

CONSEQUENCES OF CONVECTION-RADIATION INTERACTION FOR MAGNETITE-WATER NANOFLUID FLOW DUE TO A MOVING PLATE

by

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Present paper examines the boundary-layer flow of magnetic nanofluid over a radiative plate moving in a uniform parallel free stream. Water is considered as the base fluid which is being filled with magnetite-Fe₃O₄ nanoparticles. Energy balance equation is formulated with non-linear radiation heat flux. Mathematical analysis is carried out through the famous Tiwari and Das model. Similarity approach is utilized to construct self-similar form of the governing differential system. Numerical computations are made through standard shooting method. Ferrofluid velocity is predicted to enhance upon increasing the nanoparticle volume fraction which contradicts with the available literature for non-magnetic nanofluids. It is found that Fe₃O₄-water ferrofluid has superior heat transfer coefficient than pure water. Results reveal that consideration of magnetic nanoparticles in water leads to better absorption of incident solar radiations. The well-known Blasius and Sakiadis flows are also explicitly analyzed from the present model.

Key words: ferrofluid, heat transfer, numerical treatment, non-linear radiation, Blasius problem

Introduction

Traditional sources of heat transfer liquids are incapable to deliver the modern cooling requirements primarily because of their weak convective heat transfer coefficients. Researches proved that nanoparticles, generally made of metals or oxides enhance conduction and convection coefficients and hence allow for rapid higher heat transfer rates out of the coolants. Thus heat transfer and efficiency of thermal systems can be improved via nanoparticle working fluid. Owing to their enhanced thermal conductivity, nanofluids have promising applications in several areas of industry and biomedicine. These include engine cooling, cooling of transformer oil, space and defense, electronics cooling, solar water heating, nuclear reactor cooling, refrigeration, and many others. Detailed reviews about the possible heat transfer applications of nanofluids have been presented by Wang and Mujumdar [1], Saidur *et al.* [2, 3], Mahian *et al.* [4], and Kasaieian *et al.* [5]. In the past, nanofluid flow and heat transfer due to moving or stationary surfaces have been given tremendous attention by the researchers [6-18]. Magnetic

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nanofluids, known as ferrofluids, are more beneficial than the ordinary nanofluids because their thermophysical properties can be amplified through external magnets. Ferrofluids possess a form of heat transfer known as thermomagnetic convection which is beneficial when usual convection heat transfer is insufficient. Saharifi *et al.* [19] suggested using ferrite-based ferrofluids for drug delivery and hyperthermia treatment for cancer. Ferrofluids may serve to form liquid seals around the spinning drive shafts in hard disks. Some recent pertaining to the heat transfer characteristics of magnetic nanofluids can be seen through [20-24].

Study of radiative heat transfer has noteworthy importance in various branches of science and engineering. Raptis and Perdikis [25] investigated the radiation effects on visco-elastic fluid-flow over a stationary heated plate. In this work, radiation term in the heat transfer equation was linearized through the assumption of negligible temperature differences within the flow. Later, such approximation has been adopted for investigating several other boundary-layer flow problems even in recent past. Rahman and Eltayeb [26] firstly presented the heat transfer analysis through the exact expression of radiative heat flux. They investigated the flow of nano-fluid bounded by a non-linearly stretching radiative surface. Blasius flow of viscous fluid with non-linear radiation was numerically explored by Pantokratoras and Fang [27]. Mushtaq *et al.* [28] viscous flow due to a bi-directional stretching surface subjected to non-linear radiation. Flow adjacent to a stationary vertical plate influenced by non-linear thermal radiation was investigated by Pantokratoras [29]. Mustafa *et al.* [30] studied the exponentially stretched flow in the existence of non-linear radiative heat flux. In another paper, flow of power-law fluid subject to non-linear radiation was studied by Mustafa *et al.* [31].

To our knowledge, the non-linear radiation effects in ferrofluid flow have never been reported previously. Therefore, present work aims to address the consequences of non-linear radiation heat flux in the boundary-layer flow of Fe_3O_4 -water ferrofluid due to constantly moving plate in parallel free stream. Such analysis even for non-magnetic nanofluids is not yet performed. Thermal conductivity of ferrofluid is estimated through the Maxwell model which is valid for spherical nanoparticles. Computational analysis is performed through the shooting method. Graphical illustrations for the effects of embedded physical parameters are presented and discussed in detail.

