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<th>Order batching and picking in a synchronized zone order picking system</th>
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Order picking has been considered as one of the most critical operations in warehouses. In this study, we propose an analytical approximation model based on probability and queueing network theory to analyze order batching and picking area zoning on the mean order throughput time in a synchronized zone picker-to-part order picking system. The resulting model can be easily applied in the design and selection process of order picking systems.

Keywords - Order batching, order picking, applied probability, queueing

I. INTRODUCTION

Order picking – the process of retrieval stock keeping units from storage (or buffer area) in response to a specific customer request – is the most critical supply chain component for many companies [1].

The total picking time can be roughly divided into three components: set-up time at the depot, time for picking products and traveling time between different locations by walking or driving. According to [1], the traveling time takes up approximately 50% of the total order picking time. Hence, the reduction in travel time will lead to an improvement in order picking throughput time. Besides, for a warehouse with a given layout, a predetermined storage strategy and routing policy, the two major factors that impact on the order picking system performance are zoning and batching. Zoning is to divide the whole storage area into several smaller zones and send pickers to each zone to pick requested items. Batching is a preliminary operation which is to group several orders together before picking. Zoning and batching are closely related issues. However, in most literatures, they are always studied separately. Yu and de Koster [2] study the impact of order batching and zoning in a pick-and-pass order picking system. In this paper, a probability model is constructed to study the average order throughput time in a parallel picker-to-parts order picking system.

II. PROBLEM DESCRIPTION

The order picking system under consideration is a picker-to-parts narrow-aisle and ABC-class strategy synchronized zone warehouse, as shown in Fig. 1, having the following properties:

1) Each order requires a variety of items, which arrives at the warehouse according to a Poisson process. When the batch size reaches a prefixed number, those orders in this batch will be released for picking.

2) All picking is done in an active picking area constituted by several smaller picking zones utilizing bin shelving and manual order picking carts. The picking aisles are wide enough to allow a picker to change directions and also allow picking on both sides of the picking aisles.

3) Each order picker picks a single pick list at one time and there is only one picker per zone. Each picking tour begins and ends at a centralized depot where the order picker receives the pick list and deposits the picked items. For simplicity, it is assumed that the capacity of the pick bin is sufficiently large to contain all requested items to pick in this zone and stock outs never happen. Time taken to retrieve an item from a storage location is assumed to be a known constant.

4) A zone is a set of adjacent identical racks, which means one rack cannot belong to more than one zone.

5) ABC-class with across-aisle storage assignment strategy is adopted here with Return routing policy.

Fig. 2 gives a schematic illustration of the resulting queueing network including a J-node E_r / G/1 fork-join sub-network with J zones, where node 1 to node J represent the parallel J picking zones and node J+1 represents the sorting station.

III. APPROXIMATION MODEL

In this section, we first derive the first and second moments of service time of an order batch in the picking zones, and the sorting station. Then, we apply the queueing network approximation model to calculate the average throughput time of a random order in the system. The order’s travel route is as sketched in Fig.1, where the picker enters the aisle containing picks from the front cross aisle only, picks the requested item, and then leaves the aisle from the front cross aisle, and the ABC-class strategy is illustrated in Fig.1 with racks nearest to the depot representing the first class type of goods, the racks behind them representing the second class type, and so on. For such type of picking system layout, we define the following notations:

1. $b$ Number of orders in a batch;

Fig. 1. Warehouse layout.
First and Second Moments of Service Time in a Zone

First, we derive the first and second moments of the service time in a pick zone for the picking system. The travel time, \( T_{trj} \), in zone \( j \), given zone \( j \) is visited and \( q_j \) items to be picked, consists of two components: (1) travel time within the aisles, \( T_{twj} \), and (2) travel time across the aisles, \( T_{ta} \).

