A Modified Selective Mapping with PAPR Reduction and Error Correction in OFDM Systems

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Abstract — In this paper, we propose a PAPR reduction method for BPSK OFDM systems and 16-QAM OFDM systems respectively. The strategy of the algorithms is to use a correction subcode for error control and a scrambling subcode for PAPR control. The transmitted OFDM sequence is selected with minimum PAPR from a number of candidates which form a coset of scrambling subcode for each codeword of correction subcode. In a BPSK OFDM system, the PAPR reduction is obtained by combining the SLM method with binary cyclic codes. In a 16-QAM OFDM system, the PAPR reduction is achieved by combining the SLM with block coded modulation. The received signal of the modified SLM can be decoded without the need of explicit side information and the proposed scheme achieves good PAPR reduction.

Index Terms — OFDM, PAPR, SLM, BCM, binary cyclic codes, Reed-Muller codes

I. INTRODUCTION

Multicarrier modulation or orthogonal frequency division multiplexing (OFDM) has received rising interest recently and has been proposed or adopted for a number of communication systems, such as wireless local area networks and high data rate wireless transmissions. The principle of OFDM is to divide a high rate information bitstream into several parallel low rate data substreams and use these substreams to modulate a number of subcarriers by Fourier transform techniques. In spite of many advantages, one of the implementation drawbacks of OFDM is that the transmitted signal has a high peak-to-average power ratio (PAPR). Therefore, the subject of finding a computationally efficient algorithm to decrease the PAPR in OFDM systems has become an active research [1].

A number of techniques were proposed to control the PAPR of the transmitted signals in OFDM systems, such as block coding [2], [3], partial transmit sequence (PTS) [4], [5], and selective mapping (SLM) [6], [7], [8]. PTS and SLM are probabilistic methods which achieve significant PAPR reduction with only a small data rate loss; however PTS or SLM has no inherent error control compared to the use of block coding with Golay complementary sequences. Recently, Carson and Gulliver [9] and Xin and Fair [10], [11] reported a PAPR reduction method, called EC-SLM in [11], by combining the SLM technique with an error correcting code in a binary phase shift keying (BPSK) modulation. This method divides data bits into PAPR control bits and information bits and uses these PAPR bits to generate a number of candidates to represent the same OFDM signal. The transmitted OFDM signal is selected with minimum PAPR from these candidate sequences. Although this method can achieve error correction and PAPR control simultaneously, the selection of PAPR control bits has not yet been explored. Moreover, no studies have attempted to provide PAPR reduction and error control over a M-ary quadrature amplitude modulation (QAM) OFDM system.

In this paper, we examine these two problems by combining cyclic codes with SLM for BPSK OFDM and combining block coded modulation (BCM) with SLM for 16-QAM OFDM. We first consider the EC-SLM algorithm by decomposing a binary cyclic code into a direct sum of two cyclic subcodes, a correction subcode for encoding information bits and a scrambling simplex subcode for encoding PAPR bits. Equivalently, the resulting code of OFDM sequences is constructed by selecting a proper subcode from the direct sum of these two cyclic component codes. We call this modified EC-SLM as Cyclic-SLM. We then extend the EC-SLM method to a 16-QAM OFDM system, called BCM-SLM, by using a BCM code to achieve bandwidth-efficient modulation and the SLM approach to improve the PAPR performance.

The rest of the paper is organized as follows. In Section II, we review the definition of PAPR and the conventional SLM method in OFDM signals. Section III introduces the Cyclic-SLM algorithm in a OFDM system with BPSK modulation and the BCM-SLM method in a OFDM system with 16-QAM modulation. We then follow with a summary in Section IV.