Model development

Consider a laminar flow of Fe_3O_4 -water ferrofluid over a moving plate having constant velocity, U_w . The plate is coincident with the plane $y = 0$ while ferrofluid occupies the semi-infinite region $y \geq 0$. Let U_∞ be the free stream velocity. Flow is subjected to non-uniform magnetic field of strength $B(x) = B_0/x^{1/2}$ in the transverse direction. The plate is kept at constant temperature, T_w , whereas T_∞ denotes the temperature at the far field. Thus relevant boundary-layer equations governing the nanofluid flow and heat transfer with viscous dissipation and thermal radiation effects are expressed below (see [12] for details):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$\rho_{\text{nf}} \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \mu_{\text{nf}} \left(\frac{\partial^2 u}{\partial y^2} \right) - \sigma_{\text{nf}} B_0^2 (u - U_\infty) \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{\text{nf}} \left(\frac{\partial^2 T}{\partial y^2} \right) - \frac{1}{(\rho c_p)_{\text{nf}}} \frac{\partial q_r}{\partial y} + \frac{\mu_{\text{nf}}}{(\rho c_p)_{\text{nf}}} \left(\frac{\partial u}{\partial y} \right)^2 \quad (3)$$

and the boundary conditions are:

$$\begin{aligned} u = U_w, \quad v = 0, \quad T = T_w \quad \text{at} \quad y = 0 \\ u \rightarrow U_\infty, \quad T \rightarrow T_\infty \quad \text{as} \quad y \rightarrow \infty \end{aligned} \quad (4)$$

In eqs. (1)-(4), $u(x, y)$ and $v(x, y)$ denote the velocities along the x- and y-directions, respectively, $q_r = -(4\sigma^*/3k^*)\partial T^3/\partial y$ is the radiation heat flux in which $\sigma^* = 8.61 \cdot 10^{-5}$ eV/k is the Boltzman constant and k^* is the mean absorption coefficient. Further the effective density, ρ_{nf} , the effective viscosity, μ_{nf} , the effective thermal diffusivity, α_{nf} , the effective heat capacity, $(\rho c_p)_{nf}$, and effective thermal conductivity, k_{nf} , are defined:

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}, \quad \alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}}, \quad \rho_{nf} = (1-\phi)\rho_f + \phi\rho_s \quad (5)$$

$$(\rho c_p)_{nf} = (1-\phi)(\rho c_p)_f + \phi(\rho c_p)_s, \quad \frac{k_{nf}}{k_f} = \frac{(k_s + 2k_f) - 2\phi(k_f - k_s)}{(k_s + 2k_f) + \phi(k_f - k_s)} \quad (6)$$

in which ϕ denotes the volume fraction of nanoparticles and subscripts s and f indicate solid and fluid phases, respectively.

Electrical conductivity of nanofluid σ_{nf} is proposed by Maxwell [32]:

$$\frac{\sigma_{nf}}{\sigma_f} = 1 + \left[\frac{3(\sigma_s - \sigma_f)\phi}{(\sigma_s + 2\sigma_f) - (\sigma_s - \sigma_f)\phi} \right] \quad (7)$$

We consider the following transformations [6]:

$$\eta = y\sqrt{\frac{U}{2\nu_f x}}, \quad u = Uf'(\eta), \quad v = \sqrt{\frac{U\nu_f}{2x}}(\eta f' - f), \quad T = T_\infty + (T_w - T_\infty)\theta(\eta) \quad (8)$$

with $U = U_w + U_\infty$. In view of eq. (8), eq. (1) is satisfied and eqs. (2)-(4) assume the following forms:

$$\frac{1}{(1-\phi)^{2.5}} f'''' + \left(1 - \phi + \frac{\phi\rho_s}{\rho_f} \right) f f'' + \text{Ha} \frac{\sigma_{nf}}{\sigma_f} (\lambda - f') = 0 \quad (9)$$

$$\frac{1}{\text{Pr}} \left\{ \frac{k_{nf}}{k_f} + \text{Rd} [1 + (\theta_w - 1)\theta]^3 \theta' \right\}' + \left[1 - \phi + \frac{\phi(\rho c_p)_s}{(\rho c_p)_f} \right] f \theta' + \frac{1}{(1-\phi)^{2.5}} \text{Ec} f''^2 = 0 \quad (10)$$

$$\begin{aligned} f(0) = 0, \quad f'(0) = 1 - \lambda, \quad \theta(0) = 1 \\ f'(\infty) = \lambda \quad \theta(\infty) = 0 \end{aligned} \quad (11)$$

