Mathematical Modeling and Sensitivity Analysis of the Existence of Male Calico Cats Population Based on Cross Breeding of All Coat Colour Types

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Abstract

The coat color of cats is normally governed by genes found on the X chromosome in both male chromosome XY and female chromosome XX. The meiosis failure in the process of gametogenesis leads to the birth of three-colored male cats caused by an excess of the X chromosome in the male chromosome type XY. The chromosome structure of three-color male cats, called male calico cats, appeared similar to the XXY Klinefelter's syndrome in human. Mathematical modeling and investigation of the factors that influence the infrequency of male calico cats are our main objectives of this paper. In addition, we also discuss the possible contributions and strategies to overcome the scarcity of these cats. We construct a mathematical model based on a combination of genes in the chromosome that regulates the color of cat coat on the fertilization process. The mathematical model is given as a six-dimensional system of differential equations. Sensitivity analysis is used to investigate the important parameters in the existence of male calico cats. Our finding states that the probability of normal male cats meiosis is a crucial parameter in the maintenance of the existence of male calico cats. Furthermore, one of the strategies that we could recommend in maintaining the existence of male calico cats is minimizing the value of the successful meiosis probability of normal male cats.

Keywords: Male calico cats, cross breeding, sensitivity analysis, population genetics.
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1. Introduction

The African wildcats *(Felis silvestris lybica)* have become the domestic cat *(Felis silvestris catus)* that more social towards humans or other species through domestication [1], [2]. A study of social and spatial behavior in domestic cats suggests that maintaining large numbers of cats in a small area can increase the level of conflict between cats and social stress [3]. In Sweden, the number of cats that can be kept is regulated by legislation up to 15 adults or 20 young cats in one group [4].

In general, calico cats are domestic cats that have three colors, namely white, orange and black [5], [6]. This cat is not included in the rare cats category, but the male calico cats are very rare. The scarcity of male calico cats is a trigger for many myths to emerge. For example, the cat is believed to bring a lucky. The predation of the parent that causes this cat is rare is also a myth that cannot be verified. The scarcity of male calico cats is a natural phenomenon that can attract the attention of scientists to conduct further research.

For example in [7], genetic testing in domestic cats has been carried out to identify species, maintain cat health and breeding management. Research on the mechanism of the formation of Klinefelter syndrome in mammals has been discussed in [8], [9]. Mammals that have Klinefelter syndrome have different characteristics in their physical such as increased body fat mass and reduced bone mineral content [8]. This causes the need for special treatment in maintaining the health of domestic cats that suffer from Klinefelter syndrome. Klinefelter syndrome did not only occur in cats, but also in humans and other mammals such as mice or pigs [10]. Research on the existence and identification of cats with Klinefelter syndrome has been carried out mostly from various types of cats and from various regions. For example, identification of klinefelter syndrome that has been carried out in Himalayan cats as revealed in [11].

Currently, in the efforts of domestic cat breeding, breed management, species identification, a variety of genetic testing was available. This test can help clinicians and breeders of cats in knowing and determining the presence of traits that are passed on to the next generation [7]. It has been seen that genes on the X sex chromosome both in XX female and XY male cats have been shown to be regulators of coat color [12]. The chromosome separation failure

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which occurs in the gametogenesis process, so-called nondisjunction meiosis, introduces a type of tricolor male cats with XXY chromosome [13], [14], [15]. Before fertilization occurs, each cell will experience an event called gametogenesis, which is a biological process in which diploid or haploid precursor cells undergo cell division and differentiation to form adult haploid gametes. If there is a meiotic non-disjunction process, the sperm cells or ovum that usually forms haploid cells are still diploid cells, resulting in new individuals being produced experiencing an abnormality on the chromosome. The male calico cats with a sex chromosome XXY is one type of species that experiences chromosomal abnormalities, also called klinefelter syndrome, due to the excess of the X chromosome [15].

In this paper, a mathematical model is constructed based on the chromosome combination that occurs in the fertilization process. Some of the numerical simulations are given in sections 3. The sensitivity analysis for the investigation of a crucial parameter in the existence of a male calico cats is discussed in section 4. In the last section, the conclusion is presented.

2. Mathematical Model

Normal domestic cats both male and female have 38 chromosomes or 19 chromosome pairs. Sex chromosomes for normal male and female cats are XX and XY respectively. When cats have a gene for orange on one of the X chromosomes, according to the Lyon hypothesis, it can have probability non-orange descent paths, written X^OX^o [5], [6]. For modeling purposes, we assume the genetics that regulates color on the sex chromosomes are codominant, and white is not involved in genetic modification. Consequently, grouping cats based on the color of their coat phenotypically can be divided into six types, namely orange-white male cats (X^OY), black-white male cats (X^BY), male calico cats (X^OX^BY), orange-white female cat (X^OX^O), black-white female cats (X^BX^B), and tricolor female cats (X^OX^B), which are denoted as $x_1, x_2, x_3..., x_6$ respectively.