II. PRELIMINARIES

We assume in what follows that the reader is familiar with the properties of linear block codes, such as a generator matrix or a parity check matrix of linear block codes in [12]. By an encoding of \( C[n,k] \), we mean any linear bijection from \( F_2^n \) to \( F_2^m \). More precisely, a \( k \times n \) matrix \( G \) which has a basis of the code \( C \) as its row vectors is called a generator matrix of \( C \). Since a binary linear code \( C[n,k] \) is a \( k \)-dimensional subspace of \( F_2^n \), the set \( F_2^n \) of all vectors can be partitioned into \( 2^{n-k} \) disjoint cosets of \( C \):

\[
F_2^n = C \cup (e_1 + C) \cup \cdots \cup (e_{2^{k-k}-1} + C).
\]

A. PAPR IN OFDM

Given a data block of \( n \) symbols \( x = [x_0, x_1, \ldots, x_{n-1}] \) from some signal constellation, the complex envelope \( S_x(t) \) in
a OFDM system can be represented as
\[ S_k(t) = \sum_{k=0}^{n-1} x_k e^{j2\pi \frac{k}{T} t}, \quad 0 \leq t \leq T, \] (1)
where \( \frac{1}{T} \) is the bandwidth of each subcarrier. The instantaneous envelope power \( P_k(t) \) of a complex vector \( x \) is defined by
\[ P_k(t) = |S_k(t)|^2 \] and its average power \( \|x\|^2 \) is equal to
\[ \|x\|^2 = \frac{1}{T} \int_0^T P_k(t) dt = \sum_{k=0}^{n-1} |x_k|^2. \] (2)
Let \( X \) be the collection of all possible transmitted data blocks in the OFDM system. The PAPR of the transmit data block \( x \in X \) is defined as
\[ \text{PAPR}(x) = \frac{\max_{0 \leq t \leq T} P_k(t)}{P_{av}}, \] (3)
where \( P_{av} = \sum_{x \in X} \|x\|^2 p(x) \) is the average power of the set \( X \) and \( p(x) \) is the probability of \( x \). We also define the PAPR of a code \( X \) by
\[ \text{PAPR}(X) = \max_{x \in X} \text{PAPR}(x). \] (4)

The continuous PAPR of \( S_k(t) \) is approximated by its discrete \( L \)n samples \( S_k(\frac{1}{L}t_n) \), \( 0 \leq i \leq nL - 1 \), which is obtained from the \( L \)n-point IDFT of \( \{x_0, \ldots, x_{n-1}\} \) with \((L - 1)n \) zero-padding. Here we use \( L = 4 \) for numerical simulation and it is enough to estimate the continuous PAPR, as is observed in [13].

B. The conventional SLM Approach

In the conventional SLM technique, several different OFDM signals are generated by multiplying the original data block with some scrambling sequences and then the one with the minimum PAPR is selected for transmission. In terms of coding language, a binary information vector \( c = (c_1, \ldots, c_n) \in C[n, n] = F_2^n \) is first added by a number of scrambling sequences in \( Y = \{y_1, \ldots, y_{2^m}\} \) to form a set of different transmitted candidates,
\[ c + Y = \{c + y_1, \ldots, c + y_{2^m}\}, \]
i.e., a coset of \( Y \). Then the one with minimum PAPR in \( c + Y \) is selected as a transmitted signal. Since the number of information bits of \( C \) is \( k = n \), the explicit side information of \( m \) bits about the selected scrambling sequence should be transmitted reliably such that the original information block \( c \) can be recover at the receiver. The performance of PAPR reduction in SLM strongly depends on the number and the selection of the scrambling sequences in \( Y \).

III. THE MODIFIED SLM ALGORITHM

Xin and Fair [11] propose a modified selective mapping algorithm with error correcting capability and without explicit side information by using \( k \) bits for information transmission, \( m \) bits for PAPR control, and \( n - k - m \) bits for error correction. The fundamental principle of this modified SLM is to reserve some PAPR bits from information bits to generate a number of SLM candidates in the transmitter and uses these PAPR bits as implicit information to recover the original codeword without sending the explicit side information to the receiver. Equivalently, we decompose a linear code as the direct sum of two subcodes: a correction subcode for encoding information bits and a scrambling subcode for encoding PAPR bits. The transmitted signal of a resulting OFDM sequence is selected with minimum PAPR from a number of candidates which are codewords of a coset of the scrambling subcode.

