

Indexed Sovereign Debt: An Applied Framework*

Guido Sandleris

Universidad Torcuato Di Tella and Johns Hopkins University (SAIS)

Horacio Sapriza

Filippo Taddei

Rutgers Business School

Collegio Carlo Alberto

December 2008

Abstract

A number of countries have issued sovereign debt instruments indexed to real variables in recent years. This type of contracts could improve risk sharing between debtor countries and international creditors and diminish the probability of occurrence of debt crises. This paper characterizes the optimal features of real indexed sovereign debt contracts in a dynamic stochastic equilibrium framework with incomplete markets. We show that the optimal indexed debt contract should not be studied abstracting from the total portfolio of assets and liabilities of the issuing country. We also show that the optimal contract is similar to an insurance contract, and that a country can replicate it using existing instruments, in particular, a combination of international reserves and GDP-indexed bonds. Calibrating our model to Argentina's economy we find that the welfare gains from introducing indexed debt and allowing asset accumulation could be equivalent to an increase of between 0.1% and 0.5% in certainty equivalent aggregate consumption.

JEL codes: E6, F41, G15

*We thank Tito Cordella, Christine Hauser, Christian Hellwig, Eduardo Levy Yeyati, Alberto Martin, Adrien Verdelhan, Mark Wright and participants to the Pre-conference on Country Insurance at the World Bank, LACEA 2007, SED 2008, EEA 2008, ASSET 2008, LACEA 2008 for their comments and suggestions. All remaining errors are our responsibility.

1 Introduction

In March 2005 Argentina finished the debt restructuring process that followed the sovereign default and financial crisis of 2001. The restructured debt included a relatively novel component, some of the payments were linked to the evolution of Argentina's GDP. The innovation was not well received by investors at the time.¹ However, it captured the spirit of some of the proposals under discussion since the string of sovereign debt crises in Russia, Ukraine, Pakistan, Ecuador and Argentina renewed the debate on the institutional framework of sovereign credit markets and the design of sovereign debt contracts.² This paper focuses on the latter problem.

There exists consensus that indexing sovereign debt payments to real variables could in principle improve risk sharing among debtor countries and international creditors while it would diminish the probability of financial crises.³ However, there are two issues that deserve further attention: first, it is not clear what is the optimal design of real indexed sovereign debt contracts and how it depends on the total portfolio of a country; second, it is interesting to have some sense about the magnitude of the welfare effect that this type of instruments could have. This paper addresses these questions. In particular, the goals of this paper are to characterize the optimal real indexed debt contract, to evaluate whether such contract could be implemented with existing instruments and to provide a first quantitative approximation of the potential welfare gains that the introduction of real indexed debt could deliver.

In a dynamic stochastic small open economy model with limited commitment⁴ we show that the optimal indexed contract has the features of an insurance contract with payments increasing in the state of the economy, and that such a contract can be implemented through a suitable combination of non-indexed assets (such as international reserves) and standard real indexed bonds (such as GDP-indexed coupons).

Our results from calibrating the model to the Argentinian economy in the period 1983-2000 suggest that the welfare improvement of substituting standard defaultable debt with GDP-indexed

¹See Fernandez, Pernice, Streb, Alegre, Bedoya and Gonzalez (2006) or Balston and Ghezzi (2004).

²See IMF (2002) for a summary of some of these proposals.

³See Borensztein and Mauro (2002).

⁴We use the concept of limited commitment as it is usually done in the sovereign debt literature, that is, the inability of the government to commit to repay its debt.

debt in an economy with access to only non-contingent assets could be equivalent to an increase in average aggregate consumption ranging from 0.1% to 0.5% depending on the default costs.⁵

Our theoretical analysis is related to the model of sovereign lending of Eaton and Gersowitz (1981) and the quantitative adaptations of their framework by Arellano (2008) and Alfaro and Kanczuk (2008). We extend them to analyze real indexed debt. In our model the government tries to maximize the welfare of an infinitely lived risk averse representative agent facing random income shocks. In order to do so, the government can borrow from homogenous and risk neutral international creditors using one period bonds and has access to a non-contingent savings technology.

Consistently with the weak legal framework of sovereign borrowing, we assume that the government cannot credibly commit to repay its debts to foreign creditors. Every period it chooses whether to repay or default on its outstanding debts. Default entails direct costs or penalties and triggers the exclusion from credit markets.⁶ These direct costs are not appropriable by creditors and are assumed to be increasing in the level of income. The intuition behind this assumption can be related to Grossman and Van Huyck (1988) excusable and non-excusable defaults argument or to the information argument by Sandleris (2008). As the state of the economy gets worse (i.e. the income level decreases) the penalty or direct cost of defaulting is smaller since the default becomes "more excusable" or reveals less information. Thus, with non-indexed debt contracts if defaults are to occur, they are more likely to happen when the government faces low income shocks. Government borrowing will be endogenously constrained by the presence of limited commitment.

In this framework, indexed debt contracts improve welfare. They may do so through two channels. First, they improve the amount of risk-sharing between a risk-averse sovereign borrower and risk-neutral lenders; second, they reduce the probability of default and with it, the deadweight loss created by default events.

Indexed debt is welfare improving even if we take into account that non-indexed contracts also have some degree of contingency. In effect, the presence of limited commitment allows the government to default in some states and, as Zame (1993) pointed out in the context of corporate

⁵These estimates assume a discount factor of 0.8 and direct costs of defaults (or penalties) ranging from 2% to 4 % of GDP during the years (1 on average) in which the economy is excluded from credit markets.

⁶These direct costs of default or penalties have been interpreted as trade sanctions by Bulow and Rogoff (1989), as reputation spill-overs by Cole and Kehoe (1997) and as informational costs by Sandleris (2008).

debt, this introduces some degree of contingency in the otherwise uncontingent non-indexed debt contract. However, it typically does so in a costly way due to the presence of direct costs of default, including the possibility of being excluded from credit markets. Moreover, indexed debt improves welfare even considering that, through debt roll-over, the government can smooth consumption issuing non-indexed debt only. This is the case because, as long as the stochastic process of the shock is not mean reverting, the best consumption path that reborrowing can generate will follow that of the shock that affects the economy, though with a smaller dispersion. The reason is that smoothing consumption through reborrowing is affected by the wealth effects created by the shock.

The optimal indexed debt contract is one that allows the government portfolio of assets and liabilities to generate payoffs as close as possible to that of an insurance contract. In particular, the portfolio payoffs should involve positive net payments to the government from the rest of the world in the presence of bad realizations of the income shock, while it preserves non-negative returns to bondholders. A key feature of the optimal indexed debt contract is that it avoids default. The crucial ingredient for this result is the assumption that default entails no payments to creditors and a direct cost for the government (the "penalty").⁷ In a state in which the government chose to default on non-indexed debt, we can design an interest payment of indexed debt smaller than the "penalty" of default. The government would then be better off making this smaller payment and avoiding the costs of default. Creditors would also benefit, receiving a positive payment instead of nothing, and, in competitive markets, they would be willing to pay a higher bond price *ex ante*.

The assumptions that the government is risk averse and creditors risk neutral are crucial for our results. The features of the optimal indexed debt contract would be different if, for example, creditors were more risk averse than the government. Alternatively, the presence of heterogeneous creditors with different preferences over instrument types, may call for the presence of a mix of indexed and non-indexed debt in the optimal borrowing structure.

There are concerns that sovereign debt contracts indexed to real variables partially under the control of the government, such as reported GDP, may create or worsen moral hazard.⁸ In effect,

⁷All we need to assume is that there is a positive difference (deadweight loss) between what the government pays when it defaults and what creditors receive.

⁸Krugman (1988) was the first one to suggest this. In order to address this issue it has been proposed to index the contracts to variables beyond the control of the government such as commodity prices or trading partners' growth rates.

the government may have incentives to undertake less growth-oriented policies or underreport GDP growth rates if interest payments increase in GDP. However, the extent and magnitude of this moral hazard is not clear in practice.⁹ Thus, given that the only source of uncertainty in our model is shocks to income, GDP is the preferred indexation variable when we introduce indexed debt. Note also that the presence of moral hazard is not exclusive of indexed contracts. In effect, given that the costs of default are affected by the level of income, distortions on the government policies might potentially also arise with non-indexed debt too.

The few existing cases of real indexed sovereign debt contracts share common features. They usually include a threshold level of the chosen indexation variable above which payments to creditors are triggered. Then, the magnitude of their interest payments monotonically increases as the chosen indexation variable increases. Payments increasing on the chosen variable are compatible with our characterization of the optimal indexed contract. However, explaining the presence of thresholds is more challenging. We explore environments under which threshold to interest payments could be optimal. First, thresholds are compatible with the optimal debt contract if the government had access to contingent assets.¹⁰ Second, in the absence of contingent assets, thresholds are optimal if we introduce the constraint that the government cannot build a portfolio of assets and liabilities that generates net payments to the government from credit markets (excluding reborrowing). The intuition is that in those states in which the optimal "unconstrained" indexed contract calls for negative returns for creditors and net payments to the government, the best that the "constrained" contract can do is to ensure zero coupon payments to creditors. Notice that in this latter case the government can achieve *more insurance* across good states than across bad ones.

Our paper bridges two branches of sovereign debt literature that have, surprisingly, remained separate. The first strand emphasizes the presence of limited commitment in a dynamic framework. Our model is related, within this group, to Eaton and Gersowitz (1981) and, in particular, to the adaptations of their model made by Aguiar and Gopinath (2006), Alfaron and Kanczuk (2007),

⁹There are many examples of inflation indexed debt that are actively traded and, arguably, governments have a stronger incentive to underreport inflation than GDP. While lower reported inflation and GDP growth rates would lead to lower interest payments on indexed contracts, lower inflation is perceived to be a good signal while the opposite happens with lower GDP growth rates. Thus the moral hazard issue does not seem to be crucial and we will abstract from it in this paper.