The assumptions that will be used in the construction of the mathematical model are as follows: 1. Cross-breeding occurs randomly, 2. White is not involved in genetic modification, this means that white is always present in cats, 3. Gene that regulate orange and black color is codominant, 4. The effect of the competition is the same for all type of cats, 5. The cats that have more than three alleles or equal to three alleles can alive and phenotypically classified according to its color gene 6. The mortality rate for all type of cats is uniform.

In the process of gametogenesis, male cats with the X^OY and X^BY chromosomes divide to X^O, X^B and Y respectively with α as the probability for successful meiosis. As a result, the probability of failing meiosis for male cats with the X^OY and X^BY chromosome is $1-2\alpha$. Meanwhile, female cats with a chromosome of X^OX^O and X^BX^B divide to X^O and X^B respectively with the probability of successful meiosis equal to one. The female cat with X^OX^B chromosome divides into X^O and X^B with the probability of successful meiosis is ϵ. Consequently, the probability of failing to meiosis X^OX^B is $1-2\epsilon$. Calico male cats with X^OX^BY chromosome divided into X^OY with a probability of β and into X^BY with a probability of $1-\beta$. Completely, the cross between male and female cats that have different color genes on X chromosome is shown in the Table 1.

In the process of mathematical model construction, we follow the method used in [16]. The birth of cat species x_1 after Δt is obtained from Table 1 which produces offspring with the X^OY or X^OX^OY chromosomes. The X^OY chromosome can be generated from a cross between an orange-white female cat X^OX^O and orange-white male cat X^OY that produces a Y chromosome where a meioses probability is α. Then, the cross-breeding between cat species X^OX^O and X^OY that that failure meiosis, the probability failure meiosis is $1-2\alpha$, is produces offspring with sex chromosomes X^OY. Sex chromosomes X^OY or X^OX^OY are also produced by cross-breeding between X^OX^O and two species X^BY and X^OX^BY with probability meiosis α and β respectively. Furthermore, cat species with chromosome X^OY or X^OX^OY are also produced by cross-breeding between a tricolor female cat with chromosomes X^OX^B and all type of male cats that produce allele Y and X^OY. The probability parameter of successful meiosis of female cat X^OX^B is ϵ. Based on that description, the detail of birth rate for normal orange-white cats mathematically will be due to:

(a) mating between male X^OY and female X^OX^O:

$$\frac{(1-\alpha)x_1}{x_1+x_2+x_3} \times x_4;$$

(b) mating between male X^BY and female X^OX^O:

$$\frac{\alpha x_2}{x_1+x_2+x_3} \times x_4;$$

Table 1: Crossbreeding systems for orange-white male cats (X^OY), black-white male cats (X^BY), tricolor male cats (X^OX^BY), orange-white female cat (X^OX^O), black and white male cats (X^BX^B), and tricolor female cats (X^OX^B)

♂ / ♀		X^OY			X^BY			X^OX^BY	
		X^O	Y	X^OY	X^B	Y	X^BY	X^OY	X^BY
X^OX^O	X^O	X^OX^O	X^OY	X^OX^OY	X^OX^B	X^OY	X^OX^BY	X^OX^OY	X^OX^BY
X^BX^B	X^B	X^OX^B	X^BY	X^OX^BY	X^BX^B	X^BY	X^BX^BY	X^OX^BY	X^BX^BY
X^OX^B	X^O	X^OX^O	X^OY	X^OX^OY	X^OX^B	X^OY	X^OX^BY	X^OX^OY	X^OX^BY
	X^B	X^OX^B	X^BY	X^OX^BY	X^BX^B	X^BY	X^BX^BY	X^OX^BY	X^BX^BY
	X^OX^B	$X^OX^OX^B$	X^OX^BY	$X^OX^OX^BY$	$X^OX^BX^B$	X^OX^BY	$X^OX^BX^BY$	$X^OX^OX^BY$	$X^OX^BX^BY$

(c) mating between male X^OX^BY and female X^OX^O:

$$\frac{\beta x_3}{x_1+x_2+x_3} \times x_4;$$

(d) mating between male X^OY and female X^OX^B:

$$\frac{(1-\alpha)x_1}{x_1+x_2+x_3} \times \epsilon x_6;$$

(e) mating between male X^BY and female X^OX^B:

$$\frac{\alpha x_2}{x_1+x_2+x_3} \times \epsilon x_6;$$

(f) mating between male X^OX^BY and female X^OX^B:

$$\frac{\beta x_3}{x_1+x_2+x_3} \times \epsilon x_6.$$

The expectations for the number of mating events, especially for orange-white male cats, are shown by multiplication between male and female cats. The summation of mating events above indicates the birth rate of orange-white male cats x_1. In unit time, the birth rate of x_1 after Δt can be written as follows :

$$x_1(t+\Delta t) \approx x_1(t) + \Delta t \left(\frac{\gamma\,(\epsilon\, x_6 + x_4)\,((1-\alpha)\,x_1 + \alpha\, x_2 + \beta\, x_3)}{\sum_{i=1}^{3} x_i} \right),$$