A. Cyclic-SLM

Let \( C[n, k] = \langle G \rangle \) be a binary linear code used for error correction, where \( G \) is a generator matrix of \( C \). Let
\[ Y(n, 2^m) = \{y_1, y_2, \ldots, y_{2^m}\} \]
be a block code used for scrambling. In order to determine which scrambling sequence in \( Y \) is added to the original codeword, we can either transmit the explicit \( m \) bits side information or integrate these \( m \) bits as the PAPR bits along with \( k \) information bits and \( n - k - m \) parity bits. In other words, the set \( Y \) of scrambling sequences should be chosen such that any two cosets \( y_i + C \) and \( y_j + C, i \neq j \), are disjoint. The union of these cosets is a block code
\[ Z(n, 2^{k+m}) = \bigcup_{i=1}^{2^m} (y_i + C) \subset F_{2^n}. \] (5)

More precisely, in this scheme, an information vector \( u \in F_{2^k} \) is first encoded into a codeword \( c_i = u, G \in C[n, k] \); then we construct a nonlinear block code
\[ B(n, M = 2^k) = \{b_1, b_2, \ldots, b_M\} \subset Z \]
such that each vector \( b_i \) is the vector chosen from \( c_i + Y \) with the property that
\[ \text{PAPR}(b_i) = \min_{y \in Y} \text{PAPR}(c_i + y), \quad 1 \leq i \leq M. \] (6)
The resulting OFDM code \( B \) has the same rate as \( C \) and at least error correcting capability as \( Z \). The decoding at the receiver is done with two steps: first we decode the received vector \( r \) to the codeword \( b \in B \) by the decoding method of \( Z \) and then we recover from \( b = c + y \in c + Y \) to \( c \in C \) by syndrome decoding whenever \( b \) and \( y \) have the same syndrome, i.e., \( Hb = Hy \), where \( H \) is a parity check matrix of \( C \).

We can further simplify the decoding by using cyclic codes for encoding and scrambling. A proposed modified SLM algorithm using two disjoint binary cyclic codes for OFDM systems with \( 2^m - 1 \) BPSK subcarriers is shown in Fig. 1. We assume that
\[ x^n - 1 = x^{2^m-1} - 1 = s(x)t(x)g(x) \]
is a factorization over \( F_2[x] \), where \( \deg s(x) = m, \deg t(x) = k \), and \( \deg g(x) = n - k - m \). Let \( C[n, k] \) and \( Y[n, m] \) be two cyclic codes whose generator polynomials are \( s(x)g(x) \) and \( t(x)g(x) \) respectively, i.e., \( C = \langle s(x)g(x) \rangle \) and \( Y = \langle t(x)g(x) \rangle \). It is known that the intersection and sum of any two cyclic codes, \( C_1 = \langle g_1(x) \rangle \) and \( C_2 = \langle g_2(x) \rangle \), are also
cyclic codes with generator polynomials [14, p. 324]; $C_1 \cap C_2 = \langle \text{lcm}(g_1(x), g_2(x)) \rangle$ and $C_1 + C_2 = \langle \text{gcd}(g_1(x), g_2(x)) \rangle$. Since

$$ C \cap Y = \langle \text{lcm}(s(x)g(x), t(x)g(x)) \rangle = \langle x^n - 1 \rangle = \{0\} $$

and

$$ Z = C \oplus Y \quad = \langle \text{gcd}(s(x)g(x), t(x)g(x)) \rangle \quad = \langle g(x) \rangle = \bigcup_{y(x) \in Y} \langle y(x) + C \rangle, $$

we can thus use $C$ for encoding and $Y$ for scrambling to construct an OFDM code $B \subset Z$ of $2^m - 1$ BPSK subcarriers.

There are two advantages of using the cyclic codes $C$ for encoding and $Y$ for scrambling. First, any fast decoding for the cyclic code $C \oplus Y$ can be used as the first step decoding. Next, the second step decoding from $b(x) \in B$ to $c(x) \in C$ is done by multiplying $b(x) = c(x) + y(x)$ with the unique identity idempotent $e(x)$ of $C$:

$$(c(x) + y(x)) \cdot e(x) = c(x) \cdot e(x) + y(x) \cdot e(x) = c(x) + 0$$

since $y(x) \cdot e(x) \in C \cap Y = \{0\}$. Though we use binary cyclic codewords for scrambling instead of random sequences, numerical results show that with an appropriate selection of binary cyclic codes $C$ and $Y$, the code $B$ of resulting OFDM sequences has low PAPR, good Hamming distance, and high code rate.