¹⁰Furthermore, if the set of assets allows the government to complete the market, then the structure of the debt contracts may become irrelevant.

Arellano (2008) and Yue (2006) . However, we extend this framework to tackle the issue of the optimal design of real-indexed sovereign debt and its quantitative welfare effect. The second strand of literature, which is more policy oriented and best represented by Borensztein and Mauro (2004) and Durdu (2008), analyzes the issues regarding real-indexed sovereign debt contracts abstracting from a thorough modelling of its limited commitment structure.

The paper is organized as follows. Section 2 presents our baseline model. Section 3 analyzes the optimal design of real indexed debt contracts and its implementation. Section 4 analyzes the design of existing real-indexed sovereign debt instruments. Section 5 presents numerical simulations to quantify the welfare effect of real-indexed debt. Finally, Section 6 concludes.

2 The Baseline Model

2.1 Environment

Consider a small open economy in the spirit of Eaton and Gersowitz (1981) where a benevolent government tries to maximize the welfare of an infinitely lived representative agent. In each period $t = 0, 1, 2, 3, \dots$ the agent receives a stochastic endowment, $y_t(s)$, contingent on the state of the economy, $s = 1, \dots, S$ where $y_t(s) > y_t(s')$ if $s > s'$. The preferences of the representative agent are given by the following sum of instantaneous utility functions:

$$U = \sum_{t=0}^{+\infty} \beta^t E_0 [u(c_t)] \quad (1)$$

where $\beta < 1$ represents the intertemporal discount factor, $u(\cdot)$ is a strictly concave, continuous and differentiable utility function increasing in c_t , national consumption in period t , and E_0 is the time 0 expected value of the utility from consumption in future periods.

The government can issue one-period debt in international credit markets and has access to a risk free one-period international asset. In every period t , after observing the state of the economy s , the government makes two decisions in order to maximize the welfare of the representative agent: whether to repay or default on its outstanding debt with foreign creditors, and how much new debt to issue and assets to purchase.

Since here we want to focus on the welfare effect of government debt indexation, it is convenient to assume that the international asset the government can buy has a price of 1 and pays R_t^w in every state, i.e. it is risk free. It is possible to think of these assets as international reserves. Government debt instead can be indexed (or not) to the state of the economy. The promised state contingent payment *to* the rest of the world by the government portfolio of assets and liabilities can be written as:

$$b_t(s) = x_t(s)d_t - R_t^w a_t \quad (2)$$

where $b_t(s)$ is the net promised payment and $d_t \geq 0$ and $a_t \geq 0$, respectively, are its gross debt and asset position. $x_t(s)$ is the state contingent promised payment at time t of one unit of debt issued at time $t - 1$. The vector \vec{x}_t describes the full set of state contingent payments:

$$\vec{x}_t = \begin{bmatrix} x_t(1) \\ \dots \\ x_t(s) \\ \dots \\ x_t(S) \end{bmatrix}, \forall t$$

For the sake of realism we assume that the debt contract entails $x_t(s) \geq 0, \forall s$. That is, there is no state under which the government will receive a payment from its creditors on its own debt. In our set up, the standard zero coupon bond is characterized by setting $x_t(s) = 1, \forall s$. With a little abuse in definition, we will sometimes refer to $b_t(s)$ as the level of net debt of the economy. The state contingent budget constraint for the economy at time t in contingency s then becomes:

$$y_t(s) + q_t(\vec{x}_{t+1} \cdot d_{t+1}(s), R_t^w a_{t+1})d_{t+1}(s) + R_t^w a_t - a_{t+1}(s) = c_t(s) + h_t(s) \cdot x_t(s)d_t + \lambda(h_t(s), y_t(s)) \quad (3)$$

where $y_t(s)$ is the state contingent level of national income; $a_{t+1}(s)$ is the amount of assets purchased at time t and delivering at time $t + 1$; $d_{t+1}(s)$ is the amount of debt issued at time t and maturing at time $t + 1$; $q_t(\vec{x}_{t+1} \cdot d_{t+1}(s), R_t^w a_{t+1})$ is the unit price of debt issued at time t that in equilibrium depends on the total state contingent government portfolio; $\lambda(h_t, y_t(s))$ represents the direct cost (or penalty) to the country for not meeting its current payment obligations, $b_t(s)$. We assume that $\lambda(h_t(s), y_t(s))$ is a continuous and differentiable function of the level of income, $y_t(s)$. Moreover it

is a function of the choice of the country between repaying and defaulting at time t , i.e. $h_t(s)$:

$$h_t(s) = \begin{cases} 1 & \text{if the country repays} \\ 0 & \text{if the country defaults} \end{cases}$$

There is no penalty when the country pays back its debt obligations in full:

$$\lambda(1, y(s)) = 0, \forall s$$

We remain agnostic regarding the exact nature of these costs of default.¹¹ We only assume that the penalty function $\lambda(.,.)$ is monotonically increasing (but less than one to one) in the level of GDP, y . The intuition behind this assumption can be related to Grossman and Van Huyck (1988) argument about excusable and non-excusable defaults. As the state of the economy gets worse (i.e. the income level decreases) for a given level of debt, the penalty that would follow is likely to be smaller as the default would be "more excusable".¹² The following conditions formalize these assumptions:

Penalty from defaulting and income

$$0 < \frac{\partial \lambda(h, y)}{\partial y} < 1 \text{ if } h = 0 \quad (4)$$

In our setup a default choice involves a penalty (or direct cost) to be paid by the defaulting country.

While the debtor country pays in contingency s a cost equal to:

$$D(h_t, s) = \begin{cases} h_t = 1 \Leftrightarrow \text{repay} & d_t(s) \\ h_t = 0 \Leftrightarrow \text{repudiate} & \lambda(0, y_t(s)) > 0 \end{cases} \quad (5)$$

international lenders only receive:

$$L(h_t, s) = \begin{cases} h_t = 1 \Leftrightarrow \text{repay} & d_t(s) \\ h_t = 0 \Leftrightarrow \text{repudiate} & 0 \end{cases} \quad (6)$$

thus it is transparent that

$$D(h_t, s) > L(h_t, s); \text{ if } h_t(s) = 0, \forall s \quad (7)$$

¹¹As explained above, direct costs evaluated in the sovereign debt literature include trade sanctions (Bulow and Rogoff (1989)), informational costs (Cole and Kehoe (1997) and Sandleris (2008)), and legal and administrative costs.

¹²See Sandleris (2008) for an alternative interpretation of this result related to the information revealed by the default.

(7) results from the assumption that the direct costs of defaults suffered by the borrowing country are not appropriable by creditors.

Consistently with the empirical evidence, we assume that countries are *temporarily* excluded from credit markets after default. The exclusion from international credit markets is captured by setting $d_{t+1} = 0$ and $a_{t+1} = 0$ for an exogenously given period of time. This means that, when the government defaults, it cannot borrow, purchase or hold assets in all periods of credit market exclusion. In this paper we take a reduced form approach assuming that, once excluded from credit markets, the probability that the country re-enters international financial markets in any subsequent period is equal to θ . Notice that the exclusion from credit markets is particularly costly for the government as we assume that, once excluded, it cannot save.¹³ This means that the government is forced to "consume" the assets it already holds in the period in which it defaults, otherwise they will be seized.

Since international lenders are assumed to be risk neutral (or, alternatively, able to fully diversify country risk) they only consider the expected rate of return paid by the country's securities. Therefore, the country debt must guarantee an expected return equal to the world interest rate:¹⁴

$$E_{t-1} [R_t] = \frac{\sum_s |_{h_t(s)=1} \pi_t(s) \cdot x_t(s)}{q_{t-1}(s)} = R_t^w \quad (8)$$

where $\pi_t(s)$ labels the probability that contingency s realizes. Equation (8) captures the probability of repayment and the payoff to creditors. These probabilities will be endogenously determined by the equilibrium. Creditors extract no value from those states in which the country defaults.

¹³The role of this assumption, widely used in the sovereign debt literature, is to make the exclusion from future borrowing costly. As shown by Bulow and Rogoff (1989), the necessary condition for this to happen is that the government cannot replicate the payoffs of the contract from which it is excluded. Suboptimal savings as in Amador (2003) or a restricted set of assets available to the government following a default as in Kletzer and Wright (2000) and Wright (2002) would suffice for this to happen. Although the "no savings" assumption is clearly extreme, it is the simplest way to achieve this result.

¹⁴Equivalently, one may think is through the following competitive framework: international creditors observe the total amount of debt that the government wishes to issue and the interest rate it offers to pay. On the basis of this observation, creditors form expectations about the probability of repayment and default. Then, they compare the expected rate of return they receive on these bonds with the world interest rate. If the former is larger or equal than the latter, then the issuance takes place and it is randomly allocated among all applying creditors. Otherwise the issuance does not take place.