in which $\text{Pr} = (\mu_c)_f/k_f$ is the Prandtl number of pure water, $\text{Rd} = 16 \sigma^* T_\infty^3 / 3k_f k^*$ – the thermal radiation parameter, $\text{Ha} = \sigma B_0^2 / \rho_f U$ – the Hartmann number, $\text{Ec} = U^2 / c_p (T_w - T_\infty)$ – the Eckert number, and $\lambda = U_\infty / U$ – the velocity ratio parameter. It is worth noting here that $\text{Ha} = 0$ corresponds to the case of non-magnetic nanofluids which is not yet explored. eqs. (8)-(10) correspond to the classical Blasius problem when $\lambda = 1$ while Sakiadis flow problem is achieved by setting $\lambda = 0$.

Skin friction coefficient, C_f , and the local Nusselt number, Nu_x , are defined:

$$C_f = \frac{\tau_w}{\rho_f U^2}, \quad \text{Nu}_x = \frac{xq_w}{k_f (T_w - T_\infty)} \quad (12)$$

where τ_w and q_w denote the wall shear stress and wall heat flux, respectively. These are expressed:

$$\tau_w = \mu_{nf} \left. \frac{\partial u}{\partial y} \right|_{y=0}, \quad q_w = -k_{nf} \left. \frac{\partial T}{\partial y} \right|_{y=0} + q_r \quad (13)$$

Invoking variables from eq. (8) into eq. (12) one obtains

$$\sqrt{\frac{\text{Re}_x}{2}} C_f = \frac{1}{(1-\phi)^{2.5}} f''(0), \quad \frac{\text{Nu}_x}{\sqrt{2\text{Re}_x}} = -\left(k_{nf}/k_f + \text{Rd}\theta_w^3\right)\theta'(0) \quad (14)$$

where $\text{Re}_x = Ux/v_f$ is the local Reynolds number.

Numerical results and discussion

Non-linear differential system given in eqs. (9)-(11) have been evaluated numerically through standard shooting method with fifth order Runge-Kutta (RK) integration and Newton's method. First of all we transformed eqs. (8) and (9) into a system of first order equations by suitable substitutions: ($x_1 = f, x_2 = f', x_3 = f'', x_4 = \theta, \text{ and } x_5 = \theta'$). We get:

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \\ x_4' \\ x_5' \end{bmatrix} = \begin{bmatrix} x_2 \\ x_3 \\ -\left\{ [1-\phi + \phi(\rho_s/\rho_f)]x_1x_3 + \text{Ha}(\sigma_{nf}/\sigma_f)(\lambda - x_2) \right\} (1-\phi)^{2.5} \\ x_5 \\ \left(\text{Pr} \left\{ 1-\phi + \phi \left[(\rho c_p)_s / (\rho c_p)_f \right] \right\} x_1x_5 + \left[\text{Pr Ec} / (1-\phi)^{2.5} \right] x_3^2 + \right. \\ \left. + 3\text{Rd} \left[1 + (\theta_w - 1)x_4 \right]^2 (\theta_w - 1)x_5^2 \right) \\ \left. \left\{ (k_{nf}/k_f) + \text{Rd} \left[1 + (\theta_w - 1)x_4 \right]^3 \right\} \right] \end{bmatrix} \quad (15)$$

subject to the following conditions

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 1-\lambda \\ u_1 \\ 1 \\ u_2 \end{bmatrix} \quad (16)$$

First order system (16) is then integrated numerically by RK5 through a suitable choice for $f''(0)$ and $\theta'(0)$. These values are refined by employing Newton's method until the solutions for f'' and θ exponentially decay to zero with error tolerance less than 10^{-5} . The average CPU time for one set of outputs is around 1.5 second.

Table 1. Thermo-physical properties of water and magnetite-Fe₃O₄

	ρ [kgm ⁻³]	C_p [Jkg ⁻¹ K ⁻¹]	k [Wm ⁻¹ K ⁻¹]
Water	997.1	4179	0.613
Fe ₃ O ₄	5180	670	9.7

In order to investigate the physical effects of embedded parameters on the solutions, we prepared the figs. 1-8. Thermophysical properties of water and magnetite are listed in tab. 1. Figure 1 gives the velocity distribution for different values of λ when $\phi = 0$ (pure water) and $\phi = 0.2$. Fluid flows faster above the plate when larger values of λ are accounted. The pa-

parameter λ compares the free stream velocity with the velocity of the plate. The variation in velocity with ϕ appears to be non-monotonic here.