We examine the PAPR performance for an OFDM system with $n$ BPSK subcarriers, where $n = 2^m - 1$ and $5 \leq m \leq 8$. For each $m$, we choose a primitive polynomial $p(x)$ as a minimal polynomial (m.p.) for $\alpha$ to form a finite field $F_{2^m}$.

Here, we take $p(x) = x^5 + x^2 + 1$ for $m = 5$, $p(x) = x^6 + x^1$ for $m = 6$, $p(x) = x^7 + x^3 + 1$ for $m = 7$, and $p(x) = x^8 + x^4 + x^3 + x^2 + 1$ for $m = 8$. To construct an OFDM code of length $2^m - 1$ with error correcting $\epsilon$, we first choose a BCH code $Z[n, k, m, 2t + 1] = \langle g(x) \rangle$;

then a primitive minimal polynomial $s(x)$ of $\alpha^i$ from a factor of $h(x) = (x^n - 1)/g(x)$ is chosen, where $i$ is the smallest positive number such that $b(\alpha^i) = 0$. We form two cyclic codes: a correcting subcode $C[n, k] = \langle s(x)g(x) \rangle$ and a scrambling subcode

$$ Y[n, m] = \langle \frac{h(x)}{s(x)}g(x) \rangle = \langle x^n - 1 \rangle $$

which is a simplex code. The resulting code $B$ of OFDM sequences by the Cyclic-SLM algorithm is an $(n, 2^k, d \geq 2t + 1)$ block code which is a nonlinear subcode of a $t$-error correcting BCH code of dimension $k + m$.

Using this Cyclic-SLM for $m = 5$, the probabilistic PAPR at excess probability $10^{-4}$ is 5.29 dB, 5.30 dB, and 5.27 dB for a zero-error correcting OFDM code $B(31, 2^{26}, 1) \subset F_{2^{31}}^1$, a single-error correcting OFDM code $B(31, 2^{21}, 3) \subset \text{BCH}(31, 26, 3)$, and a double-error correcting OFDM code $B(31, 2^{15}, 5) \subset \text{BCH}(31, 21, 5)$ respectively, where the probabilistic PAPR of $B = F_{2}^{21}$ without SLM is 11.53 dB. Table I list the dimension, the error correction capability, the primitive polynomial $s(x)$, and the probabilistic PAPR at probability $10^{-4}$ of the resulting OFDM code $B(n = 2^m - 1, 2^k, \geq 2t + 1)$ for $m = 8$. Let $U$ be the number of scrambling sequences.

Fig. 2 presents the complementary cumulative distribution
functions (CCDF) of $U = 16, 64, 256$ for the $(255, 2^{239}, 3)$ OFDM codes in Tables I, obtained by a simplex scrambling subcode with $\alpha^7$, along with the PAPR statistics of some other OFDM codes, obtained by a random scrambling subset and some non-primitive scrambling subcodes with $\alpha^5$ and $\alpha^{13}$. Simulation results show that a primitive scrambling subcode, i.e., a simplex subcode, has better PAPR statistics than non-primitive scrambling subcodes.

**TABLE I**

SOME OFDM CODES OF LENGTH 255.