Under the assumption that items of the same class stored in the same rack are randomly distributed, the cumulative distribution function that the travel distance in a zone \( j \), given that there are \( q_j \) picks in this zone can be calculated as

\[
P(q_j \mid r) = \frac{\sum_{i=0}^{q_j} P(i \mid r)}{\sum_{i=0}^{\infty} P(i \mid r)}
\]

where

\[
P(i \mid r) = \binom{r}{i} \cdot \left( \frac{1}{M} \right)^i \cdot \left( \frac{M-1}{M} \right)^{r-i}
\]

Then the mean travel time within aisle \( k \), given \( q_j \) picks in this aisle with the farthest pick in class \( r \), is

\[
E[T_{twj} \mid q_j, r] = \sum_{i=0}^{q_j} i \cdot P(i \mid r)
\]

Therefore, the expected travel time in aisle \( k \) given \( q_j \) picks in \( k \) can simply be represented as

\[
E[T_{twj} \mid q_j, r] = \sum_{i=0}^{q_j} i \cdot P(i \mid r)
\]

The service time of a pick bin in zone \( j \) consists of four components: the setup time, the travel time, the picking time, and the conveying time to transport the picked items to the sorting station. The picking time per item, the conveyor transportation time between picking zones and sorting station and the setup time are assumed to be constants, denoted by \( t_i, tc \) and \( ts \) respectively. Then, the expected service time in zone \( j \), given \( q_j \) picks in this zone is:

\[
E[S_j \mid q_j] = E[T_{trj} \mid q_j, r] + tc + ts
\]

The expected service time in zone \( j \) given that there are \( Q \) items in a batch is therefore calculated as:

\[
E[S_j \mid Q] = \sum_{Q=0}^{Q} E[S_j \mid q_j] \cdot P(q_j, Q)
\]

where

\[
P(q_j, Q) = \binom{Q}{q_j} \cdot \left( \sum_{r=0}^{Q} \binom{r}{q_j} \cdot \left( \frac{1}{M} \right)^q \cdot \left( \frac{M-1}{M} \right)^{q-r} \right)
\]

is the probability that there are \( q_j \) items need to be picked in zone \( j \) given that there are \( Q \) items in the batch. And then, the expected service time in zone \( j \) given there are \( b \) orders in a batch can be calculated as:

\[
E[S_j \mid b] = \sum_{Q=b}^{Q} E[S_j \mid Q] \cdot P(Q, b)
\]
where $P_2(Q,b)$ is the probability that a batch has $Q$ items to be picked given that it contains $b$ orders. An order should at least contain one order line. Here, the distribution is taken as negative binomial distribution:

$$P_2(Q,b) = P(Q=b) = \binom{x-1}{\alpha-1} \left(1-\frac{1}{\alpha} \right)^x \left(\frac{1}{\alpha} \right)^b$$

where $0 < \alpha < 1$, and $\alpha > 0$ are constant parameters, and $x = 0, 1, ...$ as used in [5]. The advantage of adopting this distribution is that a large number of distributions can be approximated by adjusting $\alpha$ and $\nu$.

The second moment of service time in zone $j$ given there are $q_j$ items to be picked in that zone is:

$$E[S^2_j | q_j] = E[(T_{w_j} + T_{a_j} + \rho_t + t + tc + ts)^2 | q_j]$$

$$= E[T_{w_j}^2 | q_j] + E[T_{a_j}^2 | q_j] + 2E[T_{w_j}T_{a_j} | q_j] + (q_j + t)^2 + (tc + ts)^2 + 2q_j + t + tc + ts$$

(1)

Similarly as before, the calculation of the first term in (1), the second moment of within aisle travel time in zone $j$ assuming $q_j$ picks are can be approximated by:

$$E[T_{w_j}^2 | q_j] = E[(\sum_{i=1}^{n_j} W_i)^2 | q_j] \approx 4E[M^2 | q_j] * E[L^2 | q_j]$$

(2)

To calculate $E[M^2 | q_j]$, first notice the variance of number of aisles visited given $q_j$ picks is just the occupancy problem. Thus it can be computed as:

$$\text{Var}(M | q_j) = M_j - (1-M_j)^n - (1-M_j)^n$$

Hence, we can have:

$$E[M^2 | q_j] = \text{Var}(M | q_j) + E[M | q_j]^2$$

Similarly as calculating its first moment, the second moment of the travel time within aisle $k$, given $q_j'$ picks in the aisle with the farthest pick class being $r$, is:

$$E[T_{w}^2 | q_j', q_j', F = r]$$

$$= \int_0^{\infty} \left( \sum_{y=1}^{q_j'-1} \right) \frac{x-y}{y} + \frac{x}{y} \ dP_r(\text{travel distance in } r \leq s | q_j', F = r)$$

Then, we can show that the second moment of the travel time in aisle $k$ given that $q_j'$ picks in $k$ is:

$$E[T_{w}^2 | q_j', q_j', F = r] = \text{Var}(M | q_j) + E[M | q_j]^2$$

(3)

And the second moment of the travel time within one aisle given $q_j'$ picks in aisle $k$ is:

$$E[L^2 | q_j'] = \sum_{i=1}^{n_k} E[L_i^2 | q_j'] P[q_j' \text{ picks in aisle } k | q_j']$$

Therefore, $E[T_{w_k}^2 | q_j']$ can be approximated as in (2).