Figure 2 perceives the influence of Hartmann number on the velocity profile. The application of magnetic force in the y-direction gives resistance to the momentum transport due to which boundary-layer thickness decreases. Similar variations in f' with Hartmann number exist in Blasius and Sakiadis flows. In Blasius flow, the plate is at rest and only free stream velocity is present due to which there is inverted boundary-layer structure. An increase in magnetic field parameter opposes the momentum transport due to which boundary-layer thickness decreases. This can be observed through fig. 2 in which velocity profiles move towards the wall when Hartmann number is increased. The reduction in momentum boundary-layer thickness accompanies with an increase in the velocity near the wall region.

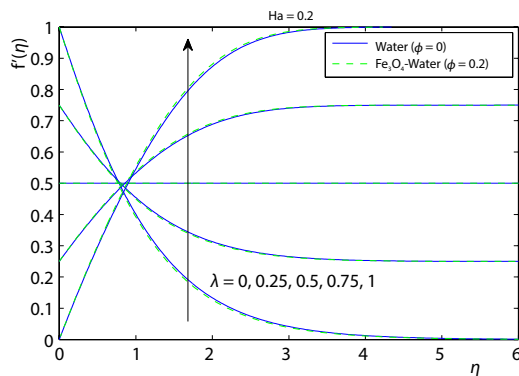


Figure 1. Effect of λ on $f'(\eta)$

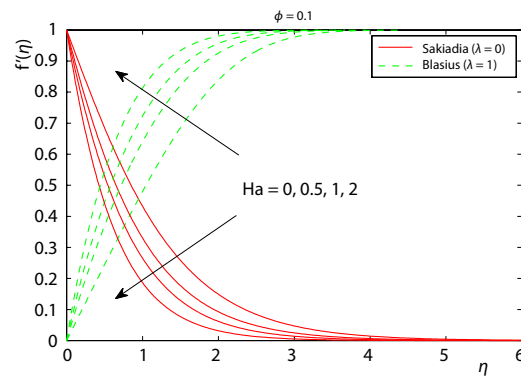


Figure 2. Effect of Hartmann number on $f'(\eta)$

In fig. 3, we plot the skin friction coefficient $1/(1 - \phi)2.5f''(0)$ against the volume fraction, ϕ , in case of both Blasius and Sakiadis flow problems. As ϕ enlarges, there is almost a linear growth in skin friction coefficient for any given value of Hartmann number. The magnitude of skin friction coefficient significantly rises when Hartmann number is incremented.

Figure 4 is presented to foresee the influence of volume fraction, ϕ , on the temperature profile. As expected, a growth in the nanoparticle volume fraction, ϕ , augments the temperature and the thickness of thermal boundary-layer. Moreover, temperature distribution in Sakiadis flow problem is larger in comparison to the Blasius flow problem.

Figure 5 captures the behavior of temperature, θ , as the radiation parameter, Rd , varies from to $Rd = 1$. Temperature θ rises sharply when Rd is increased. Temperature θ is also directly proportional to the Eckert number. For sufficiently large values of Eckert number, an overshoot in temperature profiles near the wall is observed.

The influence of temperature ratio parameter, θ_w , on temperature distribution is sketched in fig. 6. The parameter θ_w enhances the penetration depth of temperature function. In contrast to linear radiative heat transfer, here the energy balance equation involves thermal diffusivity of the form $(\alpha_{nf} + 16 \sigma^* T^3 / 3k^* k_{nf})$, which is temperature dependent. From this expression, it is evident that thermal boundary-layer will be much thicker in the non-linear radiation case when compared with the linear radiation case. When θ_w is increased, the profiles become broader and take a special S-shaped form with an inflection point.

Figure 7 gives the plot of local Nusselt number $Nu_x / (2Re_x)^{1/2}$ vs. the volume fraction ϕ . The magnitude of local Nusselt number linearly increases with ϕ only when sufficiently stronger magnetic field strength is considered. Figure 8 shows the local Nusselt number

$Nu_x/(2Re_x)^{1/2}$ as a function of Prandtl number at different values of Eckert number. We observe that magnitude of $Nu_x/(2Re_x)^{1/2}$ has direct relationship with the Prandtl number while it is inversely proportional to the Eckert number.