<table>
<thead>
<tr>
<th>$k$</th>
<th>$d$</th>
<th>$s(x)$ (m.p. of $\alpha^i$)</th>
<th>PAPR (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>255</td>
<td>1</td>
<td>without SLM</td>
<td>12.7929</td>
</tr>
<tr>
<td>247</td>
<td>1</td>
<td>$x^n + x^2 + x^3 + x^4 + 1$ ($\alpha^i$)</td>
<td>6.6287</td>
</tr>
<tr>
<td>239</td>
<td>3</td>
<td>$x^n + x^2 + x^3 + x^4 + 1$ ($\alpha^i$)</td>
<td>6.6694</td>
</tr>
<tr>
<td>231</td>
<td>5</td>
<td>$x^n + x^2 + x^3 + x^4 + 1$ ($\alpha^i$)</td>
<td>6.6208</td>
</tr>
<tr>
<td>223</td>
<td>7</td>
<td>$x^n + x^2 + x^3 + x^4 + 1$ ($\alpha^i$)</td>
<td>6.6308</td>
</tr>
<tr>
<td>215</td>
<td>9</td>
<td>$x^n + x^2 + x^3 + x^4 + 1$ ($\alpha^i$)</td>
<td>6.6229</td>
</tr>
<tr>
<td>207</td>
<td>11</td>
<td>$x^n + x^2 + x^3 + x^4 + 1$ ($\alpha^i$)</td>
<td>6.6292</td>
</tr>
<tr>
<td>199</td>
<td>13</td>
<td>$x^n + x^2 + x^3 + x^4 + 1$ ($\alpha^i$)</td>
<td>6.6230</td>
</tr>
<tr>
<td>191</td>
<td>15</td>
<td>$x^n + x^2 + x^3 + x^4 + 1$ ($\alpha^i$)</td>
<td>6.6214</td>
</tr>
<tr>
<td>183</td>
<td>17</td>
<td>$x^n + x^2 + x^3 + x^4 + 1$ ($\alpha^i$)</td>
<td>6.6314</td>
</tr>
<tr>
<td>179</td>
<td>19</td>
<td>$x^n + x^2 + x^3 + x^4 + 1$ ($\alpha^i$)</td>
<td>6.6247</td>
</tr>
<tr>
<td>171</td>
<td>21</td>
<td>$x^n + x^2 + x^3 + x^4 + 1$ ($\alpha^i$)</td>
<td>6.6322</td>
</tr>
<tr>
<td>163</td>
<td>23</td>
<td>$x^n + x^2 + x^3 + x^4 + 1$ ($\alpha^i$)</td>
<td>6.6497</td>
</tr>
<tr>
<td>155</td>
<td>25</td>
<td>$x^n + x^2 + x^3 + x^4 + 1$ ($\alpha^i$)</td>
<td>6.6721</td>
</tr>
</tbody>
</table>

**Fig. 2.** CCDF of OFDM codes of length 255.

**B. BCM-SLM**

The BCM-SLM algorithm is a combination of two approaches to improve the performance of a BCM-OFDM system: one is to use a BCM code to achieve bandwidth-efficient modulation, and the other is to use the SLM approach to improve the PAPR performance.

Let $C_1, C_2, C_3$, and $C_4$ be four binary codes of length $n$ with minimum Hamming distances $d_i$, $1 \leq i \leq 4$, respectively. A $4 \times n$ codeword array is constructed by filling the $i$th row with codewords from the $i$th component code $C_i$ as follows:

$$
\begin{pmatrix}
C_1 & C_2 & C_3 & C_4 \\
C_{1,0} & C_{1,1} & \cdots & C_{1,n-1} \\
C_{2,0} & C_{2,1} & \cdots & C_{2,n-1} \\
C_{3,0} & C_{3,1} & \cdots & C_{3,n-1} \\
C_{4,0} & C_{4,1} & \cdots & C_{4,n-1}
\end{pmatrix}
$$

Then, a complex-valued sequence of length $n$ in a 16-QAM BCM code $B$ can be formed by modulating each column of the $4 \times n$ codeword array into a 16-QAM symbol. The 16-QAM signal constellation, shown in Fig. 3, with bit labels is assigned by a set partitioning scheme in which the intra-subset squared Euclidean distances rise in the ratio $1 : 2 : 4 : 8$.