2.2 Optimization Problem and Equilibrium

The benevolent government optimizes as follows:

$$\begin{aligned}
 & \max_{c_t(s), h_t(s) = \{0,1\}, d_{t+1}(s), a_{t+1}(s); t=0, \dots, \infty} U = \sum_{t=0}^{+\infty} \beta^t E_s [u(c_t(s))] = \\
 & = \sum_{t=0}^{+\infty} \beta^t E_s \{u[y_t(s) + q_t(\vec{x}_{t+1} \cdot d_{t+1}(s), R_t^w a_{t+1})d_{t+1}(s) - a_{t+1}(s) - h_t(x_t(s)d_t) + R_t^w a_t - \lambda(h_t, y_t(s))]\}
 \end{aligned} \tag{9}$$

where the choice variables of the government - $c_t(s)$, $h_t(s)$, a_{t+1} and $d_{t+1}(s)$ - are indexed by s whenever we want to make explicit their state contingency. Our assumption on credit markets exclusion implies $a_{t+1} = d_{t+1} = 0$ with an exogenously given positive probability if the country has an unresolved default.

The utility of the representative agent in (9) implies that the government is not indifferent among all combinations of assets and liabilities that yield the same portfolio payoffs. The reason is that debt payments depend upon the level of gross debt while the default penalty depends on the level of income only. So, different levels of gross debt with the same level of net debt may entail different decisions regarding default. Therefore, as we will see in section 3.1, the design of the debt contract will be affected by the amount of assets in the portfolio. Since both assets and liabilities determine consumption through the repayment/default decision, the equilibrium market price of debt is a function $q_t(\vec{x}_{t+1} \cdot d_{t+1}(s), R_t^w a_{t+1})$ of the entire portfolio.

Our setup allows for a recursive formulation since state contingent default decisions are fully anticipated. The state variables of the recursive problem are four: the income level, $y_t(s)$, the gross amount of debt payments to be made from the country, $x_t(s)d_t$, the gross amount of payments to be received by the country, $R_t^w a_t$, and the existence of an unresolved default, $H_t = \{0, 1\}$, where $H_t = 0$ if the government has an unresolved default at time t , and $H_t = 1$ when it does not. We can write the value function for the government problem as:

$$V(H, x(s)d, a | s) = \max_{h, c, a', d'} \{u(c) + \beta \cdot E_s [V(H', x(s')d', a' | s')]\} \tag{10}$$

where we use the standard convention of omitting the time " t " label and denoting by a " $'$ " a variable referred to " $t + 1$ ". (10) can be specified in greater details. If there is no pre-existent unresolved

default, the value function if the government repays its debt is:

$$\begin{aligned} & V(H = 1, x(s)d, a | s)|_{h=1} = \\ & = \max_{a', d'} \{u(y + qd' + R^w a - x(s)d - a') + \beta \cdot E[V(1, x(s')d', a' | s')]\} \end{aligned} \quad (11)$$

while the value function if the government defaults is:

$$\begin{aligned} & V(H = 1, x(s)d, a | s)|_{h=0} = \\ & = \{u(y + R^w a - \lambda(0, y)) + \beta E[(1 - \theta)V(0, 0 | s) + \theta V(1, 0 | s')]\} \end{aligned} \quad (12)$$

where θ represents the (exogenous) probability of being able to re-enter capital markets following repudiation. Finally, the value function with a pre-existent unsettled default is given by:

$$V(H = 0, 0 | s) = u(y(s)) + \beta E[(1 - \theta)V(0, 0 | s) + \theta V(1, 0 | s')] \quad (13)$$

Note that the decision of a country to repay or default, is made period-by-period. Thus the expected value from next period onwards incorporates the fact that the government could choose to default in the future. The government default policy can be characterized by repayment and default sets. For a given level of assets and liabilities (d, a) :

$$\begin{aligned} I(y(s)) &= \{y : V(1, x(s)d, a | s)|_{h=1} \geq V(1, x(s)d, a | s)|_{h=0}\} \\ O(y(s)) &= \{y : V(1, x(s)d, a | s)|_{h=0} \geq V(1, x(s)d, a | s)|_{h=1}\} \end{aligned}$$

are the repayment and default sets respectively.

Having analyzed the agents' optimization problems we are ready to define the equilibrium of this economy:

Definition 1 *The recursive equilibrium of the economy is defined as: (i) a set of policy functions for $h(s)$ default decision, consumption allocations $c(s)$, government debt $d'(s)$, asset $a'(s)$, and repayment $I(y(s))$ and default $O(y(s))$ sets; and (ii) a bond price $q(\vec{x}'d', a')$ such that:*

1. *taking the bond price $q(\vec{x}'d', a')$ as given, default decision $h(s)$, consumption allocations $c(s)$, government international debt $d(s)$ and asset $a(s)$, repayment $I(y(s))$, and default $O(y(s))$ sets satisfy the government optimization problem;*
2. *$q(\vec{x}'d', a')$ is consistent with creditors' zero expected profit condition (8) and reflects the government default probabilities given debt $d'(s)$ issued and asset $a'(s)$ purchased by the government and its repayment $I(y(s))$, and repudiation $O(y(s))$ sets.*

2.3 Income, Debt and the Default Decision: The Case of Non-Indexed Debt

We are now ready to characterize the government repayment decision. The following lemmas explain how the repayment and default sets are affected by changes in the level of debt and assets. In this section we focus on a government issuing standard non-indexed zero coupon bonds, i.e. a debt contract with payoffs $x_t(s) = 1, \forall s, t$. For the moment we take the asset level a as given.

Lemma 2 *There exists d^* such that the government optimally defaults on its debt. Furthermore, default is also the optimal choice when $d \geq d^*$*

Proof. By (11) and (12), we may note that for a country that is in good credit standing, i.e. $H = 1$, utility when the country defaults $V(1, d, a|s)|_{h=0}$ is constant in d while $V(1, d, a|s)|_{h=1}$ is decreasing on d . Thus there always exists a level of d , d^* , at which the country is indifferent between defaulting and repaying its debt. Then, default must be optimal for any $d \geq d^*$. ■

Although the possibility of default introduces some degree of contingency in an otherwise non-contingent debt contract, it does so at the cost of a deadweight loss: the default penalty defined in (4). The presence of these penalties makes the government willing to repay its debt in some states of the world:

Lemma 3 *If for a given level of gross debt, d , the probability of default, $\sum_s \pi(s|h(s) = 0)$, is positive, then this probability increases with the level of debt.*

Proof. Let \hat{d} be such that for a given level of income \hat{y} the following condition holds:

$$V(1, \hat{d}, a|s)|_{h=0} = V(1, \hat{d}, a|s)|_{h=1}$$

We know that such \hat{d} exists from Lemma 2. Upon indifference the country was repaying at income level \hat{y} with debt payments \hat{d} . From Lemma 2 we know that for debt level $\hat{d} + \varepsilon$, $V(1, \hat{d} + \varepsilon, a|s)|_{h=0} > V(1, \hat{d} + \varepsilon, R^w a|s)|_{h=1}$, then the probability of default is increasing in the level of debt repayment. ■

We can summarize the results of the lemmas above in the following proposition that characterizes how the probabilities of default and repayment are affected by changes in the level of

debt.

Proposition 4 *If, for a given level of gross asset a , the probability of default, $\sum_s \pi(s|h(s) = 0)$, and of repayment, $\sum_s \pi(s|h(s) = 1)$, are both positive, then the probability of default is increasing in the level of debt, while the probability of repayment decreases in it.*

As in Eaton and Gersowitz (1981) the probability of default is increasing in the level of debt, and so is the equilibrium interest rate (the inverse of the market price $q(\vec{x}, d, a)$) under non-indexed debt. This means that there will be a threshold level of debt payments above which the default set will include the entire support of the income distribution. Clearly creditors will never choose to lend such amount, as they would receive no payments in all possible contingencies. The level of government borrowing is therefore restricted in equilibrium by its inability to commit to repay.

We now analyze how the repayment and default sets are affected by changes in the level of income for given asset and debt levels, a and d . We can state the following Lemma:

Lemma 5 *Given the level of asset a and debt d , if the government chooses to default at y^* it will do so when $y < y^*$, i.e. governments default when hit by bad income shocks.*

Proof. We want to compare what happens to the value of the two options (repayment and default) as income decreases. First, observe that the value function *conditional on default* (12) is differentiable with respect to the state variable y because the utility function is continuously differentiable in consumption:

$$\frac{\partial V(1, d, a|s)|_{h=0}}{\partial y} = u'(c_{h=0})[1 - \frac{\partial \lambda(0, y)}{\partial y}] > 0 \quad (14)$$

which means that if you decrease the income of a government that is entering default, the economy suffers a loss that is proportional to the level of marginal utility and compensated by the fall in the default penalty. Notice moreover that an income decrease has no effect on the continuation value in this case as an economy entering default today, will reenter the credit market at some random period in the future with no assets and no debt, irrespective of its income level today. Second, we look at the value function *conditional on repayment* (11) and find that lowering income we have two effects. The first effect is the fall in current consumption proportional to the current marginal

utility

$$u'(c_{h=1}) > 0 \tag{15}$$

while the second effect is the decrease of the continuation value under repayment because, as income decreases, the risk averse government decreases the burden on current consumption due to the fall in income by lowering asset purchase, a' , and issuing more debt, b' .

Focus now on \hat{y} , defined as the level of income at which the government is indifferent between repaying and defaulting (in the absence of this level, the proof is trivial). For any given level of current asset a and debt d , a fall in income below \hat{y} decreases the value function in case of default (12) by only (14) while the value function under repayment (11) falls by *at least* (15). But since $c_{h=1} < c_{h=0}$ at \hat{y} , then the fall in the value function conditional on default is less than the fall in the value function conditional on repayment. This implies that default is preferred to repayment. $y < \hat{y}$. ■

The intuition for this lemma is relatively simple. First, we observe that all value functions are increasing in the level of income. Then, if we find that at income y^* default is optimal, as income goes below y^* the value function of default declines less than that of repayment. The reason is that as income decreases so does the direct cost of default, and so the government continues to choose the option of default.

