DYNAMIC MECHANICAL PROPERTIES OF AGAROSE GEL BY A FRACTIONAL DERIVATIVE MODEL

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INTRODUCTION

Agarose gel has been commonly used for the cell culture of cartilage and soft tissue and also been used as phantom material for material property characterization using image techniques. The mechanical property of the agarose gel would therefore be important for such experiments and analyses. A fractional calculus approach had been proposed and applied as a method of describing the mechanical properties of viscoelastic materials. In the present study, the dynamic mechanical properties of agarose gels and its relationship with frequencies and agarose concentrations were characterized with a simple form of fractional derivative model.

METHODS

Gels with agarose concentration (weight/volume, w/v) of 2%, 3%, 4% and 5% were prepared by dissolving appropriate amount of Difco Bacto agar into the distilled deionized water. The solution was sealed and heated to 90~95 °C for 15 minutes and was magnetically stirred to homogenous. The solution was then cooled to below 35 °C for curing. After cured, gels were subject to testing immediately. 2 mm thick samples were cut from the gel and measured prior to testing (i.e. diameter, thickness). Mechanical properties of the gels were tested in frequency sweep shear mode with DMA 2980 (TA Instruments, New Castle, DE) over a frequency range of 1-20 Hz at a constant amplitude of 20 μm. Samples were compressed by 1 mm of their original thickness before testing. Complex modulus (G*), elastic modulus (G'), loss modulus (G") were recorded. Using the fractional derivative method developed by Suki et. al. (1994), the stress relaxation of the agarose gel was modeled as:

$$\boldsymbol{s}_{(t)} = QD^{\,b}[\boldsymbol{e}_{(t)}] \qquad (1)$$

where s(t) and e(t) was the stress and strain as a function of time, respectively, and Q was a viscoelastic parameter of the system. The fractional derivative operator was defined by:

$$D^{\,b}[x(t)] = \frac{1}{\Gamma(1-\boldsymbol{b})} \frac{d}{dt} \int_0^t \frac{x(\boldsymbol{t})}{(t-\boldsymbol{t})} d\boldsymbol{t} \qquad (2)$$

The order of the fractional derivative, ß, is the primary parameter in the model that completely characterizes the rheological behavior of the gels. Applying a Fourier Transform to Eq. (1), the complex modulus can therefore be modeled as:

$$G*(\boldsymbol{w}) = H\boldsymbol{v}^{\,b} + jZ\boldsymbol{v}^{\,b} \qquad (3)$$

where

$$H = Q\cos\frac{\boldsymbol{p}}{2}\boldsymbol{b} \qquad (4)$$

$$Z = Q\sin\frac{\boldsymbol{p}}{2}\boldsymbol{b} \qquad (5)$$

The magnitude of the complex modulus G* was thus modeled as:

$$|G*(\boldsymbol{w})| = \sqrt{H^2\boldsymbol{v}^{\,2b} + Z^2\boldsymbol{v}^{\,2b}} \qquad (6)$$

A custom-written MATLAB® program estimated the parameters H and Z via global multiple non-linear fitting of the G* as a function of the frequency ? (Yuan, 2000). ß, G' and G" were thereafter calculated according to Eq. (3), (4) and (5). To investigate the concentration (C) dependence of ß, we started with the well-accepted assumption (Benkherourou, 2000) that:

$$G' \propto \boldsymbol{a}C^{\,1} \qquad (7)$$

Taking the first-order derivative of G' as a function of C, and applying Eq. (4) and (5), we got:

$$\frac{\partial Q}{\partial C}(\boldsymbol{w}^{\,b}\cos\frac{\boldsymbol{p}}{2}\boldsymbol{b}) + \frac{\partial \boldsymbol{b}}{\partial C}Q(\boldsymbol{w}^{\,b}\ln\boldsymbol{w}\cos\frac{\boldsymbol{p}}{2}\boldsymbol{b} - \frac{\boldsymbol{p}}{2}\boldsymbol{w}^{\,b}\sin\frac{\boldsymbol{p}}{2}\boldsymbol{b}) \propto C^{\,1-1} \qquad (8)$$

In order for Eq. (8) to hold true for any arbitrary values of C, it requires both items on the left be proportional to $C^{?-1}$. Since G" was sufficiently small, by neglecting G", the second term in Eq. (8) led to:

$$b \propto \left| \ln C \right| \qquad (0 \leq b \leq 1, 0 \leq C \leq 1) \qquad (9)$$

