problem at the Indiana Department of Transportation (INDOT), US. In this problem, routes of total minimal length that start and end at a given depot are sought and class continuity as well as maximum route length constraints must be satisfied. INDOT is only responsible for servicing state roads, highways, and Interstates, but county roads may be traversed (while deadheading) to provide service to state roads. A fleet of homogeneous plows are based at the depot and the underlying network is assumed to be directed (an arc for each lane). When ignoring class continuity constraints, the problem can be formulated as a directed capacitated arc routing problem with vehicle capacity representing maximum route length and arc demands representing arc lengths.

The route generation procedure starts by calculating a lower bound $L_r$ on the number of routes to construct by dividing the total workload in each class (in terms of total road lengths) by the maximum distance a plow may travel when servicing that class. The user then provides $s$ seed nodes, $s \geq L_r$, with associated classes out of which feasible routes are constructed one at a time using the three-stage algorithm described in Fig. 2. Given a seed node and its class, the first stage of the algorithm constructs a feasible route made of a path from the seed node to the depot and another path in the reverse direction, without violating class continuity and maximum route length constraints. In the second stage, pairs of non-covered required arcs of opposite direction are sequentially inserted into the route as long as class continuity and maximum route length permit. Finally, in the last stage, if required arcs have not been covered, then the class continuity constraint is relaxed and the second stage is repeated by permitting class upgrading.

The three-stage algorithm was tested on a subnetwork of the Fowler subdistrict with 21 nodes, 54 arcs (all required) and three classes of roads. The routes produced by the algorithm failed to cover all required arcs of the network. However, with some manual intervention, the authors found a set of routes covering all required arcs and satisfying more constraints than the route configuration in use by the subdistrict, but having a higher deadheading cost. The authors also described two improvement methods that operate on several routes at a time. The first improvement method is an exchange heuristic that tries to reduce the distance covered by deadheading trips and better enforce class continuity and route compactness by swapping arcs between routes. The second improvement method tries to reduce the number of routes and the distance covered by deadheading trips by eliminating a route of a given class and inserting its
required arcs into other routes of the same class. The two improvement methods were applied to the existing set of routes in use by the Fowler subdistrict. The instance contained 99 nodes, 362 arcs (all required) and three classes of roads. Computational tests showed that the swap heuristic and the route elimination method reduced the distance covered by deadheading trips by 23% and 9%, respectively, over the route configuration in use by the subdistrict. Moreover, the second heuristic decreased the number of routes by more than 3%. The swap heuristic was embedded in a prototype decision support system [47].

Campbell and Langevin [48] described the commercially available vehicle routing software GeoRoute developed by the firm GIRO, based in Montreal, Canada, for postal delivery, winter maintenance, meter reading, street cleaning and waste collection applications. The GeoRoute software allows three types of winter road maintenance operations: plowing, spreading and snowblowing (for loading snow into trucks). The software can accommodate service time windows, service frequency, vehicle capacities, spreading rates, turn restrictions, street segment dependencies, and both-sides service restrictions (servicing both sides of a road segment in a single route). In addition, for spreading operations, the software can determine the number of passes required to service each road segment given the street width, the vehicle type and whether both lanes of a two-lane, two-way street (one lane each way) should be spread in a single pass or not. GeoRoute uses a two-phase method similar to the cluster first, route second method, but constructs instead one route at a time. The first phase selects a seed basic unit to initialize a cluster and allocate the basic units closest to the seed to a cluster. The second phase determines a vehicle route on the cluster with an arc routing adaptation of the GENIUS composite procedure proposed by Gendreau et al. [49] for the traveling salesman problem. The objective function is a weighted additive multicriteria function defined by the user. The user may also specify the basic units that must be serviced on the same route. GeoRoute has been implemented in Ottawa, Canada [50,51] for snow plowing and in Suffolk County,
The design of block and length pattern routes for winter road maintenance operations was studied by Gendreau et al. [53] who modeled the problem as a mixed rural postman problem with turn penalties (MRPPTP). Given a mixed graph \( G = (V, A \cup E) \) with a subset of required links \( R \subseteq A \cup E \), non-negative costs \( c_{ij} \) associated to its edges or arcs \((v_i, v_j) \in A \cup E \), and non-negative penalties associated to its turns, the MRPPTP consists of finding a minimum-cost closed chain in \( G \) containing each link of \( R \) at least once. The objective function to be minimized is a weighted combination of the deadheading travel time, the number of left turns, U-turns, straight crossings, and street changes. The authors conducted a study to properly calibrate the various weights of the objective function to obtain particular types of routes (e.g. block or length pattern). To this end, several combinations of parameter values are first tested on 30 street networks in three Montreal suburbs for each type of pattern. The MRPPTP corresponding to each experiment is solved using GeoRoute, an arc routing package developed by the Montreal-based firm GIRO. For each pattern, the combinations of parameter values are then identified based on decision rules dictated by experience. These sets of parameter values were embedded in the GeoRoute system. Computational experiments were also performed to properly calibrate the various penalty costs so as to generate length patterns for waste collection.