where α, β and ϵ are probability parameters of success in meiosis for normal male cats, male calico cats and tricolor female cats respectively. While γ is birth rate for all type of cats per unit time. Meanwhile, mortality rate and influence of competition in orange-white male cats after Δt respectively can be written as follows :

$$x_1(t+\Delta t) \approx x_1(t) - \Delta t \mu x_1, \text{ dan } x_1(t+\Delta t) \approx x_1(t) - \Delta t k x_1 \sum_{i=1}^{6} x_i,$$

where μ is mortality rate of cats per unit time and k is death rate due to logistical competition in an area with units per individual per unit time. For mortality rate and influence of logistical competition for another species are derived with same manner. We arrive at the following model that describe the dynamics for all type of cats after limiting the time series under $\Delta t \to 0$:

$$\dot{x_1} = \frac{\gamma\,(\epsilon\, x_6 + x_4)\,((1-\alpha)\, x_1 + \alpha\, x_2 + \beta\, x_3)}{\sum_{i=1}^{3} x_i} - \left(\mu x_1 + k x_1 \sum_{i=1}^{6} x_i\right) \tag{1}$$

$$\dot{x_2} = \frac{(\epsilon\, x_6 + x_5)\,\gamma\,(\alpha\, x_1 + (1-\alpha)\, x_2 + (1-\beta)\, x_3)}{\sum_{i=1}^{3} x_i} - \left(\mu x_2 + k x_2 \sum_{i=1}^{6} x_i\right) \tag{2}$$

$$\dot{x_3} = \frac{\gamma((\epsilon x_6 + x_4)((1-2\alpha)x_2 + (1-\beta)x_3) + (\epsilon x_6 + x_5)((1-2\alpha)x_1 + \beta x_3))}{\sum_{i=1}^{3} x_i} \tag{3}$$

$$+ \frac{\gamma((1-2\epsilon)x_6((1-\alpha)(x_1+x_2)+x_3))}{\sum_{i=1}^{3} x_i} - \left(\mu x_3 + k x_3 \sum_{i=1}^{6} x_i\right)$$

$$\dot{x_4} = \frac{\gamma\,\alpha\,(\epsilon\, x_6 + x_4)\; x_1}{\sum_{i=1}^{3} x_i} - \left(\mu x_4 + k x_4 \sum_{i=1}^{6} x_i\right) \tag{4}$$

$$\dot{x_5} = \frac{\alpha\,\gamma\,(\epsilon\, x_6 + x_5)\, x_2}{\sum_{i=1}^{3} x_i} - \left(\mu x_5 + k x_5 \sum_{i=1}^{6} x_i\right) \tag{5}$$

$$\dot{x_6} = \frac{\gamma\,((x_5 + (1-\epsilon)\, x_6)\,\alpha\, x_1 + (x_4 + (1-\epsilon)\, x_6)\,\alpha\, x_2)}{\sum_{i=1}^{3} x_i} - \left(\mu x_6 + k x_6 \sum_{i=1}^{6} x_i\right) \tag{6}$$

For meaningful biologically, we assume that all parameters values are positive and initial condition also $x_i(0) > 0$ for $i = 1..6$. The probability value of meiosis success parameters of all types of cats,α, ϵ, is at the interval $(0, 0.5]$ with the following interpretation: if $(\alpha, \epsilon) \to 0$ then the meiosis is going to failure, but if $(\alpha, \epsilon) \to 0.5$ then the meiosis is going to success. Meanwhile, $\beta \in (0, 1]$ with the biological interpretation: if $\beta \to 0$, the probability of meiosis to X^OY chromosome is equal to zero and to X^BY chromosome is equal to 1. But, if $\beta = 0.5$, the probability of meiois to X^OY and X^BY is equal. The model (1)-(6) is biologically and mathematically well posed in the domain,

$$\Omega = \left\{ (x_1, x_2, x_3, x_4, x_5, x_6) \in \mathbb{R}^6_+ : x_i \geq 0, i = 1..6 \text{ and } \sum_{i=1}^{3} x_i \neq 0 \right\}. \tag{7}$$

Furthermore, this domain Ω is positive invariant. Analytically it is obvious that the product between the vector field of the system (1)-(6) and the normal outside vector on the boundary that is evaluated at all points in the boundary of the nonnegative orthant of the 6-dimensional Euclidean space returns nonpositive value. This indicates that the orbit solution of the systems (1)-(6), corresponding to the initial condition that is taken in the domain Ω, remains in the interior or boundary of the domain Ω for all time $t \geq 0$.

