ADAPTIVE FUZZY SLIDING MODE CONTROL FOR UNCERTAIN NONLINEAR SYSTEMS AGAINST ACTUATOR FAULTS

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Abstract: In this paper, we propose to combine the fuzzy sliding mode control to tolerate actuator faults of unknown nonlinear systems subject to external disturbances. In particular, the idea of using adaptive fuzzy system to tolerate actuator faults of unknown nonlinear systems by approximating the system functions and the effects caused by actuator faults are avoided by the control structure. On the basis of Lyapunov stability theory its shown that the resulting adaptive closed loop system can be guaranteed to be asymptotically stable in the presence of faults on actuators and disturbances.

1 INTRODUCTION

In most practical control systems, components failures may occur at uncertain time and the size of a fault is also unknown. The faults may lead to performance deterioration or even instability of the system. Therefore, the study of designing fault-tolerant control (FTC) systems, which let the systems operate in safe conditions and with proper performances whenever components are healthy or faulted, has received considerable attention over the past two decades (Veillette et al., 1992) (Veillette, 1995) (Yang et al., 2001) (Wang.R et al., 2007) (Liao.F et al., 2002) (Wu and Zhang, 2006) (Zhang et al., 2008). The existing fault-tolerant design approaches can be broadly classified into two groups, namely passive approaches (Veillette et al., 1992) (Zhao and Jiang, 1998) and active approaches (Mao and Jiang, 2007). In the passive approaches, robust control techniques are utilized to design a fixed controller for maintaining the acceptable system stability and performances throughout normal or faulty cases.

Recently, adaptive control has been widely used to deal with actuator faults in various systems. In (Tao et al., 2004) (Boskovic et al., 1998), actuator lock-in-place (stuck at some unknown place) failures were accommodated by adaptive redundant control structure for linear systems. (Tao et al., 2004) also contains corresponding studies on some systems with known nonlinearities. (Tang et al., 2007) extended the results to MIMO parametric-strict-feedback nonlinear systems. Loss of actuator effectiveness is considered in (Ye and Yang, 2006) for linear systems in the framework of linear matrix inequality (LMI) to guarantee not only the stability, but also the robust performance of the failed system. The common advantage of these adaptive control approaches against actuator fault is that they are independent of fault detection and diagnosis (FDD).

(Boskovic and Mehra, 1999) (Boskovic et al., 2005) (Boskovic and Mehra, 2006) developed adaptive flight control based on multiple model, switching and tuning. However, the methods mentioned above require that the controlled system is known or only contains some linear unknown parameters when there is on fault.

Since it was proved that adaptive fuzzy systems are universal approximators (Wang and Mendel, 1992), and stable adaptive fuzzy control design was showed in (Wang, 1994), fuzzy logic systems (FLS) and neural network (NN) have been used to nonlinear systems, and also FTC systems. In (Poly-carpou and Helmicki, 1995), a general framework for constructing automated fault diagnosis and accommodation architectures was presented using online approximators and adaptive schemes, (Polycar-
In this paper, we propose an adaptive fuzzy sliding mode controller to tolerate actuator faults of unknown nonlinear systems with external disturbances. In contrast to the approaches developed in the literature, only one fuzzy system is used to approximate the unknown dynamics, which allows avoiding perfectly the controller singularity problem. Top deal with the external disturbances and the approximation errors, sliding mode technique is adopted. Hence, the used sliding surface has been modified such that the approaching phase is removed to overcome the knowledge of the upper bound of disturbances to guarantee the sliding condition and to efficiently eliminate the chattering phenomenon.

This paper is organised as follows: Section 2 describes the problem statement. Section 3 is dedicated to the synthesis of the proposed approach. In section 4, a simulation example demonstrates the effectiveness of the propose scheme. Finally, section 5 concludes the paper.

2 PROBLEM STATEMENT

Consider the following nonlinear system with $m$ inputs:

\[
\begin{align*}
\dot{x}_i &= x_{i+1} \\
\dot{x}_n &= f(\bar{x}) + g^T(\bar{x})u + d(t) \\
y &= x_1
\end{align*}
\]