Finally, the authors showed that it is possible to solve the MRPPTP by transforming it into an asymmetric traveling salesman problem (ATSP) based on the transformation procedure proposed by Laporte [54]. The first step of the transformation consists of replacing each edge \((v_i, v_j) \in E \) by a pair of arcs \((v_i, v_j) \) and \((v_j, v_i) \) of opposite direction and cost \( c_{ij} \). Let \( G' = (V, A \cup A') \) be the resulting directed graph and denote \( R' = \{(v_i, v_j), (v_j, v_i): (v_i, v_j) \in E \cap R \} \cup (A \cap R) \) as the subset of required arcs in \( G' \). The second step consists of transforming the MRPPTP on \( G' \) into an equivalent generalized traveling salesman problem (GTSP) on a directed graph \( H = (W, B) \). In this graph, the vertex set \( W \) has a vertex \( w_{ij} \) for each required arc \((v_i, v_j) \in R' \) and the arc set \( B \) contains an arc \((w_{ij}, w_{kl}) \) between \( w_{ij} \) and \( w_{kl} \) if the two required arcs \((v_i, v_j) \) and \((v_k, v_l) \) are not associated with the same required edge in \( G \). Let \( G'' = (V'', A_1 \cup A_2) \) be a directed graph such that \( V'' \) contains two vertices for each arc \((v_i, v_j) \in A_1 \cup A_2 \), \( A_1 \) contains an arc with cost \( c_{ij} \) for each arc \((v_i, v_j) \in A_1 \cup A' \), and \( A_2 \) contains an arc with the appropriate turn penalty for each turn in \( G \). The cost \( c_{jk} \) of an arc \((w_{ij}, w_{kl}) \in B \) is equal to the sum of the cost of arc \((v_i, v_j) \) and the length of a shortest path from \( v_j \) to \( v_k \) in \( G'' \). This cost includes all turn penalties starting from the arc terminating at node \( v_j \) and finishing with the arc emanating from node \( v_k \). Solving a MRPPTP on \( G \) then amounts to solving an asymmetric GTSP on \( H \) which consists of determining a least cost Hamiltonian circuit visiting each of several clusters at least once. Any vertex \( w_{ij} \in W \) corresponding to a required arc \((v_i, v_j) \) of \( A \cap R \) or any pair of vertices \( \{w_{ij}, w_{ji}\} \subseteq W \) corresponding to the same required edge \((v_i, v_j) \) of \( E \cap R \) defines such a cluster. The third and last step consists of transforming the asymmetric GTSP on \( H \) into an ASTP on an associated complete directed graph \( H' = (W, B \cup B') \) by means of a procedure proposed by Noon and Bean [55]. For each cluster \( \{w_{ij}, w_{ji}\} \subseteq W \) corresponding to an edge \((v_i, v_j) \) of \( E \cap R \), the arc set \( B' \) contains two arcs \((w_{ij}, w_{ji}) \) and \((w_{ji}, w_{ij}) \) of opposite direction, both with cost \(-M \), where \( M \) is a large positive constant. In addition, the cost \( c_{jk} \) of every arc \((w_{ij}, w_{kl}) \in B \) linking two clusters is replaced by the cost \( c_{ij} \) of arc \((w_{ij}, w_{kl}) \in B \). A MRPPTP defined in \( G \) can then be solved by the resolution of an ATSP defined in \( H' \).

To illustrate, consider the mixed graph \( G \) shown in Fig. 3(a), where the links of \( R \) are shown in bold lines and the numbers correspond to link costs. The construction of \( G'' \) is represented in Fig. 3(b) for the following turn penalties: 0 for a straight crossing, 1 for a right turn, 3 for a left turn, and 9 for a
U-turn. The two sets of arcs $A_1$ and $A_2$ are shown as solid lines and dashed lines, respectively, with the corresponding arc cost or turn penalty. A first transformation leads to the graph $H$ represented in Fig. 3(c). The graph $H$ has four vertices, $w_{3/2}$, $w_{2/3}$, $w_{3/5}$, and $w_{5/3}$, and three clusters $\{w_{3/2}\}$, $\{w_{2/3}\}$, and $\{w_{3/5}, w_{5/3}\}$. The numbers correspond to arc costs computed in $G'$. The graph $H$ is further transformed by first including two arcs $(w_{3/5}, w_{5/3})$ and $(w_{5/3}, w_{3/5})$ in $B'$, both with cost $-M$, where $M$ represents a very large positive number, and by replacing the cost of arcs $(w_{3/5}, w_{3/2})$, $(w_{3/5}, w_{2/3})$, $(w_{5/3}, w_{3/2})$, and $(w_{5/3}, w_{2/3})$ by the cost of arcs $(w_{5/3}, w_{3/2})$, $(w_{5/3}, w_{2/3})$, $(w_{3/5}, w_{3/2})$, and $(w_{3/5}, w_{2/3})$, respectively. The resulting graph $H'$ is shown in Fig. 3(d).

As highlighted by Laporte [54], this type of transformation induces a fair amount of degeneracy in the cost structure of the ATSP, which may limit its computational interest.

Finally, a three-stage composite heuristic was proposed by Kandula and Wright [56] for routing plows and spreaders in the state of Indiana. The heuristic takes into account class continuity and a maximum route duration for each class. In addition, both sides of a road segment must be serviced by the same vehicle. For spreading operations, the vehicle capacity constraints are given in terms of maximum route durations. Given an undirected graph, the first phase identifies a set of seed nodes in sufficient number to
respect the time limits, and then determines the maximum number of routes that can be constructed out of each seed node by means of an adaptation of the node scanning lower bound procedure introduced by Assad et al. [57] for the capacitated Chinese postman problem. A good set of seed nodes close to the depot helps to reduce the distance covered by deadheading trips. The second phase then constructs routes one at a time out of each seed node using the following greedy optimality criterion: given a partial route that ends at vertex \( v_i \), choose the non-serviced edge \((v_i, v_j)\) that fits within the time limits and maximizes the distance between \( v_j \) and the depot. If no such edge can be found, then a route containing the partial route that ends at vertex \( v_i \) is created (i.e., a route made of a shortest chain of deadheaded edges between the depot and the seed node, the partial route that ends at vertex \( v_i \), the partial route in the reverse direction to satisfy the both-sides service constraint, and a shortest chain of deadheaded edges between the seed node and the depot). An improvement procedure that tries to reduce the distance covered by deadheading trips and the number of kilometers violating the class continuity constraints without exceeding the time limits is used last. This is done by swapping edges among the routes or by transferring edges from one route to another. Comparisons with the tabu search algorithm proposed by Wang and Wright [58] for a vehicle routing problem for plowing and spreading operations (see Section 2.5) on five networks of Indiana showed that the heuristic obtained the best solutions. However, it should be emphasized that the tabu search algorithm was stopped after a given number of iterations.