3. Numerical Simulation

The Runge Kutta fourth Order Method is used to calculate the solution of the dynamical model (1)-(6). Some parameter values, such as the birth rate (γ) and the mortality rate (μ) can be calculated based on the research data. Whereas the probability of meiosis success is determined based on its definition value, namely in the interval $(0, 0.5]$. The birth rate of cat is calculated from the average cat born per unit time. Based on [17], [18], the frequency of cats giving birth in a year is normally 2-5 times with the average number of children in one birth is 5. Suppose that taking an average of giving birth to a cat in a year is 3 times with 5 children, then there are 15 kittens born per year. As a result the birth rate of a cat per month is $\gamma = \frac{15}{12}$. Meanwhile, on [19], [20] it was explained that the age of cats ranged from 12-15 years. The average age of a cat with a normal preservation level is 13 years. Consequently, we assume that the cat mortality rate per month is $\mu = \frac{1}{13 \times 12}$. The death rate due to competition k

Table 2: Notations and descriptions

Notations	Descriptions	values	unit
α	probability of successful meiosis from normal male cats	$(0, 0.5]$	non dimensional
β	probability of successful meiosis from male calico cats	$(0, 1]$	non dimensional
ϵ	probability of successful meiosis from tricolor female cats	$(0, 0.5]$	non dimensional
γ	birth rate for all type of cats	1.25	$time^{-1}$
μ	mortality rate for all type of cats	6.4×10^{-3}	$time^{-1}$
k	death rate due to logistical competition in an area	2×10^{-3}	$individual^{-1} \times time^{-1}$

is assumed to be 20% per environmental carrying capacity. If the carrying capacity is 100 individual per area per unit time (month), consequently the value of $k = \frac{0.2}{100}$ per month. The parameter values in this numerical simulation are presented in the Table 2. The numerical simulation results are shown in the Figure 1.

In Figure 1(a)-1(d), the parameter value β has been chosen equal to 0.3, assuming the probability of successful meiosis of male calico cats in the gametogenesis process on the X^OY chromosome is smaller than dividing into a X^BY chromosome. In Figure 1(a), we review the condition of the population when there is no meiosis failure in normal male cats, but meiosis failure occurs in a spesies of tricolor female cats with a probability of failure equal to $1 - \epsilon = 0.2$. The results show that black-white cat species both male and female dominate the cat population, while the number of male calico cats is very small. Similarly, when normal male and three-color female cat species did not experience meiotic non-disjunction, black-white male cat species still dominated the population, but black-white female cats decreased and three-color female cats became the most dominant (see Figure 1(b)). When meiosis failure occurs in normal male cats but does not occur in three-color female cats, the three-color female cats decline drastically, as well as black-white female cats. Meanwhile, black -white male cats still dominate the cat population (1(c)). Different phenomena occur when the probability of successful meiosis from the male calico cats to the X^OY chromosome is higher than splitting to X^BY . In this condition, orange-white male cats dominate the cat population (1(e)). But when normal male cats and three-color female cats both experience considerable non-disjunction, the male calico cats population can exist (1(f)) and can dominate the population (1(d)). This conforms to us that the failure of meiosis can lead to the dominance of the male calico cats population. To be more detailed in discussing the effects of the probability of meiosis success on the male calico cats, sensitivity analysis shortly will be discussed.

4. Sensitivity Analysis

Sensitivity analysis is carried out to determine the effect of changes in parameters toward changing solutions to systems of differential equations (1)-(6). Specifically, it is conducted to know parameter which has an important role to decrease or increase male calico cats species, then in the next, we can make the right strategy for maintenance of male calico cats. Considering the mechanism of male calico cats hybridization in genetic, we will focus on impact of uncertain parameters to the solution of the differential equations (1)-(6), i.e. the meiosis probability of normal-male cats (α),normal-female cats (ϵ) and male calico cats (β). For that purposes, we consider set of parameters $\Lambda = (\alpha, \beta, \eta)$ and the model solutions $\mathbf{X} = (x_1, x_2, ..., x_6)$, i.e. $\mathbf{X}$ is the solution of $\partial_t \mathbf{X} = f(\mathbf{X}, \Lambda)$ with initial condition $\mathbf{X}(0) = \mathbf{X}_0$. We define sensitivity function as the ratio between solution by the difference of parameters as $\Delta\Lambda \to 0$,i.e. $S(t, \Lambda) = \partial_\Lambda \mathbf{X}(t, \Lambda)$. The derivative of sensitivity function by time t is $S_t(t, \Lambda) = \partial_t \partial_\Lambda \mathbf{X}(t, \Lambda)$. Therfore, sensitivity function $S(t, \Lambda)$ is the solution of following differential equation:

$$S_t(t, \Lambda) = \partial_{\mathbf{X}} f(\mathbf{X}, \Lambda) S(t, \Lambda) + \partial_\Lambda f(\mathbf{X}, \Lambda). \quad (8)$$

where $\partial_{\mathbf{X}} f(\mathbf{X}, \Lambda)$ refers to 6×6 Jacobian matrix from (1)-(6) and $S(t, \Lambda)$ is 6×3 matrix. Furthermore $\partial_\Lambda f(\mathbf{X}, \Lambda)$ is 6×3 matrix. Consequently, the equation (8) is well defined.