3 The Optimal Indexed Debt Contract and its Implementation

3.1 On the Optimality of Indexed Debt Contracts: Main Features

In this section we characterize the optimal indexed debt contract and we compare consumption allocations in the economy under two different debt instruments - non-indexed (NID) and indexed (ID) debt . NID is the type of contract that we analyzed before, one in which promised payments are constant across contingencies, $x(s) = 1, \forall s$,¹⁵ while ID allows for state contingent promised payments on issued debt, i.e. $x(s) \neq x(s') > 0$ for at least two states s, s' . NID entails some degree of state contingent flexibility through the possibility of default: this residual contingency

¹⁵Making payments of NID equal to 1 is without loss of generality.

also builds an incentive for asset accumulation.¹⁶ The contingency of NID is enhanced by the fact that, whenever the country defaults, the cost of doing so is also state contingent. However, the degree of contingency allowed by defaults comes at the cost of the direct penalties suffered by the borrowing country, denoted by the $\lambda(\cdot)$ functional in our analysis.

ID has the potential to improve welfare. It does so through two channels: first, it increases the amount of risk-sharing between a risk-averse borrowing country and risk-neutral international lenders; second, it reduces the probability of default and so the magnitude of the direct costs that default entails for debtor countries. The crucial assumption for this second channel to operate is that the direct penalty costs - reflecting what seems realistic - constitute deadweight losses that are not appropriable by creditors¹⁷. The following Lemma formalizes the intuition that ID contracts improve welfare:

Lemma 6 *For any given level of asset a and debt d for which there exists an equilibrium non-indexed debt contract that entails default in at least one state, we can construct the indexed debt contract $x(s) > 0, \forall s$ satisfying creditors' zero profit condition, (8), and such that:*

1. *it is preferred by the government to the equilibrium non-indexed debt contract, $x(s) = 1 \forall s$;*
2. *it does not involve default in any state of the world;*
3. *if there is perfect competition among creditors, it allows debtor countries to appropriate all surplus generated by the indexed contract.*

It is instructive to go over the steps necessary to construct the optimal indexed debt contract, $x(s)$, and so we do not relegate the argument in a proof. In the following discussion, we take as given the level of asset a and debt d : since our reasoning holds for any level of the portfolio at which there is positive borrowing, doing so does not affect the generality of the argument.

Pick a state \hat{s} in which the government chose to default under the NID contract. The default has two effects that deserve our attention: the default penalty and the exclusion from credit markets. These facts can be formally studied by comparing the value functions for repayment (11) and default (12). It is once again simple to observe that the value function for repayment is a decreasing function

¹⁶In the absence of default, it would be meaningless to distinguish between asset and liabilities as one is only the negative of the other.

¹⁷See equations (6), (5) and (7) for reference.

of debt payments $x(\hat{s})d$:

$$V(1, x(\hat{s})d, a | \hat{s})|_{h=1}$$

while the value function in case of default is independent from the level of the payment to be made and depends instead on the duration of exclusion from credit markets, the size of the penalty, $\lambda(0, y(\hat{s}))$, and the level of assets held by the country, formally:

$$V(1, a | \hat{s})|_{h=0}$$

It is therefore possible to design an ID contract by picking $x(\hat{s}) > 0$ so that:

$$V(1, x(\hat{s})d, a | \hat{s})|_{h=1} = V(1, a | \hat{s})|_{h=0} \quad (16)$$

ID designed according to equation (16) makes the borrower indifferent between defaulting on its NID and repaying the ID obligations, while it is strictly preferred by international lenders because they would receive $x(\hat{s}) > 0$ instead of facing a default. However, in equilibrium, competition among creditors - equation (8) - implies that the country will be able to borrow at better terms, i.e. either at a higher price $q(\cdot)$ or decreasing its payments in some states when it was not defaulting. In any case the country welfare will be increased. It is worth observing, by (16), that the design of ID through the level of state contingent payments $x(\hat{s})$ depends on the stock of assets held by the country, a . This is due to the fact that the ability to smooth consumption depends on the country entire portfolio, not just on its liabilities. Therefore the financial design of borrowing instruments directed to facilitate consumption smoothing can not abstract from the assets held by the country.

Following the reasoning of the previous Lemma we are able to further characterize the ID contract:

Lemma 7 *For any given level of debt d and assets a , an optimal ID contract has the following features:*

1. $x(s) \geq 1$, in all states s in which the NID contract, $x(s) = 1 \forall s$, entailed no default
2. $x(s) \geq x(s')$ if $s > s'$
3. $x(\hat{s}) < 1$ and such that

$$V(1, x(\hat{s})d, a | \hat{s})|_{h=1} = V(1, a | \hat{s})|_{h=0}$$

in all states \hat{s} in which the NID contract, $x(s) = 1 \forall s$, entailed default

Proof. 1. It is easier to construct the ID contract starting from the NID contract exploiting the fact that the NID entails repayment in some contingencies and default in others. The contract designer can then exploit the fact that in all contingencies when the government is not defaulting on the NID, $s \neq \hat{s}$, we have the following inequality between (11) and (12):

$$V(1, d, a|s)|_{h=1} > V(1, a|s)|_{h=0}$$

that can be exploited to increase debt payments, in those contingencies so that $x(s) > 1$.

2. By Lemma 5, we know that default at one income level implies default at all income levels below it. Thus we have that debt payments $x(s)$ must be (weakly) increasing in income, i.e. $x(s) \geq x(s')$ if $s > s'$.

3. In all contingencies like \hat{s} , when the government is defaulting under NID, it is the case that:

$$V(1, d, a|\hat{s})|_{h=1} < V(1, a|\hat{s})|_{h=0}$$

and so debt payments under ID must be reduced with respect to NID to prevent default. Therefore, $x(\hat{s}) < 1$. ■

The previous Lemma delivers the main feature of the optimal ID contract. It is worthwhile pointing out that the ID contract so far characterized does not ensure that the full insurance consumption allocation will be attained. All it guarantees is that, by shifting payments away from contingencies with high marginal utility of consumption to contingencies where it is low, default episodes are avoided. Full insurance consumption allocation is not in general attained because the design of the ID contract is constrained by the presence of limited commitment to repay, formalized by the penalty function $\lambda(\cdot)$.

3.2 Implementing the Optimal Indexed Debt Contract: A Simple Example

In this section we provide a simple illustration of how ID is employed by the borrowing country, in conjunction with available assets, to smooth consumption. Since the government is assumed to be more risk averse than its creditors (risk neutral in our analysis), the optimal ID contract will be the one that facilitates consumption smoothing by providing the highest level of country insurance,

compatible with sustainable levels of debt. Such debt contract will be designed in such a way to generate

$$b(s) = x(s)d - R^w a < 0$$

in some state s , i.e. it requires the borrowing country to become a net recipient of funds from international credit markets in some contingencies.

The first way in which consumption smoothing could be implemented is through a standard insurance contract. This is an arrangement that asks for payments *from* the country if income is above a certain threshold, and it provides payments *to* the country if income is below the threshold. In our framework, an insurance contract would be casted by setting $x(s)$ to take both positive or negative values ($x(s) > 0$ would imply a payment from the government to international creditors, $x(s) < 0$ would imply a payment from international creditors to the government). Although this contract is simple in theory, creditors are typically reluctant to provide this kind of explicit insurance to sovereign borrowers.

An alternative way in which the optimal consumption smoothing allocation can be implemented is the one we propose and discuss in this paper: the government should build a suitable combination of non-indexed assets, e.g. international reserves, and GDP-indexed debt. This is a valid alternative to an explicit insurance contract. What matters is that the chosen portfolio of assets and liabilities generate positive and negative payments to the government in the appropriate relationship to the state of the world. This clearly is a much weaker requirement than imposing the debt contract to produce this set of payments by itself.

The following simplified example presents how the allocation obtainable through an explicit insurance contract can be implemented with a suitable combination of assets and liabilities.

Example 8 *Consider a world lasting two periods, $t = \{1, 2\}$. At $t = 1$ the country has income (GDP) equal to 20. At $t = 2$ two equally probable states of the world, Good and Bad, may happen. In the G state, GDP = 30 and in B GDP = 10. Assume that the country has no other prior source of income or wealth. A risk-averse country would like to insure itself against GDP volatility at $t = 2$ so that it always receives GDP = 20. Furthermore, assume that the world interest rate $R_w = \beta = 1$, so that the country would prefer to have constant consumption over time. If international creditors are risk neutral and financial markets are complete and competitive, in the absence of commitment problems the country could underwrite at $t = 1$ a “fair” insurance contract: the country would simply pay 10 in state G and receive 10 in state B. In such a way, it would achieve full insurance across*

states and constant consumption over time. However, as we mentioned, this kind of insurance contract is virtually absent in reality.

However, countries can shield themselves from income volatility building a portfolio of assets and contingent liabilities. Assume for simplicity that assets are non-contingent (i.e.: international reserves), but liabilities (the country debt) could be indexed to the state of the economy. The country can buy, at $t = 1$, $A > 0$ units of the asset available in international credit markets for unitary price 1. Each unit of the asset pays R_w in each state of the world at $t = 2$. The country can also issue, at $t = 1$, $D > 0$ units of debt for unitary price q . Each unit of the indexed debt pays $x(G)$ in state G and $x(B)$ in state B at $t = 2$, where $x(s)$ (for $s = G, B$) are chosen by the issuing country. In order to prevent the debt contract from becoming itself an insurance contract we assume that $x(s) > 0$ (for $s = G, B$).¹⁸ Equilibrium requires that: $R_w = E(x)/q$. Once the government chooses $x(G)$ and $x(B)$, this equation pins down the price of debt, q .