2.5. Metaheuristics

Wang and Wright [58] described an interactive decision support system, called Computer-Aided System for Planning Efficient Routes (CASPER), to assist planners at the Indiana Department of Transportation (INDOT) in the design of vehicle routes for plowing and spreading operations. The sectors are given and each of them contains exactly one depot. The system, which can accommodate service time windows, class continuity, and class upgrading, starts by calculating the number of routes to construct in a given sector for each class of roadways. Let \( G = (V, A) \) be the directed graph associated with the sector. The depot is represented by the vertex \( v_0 \). Let \( P_K = \{A_1, A_2, \ldots, A_K\} \) be a partition of \( A \) with \( A_1 \cup A_2 \cup \cdots \cup A_K = A \) and \( A_i \cap A_j = \emptyset \) for all \( i, j \in \{1, 2, \ldots, K\}, i \neq j \). For every class \( A_k \subseteq P_K \), define \( N_k, w_k \) and \( t_k \) as the number of class \( k \) routes to construct, the total workload of class \( k \) (in terms of total number of class \( k \) kilometers requiring service) and the time limit of a class \( k \) route, respectively. Thus,

\[
N_k = \left\lceil \frac{w_k}{t_k \cdot s} + dhf \right\rceil,
\]

where \( s \) is the average vehicle service speed and \( dhf \) is a nonnegative number to account for deadheading trips. For every class \( A_k \subseteq P_K \), the system builds \( N_k \) vehicle routes starting and ending at the depot using a tabu search algorithm. An initial solution is obtained by means of a route growth heuristic described in Wang [59], which is a refinement of the three-stage algorithm proposed by Haslam and Wright [46] and described in Section 2.4. In this heuristic, \( N_k \) feasible vehicle routes are first generated for every class \( A_k \subseteq P_K \) following a look-ahead procedure. Given two paths \( P = (v_i, \ldots, v_j) \) and \( P' = (v_j, \ldots, v_k) \) having a common endpoint \( v_j \) in \( G \), let \( P + P' = (v_i, \ldots, v_j, \ldots, v_k) \) denote the union of the arcs of these two paths. Let \( SP_{ij} \) be the shortest path from \( v_i \) to \( v_j \) in \( G \). Given a partial class \( k \) route \( P \) that ends at vertex \( v_i \), the look-ahead procedure selects the non-serviced required arc \((v_i, v_j)\) of class \( k \) that maximizes the number of non-serviced required arcs \((v_j, v_i)\) of class \( k \) adjacent to \( v_j \) such that the duration of the route \( P + (v_i, v_j) + SP_{j,0} \) does not violate the time limit. The remaining non-serviced required arcs are
then sequentially inserted into vehicle routes using four complementary insertion strategies that allow class upgrading and overduration. The possible resulting infeasible routes are, however, penalized by the objective function
\[ f(r) = a_1 D(r) + a_2 M(r) + a_3 C(r) \],
where 
\( D(r) \) is the total distance covered by deadheading trips in route \( r \), 
\( M(r) \) the number of minutes route \( r \) is below or above the lower or upper bound of the service time window, 
\( C(r) \) the total distance of all class upgraded road segments in route \( r \), and 
\( a_1, a_2, \) and \( a_3 \) are the corresponding penalty parameters. The four insertion strategies are defined as follows:

**Strategy 1:** Given a class \( k \) route and a vertex \( v_i \) on the route incident to a pair of non-serviced required arcs \((v_i, v_j)\) and \((v_j, v_i)\) of class \( k \), determine if the circuit \((v_i, v_j, v_i)\) can be added on the route without violating the time limit.

**Strategy 2:** Use strategy 1 by relaxing the class continuity constraint and permitting class upgrading.

**Strategy 3:** Given a non-serviced required arc of class \( k \), determine the existing class \( k \) route into which this arc should be inserted in order to minimize the objective function \( f \) without violating the time limit.

**Strategy 4:** Use strategy 3 by relaxing both class continuity and time limit constraints.

The route growth heuristic proposed by Wang [59] is summarized in Fig. 4. The insertion of an arc into a given vehicle route at Step 4 is performed by means of a procedure similar to the ADD algorithm developed by Hertz et al. [60] for the undirected rural postman problem.
Solutions violating the class continuity and time limit constraints are also allowed during the tabu search process. These infeasible solutions are penalized by the objective function $F(S) = \sum_r f_r(S)$, where $f_r(S)$ is the value of the objective function $f(r)$ assigning a value for route $r$ to any given solution $S$. A neighbor solution $S'$ is obtained from a solution $S$ by moving the service of an arc $(v_i, v_j)$ or a pair of arcs $(v_i, v_j)$ and $(v_j, v_i)$ from a route $T$ to another route $T'$ of $S$. The service of an arc or an arc pair is removed from $T$ by means of an algorithm similar to the ADD algorithm [60] while it is introduced into $T'$ using an algorithm similar to the DROP algorithm [60]. Two tabu lists $L_1$ and $L_2$ of limited length are used to register the most recent moves that have been performed during the search process. If the service of an arc $a$ (arc pair $p$) has been moved from a route $T$ to another route $T'$, then either the arc $a$ (arc pair $p$) enters the list $L_1$, and the service of $a$ ($p$) cannot be removed from $T'$ until the maximum length of $L_1$ has been reached, or the trio $(a, T, T')$ (trio $(p, T, T')$) enters the list $L_2$, and the service of $a$ ($p$) cannot be removed from $T'$ and reintroduced into $T$ until the maximum length of $L_2$ has been reached. Clearly, the tabu list $L_1$ is more restrictive (i.e., it forbids a larger collection of moves) than the tabu list $L_2$. The structure of the tabu search algorithm is described in Fig. 5.

The authors proposed to reduce the size of the neighborhood of a solution by performing service exchanges only between two routes with the worst values of the objective function $f(r)$ or by introducing the service of an arc $(v_i, v_j)$ or a pair of arcs $(v_i, v_j)$ and $(v_j, v_i)$ only into a route $T'$ whose shortest path from any vertex on $T'$ to $v_i$ or $v_j$ does not exceed a given threshold. The system allows the user to change the setting of the tabu search parameters, to modify the routes manually, to modify road segment or node attributes, and to add route configuration constraints, such as restrictions on mixing road classes and tandem servicing restrictions. A permanent tabu list can also be used to retain user-specified moves that cannot be reversed. The system was tested on data from four northern districts of Indiana [61]. On average, the system reduced the value of the objective function $F$, the distance covered by deadheading trips, and the number of routes by more than 96%, 4%, and 7%, respectively, over the routing plan in use by INDOT. The large decreases in objective function values result mainly from the CASPER routes better satisfying the service time windows. Wang et al. [61] discussed the importance of the central location of the depot (with respect to the sector to be serviced from that depot) and the compactness of the sector in achieving routes with less deadheading. The problem of partitioning a road network into compact sectors with centrally located depots for plowing and/or spreading operations was studied by Kandula and Wright [56,62] and Muyldermans et al. [63,64].
3. Vehicle routing models for snow loading

Very little work has addressed the routing of snowblowers for loading snow into trucks. A decision support system, called GeoRoute, was developed by the firm GIRO, based in Montreal, Canada, for arc routing applications including the routing of snowblowers for loading snow into trucks. The system uses a cluster first, route second method in which the route phase is an adaptation of the GENIUS algorithm proposed by Gendreau et al. [49] for the traveling salesman problem. Further details on the GeoRoute system are given in Section 2.4.