where $\bar{x} = [x_1, x_2, ..., x_n]^T$ represents the state vector, $u = [u_1, ..., u_m]^T \in \mathbb{R}^m$ is the input vector whose the component may fail during the system operation, $y \in \mathbb{R}$ is the output system, $g^T(\bar{x}) = [g_1, ..., g_m] \in \mathbb{R}^m$ and $f(\bar{x})$ are unknown continuous nonlinear functions. $d$ is the bounded external disturbance. The states $x_i \ (i = 1, ..., n)$ are measurable and the reference output $y_m$ is bounded and sufficiently derivable. This is a multiple input single output system with all the inputs contributed to a common a control object like stabilizing the closed loop system, tracking a reference signal with satisfactory performance of both. There are many such systems in our real life. The provided approach is also effective for multi input multi output systems. We only consider a simple case to simplify the presentation. The actuator faults considered in this paper is the loss of effectiveness which is modeled as follows:

\[
\begin{align*}
u_i(t) &= \rho_i v_i(t) \\
\rho_i &\in [0, 1]
\end{align*}
\]

where $\rho_i$ is the still effective proportion of the $i^{th}$ actuator after losing some effectiveness. When $\rho_i = 1$, the corresponding actuator is normal (without fault). With the actuator fault (2), the input vector can be rewritten as:

\[
u(t) = \rho \chi(t)
\]

where $\chi(t) = [v_1, ..., v_m]^T$ is the applied control vector and $\rho = \text{diag}(\rho_1, ..., \rho_m)$.

The control objective is to design a robust adaptive fuzzy sliding mode control law for the system (1) with the actuator fault (2) to ensure that all signals are bounded in the closed loop and the output $y(t)$ can track the given reference signal $y_r(t)$ as closely as possible despite the presence of uncertainties, external disturbances and actuator faults. From the fault model (2), it is reasonable that there is at least one actuator still active for the control purpose. In this case, we propose to use a proportional actuation structure as follows (Ping and Yang, 2008):

\[
\chi(t) = \beta v_0(t)
\]

where $\beta = [b_1, ..., b_m]^T$ represents the matrix of proportional actuation and $v_0(t)$ the proposed robust...
adaptive fuzzy sliding mode control law. Using equations (3) and (4), the system (1) will be described by:

\[
\begin{aligned}
\begin{cases}
x_i &= x_{i+1} & 1 \leq i \leq n-1 \\
x_n &= f(\chi) + g^T(\chi)\psi \nu_0(t) + d(t) \\
y &= x_1
\end{cases}
\]

(5)

For this, the following assumptions are needed:

**Assumption 1**: System (1) is constructed such that despite the loss of actuator effectiveness according to (2), the system still be forced.

**Assumption 2**: The external disturbance \(d(t)\) is assumed to be bounded, i.e., there exists a positive unknown constant \(\chi\) such that: \(|d| < \chi\).

The proposed control scheme combines fuzzy logic for approximation and sliding mode for robustness to attain the control objectives.

3 Proposed approach

3.1 Fuzzy logic system

An fuzzy logic system (FLS) consists of four parts: the knowledge base, the fuzzifier, the fuzzy inference engine manipulating fuzzy rules, and the defuzzifier (Wang, 1994). The knowledge base for the FLS comprises a collection of fuzzy IF-THEN rules. The fuzzifier maps a real point in the input space (measurement of the systems state) to a fuzzy set. In general there are two possible choices of this mapping, namely singleton or non-singleton. In this paper, we use the singleton fuzzifier mapping. The fuzzy inference engine performs a mapping from fuzzy sets of the input to fuzzy sets in the output space, based on the fuzzy IF-THEN rules (in the fuzzy rule base) and the compositional rule of inference. The defuzzifier maps fuzzy sets in the output space to a crisp point in this space; in this study we use the centre-average defuzzifier mapping (Wang, 1994).

The output of a multi input single output FLS with centre-average defuzzifier, product inference, and singleton fuzzifier are of the following form:

\[
y(\chi) = \frac{\sum_{i=1}^{m} y_i \cdot (\prod_{j=1}^{n} \mu_i(x_j))}{\sum_{i=1}^{m} (\prod_{j=1}^{n} \mu_i(x_j))}
\]

(6)

where \(\mu_i(x_j)\) represents the membership degree of the input \(x_j, y_i\), the conclusion constant corresponding to the \(i^{th}\) rule and \(m\) the number of used fuzzy rules.

The output of the FLS can be rewritten on the following vectorial form (Wang, 1994):

\[
y(\chi) = \psi^T \phi(\chi)
\]

(7)

where \(\psi = [y_1,...,y_m]^T\) represents the vector of the adjustable parameters and \(\phi(\chi) = [\prod_{i=1}^{m} \mu_i(x_j),...,\prod_{i=1}^{m} \mu_i(x_j)]^T\) the regressor vector.