We calculated the following locally sensitivity : (i) the sensitivity of the α parameter to the $\mathbf{X}$ solution (notated as $S_\alpha \mathbf{X}$), (ii) the sensitivity of the β parameter to the $\mathbf{X}$ solution ($S_\beta \mathbf{X}$), and (iii) the sensitivity of the ϵ parameter to the $\mathbf{X}$ solution ($S_\epsilon \mathbf{X}$). The computation is conducted around $\Lambda^* = (\alpha^*, \beta^*, \epsilon^*) = (0.5, 0.3, 0.7)$. Meanwhile, the other parameter values are the same as in the Table 2. After numerical computations, the solutions of differential equation (8) can be seen in Figure 2.

The sensitivity of the parameters to the change of solution x_1 is illustrated in Figure 2-(a). Here is described that the probability of successful meiosis from male calico cats (β) is the most sensitive parameter toward solution changes x_1. The change in the value of this parameter is directly proportional to the number of changes in the cat species x_1. This means that if the probability change of successful meiosis on the male calico cats is large,

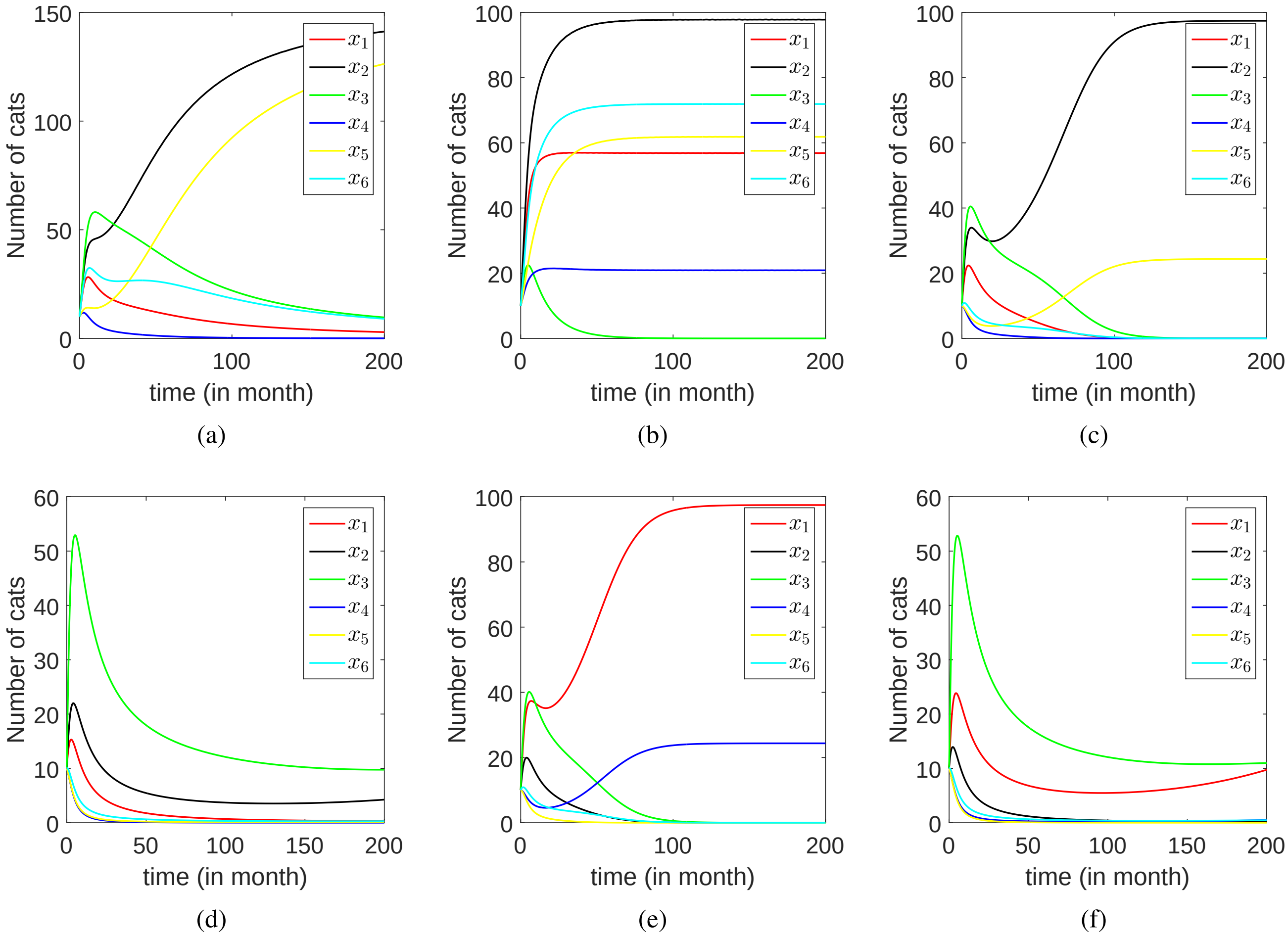

Figure 1: Numerical simulation for dynamical equation (1) - (6) using paramaters values in Table 2 with probability meiosis values are different,i.e. (a) $(\alpha, \beta, \epsilon) = (0.5, 0.3, 0.3)$, (b) $(\alpha, \beta, \epsilon) = (0.5, 0.3, 0.5)$, (c) $(\alpha, \beta, \epsilon) = (0.2, 0.3, 0.5)$, (d) $(\alpha, \beta, \epsilon) = (0.1, 0.3, 0.1)$, (e) $(\alpha, \beta, \epsilon) = (0.2, 0.8, 0.5)$, (f) $(\alpha, \beta, \epsilon) = (0.1, 0.8, 0.1)$.

then the cat species x_1 will be increase, and vice versa. But at the same time, increasing of β parameter is inversly proporsional to the cat species x_2, as shown in the Figure 2-(b). Consequently, the effect of β increasing can decrease male cats with black colour.