Gilbert [65] proposed a model and a heuristic method for the snowblower routing problem for loading snow into trucks in the city of Montreal, Canada. Considering a given sector with its fixed depot location and the number of possible workdays to complete snow loading operations measured from the end of the storm, the problem consists of designing a single snowblower route that starts and ends at the depot for each workday, so that the large volumes of snow pushed to the sides of the roadways and sidewalks are loaded into trucks by the snowblower for hauling to disposal sites, while satisfying some side constraints such as service hierarchy and special restrictions put into effect during snow loading. For example, the city bans snow loading on both sides of a roadway during the same time interval and on high-class roadways during the AM and PM peak periods. Also, the right-hand side of a high-class one-way street must be serviced before its left side. Finally, the side of a high-class two-way street carrying the heaviest traffic during the AM peak or during the PM peak must be serviced before the other side. The objective is to minimize the total deadhead travel time over the given set of workdays.

The arc routing problem is formulated as a node routing problem with nodes representing sides of roadways to be serviced by the snowblower. Consider a mixed graph $G = (V, E \cup A)$ in which the two link sets $E$ and $A$ are used to represent two-lane, two-way streets (one lane each way) and one-way streets, respectively. The depot is represented by the vertex $v_0$. Associated with every edge or arc $(v_i, v_j)$ are two lengths or traversal times: $s_{ij}$ is the time of servicing edge or arc $(v_i, v_j)$ and $d_{ij}$ is the traversal time of deadheading edge or arc $(v_i, v_j)$. The graph $G$ is augmented by adding a copy $(v_i', v_j')$ of each arc $(v_i, v_j)$ in $A$ with lengths $s_{ij}$ and $d_{ij}$ to model one-way streets requiring service on each side. Similarly, let $G' = (V, A \cup A')$ be a directed graph constructed from the multigraph $G$ by replacing each edge $(v_i, v_j)$ in $E$ with a pair of arcs $(v_i, v_j')$ and $(v_j, v_i')$ of opposite direction and lengths $s_{ij}$ and $d_{ij}$ to model two-lane, two-way streets requiring separate service on each side. The arc routing problem on $G'$ is transformed into an equivalent node routing problem on a complete directed graph $H = (W, B)$ where the vertex set $W$ has a vertex $w_{ij}$ for each arc $(v_i, v_j)$ in $A \cup A'$. The cost $t_{jk}$ of an arc $(w_{ij}, w_{kl}) \in B$ is equal to the traversal time of a shortest path from $v_j$ to $v_k$ in $G'$. Solving a directed Chinese postman problem on $G'$ then amounts to solving an asymmetric traveling salesman problem on $H$.

In Montreal, the number of possible workdays to complete snow loading operations measured from the end of the storm depends on total snow accumulation. For example, a sector must be cleared within 4 days for a snow accumulation of less than 20 cm, within four and a half days for an accumulation of 20–25 cm, and within 5 days for 25 cm of snow or more. Let $D$ be the set of possible workdays to complete snow loading operations. Every workday is divided into a given number of periods of work, which is dependent on parking restrictions. For every workday $d \in D$, define $P(d)$ as the set of periods of work associated with workday $d$. The sets $P'(d)$ and $P''(d)$ contain all periods of work associated with workday $d$ except the first and last period, respectively. Let $P_K = \{P_1(d), P_2(d), \ldots, P_K(d)\}$ be a partition of $P(d)$ with $P_1(d) \cup P_2(d) \cup \cdots \cup P_K(d) = P(d)$ and $P_i(d) \cap P_j(d) = \emptyset$ for all $i, j \in \{1, 2, \ldots, K\}, i \neq j$. For every vertex $w_{ij} \in W$ corresponding to an arc $(v_i, v_j)$ of $A \cup A'$, define $D_{ij} \subseteq D$ as the subset...
of workdays for which the arc \((v_i, v_j)\) can be serviced. For every workday \(d \in D\), for every period of work \(p \in P(d)\), and for every vertex \(w_{ij} \in W\) corresponding to an arc \((v_i, v_j)\) of \(A \cup A'\), define the binary constant \(a_{ij}^{pd}\) equal to 1 if and only if arc \((v_i, v_j)\) can be serviced during period \(p\) of workday \(d\). The subsets of workdays and periods of work for which an arc can be serviced are defined based on the hierarchy of the network and the snow loading regulations mentioned above. For every workday \(d \in D\) and for every period of work \(p \in P(d)\), define \(T_p\) as the time limit of period \(p\). For every workday \(d \in D\) and for every period of work \(p \in P'(d)\), let \(p^-)\) be the period of work preceding period \(p\) of workday \(d\). For every workday \(d \in D\), for every period of work \(p \in P(d)\), and for every vertex \(w_{ij} \in W\) corresponding to an arc \((v_i, v_j)\) of \(A \cup A'\), let \(y_{ij}^{pd}\) be a binary variable equal to 1 if and only if vertex \(w_{ij}\) is visited during period \(p\) of workday \(d\). For every workday \(d \in D\), for every period of work \(p \in P(d)\), and for every arc \((w_{ij}, w_{kl}) \in B\), let \(x_{ij,kl}^{pd}\) be a binary variable equal to 1 if and only if arc \((w_{ij}, w_{kl})\) is traversed during period \(p\) of workday \(d\) in the optimal solution.