The second moment of the cross-aisle travel time given $q_j$ picks in zone $j$, $E[T_{a_j}^2 | q_j]$ can be calculated as two conditions according to whether the number of aisles is odd or even. Similarly as the analysis in calculating $E[T_{a_j}^2 | q_j']$, we have:

$$E[T_{a_j}^2 | q_j, M_j \text{ is even}]$$

$$= \frac{4W_j^2}{v^2M_j^2} \left[ \sum_{i=1}^{(M_j-1)/2} \sum_{P=1}^{(M_j-1)/2} (k+k)^{\nu} - 4 \sum_{i=1}^{(M_j-1)/2} (k+M_j-1)^{\nu} + 2 \sum_{i=1}^{(M_j-1)/2} \right]$$

$$+ 2 \sum_{i=1}^{(M_j-1)/2} k^{\nu}(M_j-1)^{\nu}$$

As for the product term $E[T_{w_j} \cdot T_{a_j} | q_j]$ in (1), by assuming $T_{w_j}$ and $T_{a_j}$ are independent, we have:

$$E[T_{w_j} \cdot T_{a_j} | q_j] = E[T_{w_j} | q_j] * E[T_{a_j} | q_j]$$

Therefore, $E[S^2_j | q_j]$ can be computed as stated in (1). Then, the second moment of service time in zone $j$ given that a batch consists of $b$ orders can be calculated as:

$$E[S^2_j | b] = \sum_{j=1}^{n} E[S^2_j | q_j] P(b | q_j) P(Q \in b)$$

B. Mean and SCV of Maximum Service Time in Pick Zones

It is difficult to model job service time in fork-join synchronization models analytically. Indeed, to date, exact analytical results exist only for the mean response time of a two server system consisting of homogeneous $M/M/1$ queues [6]. In the previous section, we have derived the first and second moments of the service time in each zone node. Now, we consider $P$ picking zones as one service node in a simple sequential queue, by using the derived approximation formula below to approximate the first and second moments of maximum service time among these zones.

Our approach is inspired by [7] for approximating the mean of the maximum of multiple random variables in a split-merge queue. In this queue, a job splits into a number of subtasks which are serviced in parallel. Only when all the subtasks finish servicing and rejoin can the next job split and start servicing. Hence, the fork-join time of the task is the maximum of the subtasks’ processing times.

For generally distributed random variables, similarly as Harrison and Zertal [7], we derive the first moment of the maximum of independent, non-negative random variables with first moment $E(e_i) = (e_1, ..., e_n), \ a = (\alpha_1, ..., \alpha_n)$, by the function $\Phi(n, a, E)$, defined by the recurrence for $k = 2, ..., n$ with $\Phi(1, a, E) = a^{-1}$

$$\Phi(k, a, E) = E(s + \sum_{i=1}^{k} a_i)$$

where $L_\nu(a, s) \equiv \sum_{i=1}^{n_k} \alpha_i L_\nu(a_i, s)$ is the Laplace form of the probability density function of the maximum of $m$ exponential random variables with parameters $a = (a_1, ..., a_n)$ and $a_i = (a_\alpha_1, a_\alpha_2, a_\alpha_3, ..., a_\alpha_n)$.