More precisely, a 16-QAM BCM codeword $b = [b_0 b_1 \ldots b_{n-1}] \in B$ can be written as

$$
b_k = \left( \frac{1}{\sqrt{2}} j^{c_1 k + 2 c_2 k} + \sqrt{2} j^{c_3 k + 2 c_4 k} \right) e^{j \pi/4}, \quad 0 \leq k \leq n-1,
$$

in terms of these four binary component codewords $c_i \in C_i$. The minimum squared Euclidean distance between any two codewords in $B$ is known as [15]

$$
D_E^2 = \min_{1 \leq i \leq 2} d_i \cdot 2^{i-1} \cdot 2^2, \quad (7)
$$

where $E$ is the minimum Euclidean distance between signal points. To increase the code rate and balance the minimum squared Euclidean distance of a resulting 16-QAM BCM code, we require $d_1 = 2d_2 = 4d_3 = 8d_4$. For example, we can take the component code $C_i = \text{RM}(i, m)$, the $i$th order Reed-Muller code, of length $n = 2^m$ generated by the Boolean monomials of degree at most $i$ [12]. For example, the second order Reed-Muller code is generated by

$$
\text{RM}(2, m) = \{1, x_1, \cdots, x_m, x_1 x_2, \ldots, x_m x_{m-1} x_m\}.
$$

A generator matrix $G$ of a 16-QAM BCM code can be written as

$$
G = \begin{pmatrix}
G_1 & 0 & 0 & 0 \\
0 & G_2 & 0 & 0 \\
0 & 0 & G_3 & 0 \\
0 & 0 & 0 & G_4
\end{pmatrix}, \quad (8)
$$

**Fig. 3.** Set partitioning for 16-QAM constellation.
where $G_1$, $G_2$, $G_3$, and $G_4$ are the generator matrices of the component code $C_1, C_2, C_3$, and $C_4$, respectively. First, we choose the subcode of the first order Reed-Muller code except the zero order Reed-Muller code, $\text{RM}(1, m) \setminus \text{RM}(0, m) = \langle G_Y \rangle$, as the encoder of PAPR control bits. Then, we divide the input bits of $G_4 = G_X \cup G_Y$ into the information bits with generator matrix $G_X$ and the PAPR control bits with generator matrix $G_Y$. After encoding the PAPR control bits, we can get a number of binary scrambling sequences from the PAPR subcode $\langle G_Y \rangle$. We add these scrambling sequences with the codeword generated by the generator matrix $G_X$ and then use these resulting sequences with the codewords from the component code $C_1, C_2,$ and $C_3$ to form a number of $4 \times n$ codeword arrays. Finally we can get a number of 16-QAM candidate sequences by mapping these codeword arrays into the 16-QAM sequences and the one with minimum peak power is transmitted.

Our aim here involves a trade-off among the PAPR performance, the data rate, and the squared Euclidean distance in a 16-QAM BCM-SLM OFDM system. We examine the PAPR performance for our proposed algorithm for a 16-QAM OFDM system with $n$ subcarriers, where $n = 2^m$ and $4 \leq m \leq 7$. In simulation results, we focus on the components codes of $\{\text{RM}(2, m), \text{RM}(3, m), \text{RM}(4, m), \text{RM}(5, m)\}$ to form a 16-QAM BCM-OFDM system. The minimum square Euclidean distances of these resulting 16-QAM BCM-OFDM codes is $2^{m-2}$. To compute the average power of a 16-QAM BCM-OFDM system, we first average the power of a transmitted sequence in each period, and then we average all of the average power of transmitted sequences in $B$.

Fig. 4 shows the CCDFs of PAPR in a 16-QAM BCM-SLM OFDM system with $n = 128$ subcarriers and with minimum squared Euclidean distance 32. We compare the PAPR statistics of the modified BCM-SLM using Reed-Muller (RM) codes as component codes with original SLM which has no error correction capability for $U = 16, 32, 64,$ and 128 scrambling sequences. Numerical results show that RM codes achieve good performance in PAPR reduction than conventional SLM. Moreover, the advantage of this modified BCM-SLM is to integrate the PAPR reduction with error correction so that we do not need to use another error correcting code to transmit these PAPR control bits explicitly.

IV. Conclusion

We considered in this paper the reduction of peak-to-average power ratio in OFDM systems with BPSK and 16-QAM modulation. In Cyclic-SLM over BPSK, we have shown that a cyclic simplex code is a good candidate for a scrambling subcode and simulation results show that the resulting OFDM codes achieve good performance in PAPR reduction. In BCM-SLM over 16-QAM, we combine a BCM code with SLM to achieve bandwidth-efficient modulation and reduce the PAPR statistics.

REFERENCES