If the end location of one period of work of a given workday (except the last period) and the start location of the next period of work of the same workday do not coincide, then the snowblower must use the transportation network to go from one location to another. In order to model the additional deadheading time incurred by the snowblower, Gilbert suggested extending \(H\) by adding an artificial vertex \(w_f\) representing the transfer point between two consecutive periods of the same workday. Two arcs of opposite direction are included between a vertex \(w_{ij} \in W\) and the transfer vertex \(w_f\). A vertex \(w_0\) representing the depot is also added to \(H\) and linked to each other vertex in \(W\) with a pair of arcs of opposite direction. Let \(H' = (W \cup \{w_0, w_f\}, B \cup B_1 \cup B_2)\) be the extended complete directed graph where \(B_1 = \{(\{w_0, w_{ij}\}, (w_{ij}, w_0): w_{ij} \in W\}\) and \(B_2 = \{(w_f, w_{ij}), (w_{ij}, w_f): w_{ij} \in W\}\). For each workday \(d \in D\), the transfer vertex \(w_f\) must be visited \(|P(d)| - 1\) times along the route, and the depot vertex \(w_0\) must be visited at the beginning and at the end of the first and last period of the workday, respectively. For every workday \(d \in D\), for every period of work \(p \in P(d)\) \(\setminus P'(d)\) \((p \in P(d) \setminus P''(d))\), and for every arc \((w_0, w_{ij}) \in B_1\) \(((w_{ij}, w_0) \in B_1)\), let \(x_{0,ij}(x_{ij,0})\) be a binary variable equal to 1 if and only if arc \((w_0, w_{ij})\) \(((w_{ij}, w_0))\) is traversed during period \(p\) of workday \(d\) in the optimal solution. Finally, for every workday \(d \in D\), for every period of work \(p \in P'(d)\) \((p \in P'(d) \setminus P''(d))\), and for every arc \((w_f, w_{ij}) \in B_2\) \(((w_{ij}, w_f) \in B_2)\), let \(x_{ij,f}(x_{ij,f})\) be a binary variable equal to 1 if and only if arc \((w_f, w_{ij})\) \(((w_{ij}, w_f))\) is traversed during period \(p\) of workday \(d\) in the optimal solution.

We present here a slightly modified version of the Gilbert nonlinear 0–1 integer programming model for the snowblower routing problem. (We eliminate some variables used by Gilbert to clarify the interpretation of results).

Minimize

\[
\sum_{d \in D} \sum_{p \in P(d)} \sum_{w_{ij} \in W} \sum_{w_{kl} \in W} t_{ij}x_{ij,kl}^{pd} + \sum_{d \in D} \sum_{p \in P'(d)} \sum_{w_{ij} \in W} \sum_{w_{kl} \in W} t_{ij}x_{ij,f}^{pd} \quad (3.1)
\]

subject to

\[
\sum_{w_{kl} \in W \cup \{w_0\}} x_{ij,kl}^{pd} = y_{ij}^{pd} \quad (d \in D, p \in P(d) \setminus P''(d), w_{ij} \in W), \quad (3.2)
\]

\[
\sum_{w_{kl} \in W \cup \{w_f\}} x_{ij,kl}^{pd} = y_{ij}^{pd} \quad (d \in D, p \in P''(d), w_{ij} \in W), \quad (3.3)
\]
\[
\sum_{wkl \in W \cup \{w_0\}} x_{klij}^p d = y_{ij}^p d \quad (d \in D, p \in P(d) \setminus P'(d), w_{ij} \in W),
\]

\[
\sum_{wkl \in W \cup \{w_f\}} x_{klij}^p d = y_{ij}^p d \quad (d \in D, p \in P'(d), w_{ij} \in W),
\]

\[
\sum_{w_{ij} \in W} x_{ij,0}^p d = 1 \quad (d \in D, p \in P(d) \setminus P'(d)),
\]

\[
\sum_{w_{ij} \in W} x_{ij,f}^p d = 1 \quad (d \in D, p \in P'(d)),
\]

\[
\sum_{w_{ij} \in W} x_{ij,0}^p d = 1 \quad (d \in D, p \in P''(d)),
\]

\[
\sum_{w_{ij} \in W} x_{ij,f}^p d = 1 \quad (d \in D, p \in P''(d)),
\]

\[
\sum_{d \in D} \sum_{p \in P(d)} s_{ij} y_{ij}^p d \leq a_{ij}^p d \quad (d \in D, p \in P(d), w_{ij} \in W),
\]

\[
\sum_{p \in P_k(d)} y_{ij}^p d + y_{i'j}^p d \leq 1 \quad (d \in D, P_k(d) \in P_K, (v_i, v_j), (v_{i'}, v_{j'}) \in A),
\]

\[
\sum_{p \in P_k(d)} y_{ij}^p d + y_{ji}^p d \leq 1 \quad (d \in D, P_k(d) \in P_K, (v_i, v_j), (v_j, v_i) \in A'),
\]

\[
\sum_{w_{ij} \in W} s_{ij} y_{ij}^p d + \sum_{w_{ij} \in W} \sum_{w_{kl} \in W} t_{jkl} x_{ijkl}^p d \leq T_p \quad (d \in D, p \in P(d) \setminus P'(d)),
\]

\[
\sum_{w_{ij} \in W} s_{ij} y_{ij}^p d + \sum_{w_{ij} \in W} \sum_{w_{kl} \in W} \left( t_{jkl} x_{ijkl}^p d + t_{jkl} x_{ij,f}^p d x_{f,kl}^p d \right) \leq T_p \quad (d \in D, p \in P'(d)),
\]

The nonlinear objective function (3.1) minimizes the total deadheading traversal time. Constraints (3.2)–(3.5) require that the snowblower enters and leaves each vertex associated with a side of a roadway during the same workday and the same period of work. Constraints (3.6) and (3.7) ensure that the snowblower
leaves and enters the depot during the first and last period of each workday, respectively. Analogously, constraints (3.8) and (3.9) ensure that exactly one arc leaves and enters the transfer vertex during each period of a given workday except the first and last period, respectively. Constraints (3.10) require that each vertex associated with a side of a roadway be visited during exactly one of the appropriate workdays. Constraints (3.11) state that a vertex associated with a side of a roadway can be visited during a period of work only if it is allowed. Constraints (3.12) and (3.13) ensure that only one side of a one-way and two-way street can be serviced during each subset of periods of work associated with a given workday, respectively. Constraints (3.14) and (3.15) are the time limit restriction for the first period of work and for each period of work except the first period associated with a given workday, respectively, whereas constraints (3.16) are the subtour elimination constraints. Finally, all variables are restricted to be binary.