Note that in the absence of commitment problems, the country can not design the indexed-debt contract (i.e. choose $x(G)$ and $x(B)$) and determine the levels of debt, D , and asset holding, A , independently if it wants to achieve full insurance. First, if we want to compare our portfolio to the insurance contract mentioned above, we have to focus on the portfolios that do not decrease (increase) the country income at $t = 1$:

$$(1) \quad qD - A = 0$$

Among these portfolios, we focus on those that provide full insurance against GDP volatility at time 2:

$$(2) \quad A - x(G)D = -10$$

$$(3) \quad A - x(B)D = +10$$

The government has 4 choices: the level of debt, D , the level of assets A , and the structure of the contract, $x(G)$ and $x(B)$. The market price of the debt contract, q , is determined by the no arbitrage condition:

$$(4) \quad 1 = E(x)/q$$

Given that we have only equations (1), (2) and (3), the government can choose independently only two of its variables if it wants to achieve the full insurance allocation. The other variables will need to be set at the levels determined by the above system of equations. For instance: if the government chooses the structure of the debt contract (i.e. $x(B)$ and $x(G)$), then the amount of debt and assets, and the price of the contract will have to be determined according to the above equations. Equivalently, if the level of assets and debt are chosen by the government, the structure of the debt contract and the price of the contract will be endogenously determined by the full insurance allocation. Note that the insurance contract is a special case of the system of equations in which $x(B) = -10$, $x(G) = +10$, $D = 1$ and $A = 0$. However, the full insurance allocation can also be achieved even if we restrict $x(s) > 0$ or $x(s) > 1$ for $s = B, G$. For example, if $x(B) = 1$ and $x(G) = 2$, then $A = 30$ and $D = 20$.

¹⁸ Assuming $x(i) \geq 0$ implies that creditors may receive a negative return (though with a positive payment) in some states. We could assume $x(i) \geq 1$ so that creditors always get at least the nominal value of the contract and the results would remain unchanged.

As this example makes clear, since explicit insurance contracts are unavailable in reality at the current level of financial sophistication, the government can replicate the insurance contract allocation through an adequate choice of its portfolio of assets and liabilities. This choice must include a suitable combination of risk-free assets and real-indexed debt.

The reasoning of this example could be easily extended to economies with limited commitment, like the ones studied in the general model of this paper. The presence of limited commitment though would typically allow the government to achieve less than perfect consumption smoothing in equilibrium. This is in fact what we find in the quantitative exercise that we conduct in the last section of the paper. Before turning to the quantitative exercise, we briefly discuss the optimality of the *existing* ID contracts in the subsection that follows.

3.3 On Existing Real Indexation Rules for Sovereign Debt

Our model can also help to evaluate some of the proposals regarding alternative indexation rules for sovereign debt contracts. There are only two limitations to keep in mind. First, we have abstracted from moral hazard in our analysis of indexed debt. In this perspective the only source of uncertainty for borrowing countries is income shocks. It follows therefore that, in our framework, GDP may be employed as the optimal indexation variable when we discuss indexed debt.¹⁹ Second, it is important to emphasize that our analysis is performed under the assumption that creditors are homogenous and risk neutral. Alternative assumptions on this regard may affect the welfare implications that we obtain. For instance, in the unlikely scenario that creditors were more risk averse than the government, NID might become the best option for the government. Alternatively, in the presence of heterogenous creditors with different preferences over instruments types, a mix of ID and NID might characterize the optimal borrowing structure.

The few existing cases of real indexed sovereign debt contracts share common features. They usually include, as in the Bulgaria and Argentina GDP-indexed bonds, a threshold level of the chosen variable above which payments are triggered, and the magnitude of the payments increases on the chosen variable after that threshold.²⁰

¹⁹In our framework it is equivalent to index the contract to GDP and to the state of the economy.

²⁰See Appendix for a brief survey of real-indexed sovereign debt contracts.

As Lemma 7 shows, the optimal real indexation rule does not involve thresholds.²¹ In fact, if the space of the shock is continuous, it requires continuous increases in payments as the state of the economy improves.

An interesting issue to analyze is under what conditions the presence of these thresholds levels are optimal. In order to study this issue we focus on the case of the Argentina's GDP warrant. The threshold level of income that triggers payments of the variable component in Argentina's GDP warrant involves two conditions:

- i. real growth rate of GDP in the period higher than the trend growth rate of GDP (approximately 3.5%);
- ii. GDP of the period higher than trend GDP (calculated with a real annual growth rate of approximately 3.5% starting from the initial year)

The "constrained" optimality of each of these conditions and whether both are necessary depends on the stochastic process that Argentina's income follows and the set of available assets. First note that the presence of these thresholds would be irrelevant if the government has access to contingent assets that allow it to span the whole state space of the shock. Assuming that this is not the case, we will consider two alternative stochastic processes for the shock. Assume first that shocks to income are i.i.d. and normally distributed around a real growth rate of 3.5% (i.e. this assumption implies that shocks are permanent). Then the first condition is not optimal if the government can hold non-contingent assets. The intuition is simple. As shocks are i.i.d. around a 3.5% real growth rate, then, regardless of the previous outcome, the growth rate should be expected at 3.5%. Any growth rate below that would then entail net payments from creditors to the government under an optimal insurance contract, with payments inversely related to the growth rate. So, the optimal portfolio of debt and assets should replicate this. Assuming, as we do, that assets are non-contingents, then payments on the debt should not be equal to zero for all states below the average.

However, if the government is unable to hold assets, then the best that can be done is to make debt payments equal to 0 in those states in which a payment from creditors to the government was

²¹The presence of limited commitment may make full insurance unfeasible though.

optimal, which is precisely what the first condition does. Assume instead that the trend growth rate of income is 3.5% and that shocks are temporary - at the business cycle frequency for instance -, normally distributed around the trend. Under these assumptions the second condition would be "constrained" optimal if the government cannot hold assets, as long as it does not entail default. It would entail zero payments any time income is below its expected level. In conclusion, it is not clear that including both conditions simultaneously delivers the constrained optimal contract.

4 The Welfare Effects of Real Indexation: Quantitative Results

In this section we present a quantitative exercise to provide (1) a first estimation of the welfare effects of the introduction of indexed debt (ID) contracts and (2) an illustration of how the government optimally builds his portfolio to attain the maximum amount of consumption smoothing. Our exercise will compare the welfare of the economy under two type of debt arrangements: one in which the country issues standard zero coupon bonds so that the face value of its payments are constant across states, and another in which it issues indexed debt where coupon payments are indexed to the state of the economy and monotonically increasing in its endowment. Under both debt arrangements the government can also purchase an international asset that makes non contingent interest payments. Although we do not assume that the country issues the optimally designed debt contract, the indexed instrument used in the numerical exercise carries the most important qualitative features we derived in the paper about the optimal contract.

Argentina is the benchmark economy in the quantitative exercise, since it represents the major case of sovereign default in recent history and is also the country that issued GDP-indexed sovereign debt. The GDP statistics for Argentina correspond to the seasonally adjusted quarterly real GDP series for 1983.1-2000.2 obtained from the Ministry of Economy and Production (MECON) of Argentina. The series is filtered with the Hodrick-Prescott procedure. Output, debt and private consumption are expressed as a percentage of GDP.

4.1 Calibration

We choose the parameters based on existing empirical work about emerging markets. Otherwise parameters are set to mimic empirical regularities of the Argentinean economy. The per period utility function of households is assumed to have a standard CRRA specification:

$$U(c) = \frac{(c)^{1-\sigma}}{1-\sigma} \tag{17}$$

with a standard value for the risk aversion coefficient, i.e. $\sigma = 2$. The quarterly risk-free world interest rate is set at 1%. Consistent with the findings of Gelos, Sahay and Sandleris (2003) for the 90s, the reentry probability is set to 0.2, which corresponds approximately to one year of exclusion from international credit markets after a default event. The discount factor β is set at 0.95, larger than Alfaro and Kanczuk (2007) work with a value of 0.5, Aguiar and Gopinath (2006) use a value of 0.8. Yue (2006) considers a value of 0.74. A high degree of impatience has been linked to political factors in emerging economies (see, for instance, Cuadra and Sapriza (2006) and Hatchondo, Martinez and Sapriza (2006)).

The parameters characterizing the income process in the model are set to approximately match the observed volatility of 4.08% of the cyclical component of GDP in Argentina for the period under study. The output follows an AR(1) process with an autocorrelation coefficient of $\rho = 0.9$ and $\sigma_z = 3.4\%$. The mean of log output equals 0 so that average detrended output in levels is standardized to one. We solve the model numerically using the discrete state-space method. Each period represents a quarter. The process of output is approximated by a discrete first order Markov chain with 25 values using Hussey and Tauchen's (1991) procedure.

We assume as a benchmark that when the country defaults, it experiences a one period output loss, λ , of 8%²² with a structure following Arellano (2008). Arteta and Hale (2006), Sandleris (2008) and Dooley (2000) among others, provide a rationale for the loss of output when countries face debt crises. The complete model parameterization is shown in Table 2.

The indexation rule corresponds to an increasing function of output for income levels below the

²²This is approximately equivalent to an average output loss of 2% while in autarky, given that on average country remain in autarky for 4 quarters.

mean, with a range from 10% of the bond face value for lowest income to 140% for the highest value of income. For income realizations above the mean, the country pays more than the face value. The welfare comparison between the indexed and non-indexed bond economies are made in terms of certainty equivalent consumption, following the methodology of Lucas (2003): we compare the certainty equivalent consumption calculated under both debt arrangements and then obtain the difference between the two as a measure of the welfare gain of indexed debt.