In Figure 2-(c), The probability of successful meiosis for all types of cats are inversely proportional to solution change of x_3. This means that the increase in the parameters of the probability of successful meiosis can cause scarcity in male calico cats species x_3. This phenomenon indicates to us that the increasing population of male calico cats can occur when meiosis fails. Furthermore, from the three parameters, the parameters that give the most influence to change the solution of x_3 is the probability parameter of successful meioses of normal male cats α. These results confirm that the failure of meiosis on normal male cat can cause an abundance of male calico cats. In contrast, the increased of the probability of successful meiosis on normal male cat will result in the scarcity of male calico cats. The successful probability of meiosis from the tricolor female cat (ϵ) is the second sensitive parameter after the parameter of α. Basically, the failure of meiosis of all types of cats both male and female can have a positive effect on the existence of male calico cats. If this scarcity of male calico cats occurs in nature, the strategy we can recommend in overcoming the scarcity of that cat populations is to reduce or inhibit the success of

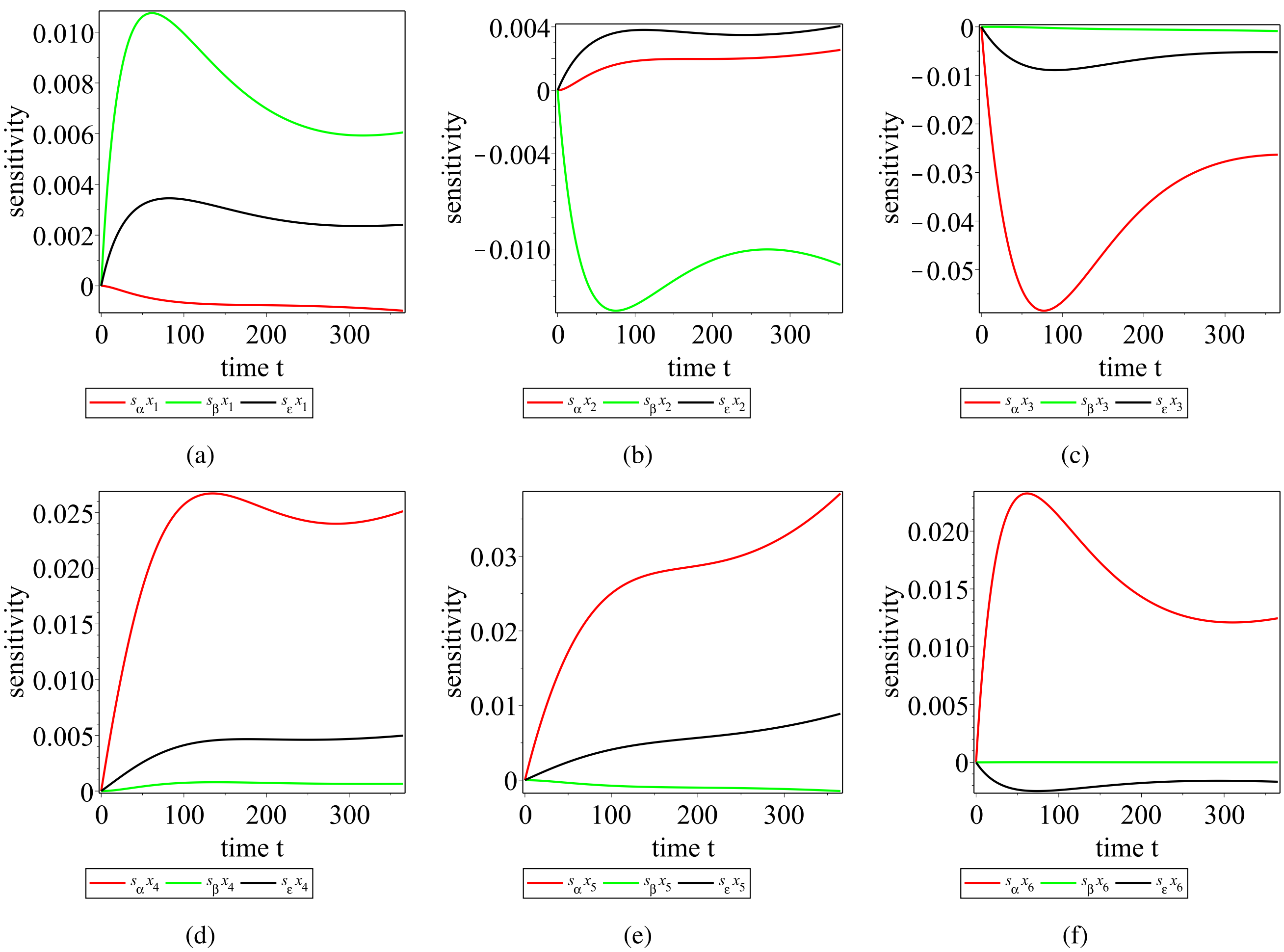

Figure 2: Numerical approximates for the time-dependent sensitivity states around Λ^* using the other parameters value is same as in Table 2. (a) sensitivity Λ to x_1, (b) sensitivity Λ to x_2, (c) sensitivity Λ to x_3, (d) sensitivity Λ to x_4, (e) sensitivity Λ to x_5, (f) sensitivity Λ to x_6.

meiosis on the hybridization.