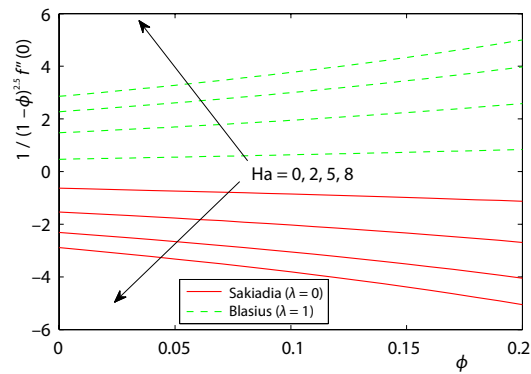


Figure 3. Effect of λ , Hartmann number and ϕ on C_f

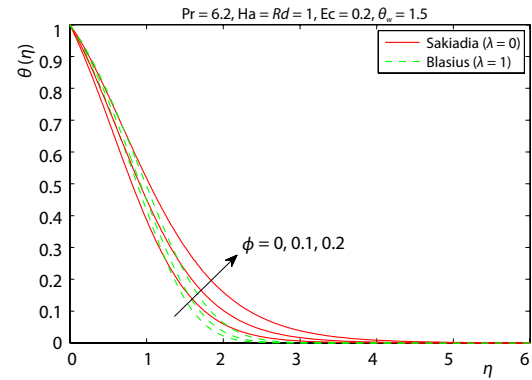


Figure 4. Effect of ϕ on $\theta(\eta)$

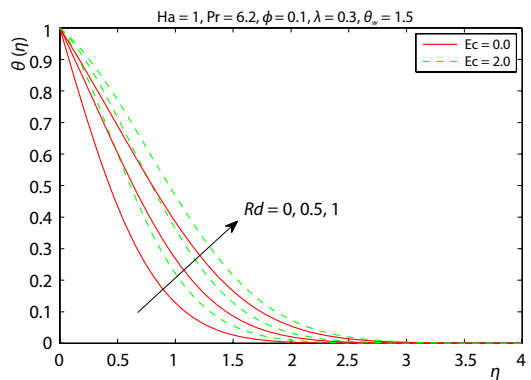


Figure 5. Effect of Rd and Eckert number on $\theta(\eta)$

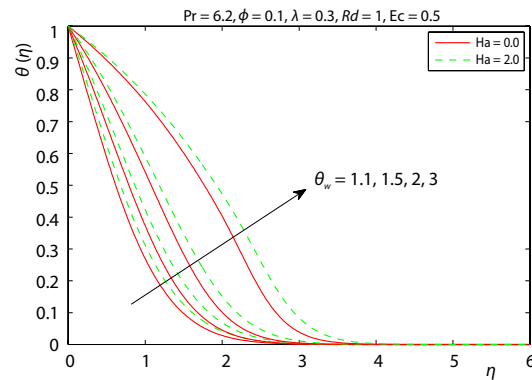


Figure 6. Effect of Hartmann number and θ_w on $\theta(\eta)$

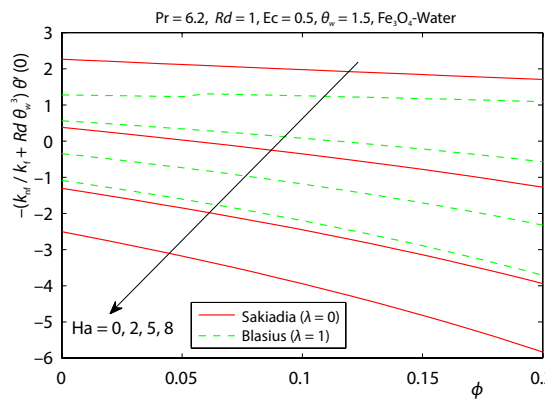


Figure 7. Effect of λ , Hartmann number and ϕ on Nusselt number

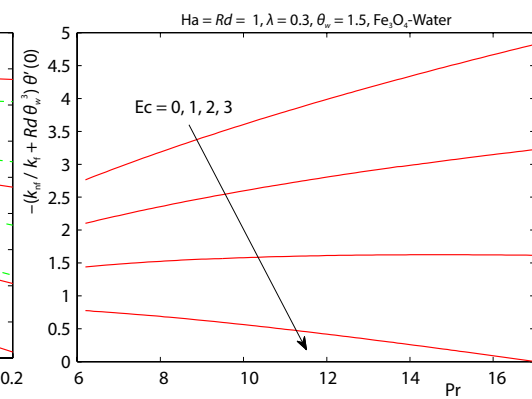


Figure 8. Effect of Prandtl and Eckert number on Nusselt number

Numerical results of local Nusselt number $Nu_x / (2Re_x)^{1/2}$ for different values of λ , θ_w , and Rd are provided in tab. 2. Interestingly, a drastic rise in wall heat flux is observed when the velocity ratio parameter λ is varied from $\lambda = 0$ to $\lambda = 0.5$. Moreover, both temperature ratio parameter and radiation parameter support the heat transfer rate from the plate.