4.2 Simulation Results

This section describes some key quantitative properties of the model and presents the welfare comparison results derived from the indexed vs. non indexed debt economies. A sensitivity analysis is conducted with respect to key parameter values like the harshness of the output penalty and time-discounting. As observed in the data, the incentives to default in the model are higher for highly indebted countries and default risk is countercyclical.

Figures 1 and 2 show the default regions for the calibrated economy under both debt arrangements, i.e., the combinations of income shocks and foreign debt levels for which default is the optimal decision. Given an income realization, if default is optimal for a certain level of foreign debt, it will also be optimal for all higher levels. Analogously, if repayment is optimal for a given debt level, it will be optimal for all lower levels. Thus, for each realization of the income process, default incentives are stronger for highly indebted governments. This result arises because the value of paying back and remaining in good credit standing is strictly decreasing in foreign debt, while the value of defaulting and going to autarky does not depend on the amount of foreign debt but only on income. Thus, if default is the optimal decision for some level of net debt b given s , then the value of honoring the debt contract is lower than the benefit of defaulting. A higher amount of foreign debt increases the cost of repaying without affecting the cost of default. Then, for each value of s , there exists a threshold $b^*(s)$ for which the value of not default is equal to the value of default. For all levels of net debt lower than $b^*(s)$ the government optimally honors its debt.

Comparing Figure 1 and Figure 2 we can also observe that the default regions under non-indexed and indexed debt take the expected shapes. First, default is more likely with non indexed debt, i.e. a lower amount of debt can be supported for a given level of income. Second, the default

choice tends to be less income dependent with a well designed indexed debt contract. The shape of the default region under indexed debt is not a perfect rectangle, as it would be with the optimal contract, because our indexed debt contract in the numerical exercise is only a proxy of the optimal one.

As expected, for a given income realization the discount bond price schedule (not displayed) is a decreasing function of foreign debt. For small levels of foreign debt, the government always pays back its debt, so it borrows from international markets at the world risk free interest rate. In this debt range, the bond price is simply the inverse of the gross risk free rate. However, as the foreign debt increases, at a certain level the bond price starts to decrease because the incentives to default are stronger for highly indebted governments. At a sufficiently large debt level the government always defaults regardless of the output realization, so the probability of default is one and consequently the bond price is zero. The bond price schedule also illustrates that for all levels of debt, the bond price is lower (the interest rate is higher) when the economy is hit by an adverse output realization. This result derives from the presence of incomplete asset markets in the model. This market structure makes defaulting on foreign debt more attractive in bad times when output is low since the repayment of non contingent loans is more costly in recessions. Since productivity shocks are persistent, lending resources to the government in times of low output involves a higher default risk. Thus, risk neutral lenders are willing to lend resources to the government by charging a higher interest rate.

Figures 3 and 4 depict the corresponding portfolio choices (asset purchased and debt issued by the country produced by simulating the benchmark economy (defined in Table 1) for 5000 periods under both debt arrangements. Figure 5 and 6 display optimal portfolio choices for an economy where the β patience rate is increased to 99%. The qualitative message of these figures is that in our simulated economy a country will indeed choose optimally to issue debt *and* purchase international assets. It is interesting to notice that, although small in the benchmark case, there is a slight increase in the amount of international assets a country wishes to accumulate when we have a well designed indexed debt contract. This is not in contradiction with Alfaro and Kanczuk (2006) who find that it is optimal for countries not to hold international reserves (assets). The reason for our different result is quite interesting and not immediately obvious: assets matter because, in

conjunction with the designed indexed debt, they provide the optimal stream of payment *to* and *from* the country that stabilizes consumption as much as possible across contingencies. When the country issues non indexed debt, assets matter only in the contingencies when it defaults. As default events are relatively rare in our economy, even with non indexed debt, the rationale to purchase assets is lower with non indexed than with indexed debt. That is precisely what we find.

As shown in Table 2, the welfare gain from indexation for the benchmark economy range from 0.1% to 0.5% in terms of certainty equivalent consumption. Our simulated economy would be able to achieve a constant level of consumption that is 0.1-0.5% higher (in perpetuity) with indexed debt. The main effect behind this relatively big improvements in welfare is the fact that with well designed indexed debt contracts, the frequency of default falls substantially, and this generates a large increase in the amount of debt that can be supported in equilibrium. As a result, a relatively impatient government is able to front load consumption. Consistently, with this argument the welfare gain is increased for more impatient economies. The intuition is that the gain from avoiding defaults through an indexed debt contract becomes larger because more impatient economies default more often, especially with non indexed debt.

Note that the welfare gains we present are the result of comparing an economy with no indexing ability whatsoever, and one with well designed indexed debt. In reality, even with standard zero coupon bonds countries do have some degree of state contingency even without defaulting as debt is issued in different currencies, or at different maturities (see Buera and Nicolini (2004)). In addition, other contracts can be used for insurance such as commodity price derivatives. This means that the welfare gains that we found constitute probably an upper bound for the real welfare gains that indexed debt could generate.

5 Conclusions

Proposals on real indexation of sovereign debt contracts have been around for some time. Furthermore, a number of countries have issued these type of instruments. However, research on the characteristics of the optimal real indexed debt contract in a dynamic equilibrium framework is scant, which makes it extremely difficult to formulate policy recommendations regarding the design

of these contracts. Furthermore, little has been done in terms of quantifying the welfare effects that they may generate. This paper fills such void. We (qualitatively) characterize the optimal real indexed debt contract in a dynamic equilibrium framework showing that the optimal indexed debt contract should generate payoffs, for the country portfolio as a whole, similar to those of an insurance contract with payments increasing in the state of the economy.

We show how these payoffs can be implemented with a suitable combination of non-contingent assets, such as international reserves, and indexed debt contracts, such as GDP coupons. We evaluate the design of existing indexed debt contracts with the optimal indexed debt contract that we characterize in our model. Existing indexed debt contracts show payments increasing in the state of the economy as the optimal debt contract does, but they also include minimum threshold levels of the chosen real variable that trigger the payments. We argue that this last feature is in general suboptimal as it reduces the amount of insurance that can be achieved.

We also quantify the welfare effects of real indexed debt contracts by calibrating our model to the Argentinean economy. We compare a scenario in which the country can only issue non-indexed debt with one in which only indexed debt can be issued (in both cases assuming that the country can also purchase non-contingent assets) and we show that welfare gains with indexed debt are equivalent to an increase of between 0.1% and 0.5% in aggregate average consumption. The magnitude of the welfare gains is positively correlated with the direct cost caused by a default and negatively related to the time discount factor.

Our analysis has relevant policy implications for the design of real indexed debt contracts. First, and differently from the existing literature on sovereign borrowing, it makes clear that debt design and default choices should be analyzed under a portfolio approach, i.e. taking into account that countries not only issue debt but at the same time purchase assets in international markets. Furthermore, our study provides the first model-based quantitative assessment of their welfare effects. However, the latter is done under a certain set of restrictions such as not allowing for indexed and non-indexed debt to coexist or limiting the set of assets available to the country to build its portfolio to a single non-contingent international asset. Finally, we disregard the issue of moral hazard that has received some attention in the literature on indexed debt. Relaxing these assumptions is part of our ongoing research project on the topic of real indexed sovereign debt, as

it could help improve our quantitative assessment and may bring new insights on the design of real indexed debt contracts.

6 References

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7 Appendix

7.1 Algorithm

1. Assume an initial function for the price of the bond $q_0(a', xb', s)$. Use the inverse of the risk free rate.

2. Use q_0 and the initial values of the value functions (eg. start with 0 matrices) to iterate on the Bellman equations and solve for the value functions and the policy functions for assets and default.

3. Given the default function and the default sets, update the price of the bond using the following equation:

$$q_{t-1}(s) = \frac{\sum_s |_{h_t(s)=1} \pi_t(s) \cdot x_t(s) + \sum_s |_{h_t(s)=\varphi_t(s)} \pi_t(s) \cdot \varphi_t(s) x_t(s)}{R^w} \quad (18)$$

4. Use the updated price of the bond to repeat steps 2 and 3 until the following condition is satisfied:

$$\max \{q_i(a, xb', s) - q_{i-1}(a, xb', s)\} < \epsilon \quad (19)$$

where i represents the iteration and ϵ is a small number.

7.2 Tables and Figures

Table 1. Benchmark Parameter Values		
Discount Factor	β	0.95
Risk Aversion	σ	2
Re-entry Probability	θ	0.20
Output Loss	λ	8%
Risk Free Interest Rate	r_f	0.01
Autocorrelation coefficient of innovations	ρ	0.90
Standard deviation of innovations	σ_z	0.034

Table 2. Welfare Comparizon: GDP-Indexed Vs Standard Debt

(% Gain in Terms of Certainty Equivalent Consumption)		
	Discount Factor	
Default (Output) Penalty	0.8	0.95
<i>a la</i> Arellano (2004)		
8 %	0.51%	0.11%
4%		0.13%
Average credit markets exclusion: 4 quarters		

7.3 A Brief Survey of Real-Indexed Sovereign Debt contracts

Proposals to index sovereign debt payments to real variables have been around for nearly 25 years when the debt crisis of the 1980s triggered interest in the issue. Around that time, Bailey (1983) suggested that debt should be converted into claims proportional to exports, and Lessard and Williamson (1985) made the case for real indexation of debt claims. A few years later, Shiller (1993) discussed the importance of creating “macro markets” for perpetuities linked to GDP.