We now derive an approximation for the second moment of the maximum of a set of independent, non-negative random variables. First, consider $T$
= \max(T_1, T_2) \) for two non-negative exponential random variables \( T_1, T_2 \) with parameters \( \lambda_1, \lambda_2 \) respectively. Then,
\[
E[T^2] = E[T_1^2] + P(T_1 > T_2)E[T_1^2 - T_1^2 | T_1 > T_2] = E[T_1^2] + 2E[\alpha_1^2]/\lambda_1^2.
\]

Hence, the second moment of the maximum of \( n \) independent non-negative random variables can be approximated by \( \Phi^2(k, a, E) \) defined by the recurrence, for \( k = 2, \ldots, n \), with \( \Phi^2(1, a_1, E_1) = E \) as
\[
\Phi^2(k, a, E) = \sum_{i=1}^{k-1} \Phi^2(k-i, a_i, E_i) + (2E - e^k)\Phi^2(0, a_k, E_k).
\]

Therefore, the first and second moments of maximum service time in zone \( j \) given \( b \) orders in a batch, \( E[SE_j^2 | b] \) and \( E[SE_j | b] \), can be expressed by
\[
E[SE_j^2 | b] = \Phi(M_j, a, E), \quad E[SE_j | b] = \Phi(M_j, a, E)
\]
where \( \alpha = (E[SE_j^2 | b], \ldots, E[SE_j^2 | b]') \) \( E = (E[SE_j | b], \ldots, E[SE_j | b]) \).

Hence, the squared coefficient of variation (SCV) of the maximum service time among all zones, given that there are \( b \) orders in a batch can be calculated by
\[
c_{p0}^{b} = (E[SE_j^2 | b] - E[SE_j | b]^2)/E[SE_j | b]^2.
\]

C. Mean and SCV of Service Time at the Sorting Station

Since orders are baked before picking, they need to be sorted again into the original orders and then sent for packing and transportation. Here, at the sorting station, the service time of a batch is modeled as a constant setup time plus a sorting time and the sorting time per item is also a constant. Then, the service time for sorting a batch containing \( Q \) items can be represented as
\[
SE = sc + s_i, Q
\]
where \( sc \) is the setup time and \( s_i \) is the sorting time per item.

Hence, the first moment of the service time of sorting a batch containing \( b \) orders is
\[
E[SE] = sc + s_i \sum_{Q=Q}^{Q} P(Q, b) = sc + s_i \cdot E[Q]
\]
And the second moment of the service time of a batch containing \( b \) orders is
\[
E[SE^2] = sc^2 + 2scs_i + E[Q] + s_i^2 \cdot E[Q^2]
\]
where
\[
E[Q^2] = \sum_{Q=Q}^{Q} Q^2 \cdot P(Q, b)
\]
Therefore, the SCV of the service time at the sorting station, given \( b \) orders in a batch can be expressed as
\[
c_{p0}^{b} = (E[SE^2] | b) - E[SE] | b^2)/E[SE] | b^2.
\]

D. Mean Throughput Time of \( b \) Orders in the System

The calculation of mean throughput time of \( b \) orders in the parallel zoning order picking system is based on the open queueing network approximation model described in [8]. Fig. 3 is the simplified 2-stage tandem queue, after considering all \( J \) zones as one service node (node \( p \)). Each node operates like a \( E | G/1 \) queueing system. The approximation analysis uses two parameters, the mean service time and the SCV, to characterize the arrival process and the service time at each node.

![Fig. 3. Simplified queuing network of the order picking system.](image)

For the arrival process of each node, node \( p \) and node \( s \), the parameters are \( \lambda_i, i = p, s \), the arrival rate, and \( c_{p0}^{s}, i = p, s \), the SCV of the inter-arrival times of each node. Since order customers arrive at the system according to a Poisson process with rate \( \lambda_i \). After batching \( b \) orders, the batched orders are sent to the parallel zones for picking. Therefore, the input of the system has an Erlang distribution with parameters \( (b, \lambda_i) \). The input rate for the system is \( \lambda_i = \lambda/b \). The SCV of the input inter-arrival time is \( c_{p0}^{b} = 1/b^2 \).

In step 1, we calculate the arrival rates and the utilizations of each node. Here, since there is exactly one determinant path from node \( p \) to \( s \), we simply have \( \lambda_i = \lambda_i = \lambda/b \) and \( \rho_i = \lambda_i/\mu_i = \lambda_i E[SE_j | b] \), for \( i = p, s \).