According to the universal approximation theorem (Wang, 1994), there exists an optimal fuzzy system in the form (5) such it approximates uniformly an unknown continuous function \(h(\chi)\) on a compact set for any approximation accuracy:

\[
h(\chi) = \psi^T \phi(\chi) + \varepsilon
\]

(8)

where \(\varepsilon\) is a very small positive constant.

3.2 Sliding mode control

To attain the desired objectives, we propose to use a sliding mode control. This choice is motivated by the fact that sliding mode allows to maintain the tracking performances in presence of both structural uncertainties and external disturbances (Slotine and Li, 1991). For this, we consider the following sliding surface:

\[
S(t) = e^{(n-1)}(t) + \sum_{i=1}^{n-1} \lambda_i e^{(i-1)}(t)
\]

(9)

where \(e(t) = y(t) - y(t)\) denotes the tracking error and \(e^{(i)}(t)\) its \(i^{th}\) time derivative. The constants \(\lambda_i\) are chosen such the corresponding polynomial roots are stable (Slotine and Li, 1991). Using the sliding surface \(S(t)\) in this actual form presents two major drawbacks: (i) during the reaching phase, the system is sensitive to uncertainties and external disturbances, which provokes chattering phenomenon in the neighborhood of the sliding surface. (ii) Choosing big values of the slopes which allows reducing the reaching phase but requires an important starting energy, and small values give a slow response. So, it is necessary to find a trade-off between the starting energy and the time response (Hussain et al., 2010). To overcome this problem, we propose to use a modified the sliding surface allowing to suppress the reaching phase, and hence the system will be at \(t = 0\) on the surface \((S(t) = 0)\). In this case, the sliding surface will be defined as follows:

\[
S(t) = e^{(n-1)}(t) + \sum_{i=1}^{n-1} \lambda_i e^{(i-1)}(t)
\]

\[
-\frac{2}{n} \left[ \frac{n}{2} - \arctg(t) \right] \left( e^{(n-1)} + \sum_{i=1}^{n-1} \lambda_i e^{(i-1)} \right)
\]

(0)

\[
e^{(n-1)}(t) + \sum_{i=1}^{n-1} \lambda_i e^{(i-1)}(t) + S_0
\]

(10)
If \( f(x) \) and \( [g^T(x)p] \) are well known, the control law can be given as:

\[
v_0 = [g^T(x)p]^{-1} \left[ -f(x) + y^{(n)} + \sum_{i=1}^{n-1} \lambda_i e^{(i)}(t) \right] + [g^T(x)p]^{-1} [k_d \text{sign}(S(t))]
\]

(11)

Where \( k_d \) is a positive constant chosen such that: \( S(t)S(t) < 0 \).

However, the dynamics of the system studied in our paper are unknown which makes the use of this control law impossible. To resolve this problem, one can use direct adaptive fuzzy controller or an indirect adaptive fuzzy controller. In the direct scheme, the control law is approximated by a fuzzy system. Nevertheless, the adaptation laws used are very complicated to avoid the singularity problem. In this work, we propose to approximate the unknown terms using only one fuzzy system under the constraint that the robustness of the closed loop system is guaranteed and the number of the involved parameters in the control design is reduced.

### 3.3 Control Law Synthesis

This section is dedicated to the synthesis of the proposed approach.

Using (5), the time derivative of the sliding surface (10) is given by:

\[
S(t) = e^{(n)} + \sum_{i=1}^{n-1} \lambda_i e^{(i)}(t)
\]

and

\[
S(t) = y^{(n)} - f(x) - [g^T(x)p]v_0 - d + \sum_{i=1}^{n-1} \lambda_i e^{(i)}(t)
\]

(12)

If we muster all the unknown parameters in one, the above expression can be rewritten as:

\[
S(t) = y^{(n)} - f_d(x) - [g^T(x)p]v_0
\]

(13)

where \( f_d(x) = f(x) + d - \sum_{i=1}^{n-1} \lambda_i e^{(i)}(t) \).

According to assumption 1, we have \([g^T(x)p] \neq 0\). So, it can be positive or negative. We assume in this work, that there exists a positive constant \( g_0 \) such that: \([g^T(x)p] > g_0 > 0\). Furthermore, the function \( f_d(x) \) is unknown. To attain the control objectives, we propose to use a fuzzy system \( \frac{1}{2}[g^T(x)p] \) to approximate. We define a new variable \( \alpha \) such that: \( \alpha = g_0^{-1} \|\psi\|^2 \).

According to (8), we define the approximation error as: \( \hat{\alpha} = \alpha - \alpha \) whose time derivative is given by: \( \dot{\hat{\alpha}} = -\hat{\alpha} \).