The recent string of sovereign debt crises in Russia, Ecuador, Pakistan, Ukraine and Argentina generated a second wave of interest in contingent sovereign debt contracts for emerging countries. Haldane (1999), Daniel (2001) and Caballero (2003) suggested that countries would benefit from issuing debt indexed to some relevant commodity price. Drèze (2000) suggested the use of GDP-indexed bonds as part of a strategy to restructure the debt of the poorest countries and Borensztein and Mauro (2002, 2004) tried to revive the case for GDP-indexed bonds for emerging countries.²³

The basic idea behind all these proposals is to use indexed sovereign debt as a way to improve risk sharing between debtor countries and international creditors and, in doing so, to reduce the probability of occurrence of debt crises. One important difference across proposals is whether they suggest indexing the debt instruments to variables partially under the control of the government or beyond it. While indexation to broader measures such as GDP or exports that are partially under

²³See Borensztein and Mauro (2002) and Borensztein et al. (2004) for a detailed analysis of indexing proposals for sovereign debt.

the control of the government would likely provide greater insurance benefits, potential investors might be concerned about the authorities' incentives to tamper with data or undertake less growth-oriented policies. These concerns regarding the potential risks of moral hazard were first discussed in this context by Krugman (1988) and Foot, Scharfstein and Stein (1989).

It is not clear how relevant are these moral hazard issues in reality. However, if they were, the option of indexing debt contracts to commodity prices outside the control of the government would only be useful to a small group of emerging countries as GDP growth is poorly correlated with these variables for most of them.

Although more than 20 developing and developed countries have issued inflation indexed debt, the experience with bonds indexed to real variables is much more limited. Table 1 summarizes the experiences with each of them.

Table 1: Countries Issuing Real Indexed Bonds

Type	Country
GDP-Indexed Debt	Costa Rica (1990), Bosnia and Herzegovina (1990s), Bulgaria (1994), Argentina (2005)
Commodity Indexed Debt	US (1864), France (1970s), Mexico (1990s), Nigeria (1990s) Venezuela (1990s)

Source: Borensztein et al. (2004)

7.3.1 Argentina GDP-warrant

The most recent experience with sovereign bonds indexed to real variables is Argentina's GDP-warrant. In March 2005, Argentina finished its debt restructuring process that followed the default and financial crisis of 2001. Each new bond issued in the restructuring included a unit of GDP-linked warrants. These warrants were tied to the bonds for the first 180 days, and became detachable thereafter.

Given the magnitude of the restructuring, Argentina's GDP-warrants are the first sovereign debt instruments indexed to real variables for which there is a sizable market. The GDP-linked securities have a notional amount equal to the corresponding defaulted debt tendered and accepted in the 2005 restructuring, converted to the corresponding currency using the exchange rate as of

December 31, 2003 (roughly US\$62 bn, of which AR\$88 bn, US\$14.5 bn external and US\$2.9bn internal, EUR11.9 bn).

Payments on the GDP-linked securities take place only if the following three conditions are met:²⁴

(i) Actual real GDP exceeds the base case GDP for each reference year

(ii) Annual growth in actual real GDP is higher than the growth rate in the

base case GDP for the reference year (base case GDP real annual growth rate is 3.5% per year initially and gradually converging to 3%)

(iii) Total payments made on the security do not exceed the payment cap

of 48% of the notional amount during the life of the security.²⁵

Whenever these three conditions are met the formula used to calculate the payments for each notional unit of the warrants is the following:

$$Payment = 0.05 \cdot ExcessGDP \cdot UnitofCurrencyCoefficient$$

where Excess GDP is the amount by which actual real GDP converted into nominal GDP using the GDP deflator exceeds the base case nominal GDP, and the Unit of currency coefficient is defined as: USD: $1/81.8 = 0.012225$, Euro: $1/81.8 \cdot (1/0.7945) = 0.015387$, ARS: $1/81.8 \cdot (1/2.91750) = 0.004190$.

As GDP data is usually published with a lag of a couple of months and it is usually revised in the following months, payments are calculated on the November following the relevant reference year, and made effective a month later. This creates a lag of a year between the economic performance that might trigger a payment and the payment itself. Thus, potentially, troublesome situations may arise for the government. For example, assume that after a year of very high growth that meets all the conditions for the payment, an adverse shock drives the economy into a recession. The lag in

²⁴In all cases calculations are based on the data published by Argentina's Bureau of Statistics (INDEC).

²⁵The GDP-warrants expire when the 48 cents per dollar cap is reached, and no later than December 15th, 2035.

the payments of the GDP warrant implies that the government will have to make a large payment precisely during the recession.

Trading of GDP-warrants began in a “when and if” market before they were detached. In May 2005 the “consensus” value of the GDP-warrant with dollar coupon among investors was 2 cents per dollar. In July 21st, the first available date with data from the “when and if” market, the bid price for the GDP warrants was 3 cents per dollar coupon (50% increase in two months). Furthermore, by the end of 2005 the Argentina’s outstanding growth rates and a better understanding of the instrument led the markets to reevaluate their assessment of the value of the GDP-warrant. The price almost tripled when compared with the consensus value upon issuance. By the end of 2006 its price reached 13 cents per dollar coupon, six times higher than the “consensus” value at the time of the exchange and four times higher than the first available trading price. This remarkable increase in price has caught the attention of investors, authorities and the public in Argentina.

The first payment of the GDP-warrant took place in December 2006 and amounted to US\$387 mn. In fact, given current consensus forecast for GDP growth in Argentina in the next couple of years, payments on the indexed component are expected to triple in the next two years. In 2008 payments of the indexed component are expected to be roughly equivalent to total coupon payments (plus capitalization) on the three new bonds issued in the restructuring considered together.

Summarizing, despite little interest shown initially by investors, the market for Argentina’s GDP-warrant took off fueled by the excellent performance of Argentina’s economy in recent years. This is good news for GDP-indexed bonds as it would be the first successful case of such an instrument. However, it is inevitable to wonder whether it was a good idea to include them in the exchange from the point of view of Argentina, given that they did not seem to have any significant impact in the level of acceptance of the proposal.

7.3.2 Brady bonds with Value Recovery Rights (VRRs)

Some years before Argentina’s experience with GDP-warrants, a handful of emerging market economies issued bonds with elements of real indexation in the past. Various Brady bonds issued by Bosnia and Herzegovina, Bulgaria, Costa Rica, Mexico, Nigeria, and Venezuela in exchange

for defaulted loans in the early 1990s included Value Recovery Rights (VRRs). The VRRs were designed to provide the banks with a partial recovery of value lost, as a result of the debt and debt-service reduction contemplated in the Brady exchange, in the event of a significant increase in the debtor country's capacity to service its external debt. Mexico's VRRs, for example, provided for the possibility of quarterly payments, beginning in 1996, based upon certain increases in the price of oil.

Brady bonds issued by Bulgaria, Bosnia & Herzegovina and Costa Rica contained elements of indexation to GDP. In the case of Bulgaria for example, its Discount Brady bonds had a component named Additional Interest Payments (AIP) that was indexed to GDP. The AIP was triggered when two conditions were met: (1) Bulgaria's GDP surpasses 125% of its 1993 level, and (2) there is a year-over-year increase in GDP. For these years (not including the year in which the threshold is reached) the semi-annual interest supplement was defined as half of that year's GDP growth. The outlays themselves were scheduled to occur "as soon as practically possible" and were to coincide with regular interest payment dates. The AIP were not warrants, detachable or otherwise, though they were intrinsically equivalent.

Bulgaria's GDP-linked bond was generally viewed as a failure as a result of two factors. First, the bonds were callable at par. This meant that the government could decide to repurchase the bonds rather than pay out when faced with onerous GDP-linked payments, and as a result investors would miss out on the lucrative upside. In fact this is exactly what happened. A second problem with Bulgaria's bonds was that the conditions were fairly vague. In effect, "GDP" itself was not well defined, so it was open to interpretation the exact measure of GDP to be used. The government made use of this ambiguity for a while choosing definitions of GDP that prevented the AIP from being triggered.

Bosnia and Herzegovina's GDP-linked Brady bonds included additional interest payments whenever GDP growth rates exceeded for two years a predetermined growth rate and GDP per-capita rise above \$2,800 (1997 USD, adjusted by German CPI). These bonds have also suffered with problems in the definition of GDP and their trading activity has been very limited according to Bear Stearns.

In general, the experience with VRRs has not been positive. Indexation formulas were complex and often ambiguous. There were restrictions on their tradeability and many times were not detachable, and some of the bonds were callable.

7.3.3 Commodity-Linked Bonds

The main advantages of bonds indexed to commodity prices are that the data is available without a time lag and is not subject to manipulation by governments. Compared with GDP-linked bonds, however, their main disadvantage is that for most countries the correlation between economic performance and commodity prices is relatively low.

Bonds whose repayments are indexed to commodity prices have been used, although rarely, since the 1700s. In 1782, the State of Virginia issued bonds linked to the price of land and slaves. In 1863 the Confederate States of America issued “cotton bonds”, whose payments increased with the price of cotton. “Gold clauses,” effectively indexing payments to the price of gold, were widespread in the United States in

the 19th century through 1933. France also experimented with gold-price-indexed bonds, the “Giscard,” in 1973, but the losses caused by the depreciation of the French Franc caused the government to cease offering this instrument.