The meiosis failure of all types of cats actually harmed the existence of normal orange-white female cat species. This phenomenon can be seen in the Figure 2-(d) which shows that the changes in the probability of successful meiosis of the all types chromosomes of cats have a positive effect on the solution of x_4. Furthermore, the most sensitive parameter for changing the solution of x_4 is the parameter of α, followed by the ϵ parameter. While the effect of changing the parameter β is very little effect on the change in the solution of x_4. These results show that the successful meiosis of normal cats chromosome can contribute to maintaining the existence of orange female cat species but can eliminate the presence of male calico cats.

The success of meiosis in normal male cats not only maintain the existence of an orange-white female cat species x_4 but also black-white female cat and tricolor female cats. This phenomenon can be seen in Figure 2-(e) and 2-(f) respectively. That figures can explain to us that the parameters α is a very crucial for maintaining the existence of black-white female and tricolor female cats. When the value of α increases in the interval of its domain, the value of x_5 and x_6 are also increase, and vice versa.

In Figure 2-(e), it can be seen that the parameter of β has a negative effect on the x_5 solution. This shows that the

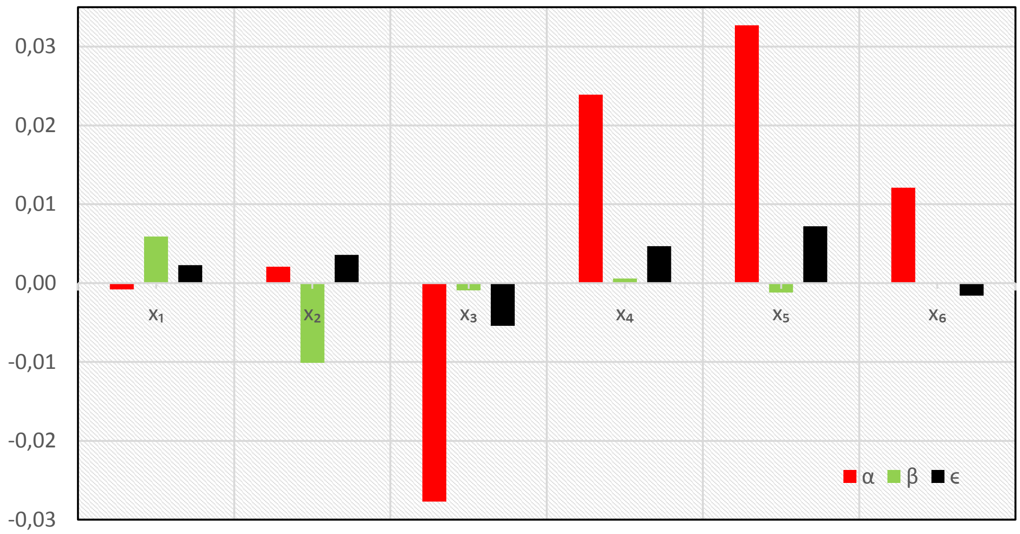

Figure 3: Sensitivity index after arrive at equilibrium

probability of successful meiosis from the chromosome of male calico cats X^OX^BY to X^OY has a role in reducing the black-white cat species. However, in tricolor female cats, it is not effect as shown in Figure 2-(f). Based on the results of the sensitivity analysis, the sensitivity index after arrive at equilibrium conditions can be seen in Figure 3. The figure shows us that the most sensitive parameter is the alpha parameter. The reduced α value of the reference value $\alpha^\star$ causes the calico male cats to increase, but conversely an increase in the value of the α parameter from the reference value $\alpha^\star$ can cause black-white male cats x_5 to increase.