**Proposition** The control law

\[
v_0 = M_0.S(t) + \frac{\alpha}{2}\beta \phi(x)^T \phi(x)S(t)
\]

(14)

with

\[
\dot{\alpha} = \frac{\gamma}{2}\beta \phi(x)^T \phi(x)S(t)
\]

(15)

guarantees the stability and the robustness of the closed loop system in presence of actuators faults. It ensures also the boundedness of all the involved signals.

**Proof**

According to (8), (13) and (14), using the fact that the reference signal \( y^{(n)} \) yields to:

\[
S(t)S(t) \leq \frac{g_0 \phi(x)^T \phi(x)S(t)}{2\beta} + \frac{\beta^2}{2} + \frac{g_0 S^2(t)}{\eta^2} + \frac{\eta^2(c^2 + \chi^2)}{2g_0}
\]

(16)

where \( \eta \) and \( \chi \) two positive constants.

To prove the stability, we consider the following Lyapunov function:

\[
V_L = \frac{1}{2} S^2(t) + \frac{g_0}{2\beta} \alpha^2
\]

(17)

Using equations (15) and (16), the time derivative of (17) becomes:

\[
\dot{V}_L \leq \frac{1}{2} \left[ \frac{\beta^2 + \eta^2(c^2 + \chi^2)}{g_0} - g_0 \sigma \alpha^2 \right] + \frac{\eta^2(c^2 + \chi^2)}{2g_0}
\]

(18)

Let \( a_0 = \frac{1}{2} \left[ \frac{\beta^2 + \eta^2(c^2 + \chi^2)}{g_0} - g_0 \sigma \alpha^2 \right] \) and \( b_0 = \min(2g_0 M_0, \eta \sigma) \). Then, the time derivative of \( V_L \) is given by:

\[
\dot{V}_L \leq a_0 + b_0 V_L(0)
\]

(19)

which implies

\[
V_L(t) \leq V_L(0) \exp(a_0t) + \frac{b_0}{a_0} \forall t \geq 0
\]

(20)

Hence, the Lyapunov function converges toward a bounded value \( \frac{b_0}{a_0} \). This implies that all the involved signals are bounded. Furthermore, we can have \( \lim S(t) \leq \frac{b_0}{a_0} \), which ensures the convergence of the tracking error to zero (Wang, 1994).

### 4 Simulation and Results

In this section, the presented adaptive fuzzy fault tolerant controller is applied to a nonlinear system
with the actuator faults described as (2). Example: We consider that after transformation, the nonlinear system can be written as the following from which has redundancy actuation structure.

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{5 \sin x_1 - 0.02 \varepsilon_1 \cos(x_1) \sin(x_1)}{3 - 0.02 \cos^2 x_1} + \frac{\cos^2 x_1}{3 - 0.02 \cos^2 x_1} u_1 \\
&\quad + \frac{2 \cos^2 x_1}{3 - 0.02 \cos^2 x_1} u_2 + d
\end{align*}
\]

(21)

Where the actuators of \( u_1 \) and \( u_2 \) are the control inputs, \( d = 0.1 \sin(2t) \) represents the external disturbances. The evolution of the actuators effectiveness \( \rho = \text{diag}(\rho_1, \rho_2) \) is given by figure (1). In order to control system (21), control law is applied with the following simulation parameters: \( m = 5 \times 5 = 25 \) rules for the fuzzy logic system, with \( \rho = 0.01, \gamma = 4.250 \) and initial values \( \alpha = 0.25 \). Figures 2 and 3 give the simulation results for regulation problem and 4-5 those of tracking of a sinusoidal reference signal. We can see the convergence of the states to their respective reference signals despite the presence of both effectiveness loss (figure 1) and external disturbances.

\[\text{Figure 1: Evolution of the state variables}\]

\[\text{Figure 2: Evolution of the state variables}\]

\[\text{Figure 3: control signals}\]

\[\text{Figure 4: Evolution of the state variables and their reference signals}\]

\[\text{Figure 5: control signals}\]

\[\text{5 CONCLUSIONS}\]

In this paper, a fuzzy sliding mode approach for fault tolerant control problem for an uncertain perturbed nonlinear system is studied. To overcome the problem of unknown dynamics, only one adaptive fuzzy system has been used. Furthermore, the sliding surface has been modified to suppress the reaching phase and hence improve the robustness of the closed loop system. The global stability has been established in the sense of Lyapunov. Many simulations have presented to show the good performances despite the presence of actuator failures. As future work, the case of actuator lock-in-place will be also treated and the extension of this approach to multi-input multi-output will be studied.
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