Gilbert proposed to solve the model using a heuristic approach that first initializes feasible partial routes one at a time by inserting higher priority vertices into the first periods of work, and then balances out the workload of the partial routes by filling the emptiest initial partial routes with lower priority vertices, mainly in the last workdays. The heuristic is described in Fig. 6.

The second insertion strategy operates as a parallel insertion method and is reminiscent of the parallel-insert algorithm proposed by Chapleau et al. [66] for the capacitated arc routing problem. Other orderings of the two insertion strategies can be implemented to generate other procedures. The insertion of a vertex into a given partial route is performed by means of the cheapest insertion procedure [67] for the traveling salesman problem. Given a partial route $C_{pd}$, the arc $(w_{ij}, w_{kl})$ in $C_{pd}$ and the vertex $w_{mn}$ not in $C_{pd}$ are first chosen such that $t_{jm} + t_{nk} - t_{jk}$ is minimal, and then $w_{mn}$ is inserted between $w_{ij}$ and $w_{kl}$. However, this rule cannot be used to choose the first vertex to insert into $C_{pd}$ if no vertex is visited during the two periods preceding and succeeding period $p$. Then, the vertex $w_{ij}$ that maximizes the length $s_{ij}$ is inserted into $C_{pd}$. In addition to the two complementary insertion strategies, Gilbert also suggested reoptimizing the partial route $C_{pd}$ for which the set of vertices $W_{pd}$ has just become empty. This is achieved by using an adaptation of the Carpaneto and Toth [68] algorithm for the asymmetric traveling salesman problem (ATSP). For small instances (about 15 vertices or less), the ATSP was solved exactly. Otherwise, the algorithm was stopped. The heuristic was tested on actual data from one district in the city of Montreal, Canada, involving 470 vertices that represent sides of roadways to be serviced by the snowblower.
In order to reduce the size of the problem, these vertices are aggregated into 122 geographic zones that contain a collection of neighboring sides of roadways. The snowblower routing plan was obtained in two minutes and had a smaller distance covered by deadheading trips than the plan used by the city.

4. Fleet sizing and replacement models

Relatively little work has been accomplished for determining the sizes and replacement schedules of fleets for winter road maintenance vehicles. Large fleets are required to clear the roadways and sidewalks promptly to allow safer travel, but expenditures increase as fleet size increases. Thus, the tradeoff between minimizing the completion time for winter road maintenance and minimizing the expenditures is key. In vehicle fleet replacement, the important tradeoff is between minimizing maintenance costs for keeping old vehicles and minimizing acquisition costs for new, perhaps more efficient, vehicles. At the strategic and tactical planning levels, replacement schedules specify the sequence of vehicles to replace in the future and the length of time each vehicle in the sequence is to be kept in service. At the operational and real-time levels, replacement schedules specify whether to keep an existing vehicle or to replace it immediately with a new vehicle.

Optimization and analytical models for vehicle fleet sizing and replacement in the context of winter road maintenance are reviewed in this section. Models for fleet sizing are discussed first, followed by an optimization model to determine replacement schedules of old vehicles by new vehicles. The characteristics of these models are then summarized in Table 3.

4.1. Fleet sizing models

The combined optimization models of Kandula and Wright [56,62], Labelle et al. [8], and Hayman and Howard [69] reviewed in the first [2], second [3], and third parts of the survey [1], respectively, are an attempt at integrating fleet sizing with other components of winter road maintenance operations such as sector design, depot location, and snow disposal assignment. However, these closely interdependent problems are most often treated separately: strategic and tactical plans are developed first, followed by fleet sizing to determine the number of plows, spreader vehicles, and trucks for the planned sectors and depots. Fleet sizing problems can be grouped into two classes according to the winter road maintenance operation involved: fleet sizing for snow plowing and truck fleet sizing for hauling snow to disposal sites. An optimization model for the snowplow fleet sizing problem is first reviewed, followed by two analytical models for the truck fleet sizing problem.

4.1.1. Fleet sizing models for snow plowing

Hayman and Howard [69] treated a snowplow fleet sizing problem in which the number of plows dispatched from depots to clear the roadways is determined so as to minimize operational and depot depreciation costs, while satisfying a specified level of service for each road class. The number of snowplows for high-class roadways must be large enough to limit the average snowfall accumulation to some critical depth, whereas maximum service times are imposed on every other road class. The problem is formulated as a linear integer programming problem. Let $I$ be the set of vehicle depots and let $J$ be the set of roadways to be serviced. For every depot $i \in I$ and for every roadway $j \in J$, let $x_{ij}$ be a non-negative integer variable representing the number of snowplows based at depot $i$ to clear roadway $j$, and let $b_{ij}$
Table 3
Characteristics of fleet size and replacement models

<table>
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and \( d_{ij} \) be the unit time cost in traveling from depot \( i \) to roadway \( j \), and the distance from depot \( i \) to the centroid of roadway \( j \), respectively. For each depot \( i \in I \), define \( a_i \) as the cost of deployment of the snowplows to depot \( i \) including the depot depreciation cost applied to each snowplow based at depot \( i \). For every roadway \( j \in J \), let \( d_j, l_j, p_j, \) and \( t_j \) be the average distance from a depot to roadway \( j \), the length of roadway \( j \), the number of plow passes required to clear roadway \( j \), and the allowable time for plowing roadway \( j \) calculated from the beginning of the snowfall, respectively. Let also \( s, r \) and \( d \) be the average vehicle speed, the rate of snowfall and the critical snow depth, respectively. Finally, if we let \( A \subseteq J \) be a set of high-class roadways, then the formulation is as follows:

**Minimize**

\[
\sum_{i \in I} \sum_{j \in J} a_i x_{ij} + \sum_{i \in I} \sum_{j \in J} \left( b_{ij} \frac{d_{ij}}{s} \right) x_{ij} \quad (4.1)
\]

subject to

\[
\sum_{i \in I} x_{ij} \geq \frac{p_j l_j r}{s \cdot d} \quad (j \in A), \quad (4.2)
\]

\[
\left( \frac{p_j l_j}{\sum_{i \in I} x_{ij}} \right) \leq t_j - \frac{\left( d_j \right)}{s} \quad (j \in J \setminus A), \quad (4.3)
\]

\[
x_{ij} \geq 0 \text{ and integer} \quad (i \in I, j \in J). \quad (4.4)
\]

The objective function (4.1) minimizes the sum of all depreciation and operational costs. Constraint set (4.2) imposes a minimum number of plows to clear each high-class roadway. The specification for high-class roadways requires that the snowfall should not be allowed to accumulate beyond the critical depth \( d \). The snow will accumulate to \( d \) at time \( d r \). The distance a plow will travel during this time is \( s d r \). If a group of plows are to follow one another down a roadway and are spaced according to \( s d r \), then the maximum snow accumulation between them would be \( d \). This corresponds to the service level desired. Therefore, the required number of plows to clear each high-class roadway is given by the right-hand side of (4.2). Brohm and Cohen [70] gave a similar expression to calculate the number of vehicles required to plow a roadway segment with a specified level of service and to estimate the number of spreader vehicles required. If we assume that the plowing of a lower-class roadway may be equally divided among the plows dispatched from all depots, then constraint set (4.3) ensures that the total time available to plow each lower-class road is respected. Finally, all variables \( x_{ij} \) naturally assume non-negative integer values. In order to reduce the size of the problem, some \( x_{ij} \) variables are discarded if the total time available to plow lower-class road \( j \) from vehicle depot \( i \) is higher than a specific threshold. Tests were performed on a real problem involving 15 potential depot sites and 41 roadway sections. The LP relaxation of the model (4.1)–(4.4) was solved using the simplex algorithm and the total number of plows required was rounded up to the nearest integer value.

Ungerer [71] described a simulation model to help planners in determining the number of snowplows to suit regional winter maintenance demands, operating costs and the desired service level. Inputs of the model are the numbers defining the cumulative probability function of the number of snowstorms per year of each type, the number of annual simulation runs, the level of service, the target storm type for which the number of snowplows available is sufficient to satisfy performance demands, the highway lane-kilometers, and the highway classification, as well as various costs associated with purchasing.
and maintaining snowplows and rentals if not enough equipment is available to meet the desired level of service. Monte Carlo simulation is used to generate distributions of snowplow requirements and associated costs per year for each storm type. Insights for assigning the required number of snowplows annually are provided by varying the input values of the model and analyzing the resulting snowplow number and operational cost distributions and the various tradeoffs.

4.1.2. Fleet sizing models for hauling snow to disposal sites

In determining the fleet size for snow hauling trucks in a sector, one usually wants to balance the fixed and variable costs for the trucks, and the length of time for the snow loading and hauling operations. A detailed analysis of snow loading and hauling operations was performed by the Bureau of Management Consulting, Transport Canada [36] who proposed two analytical models to determine the size of a homogeneous fleet of trucks assigned to a snowblower. The first model estimates the number of trucks required by a continuously operating snowblower. This number corresponds to one plus the quotient of the round trip travel time between the snowblower and the disposal site, and the time for filling a truck with snow. The round trip travel time for a truck includes the average time required to travel from the snowblower to the disposal site and back to the blower, and the average time to unload a truck at a disposal site. This model tends to underestimate the real truck fleet size required to allow continuous operation of a snowblower since it does not include time for queuing alongside a snowblower or at a disposal site, nor does it include variability in travel time or unloading time.

The second model is based on a two-stage cyclic closed queuing system with transit times developed by Posner and Bernholtz [72]. The model is a finite closed system with two stations in which the time taken to move from one station to the next is assumed to be a random variable with a general distribution. At each station, the service times are independent, exponentially distributed random variables with expected service times that may be arbitrary functions of the number of units at that station. In the snowblower—truck—disposal site cyclic closed system shown in Fig. 1, the two stations represent the snowblower and the disposal site, and trucks circulate cyclically between them. The two stage cyclic closed queuing model of Posner and Bernholtz [72] was used to compute the marginal probability \( P \) that a snowblower never becomes idle for various numbers of trucks in the system given known loading and unloading times as well as truck speed. This marginal probability then serves to evaluate alternative fleet sizes. Let \( C_b \) and \( C_t \) be the hourly cost for a snowblower and the hourly cost for a truck, respectively. Let \( m \) be the total number of trucks in the snowblower—truck—disposal site system. The total operating cost \( C \) of the system is given by (4.5).

\[
C = \frac{C_b + C_t m}{P}. \tag{4.5}
\]

Setting \( m \) in (4.5) to a very small number minimizes the total cost whereas setting \( m \) equal to a very large number minimizes the completion time. As \( m \) varies between these extremes, candidate compromise fleet sizes can be identified. The accuracy of the model was validated using historic data from one Canadian city. Tests showed that the model was useful in analyzing a variety of scenarios related to the modification of loading and unloading times as well as truck speed and truck capacity.

However, the two-stage closed queuing model with transit times involves certain restrictive assumptions. For a first-in-first-out loading and unloading discipline, loading and unloading times are required to be exponentially distributed. It is also assumed that trucks transit instantaneously from the snowblower to the disposal site or inversely. To relax these assumptions, Chugha and Posner [73] proposed a two-stage
cyclic queue model with general service time distributions of the Erlang class and with time lags between stations. The authors developed a good estimate of the utilization of a station defined as the proportion of time that a station is busy over a long period of time. This estimate could serve in the context of winter road maintenance to study disposal site utilizations by developing operating cost characteristics which could then be used for solving the snow disposal site location problem.