5. Conclusion

We construct a dynamical model for all types of cat species by considering the process of color inheritance. Sensitivity analysis is used to find out the most influential parameters for the existence of all cat species, especially in male calico cats. The sensitivity analysis and numerical simulations results show that the probability of meiosis successful value from normal male cats is closely related to the existence of black-white female cats and male calico cats. The difference is that the effect on black-white female cats is directly proportional, while on male calico cats is inversely proportional. Based on the result, minimizing the probability of meiosis success parameter from normal male cats is one strategy that our recommendation for increasing the abundance of male calico cats species. Consequently, the species of black-white female cats will be declined. However, if the presence of male calico cats is considered a disability in cats, then the strategy is to increase the probability of meiosis success in all types of male or female cats. The challenge for us is determining the magnitude of the optimal probability of meiosis successful value for normal male cats so that the population of male calico cats and black-white female cats can coexist.

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References

[1] Macdonald, D.W., Yamaguchi, N. and Kerby, G., 2000. Group-living in the domestic cat: its sociobiology and epidemiology. *The domestic cat: the biology of its behaviour*, 2, pp.95-118.

[2] Driscoll, C.A., Menotti-Raymond, M., Roca, A.L., Hupe, K., Johnson, W.E., Geffen, E., Harley, E.H., Delibes, M., Pontier, D., Kitchener, A.C. and Yamaguchi, N., 2007. The Near Eastern origin of cat domestication. *Science*, 317(5837), pp.519-523.

[3] Loberg, J.M. and Lundmark, F., 2016. The effect of space on behaviour in large groups of domestic cats kept indoors. *Applied Animal Behaviour Science*, 182, pp.23-29.

[4] SJVFS, 2008:5, 2008. Statens Jordbruksverks freskrifter Och Allmnna rd Omhllande Av Hund Och Katt. [Title Translation: The Swedish Board ofAgricultures Regulations and Recommendations on the Keeping of Dogs andCats], http://www.jordbruksverket.se/download/18.26424bf71212ecc74b080001024/1242046873606/2008-005.pdf.(accessed09.25.19).

[5] Centerwall, W.R. and Benirschke, K., 1973. Male Tortoiseshell and Calico (TC) Cats: Animal models of sex chromosome mosaics, aneuploids, polyploids, and chimerics. *Journal of Heredity*, 64(5), pp.272-278.

[6] Centerwall, W.R. and Benirschke, K., 1975. An animal model for the XXY Klinefelter's syndrome in man: tortoiseshell and calico male cats. *American journal of veterinary research*, 36(9), pp.1275-1280.

[7] Lyons, L.A., 2012. Genetic testing in domestic cats. *Molecular and cellular probes*, 26(6), pp.224-230.

[8] Wistuba, J., 2010. Animal models for Klinefelter's syndrome and their relevance for the clinic. *MHR: Basic science of reproductive medicine*, 16(6), pp.375-385.

[9] Bamber, R.C., Bamber, R.C., Herdman, E.C. and Herdman, E.C., 1927. The inheritance of black, yellow and tortoiseshell coat-colour in cats. *Journal of Genetics*, 18(01).

[10] Harvey, M.J.A., 1968. A male pig with an XXY/XXXY sex chromosome complement. *Reproduction*, 17(2), pp.319-324.

[11] Pyle, R.L., Patterson, D.F., Hare, W.C.D., Kelly, D.F. and Digiulio, T., 1971. XXY sex chromosome constitution in a Himalayan cat with tortoise-shell points. *Journal of Heredity*, 62(4), pp.220-222.

[12] Pedersen, A.S., Berg, L.C., Almstrup, K. and Thomsen, P.D., 2014. A tortoiseshell male cat: chromosome analysis and histologic examination of the testis. *Cytogenetic and genome research*, 142(2), pp.107-111.

[13] Hageltorn, M. and Gustavsson, I., 1981. XXY-trisomy identified by banding techniques in a male tortoiseshell cat. *Journal of Heredity*, 72(2), pp.132-134.

[14] Leaman, T., Rowland, R. and Long, S.E., 1999. Male tortoiseshell cats in the United Kingdom. *Veterinary record*, 144(1), pp.9-12.

[15] Nicholas, F.W., 2009. *Introduction to veterinary genetics*. John Wiley & Sons.

[16] Suandi, D., Wijaya, K.P., Apri, M., Sidarto, K.A., Syafruddin, D., Gtz, T. and Soewono, E., 2019. A one-locus model describing the evolutionary dynamics of resistance against insecticide in Anopheles mosquitoes. *Applied Mathematics and Computation*, 359, pp.90-106.

[17] Prescott, C. W., 1973. Reproduction patterns in the domestic cat. *Australian veterinary journal*, 49(3): 126-129.

[18] Jemmett, J.E. and Evans, J.M., 1977. A survey of sexual behaviour and reproduction of female cats. *Journal of Small Animal Practice*, 18(1), pp.31-37.

[19] Nutter, F.B., Levine, J.F. and Stoskopf, M.K., 2004. Reproductive capacity of free-roaming domestic cats and kitten survival rate. *Journal of the American Veterinary Medical Association*, 225(9), pp.1399-1402.

[20] Cozzi, B., Ballarin, C., Mantovani, R. and Rota, A., 2017. Aging and veterinary care of cats, dogs, and horses through the records of three university veterinary hospitals. *Frontiers in veterinary science*, 4, p.14.