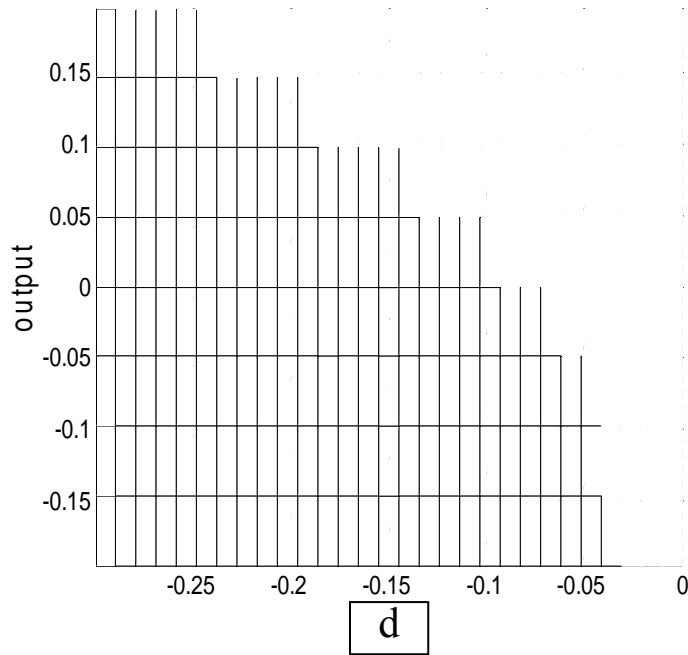
Oil-backed bonds appeared in the financial markets during the late 1970s. Mexico is considered the first country to offer oil-linked bonds in April 1977. The “Petrobonos” were issued domestically on behalf of the government by NAFINSA, a development bank owned by the Mexican government. They had a relatively active domestic secondary market developed in which most investors were Mexican. The bond promised to pay an annual rate of 12.65823% and had a three years maturity. Upon maturity, the Petrobonos were redeemed at a value equal to the maximum of the face value or the market value of the referenced units of oil plus all coupons received during the life of the bond.

Other countries and private companies have also experimented with commodity-linked bonds. For instance, Venezuela issued oil linked-bonds as part of its Brady agreement. India issued oil linked bonds to oil companies in April 1998 in payment for debts it had incurred by receiving oil

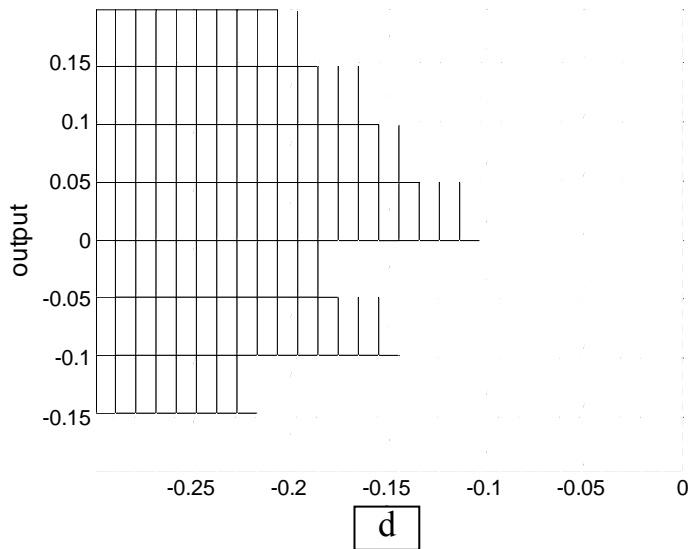
products below market cost. Malaysia accepted a loan from Citibank indexed to palm oil. More recently, loans combined with protection (through swaps) from commodity price fluctuations have also been made available by the World Bank to member countries, beginning in September 1999, though interest has thus far been limited.

Default Region

Benchmark Economy ($\beta = 95\%$ $\theta = 20\%$ Default Penalty a la Arellano (2004) = 8%))



Non indexed Debt - Figure 1

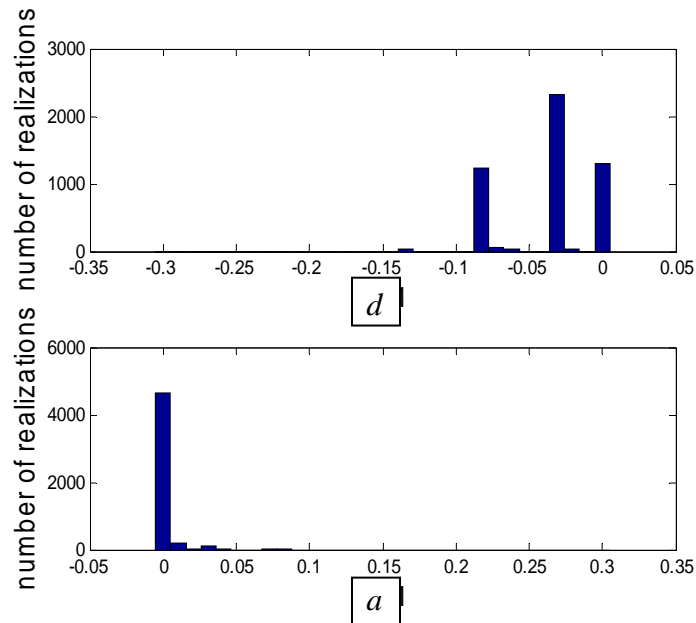


[Figure: Indexed Debt - Figure 2]

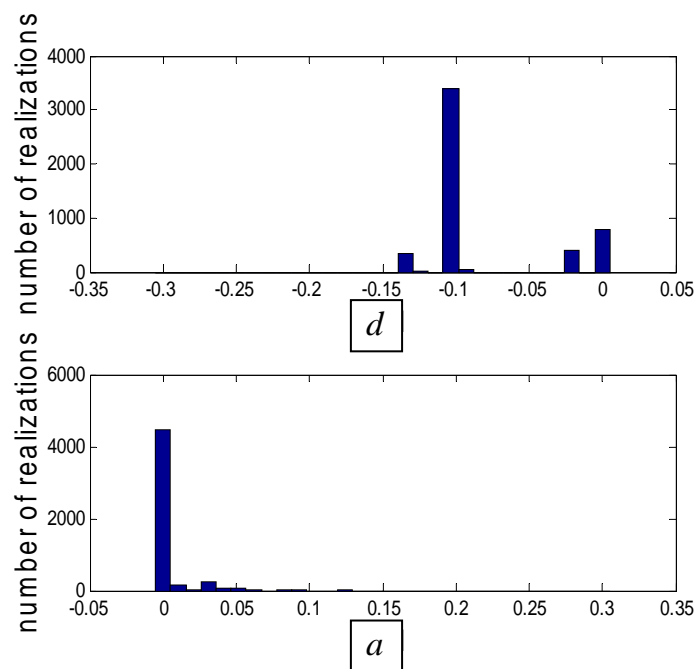
Optimal Country Portfolios

Number of Periods (over 5000 simulated periods) in which the Country Undertakes the Displayed Asset/Debt Position

- Benchmark Economy ($\beta = 95\%$ $\theta = 20\%$ Default Penalty a la Arellano (2004) = 8%)

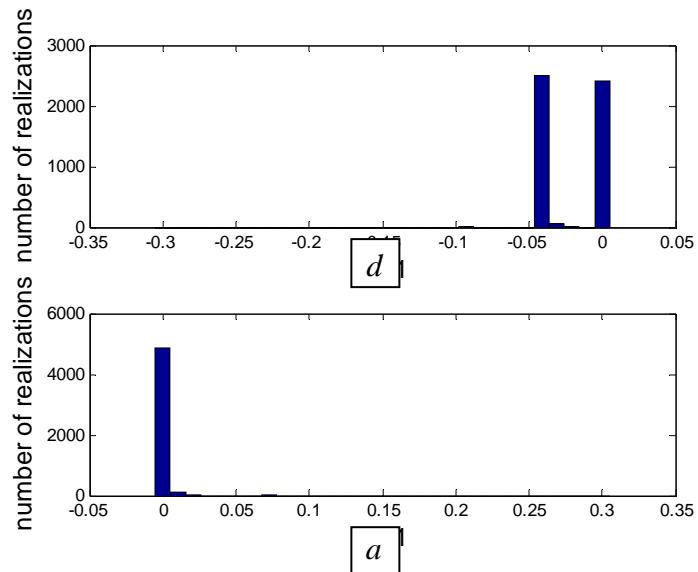


Non Indexed Debt - Figure 3

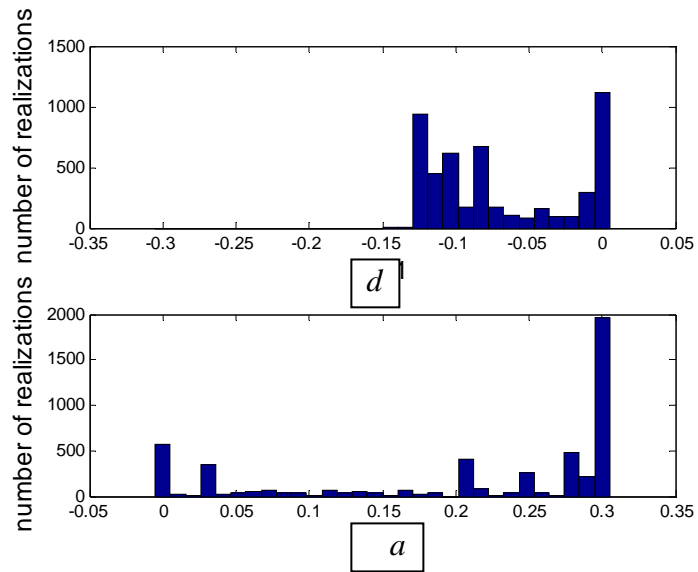


Indexed Debt - Figure 4

- Benchmark Economy ($\beta = 99\%$ $\theta = 20\%$ Default Penalty a la Arellano (2004) = 8%)



Non Indexed Debt - Figure 5



Indexed Debt - Figure 6