4.2. Vehicle fleet replacement models

Very little work has been published concerning fleet replacement for winter road maintenance. The Bureau of Management Consulting, Transport Canada [36] proposed a basic replacement model to determine a cost minimizing replacement schedule for snow and ice control vehicles, where cost is measured by estimates of the operating, maintenance, and net replacement costs. Equations are derived for these costs. The problem is formulated as a nonlinear programming problem. Let $x$ be a non-negative variable representing the number of cumulative hours utilized by a vehicle of a homogeneous fleet. Define $Q$, $r$ and $p$ as the initial purchase cost of the vehicle, the average annual rate of replacement of a vehicle in the fleet, and the average annual number of hours utilized by a vehicle in the fleet, respectively. Define also $A$, $B$, $C$ and $D$ as four constants whose values can be determined by regression with a suitable set of data. The formulation is as follows.

\[
\text{Minimize} \quad \frac{Ax^B + Q(1 + r)x^p}{x} - C e^{-Dx/p},
\]

subject to

\[
x > 0.
\]  

The objective function (4.6) minimizes the hourly cost for the utilization of the vehicle during $x$ cumulative hours. This is the sum of the operating and maintenance costs for the utilization of a vehicle in the fleet during $x$ cumulative hours and the net replacement cost divided by the number of cumulative hours utilized by the vehicle. The net replacement cost is defined as the difference between the cost of acquiring a new vehicle after $x$ cumulative hours of utilization of the vehicle and the resale value of a vehicle in the fleet after $x$ cumulative hours operated. Computational experiments on historic data from a Canadian city showed that the model was useful in determining replacement schedules for snowplows, sidewalk snowplows and snowblowers. In addition to deciding when to replace individual vehicles, the model can also be used to make replacement scheduling decisions for a homogeneous fleet of vehicles having the same acquisition and operating cost structures. The authors discussed the case where newer vehicles have different acquisition or operating cost structures than older less sophisticated vehicles. Computational results indicated that the model can still be used to find the optimal buy and sell policy for a newer or an older vehicle by calibrating the operating, maintenance, and net replacement costs through regression analysis.

Computerized systems have also been developed to help planners in determining the sizes or replacement schedules of fleets for winter road maintenance vehicles. Such systems were described, for example, by Hammond [74] and Nielson [75].
5. Conclusions

Winter road maintenance operations involve a host of system design and vehicle routing problems that can be addressed with operations research techniques. This paper is the last part of a four-part survey of optimization models and solution algorithms for winter road maintenance. It addresses vehicle routing, fleet sizing, and fleet replacement models for plowing and snow disposal operations. (The two first parts of the survey [2,3] discuss system design models for winter road maintenance operations. The third part of the review [1] addresses vehicle routing, depot location, and crew assignment models for spreading operations.)

Vehicle routing problems for winter road maintenance are very difficult and site specific because of the diversity of operating conditions influencing the conduct of winter road maintenance operations and the wide variety of operational constraints. Hence, all algorithms developed for the routing of vehicles for winter road maintenance are heuristics. Early models were generally solved with simple constructive methods for undirected and directed versions of the Chinese postman problem, and used simulation models to evaluate benefits. Implementation details and operational constraints were rarely considered. One then witnessed a gradual consideration of more realistic vehicle routing problems and a gradual introduction of local search techniques. While some recent models are solved with composite heuristic methods, which blend route construction and improvement algorithms, others are solved using metaheuristics, which have proven to be very effective for several classes of discrete optimization problems.

However, even though recently proposed models tend to incorporate many of the characteristics of applications arising in practice, they have not yet been widely implemented. A 1995 survey in Minnesota reported that only a single agency out of 414 jurisdictions (counties, cities, townships) used a computerized routing software for snow and ice control [76]. This gap between theory and practice may be reduced with the documentation of recent successful routing software packages for winter road maintenance. The CASPER system and the GeoRoute Municipal package are two illustrative examples of this progress. The factors explaining the success of these routing software packages are discussed by Campbell and Langevin [48].

Although there has been some work on vehicle routing problems for spreading and plowing operations, there is almost no research on the routing of snowblowers and trucks for snow loading and hauling operations. Most models for the routing of trucks for hauling snow to disposal sites are based on a simple cyclic closed system based on a static allocation of trucks. A more sophisticated approach that dynamically redeployed trucks between different snowblowers according to changing needs would be worth exploring. For the snowblower routing problem, composite methods, such as the one embedded in the GeoRoute package, are promising optimization approaches.

There are strong interactions between the various winter road maintenance problems of routing of spreaders, plows, snowblowers, and trucks, locating disposal sites, designing sectors, assigning sectors to disposal sites, assigning crews, and managing vehicle fleets. However, models that take all these aspects of winter road maintenance into consideration get extremely complex if not simply intractable. The traditional approach has thus been to deal separately and sequentially with each problem. Very frequently, disposal sites are first located, sectors are then designed and assigned to disposal sites, and routes are determined last. Since the quality of the routes produced in each sector is highly dependent on the quality of the configuration of the sectors, this approach obviously leads to suboptimal routing decisions. As highlighted by Ghiani and Laporte [77], a better sequential approach could consist of designing the routes first and locating facilities last. Several researchers have employed this second
approach for the solution of location-arc routing problems. Another direction worth pursuing involves the use of multiobjective analysis to assist planners in making fleet sizing decisions. In particular, in determining the truck fleet size for hauling snow to disposal sites, the tradeoff between minimizing the fixed and variable costs for the trucks and minimizing the length of time for the snow loading and hauling operations remains largely unexplored.

Finally, technology now available provides new opportunities for the planning of vehicle routing for winter road maintenance. For example, road weather information systems could lead to real-time vehicle routing to treat only those areas in need at a particular time. Truck mounted pavement sensors could help better determine when and how to treat a road and electronic spreader controls on trucks can adjust the amount of materials being spread based on vehicle speed. Automatic vehicle location using positioning system technology could permit real-time reallocation of vehicles and crews, real-time status reports to inform road users of current operations and road conditions (e.g., spread or not, plowed or not, etc.), and to identify unauthorized travel. Many winter maintenance management systems that include one or more of these technologies have been developed. However, to fully achieve the benefits promised by these new technologies, further developments are required to integrate them with optimization techniques for the planning of vehicle routing for winter road maintenance. The greater power and sophistication of road weather information systems, weather forecasting services, winter road maintenance equipment, geographic information systems, global positioning systems, communication systems, computer systems, and optimization techniques can now be merged to address the full scope of vehicle routing for winter road maintenance.

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