In the next step, we calculate the SCV of the inter-arrival times at each node, which is done iteratively in three phases: Merging, Flow and Splitting. For the first phase, Merging, several arrival processes to each node are merged into a single arrival process. In the Flow phase, the SCV of the inter-departure times, \( c_{p0}^{s} \), depends on the SCV of the inter-arrival times, \( c_{p0}^{b} \), and that of the service time, \( c_{p0}^{s} \). Several authors suggest different approximation formulae. Here, we use the one proposed in [9]. For the Merging, since there is only one determinant path from node \( p \) to \( s \) and no external arrival process, the SCV of the inter-arrival times is \( c_{p0}^{b} = c_{p0}^{b} \), \( \lambda_i = \lambda_i = \lambda/b \), and the SCV of the inter-departure times can be written as
\[
c_{p0}^{s} = -1 + \lambda_i \rho_i (c_{p0}^{b} + 1)/\mu_i + (1 - \rho_i)(c_{p0}^{b} + 1 + 2\rho_i) \quad i = p, s.
\]

In the last phase to split the departure process, we apply the formula \( c_{p0}^{s} = c_{p0}^{b} \).

The iteration of step 2 starts with phase 1, the initial values are \( c_{p0}^{b} = 1 \) and stops when \( c_{p0}^{s} \) converges.

With the inter-arrival rate \( \lambda_i \), SCV of inter-arrival time \( c_{p0}^{b} \), and the service time parameters \( E[SE_j | b] \) and \( c_{p0}^{s} \), the expected waiting time of a batch containing \( b \) orders at each node can be approximated by the Krämer-Langenbach-Belz formula [9], which is given by
\[
E[W | b] = \frac{\rho_i/\mu_i}{1 - \rho_i} \frac{c_{p0}^{b} + c_{p0}^{s}}{2} \cdot G_{KLB}
\]
with
\[
G_{KLB} = \begin{cases} \frac{2 - 3\rho_i}{3} \left( c_{p0}^{b} + c_{p0}^{s} \right)^2, & 0 \leq c_{p0}^{s} \leq 1 \\ \frac{1 - \rho_i}{c_{p0}^{b} + c_{p0}^{s}} \left( c_{p0}^{b} + c_{p0}^{s} \right)^2, & c_{p0}^{s} > 1 \end{cases}
\]

The expected sojourn time of a batch containing \( b \) orders at node \( i \) can be expressed as
\[
E[T_i | b] = E[W | b] + E[SE_j | b] \quad i = p, s.
\]

Hence, the throughput time of a batch containing \( b \) orders is the sum of the expected sojourn time at each node.
The simulation model was run 100,000 times for the parameters of the warehouse in picking system with 3 accuracy in estimating the expected throughput times. Since the arrival of orders is a Poisson process, if \( t_n \) is the time of the \( n \)th arrival, then \( t_n \) follows an Erlang distribution. Hence, for the \( n \)th arrival of a Poisson process with parameter \( \lambda \), the expected arrival time is \( \frac{n}{\lambda} \). Accordingly, the expected waiting time to form a batch of \( b \) orders can be estimated as

\[
E[W | b] = \sum_{r=1}^{b} \frac{(r-1)}{\lambda b}
\]

Therefore, the mean throughput time of a random order in the system is given by

\[
E[T | b] = E[W | b] + E(T | b)
\]

### E. Mean Throughput Time of a Random Order

The mean throughput time of an arbitrary order in the system consists of two components: the expected waiting time for the order to form a batch and the mean throughput time of picking and sorting in the system. The impact of order batching on mean order picking throughput time in the system. It can be seen from Fig. 4 that the approximate model provides sufficient accuracy in estimating the expected throughput times.

### IV. CONCLUSION

In this paper, we developed an analytical probability model based on an open queueing network approximation model to evaluate the performance of a synchronized picker-to-parts order picking system with online order arrivals. Firstly, the first and second moments of service times of each pick zone are derived and then an approximation formula is used to obtain the first and second moments of the maximum service time among all zones. After that, the order picking system is modeled as a 2-stage tandem queueing network with \( E / G / 1 \) queues on each node. This study extends previous work by considering the synchronized zoning issue and the ABC-class storage strategy, which are widely used in practice. This paper also takes the subsequent sorting operation after picking into consideration.

Although this study is particularly aimed at synchronized zone order picking, it considers mainly common characteristics of many order picking systems. It is possible to extend our model to different order arrivals, zoning layout, storage strategies and routing policies, where the open queueing network approximation model can still be applied.

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