RESULTS

The fitted data shows that G* and G' of the gel increase slightly with frequency (Figure 1 and 2) and ß is within the range of 0.025 to 0.03. These are consistent with the previous study by Ross-Murphy el. al. (1993). A curve fitting between ß and C is shown in Figure 3.
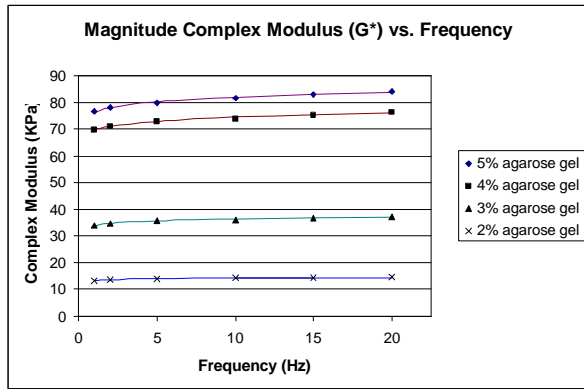

Figure 1. Magnitude of the complex modulus vs. frequency for agarose gel of different concentrations
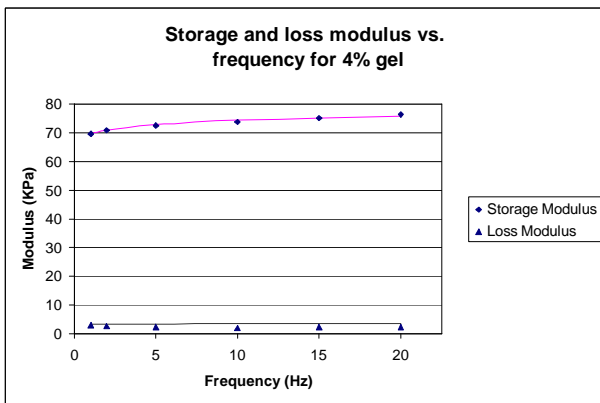

Figure 2. Elastic modulus and loss modulus as a function of frequency for 4% agarose gel

DISCUSSION

A theoretical basis of fractional derivative to the viscoelasticity of polymers was provided by Rouse's theory (Bagley, 1983) predicting the macroscopic mechanical properties of long, coiled, and entangled polymer molecules in a Newtonian solution. Rouse's theory leads to:

$$G(w) = [\frac{3}{2}(m_0 - m_s)nkT]^{1/2}(jw)^{1/2} \qquad (10)$$

The fractional power leads to a fractional derivative when Eq. (10) is transformed into time domain. (Bagley, 1983)

The model used in the present study showed that parameter ß primarily determines the viscoelastic properties of the gel. Indeed, for a given agarose concentration, when ß? 0, the fractional derivative

model reduces to a pure elastic material; when ß? 1, the model can be simplified to a pure viscous material. For any intermediate values of ß, the model will exhibit a viscoelastic material property.
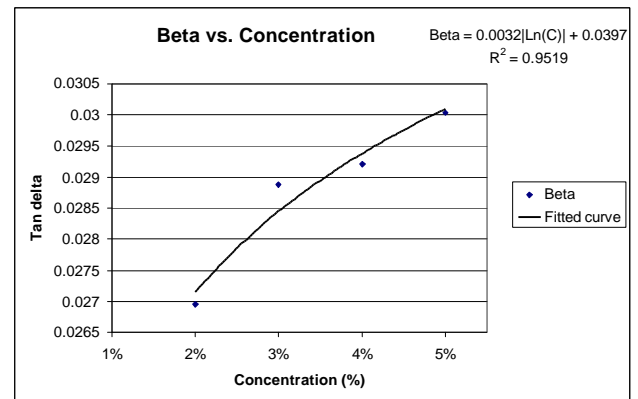

Figure 3. ß as a function of agarose concentration with a curve fitting between ß and |ln C|

Such fractional derivative model can provide information in tissue culture studies employing dynamic pressurization (Saris, 2000). The fact that ß=0.025 ~0.03 indicated that the elasticity of the agarose gel significantly prevails its viscosity. G' and G" of the agarose gel is only slightly frequency-dependent, verifying its usage in dynamic pressure stimulation in tissue engineering. Such model may also contribute to magnetic resonance elastography (MRE) phantom study. MRE has been proposed as a novel method of measuring the stress distribution within the object. Agarose gel, due to its good elastic property and availability, has been selected as the phantom material. The conversion of shear wavelength to the elastic or viscous modulus can not be well studied without establishing an analytical model for the tested material. Our fractional derivative model, when expended to higher frequency (150~500 Hz), may provide a fast method to validate the current MRE wavelength-modulus conversion algorithm.

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