Concluding remarks

A numerical study for the boundary-layer flow of magnetite-water ferrofluid over a constantly moving horizontal plate with parallel free stream is presented. Numerical results are derived by shooting method. The major aspects of this work are outlined below:

- Skin friction coefficient in Fe3O4-water ferrofluid is much larger in comparison to pure water.
- Different from the non-magnetic nanofluids, the velocity of the ferrofluid has non-monotonic relationship with ϕ .
- Both magnetic field effect and free stream velocity serve to enhance convective heat transfer coefficient of ferrofluid.
- The parameter λ has a dual behavior on the hydrodynamic boundary-layer. When $\lambda = 1$, the velocity profile f' is a straight line.
- Temperature θ and heat transfer rate from the plate are directly and non-linearly proportional to the radiation parameter.
- When wall to ambient temperature ratio, θ_w , is increased, temperature profile changes from normal shape to S-shaped form with a point of inflection, and
- The well-known Blasius and Sakiadis flow problems can be obtained as the special cases of present model.

Nomenclature

$B(x)$ – magnetic field strength, $[Nm^{-1}A^{-1}]$
 C_f – skin friction coefficient
 c_p – specific heat, $[Jkg^{-1}K^{-1}]$
 Ec – Eckert number
 f – dimensionless x-components of velocity
 Ha – Hartmann number
 k – thermal conductivity, $[Wm^{-1}K^{-1}]$
 Nu_x – local Nusselt number
 Pr – Prandtl number
 q_r – Rosseland radiative heat flux, $[Wm^{-2}]$
 q_w – wall heat flux, $[Wm^{-2}]$
 Rd – radiation parameter
 Re – local Reynolds number
 T – fluid temperature, $[K]$
 T_w – wall temperature, $[K]$
 T_∞ – ambient fluid temperature, $[K]$

U – positive constant, $[ms^{-1}]$
 U_w, U_∞ – velocity of the stretching sheet and far field, respectively, along x-directions, $[ms^{-1}]$
 (x, y) – 2-D Cartesian co-ordinate system, $[m]$
 u, v – velocity components along the x-, y-directions, $[ms^{-1}]$
 $'$ – 1st order derivative with respect to η
 $''$ – 2nd order derivative with respect to η
 $'''$ – 3rd order derivative with respect to η

Greek symbols

α – thermal diffusivity, $[m^2s^{-1}]$
 η – similarity variable
 θ – dimensionless temperature
 θ_w – temperature ratio parameter

Table 2. Numerical values of local Nusselt number $Nu_x / (2Re_x)^{1/2}$ when $Ha = 1, Pr = 6.2, Ec = 0.5,$ and $\phi = 0.2$

λ	θ_w	Rd	$-(k_{mf} / k_f + Rd\theta_w^3) \theta'(0)$
0	1.5	1	-0.0062596954
0.2			1.9233832
0.5			2.8784479
1			0.17352702
0.5	1.1		2.4715736
	1.5		2.8785745
	2		3.5724593
	2.5		4.3961313
	1.5	0	1.9517906
		0.5	2.4641607
		1	2.8785745
		1.5	3.2432484

λ	– velocity ratio parameter	ϕ	– nanoparticle volume fraction
μ	– dynamic viscosity, [$\text{kgm}^{-1}\text{s}^{-1}$]	<i>Subscript</i>	
ν	– kinematic viscosity, [m^2s^{-1}]	<i>s</i>	– nanoparticle material
ρ	– density, [kgm^{-3}]	<i>f</i>	– base fluid
σ	– electrical conductivity, [$1\Omega^{-1}\text{m}^{-1}$]	<i>nf</i>	– nanofluid
τ_w	– wall shear stress, [$\text{kgm}^{-1}\text{s}^{-2}$]		

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